velocity for the whole interval of 30 s for which data are available is given by the slope of the chord OC, or 2.7 m s^{-1}, whereas the average speed over the 30 s is 14 m s^{-1}, the total distance traveled being 420 m.

If the velocity is measured from the slope of the tangent line at a series of times, a velocity-time curve can be constructed; this is shown in Fig. 3.6(b). The velocity is seen to increase to a maximum value of 25 m s^{-1} at $t = 10$ s corresponding to the point A on the space-time curve. It then decreases to zero at $t = 18$ s, when the change of direction occurs. Thereafter the velocity is negative, since the motion is now from right to left. Its magnitude increases until at the point C it reaches a value of -33.3 m s^{-1}. Beyond that there is no information. The acceleration at any time is given by the slope of the tangent line to this velocity-time curve. This may be measured for a series of times and the result may be plotted as in Fig. 3.6(c). The acceleration is seen to be $+4$ m s^{-2} at $t = 0$, and then to decrease to zero at $t = 10$ s. Thereafter the acceleration is negative until the end. Its magnitude reaches a maximum at $t = 18$ s, where the acceleration is -4.8 m s^{-2}. The average acceleration over the whole 30 s is obtained from the slope of the chord OC of the velocity-time diagram: it is -1.1 m s^{-2}. The important thing to realize is that the process of getting the acceleration-time curve from the velocity-time curve is exactly analogous to the process of getting the velocity-time curve from the space-time curve.

3.7 WORKING BACKWARDS: INTEGRATION

Consider a situation where one is presented with a velocity-time curve, obtained, perhaps, by reading the speedometer of a car at set intervals during a trip. Figure 3.7 shows a possible form of such a curve. How can one obtain from this curve the displacement of the automobile from starting? Consider two instants t_A

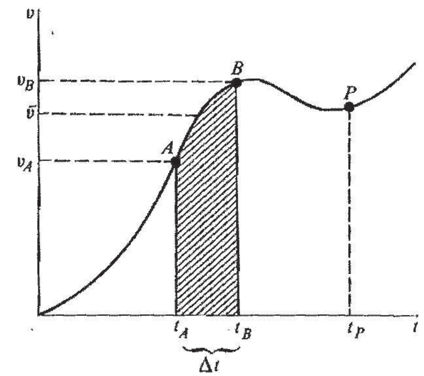


Fig. 3.7. A general velocity-time diagram with the construction for obtaining the displacement.

and t_B, where the interval $t_B - t_A$ may be denoted by Δt. If $\bar{v}$ is the average velocity over the interval Δt, then the displacement in that interval is just $\bar{v} \cdot \Delta t$. But what value are we to take for $\bar{v}$? If v_A and v_B are the velocities corresponding to the instants t_A and t_B, then clearly v_A is less than $\bar{v}$ and v_B is greater than $\bar{v}$ for the particular situation illustrated. But if the interval Δt is made smaller and smaller, the difference between v_A and v_B also becomes less and less, and either of them may be taken as representing $\bar{v}$ without appreciable error. All three products $v_A \cdot \Delta t$, $v_B \cdot \Delta t$, and $\bar{v} \cdot \Delta t$ are indistinguishable in the limit as Δt tends to zero, and they are all equal to the area of the vertical strip shown shaded in Fig. 3.7.

Finally the total displacement of the automobile up to the point P corresponding to a total time t_P from starting is obtained by adding up all these elementary strips. The procedure is as follows. On the velocity-time graph, a whole series of closely spaced vertical lines are drawn which divide the area under the curve into a large number of areas like that shown in Fig 3.7, but with very much smaller bases. The value of the velocity at the top of each little strip is read off (and it does not matter in the least which point on the curved portion is used) and this velocity is multiplied into the base to give that little area. All these little areas are then added up, and the sum clearly gives just the whole area under the curve between $t = 0$ and $t = t_P$. The conclusion is that the total displacement of an object in time t from starting is given by the area under the velocity-time curve between the origin and time t. If the area under the curve lies below the time axis, the corresponding displacement is negative since this corresponds to an object traveling from right to left. A velocity-time curve for such a situation is shown in Fig. 3.8. The area under the positive portion of the curve would be found first, and then the area under the negative portion. The total displacement in time t_P is then just the algebraic sum of these. If the areas are considered both to be positive, then the sum would give the total *distance* traveled instead of the displacement.

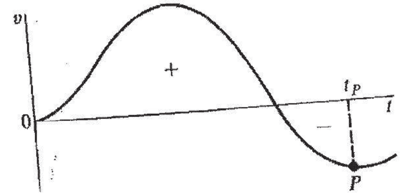
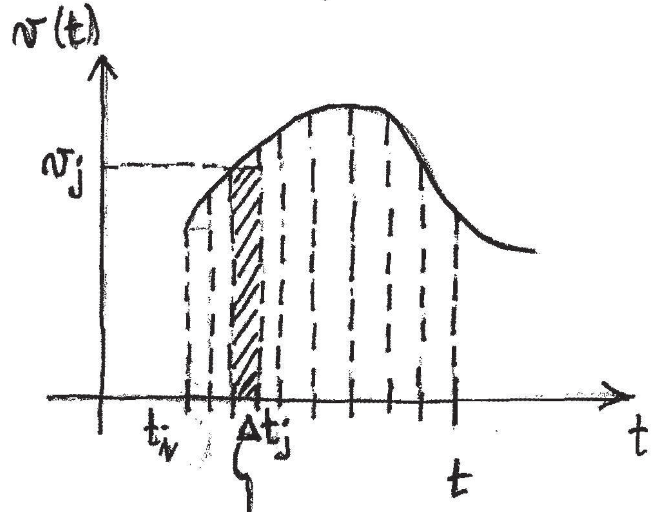


Fig. 3.8. A velocity-time graph showing positive and negative areas.

The process of finding the displacement from the velocity-time curve is known as integration and the displacement is said to be the *integral* of the velocity. It should be clear that the velocity is the integral of the acceleration in exactly the same way. Considering Fig. 3.6 again: the process of differentiation takes us from diagram (a) to (b) and then to (c); the process of integration takes us from (c) to (b) and then to (a).

ovvero $x(t) - x(t_i)$

(*)



se faccio tendere a zero questi intervallini di tempo Δt_j e sommo sulle aree dei vari rettangolini:

$$\sum_j v_j \cdot \Delta t_j \longrightarrow \int_{t_i}^{t} v(t)\,dt ,$$

ovvero nel limite $\Delta t_j \to 0$ la sommatoria corrisponde all'integrale; l'integrale rappresenta quindi l'area sottesa dalla curva tra t_i e t.

L'integrale

$$\int_{t_i}^{t} v(t)\,dt = \int_{t_i}^{t} \frac{dx}{dt}\,dt = x(t) - x(t_i)$$

da lo spostamento.