le 2. An object of mass 10 kg is subjected simultaneously to two constant a force $\mathbf{F}_1$ of magnitude 5 N towards the North, and a force $\mathbf{F}_2$ of magnitude owards the East. What is the acceleration of the object?

m. The unbalanced force acting on the object is $\mathbf{F}_1 + \mathbf{F}_2$, and this vector illustrated in Fig. 5.2. The magnitude of the sum is given by

$$F^2 = (5\text{ N})^2 + (12\text{ N})^2 = (13\text{ N})^2,$$

$$\therefore F = 13\text{ N}.$$

'orce $\mathbf{F}$ acts along the line OP at an angle θ North of East as shown, where

$$\tan\theta = 5/12 = 0.417.$$

efore $\theta = 22° 37'$. The acceleration is also directed along OP, and is of nitude

$$a = F/m = 13\text{ N}/10\text{ kg} = 1.3\text{ m s}^{-2}.$$

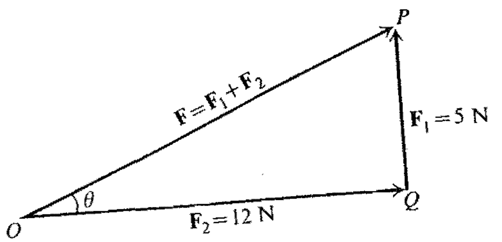


Fig. 5.2. Illustrating Example 2.

xample 3. Two sliders on an air track are fitted with magnets arranged in such a ay that the sliders repel one another. The sliders are pushed together until they re nearly in contact and are then released. One slider, of mass 0.8 kg, is observed o move away with an initial acceleration of 3 m s^{-2}. What is the initial acceleration f the other slider if its mass is 0.6 kg?

Solution. We do not need to know anything about magnetism to solve this probem: all that is necessary is to realize that with magnets present the sliders will interact with one another, and that Newton's third law will apply to this interaction. From Eq. (5.5) we get

$$\mathbf{F}_1 = -\mathbf{F}_2, \quad \text{and so} \quad m_1\mathbf{a}_1 = -m_2\mathbf{a}_2.$$

The negative sign tells us that the second slider moves off with an acceleration in a direction opposite to the acceleration of the first slider, and the magnitude of the

5.4 THE FORCE OF GRAVITY

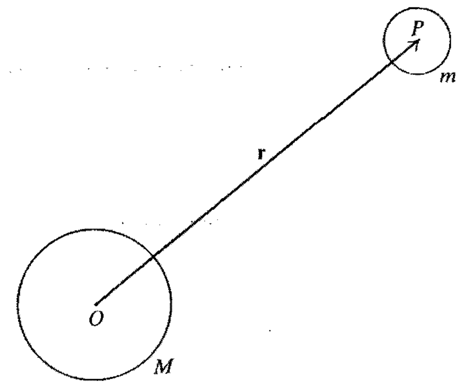
All of Newton's efforts in mechanics were directed toward the aim of explaining the motions of the planets round the sun and of the moon round the earth. His supreme achievement lay in the recognition that the force that caused an apple to fall to the ground and the force that kept the moon in place in its orbit round the earth were only different manifestations of one universal force. The form of this force necessary to explain the known facts about the motion of the planets and the moon was worked out logically, and his law of universal gravitation was formulated as follows.

> *Every particle in the universe attracts every other particle with a force that varies directly with the product of their masses and inversely with the square of their distance apart.*

To be of much use this formulation, which is in terms of particles, must be supplemented by a second law so that it can be applied to the sun, the earth, and the moon. This second law may be stated as follows.

> *In its external gravitational action, a spherical body with a spherically symmetric distribution of mass acts as if its mass were concentrated at its center.*

This second law does not have the same status, of course, as the law of gravitation itself, since it can be shown to follow from it. Nevertheless, it is convenient to state it overtly, since the proof is too difficult for an introductory book such as this one. Note that it is not enough for the body to be spherical. Thus a hemisphere of lead and a hemisphere of wood of the same radius can be joined face to face to form a sphere, but in such a case the sphere would not act as if all the mass were concentrated at the center, since the distribution of mass would clearly not have spherical symmetry. On the other hand, the earth has a much higher density in its core than near the surface, but the variation of density only depends on the distance from the center of the earth and so the mass distribution has spherical



symmetry. Figure 5.3 shows two spheres with centres at O and P. The center P is considered to be located with respect to O as origin by the position vector $\mathbf{r}$. Then, the force which sphere O exerts on sphere P is given mathematically as

$$\mathbf{F}_{OP} = -G\frac{Mm}{r^3}\mathbf{r}, \quad (*) \tag{5.9}$$

where M and m are the respective masses. The force which sphere P exerts on sphere O is $\mathbf{F}_{PO} = -\mathbf{F}_{OP}$, by the third law. The negative sign in Eq. (5.9) is necessary, since the force is attractive, i.e. from P to O in the direction opposite to $\mathbf{r}$. The constant of proportionality in Eq. (5.9) is called the *gravitational constant* and has the value $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} (or m^3 kg^{-1} s^{-2}). It is measured with a torsion balance entirely analogous to that used to verify Coulomb's law in electrostatics (cf. Section 22.4), but the experiment is difficult to perform and G is the least accurately known of all the constants of nature.

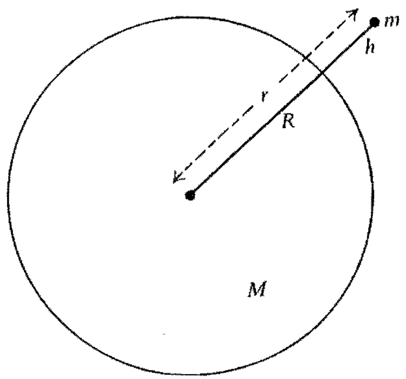


Fig. 5.4. A small body and the earth exerting gravitational forces on each other.

Consider now a mass m near the surface of the earth. The gravitational force exerted on it by the earth has a magnitude GMm/r^2, where M is here the mass of the earth. The correct value of r to be substituted into this expression for the force is

$$r = R + h,$$

where R is the radius of the earth and h is the height of the object above the earth, as shown in Fig. 5.4. Since the radius of the earth is 6.37×10^6 m, the difference between r and R is negligible for all ordinary heights h. So the force which the earth exerts on the object is essentially GMm/R^2, and this, by the second law, will cause the object to accelerate towards the surface of the earth with an acceleration g. Thus

$$F = G(Mm/R^2) = mg, \tag{5.10}$$

where, for simplicity, the equation has been written in terms of the magnitudes of the vectors only. This simplification will rarely cause any ambiguity. The mass of the object is seen to cancel out from Eq. (5.10), giving

$$g = GM/R^2. \tag{5.11}$$

This is the gravitational acceleration of any object near the earth, and is seen to be a constant. Its value is 9.8 m s^{-2}. In fact, g varies slightly over the surface of the earth for two main reasons: one, the radius of the earth at the equator is greater than the radius at the poles; two, the earth is spinning about its axis. The maximum difference in g between any two points on the earth only amounts to about 0.05 m s^{-2}, and this is negligible for most purposes. At any one place on earth the gravitational acceleration is rigorously independent of the mass of the object, and this is the justification for the use of the beam balance for comparisons of mass.

The force exerted on the object by the earth is called the *weight* of the object, and is denoted by $\mathbf{W}$, or more simply by W when the vector nature of this force need not be taken into account. By Eq. (5.10)

$$W = mg. \tag{5.12}$$

The reaction to this force, in the sense of the third law, is an equal and opposite force exerted by the object on the earth, and acting at the center of the earth. Thus every time an apple falls to the ground, the earth accelerates upwards toward the apple. Fortunately, the enormous mass of the earth renders this real mechanical effect of no practical importance. It must be stressed again that the question of how the earth exerts its force on an object over the intervening distance (and vice versa) is left open. The physicist is satisfied with the quantitative law of gravitation expressed in Eq. (5.9) without worrying about the philosophical implications of action at a distance.

Example 1. What is the acceleration of gravity at a height of 1000 km above the surface of the earth?

Solution. By Newton's second law, the acceleration of gravity at any height is the force exerted on any object at that height divided by the mass of the object. Let us denote it by $\mathbf{g}'$ to distinguish it from $\mathbf{g}$, the acceleration of gravity at the surface of the earth. Then the magnitude of $\mathbf{g}'$ is given by

$$g' = \frac{GM}{r^2} = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+h}\right)^2.$$

This equation tells us the manner in which g' varies with height. Using the value of R in the text and $h = 10^6$ m, we get

$$g' = 9.8 \text{ m s}^{-2}\left(\frac{6.37}{7.37}\right)^2 = 9.8 \text{ m s}^{-2} \times 0.747 = 7.3 \text{ m s}^{-2}.$$

Even at this great height the gravitational acceleration is still very large.

(*) $\vec{F}_{OP}$ si può anche scrivere come $\vec{F}_{OP} = -\frac{GMm}{r^2}\hat{r}$
(vedi lezione in classe)