Why gravity waves of inflation are important?

- A smoking gun of a period of inflation in the early universe

- The amplitude of the inflationary gravity waves probes the ***energy scale of inflation***

$$V^{1/4} = 1.06\times 10^{16}\, GeV \left(\frac{r}{0.01}\right)^{1/4}$$

GUT SCALE

- ***a detection would provide a firm observational link to physics of the early universe, characterized by energies never achievable in labs.***

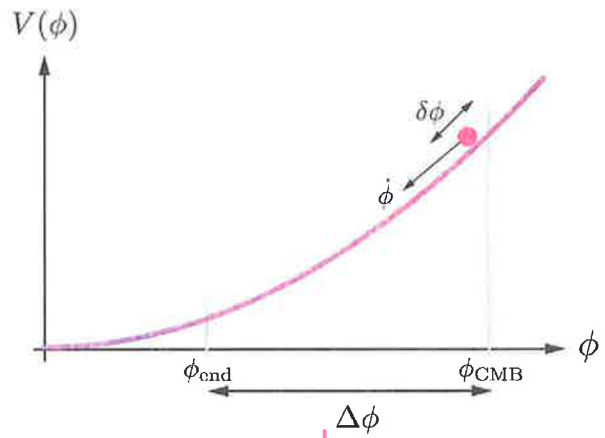
- ***a detection of r (or constraints on r) give information on the excursion of the inflaton field*** *during inflation (in the observable window)*

- inflationary gravity waves generate a ***unique*** imprint into the CMB ***polarization pattern (the so called B-mode of CMB polarization).***

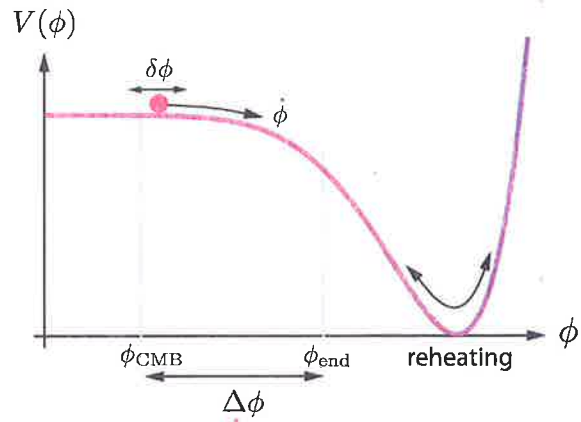
Classifying inflationary models

Roughly speaking: ``Large field" models can produce a high level of gravity waves;
``small field" models produce a low level of gravity waves

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_\zeta} = \epsilon$$



``Large field'' like potential



``Small field'' like potential
predict ε very very small

Before seeing why is that...........

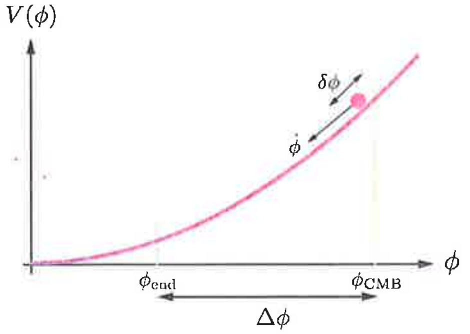
Let us first compute the number of e-foldings between a generic time during inflation and the end of inflation *in terms of the scalar field*

$$N = \int_{t}^{t_f} H dt \simeq H \int_{\phi}^{\phi_f} \frac{d\phi}{\dot{\phi}} \simeq -3H^2 \int_{\phi}^{\phi_f} \frac{d\phi}{V_{,\phi}} \simeq -8\pi G \int_{\phi}^{\phi_f} \frac{V}{V_{,\phi}} d\phi$$

For example: the scales of wavenumber k relevant for CMB cross the horizon during inflation N_k e-foldings before the end of inflation given by

$$N_k = \int_{t_k}^{t_f} H dt \simeq -8\pi G \int_{\phi(t(k))}^{\phi_f} \frac{V}{V_{,\phi}} d\phi$$

N_k must be of the order of 60



"Large field" models can produce a high level of gravity waves (r>0.01)

"Small field" models produce a low level of gravity waves (r<0.01)

Take the previous formula

$$N_{CMB} \simeq -8\pi G \int_{\phi_{CMB}}^{\phi_f} \frac{V}{V_{,\phi}} d\phi \sim (8\pi G)^{1/2} \int_{\phi_{CMB}}^{\phi_f} \frac{d\phi}{\epsilon^{1/2}} \sim \frac{1}{M_{pl}} \frac{1}{\epsilon^{1/2}} \int_{\phi_{CMB}}^{\phi_f} d\phi$$

Then**

$$\epsilon^{1/2} \sim \left(\frac{\Delta\phi}{M_{Pl}}\right) \frac{1}{N_{CMB}}$$

The precise relation one obtains is

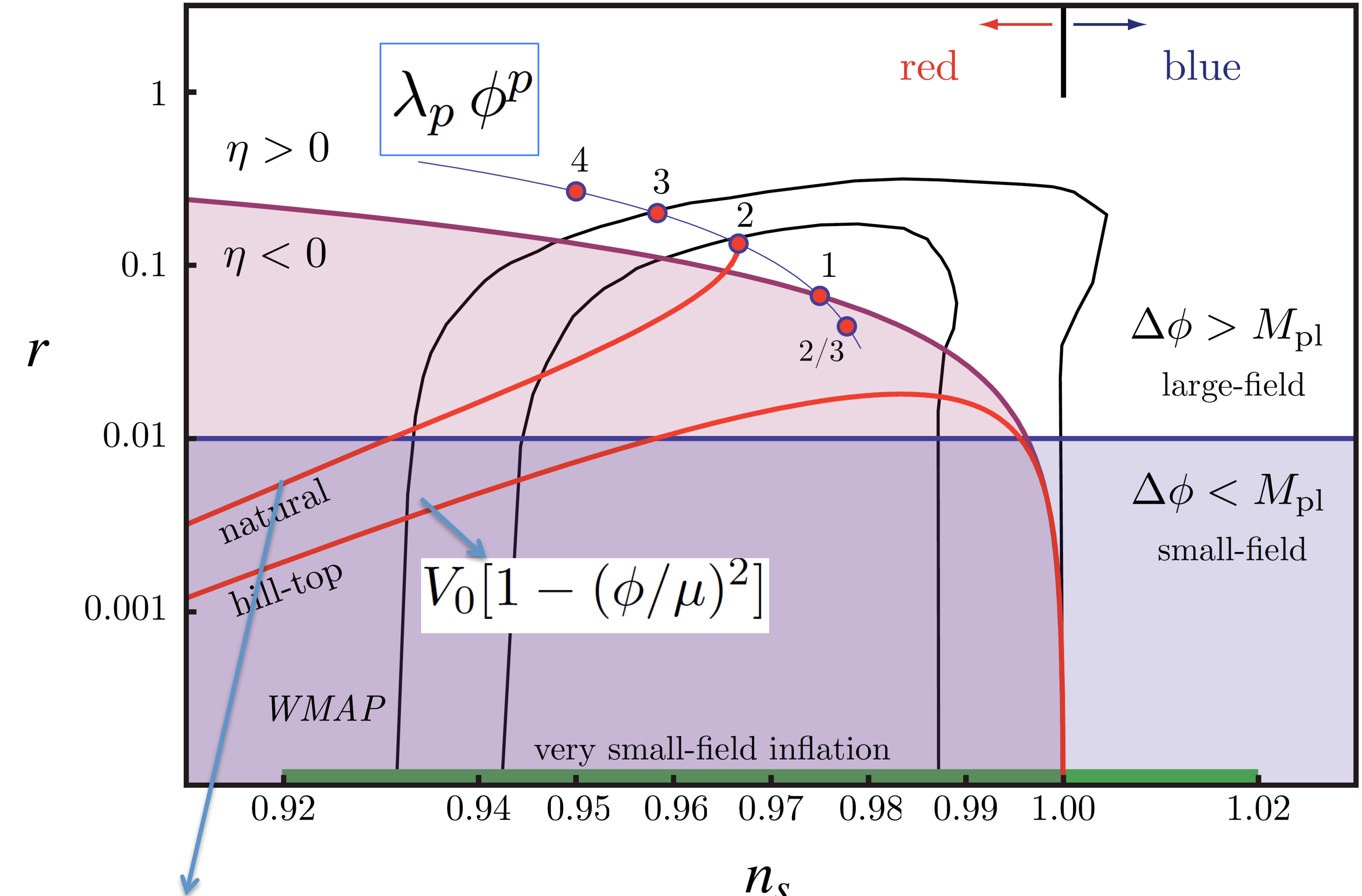
$$\frac{\Delta\phi}{M_{\rm Pl}} \simeq \left(\frac{r}{0.01}\right)^{1/2}$$

but remember the tensor-to-scalar ratio $r \sim \epsilon$

So the bigger is the field excursion during inflation the bigger is the amplitude of the gravity waves

**: Per arrivare a questo risultato si puo` anche partire dalla formula sulla escursione del campo scalare che e` stata ricavata alla fine delle note del file chiamato "Blocco2.pdf".

``Trajectories'' of models in the (r-n_s) space



Natural inflation

shift symmetry slightly broken $V_0[1-\cos(\phi/\mu)]$