General Introduction: the issue of the “initial conditions”

The issue of the ``initial conditions''

- We know that the universe is expanding (and right now the expansion is accelerated!!).
  Observational probes of the standard Hot Big-Bang model:
  Hubble diagram, Big-Bang nucleosynthesis, the Cosmic Microwave Background (CMB) with a blackbody spectrum

- On very large scales (say > 100 Mpc) we observe, on average, a smooth (i.e. homogeneous and isotropic) universe

- But homogeneity and isotropy is just a zero-order approximation: the real physical universe we observe is characterized by deviations from homogeneity and isotropy (and equilibrium): these deviations are the different structures we observe e.g.: galaxies, clusters of galaxies, anisotropies in the temperature and polarization of the CMB.

- So one of the main aspect I will focus on in this course is the following:
  all these structures need some initial conditions to start from and to grow subsequently by gravitational instability.

The issue of the ``initial conditions''

- By initial conditions here I mean: some perturbations in the matter/energy density on cosmological scales (1-10^4 Mpc) at some early time/large scale limit (before primordial nucleosynthesis)

$$\frac{\delta\rho}{\rho}|_{\text{initial}} = \frac{\rho(\mathbf{x}, t_i) - \bar{\rho}(t_i)}{\bar{\rho}(t_i)}$$

- Then the questions we will address are:

  What is this initial time/epoch?
  What typical energies we are referring to?
  Is there a ***dynamical*** mechanism that provides such primordial density perturbations?
  How do we exactly characterize the evolution of these primordial density perturbations?
  Is there a link between this early epoch and the mechanism of baryogenesis/dark matter particle production?

The issue of the ``initial conditions''

For example:

density perturbations in non-relativistic matter (during a matter dominated universe) grow on "small scales" (at the linear level) as

$$\delta_m(t) \propto a(t)$$

namely

$$\delta_m(t) = \delta_i \left(\frac{a(t)}{a_i} \right)$$

What is the mechanism that set up the initial density perturbations?

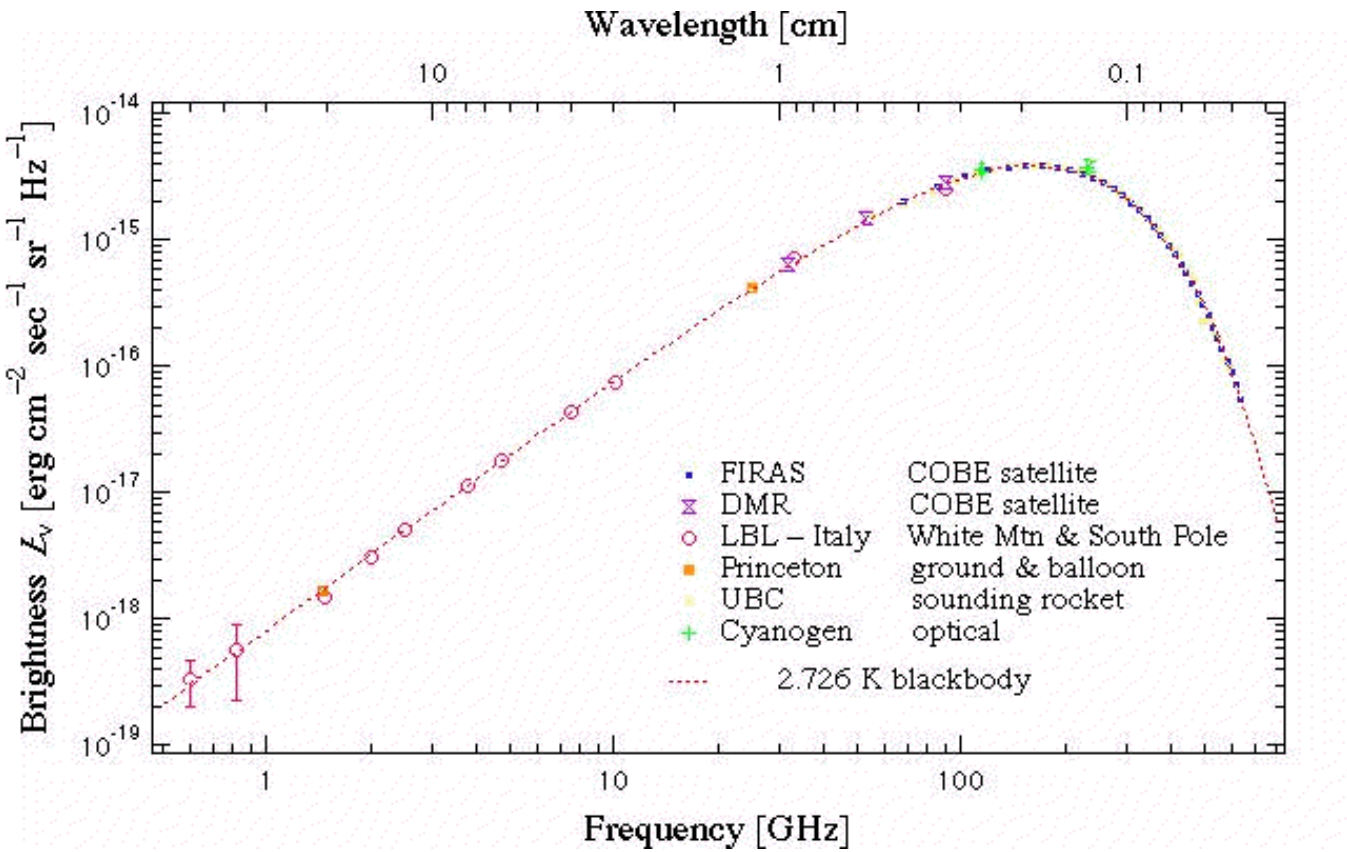
The “smooth’’ isotropic universe

THE CMB INTENSITY SPECTRUM

Best fit T_0=2.725 ± 0.002K (95% CL)
No spectral distortions detected
(apart y-distortions from Sunyaev-Zel’dovich effect)

A ``perfect’’ blackbody spectrum

$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$



The ``smooth'' isotropic universe

A ``perfect'' blackbody spectrum $$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$

$$\rho_\gamma = \frac{4\pi}{c}\int d\nu I_\nu = \frac{8\pi h}{c^3}\int d\nu \frac{\nu^3}{e^{\frac{h\nu}{kT}}-1} \longrightarrow \rho_\gamma = \frac{\pi^2}{15}T^4$$

(I use c=k=ħ=1)

$$n_\gamma = \frac{4\pi}{c}\int d\nu \frac{I_\nu}{h\nu} = \frac{8\pi}{c^3}\int d\nu \frac{\nu^2}{e^{\frac{h\nu}{kT}}-1} \longrightarrow n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3 \simeq 422\,\mathrm{cm}^{-3}$$

(If $T=T_0=2.725$ K)

N.B.:

$$\rho_{0crit} = \frac{3H_0^2}{8\pi G} = 1.88\,h^2\,10^{-29}\mathrm{g\,cm}^{-3} \simeq 8.1\times 10^{-11}h^2(\mathrm{eV})^4$$

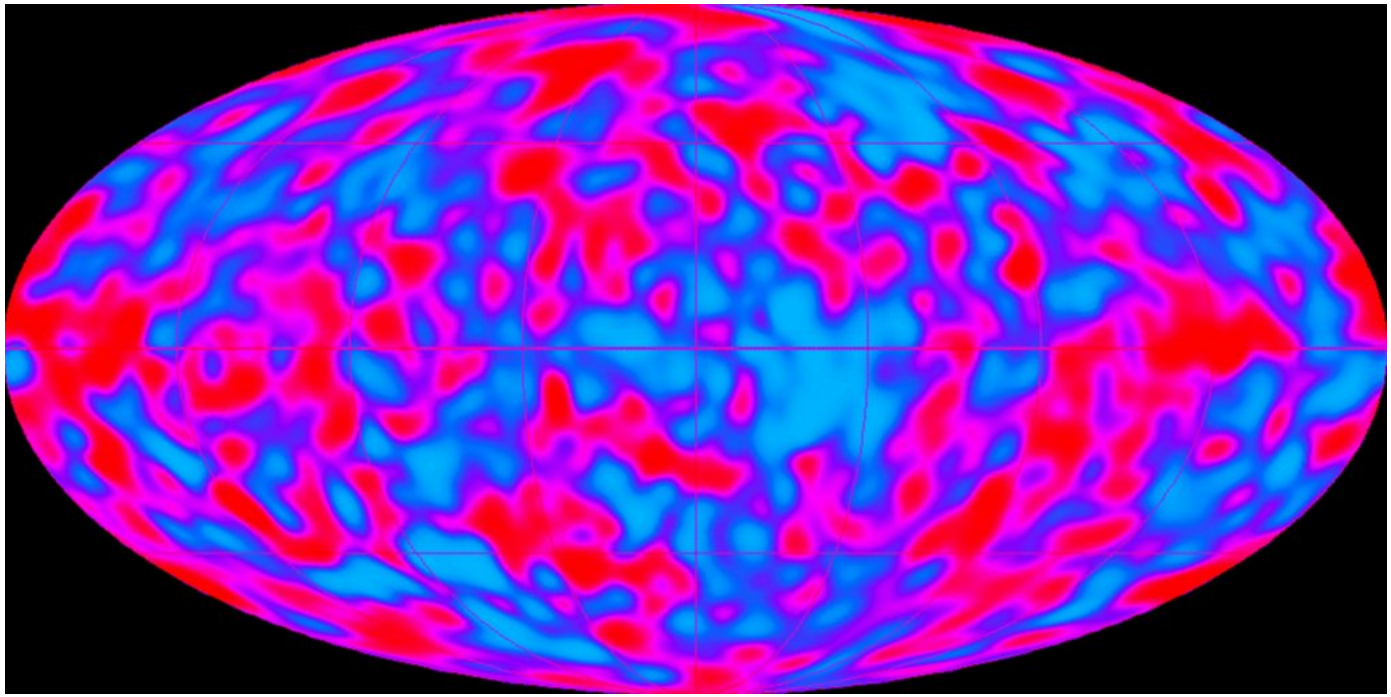
$$H_0 = 100\,h\,\mathrm{km\,sec}^{-1}\,\mathrm{Mpc}^{-1} = 2.133\times 10^{-33}\,h\,\mathrm{eV}$$

$$H_0 = 67.4 \pm 0.5 \quad \mathrm{km\,s}^{-1}\,\mathrm{Mpc}^{-1}$$ ***(from the Planck satellite 2018 measurements)***

$$\Omega_\gamma = \frac{\rho_{0\gamma}}{\rho_{0crit}} = \frac{\pi^2}{15}(2.725\,K)^4\frac{1}{8.1\times 10^{-11}h^2\mathrm{eV}^4} \simeq 2.47\times 10^{-5}h^{-2}$$

The ALMOST ``smooth'' isotropic universe

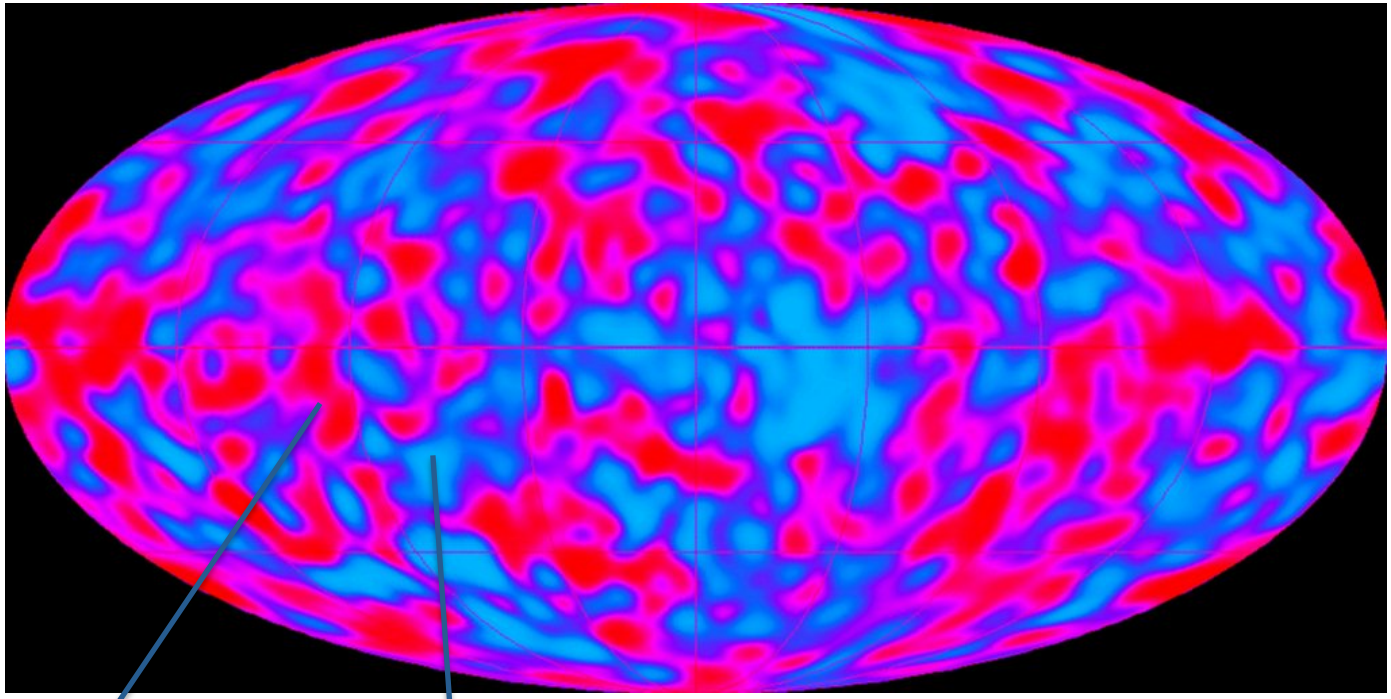
Full-sky map of CMB temperature anisotropies from COBE-DMR satellite (1994)



Intrinsic CMB temperature anisotropies (angular resolution ≈7°)

$$\frac{\Delta T}{\overline{T}}(t_0, \mathbf{n}) \equiv \frac{T(t_0, \mathbf{n}) - \overline{T}}{\overline{T}} \simeq \frac{35\,\mu K}{2.725 K} \simeq 10^{-5}$$

The ALMOST ``smooth'' isotropic universe



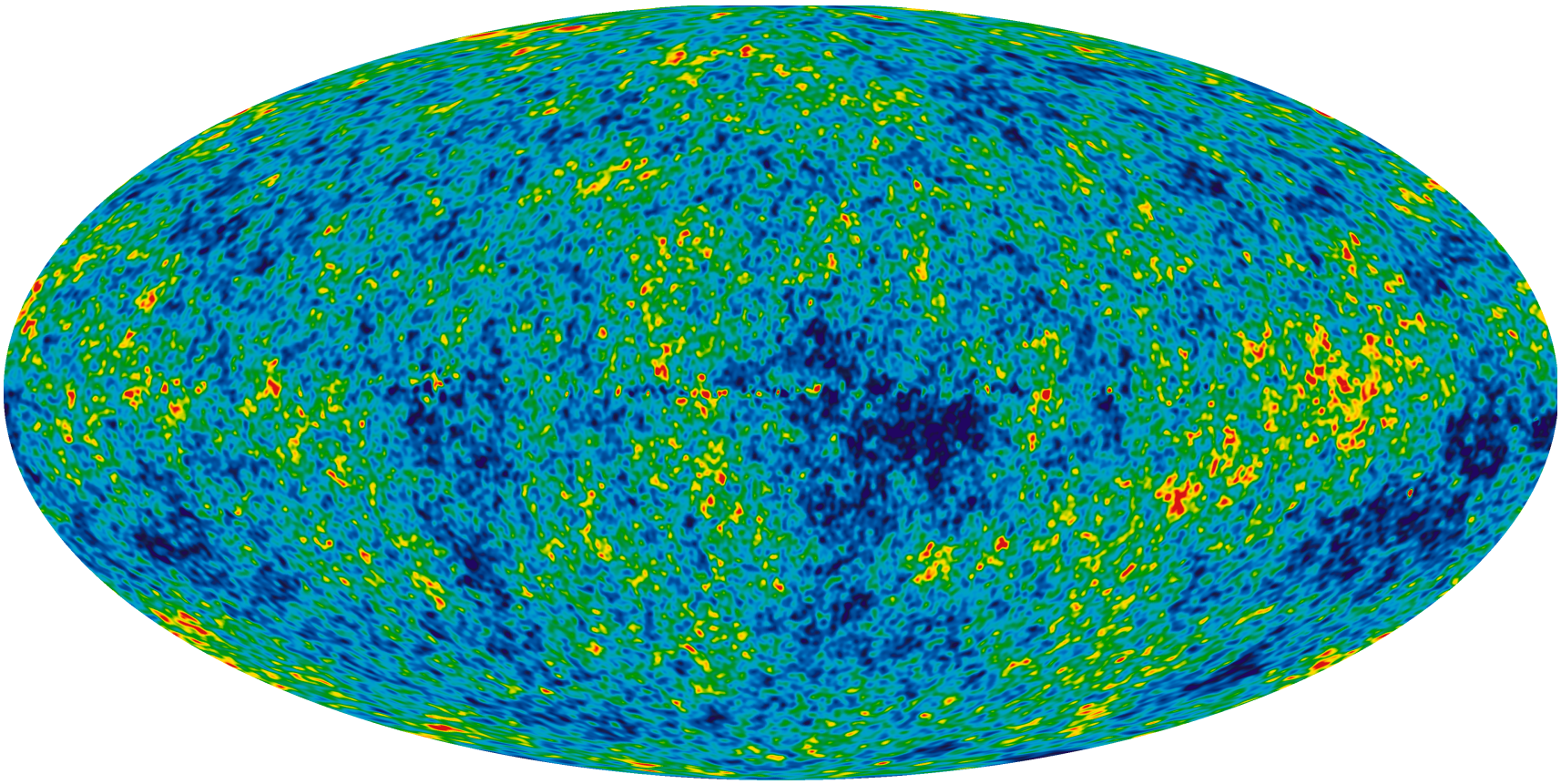
T=2.7251 K

T=2.7252 K

$$\frac{\Delta T}{T} \sim \frac{\delta\rho}{\rho}\Big|_{\text{initial}} \sim 10^{-5}$$

The ALMOST ``smooth'' isotropic universe

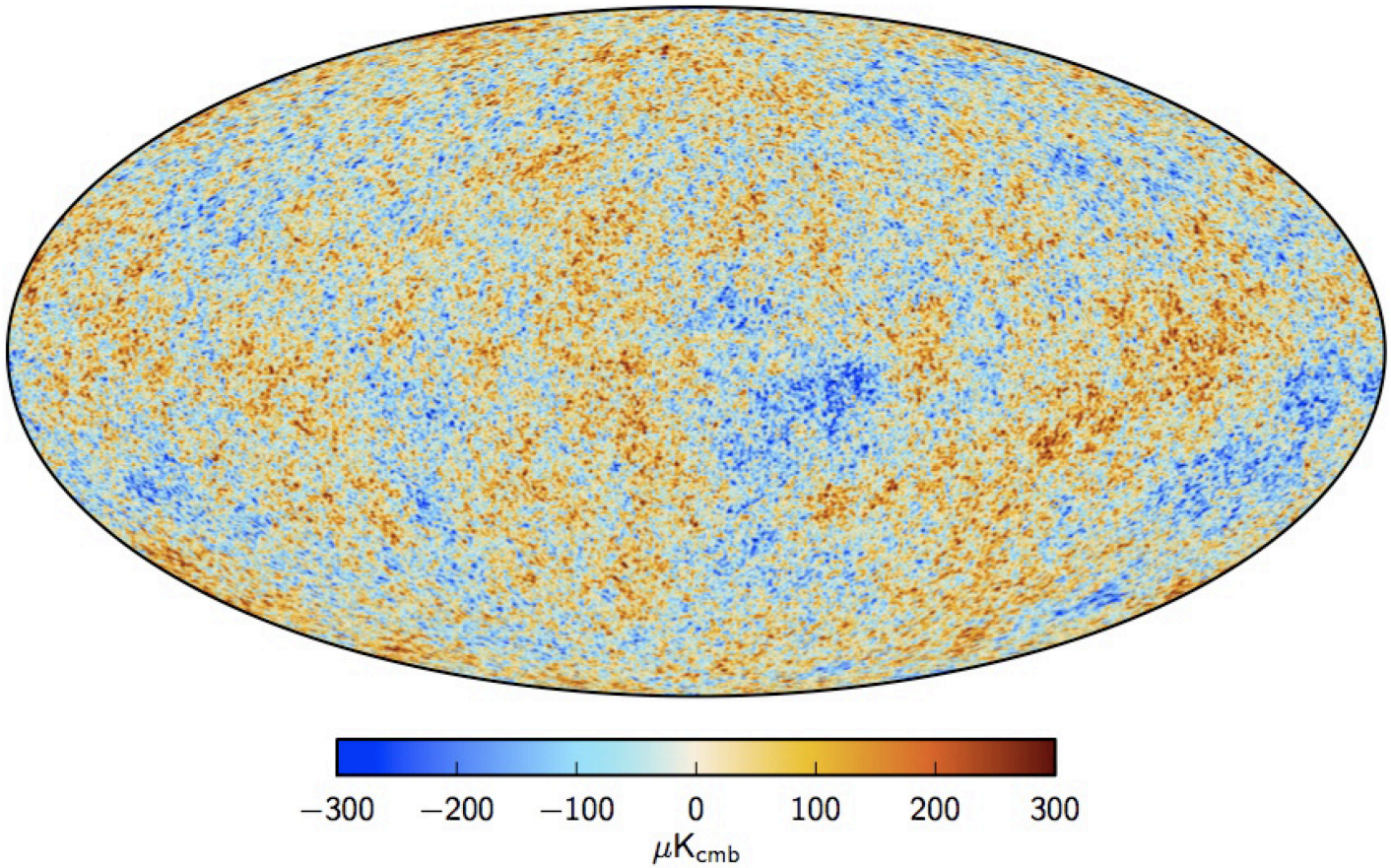
Full-sky map of CMB temperature anisotropies from the WMAP satellite (2012)



Angular resolution ~12 arcminutes

The ALMOST ``smooth'' isotropic universe

Full-sky map of CMB temperature anisotropies from the *Planck* satellite (2015)

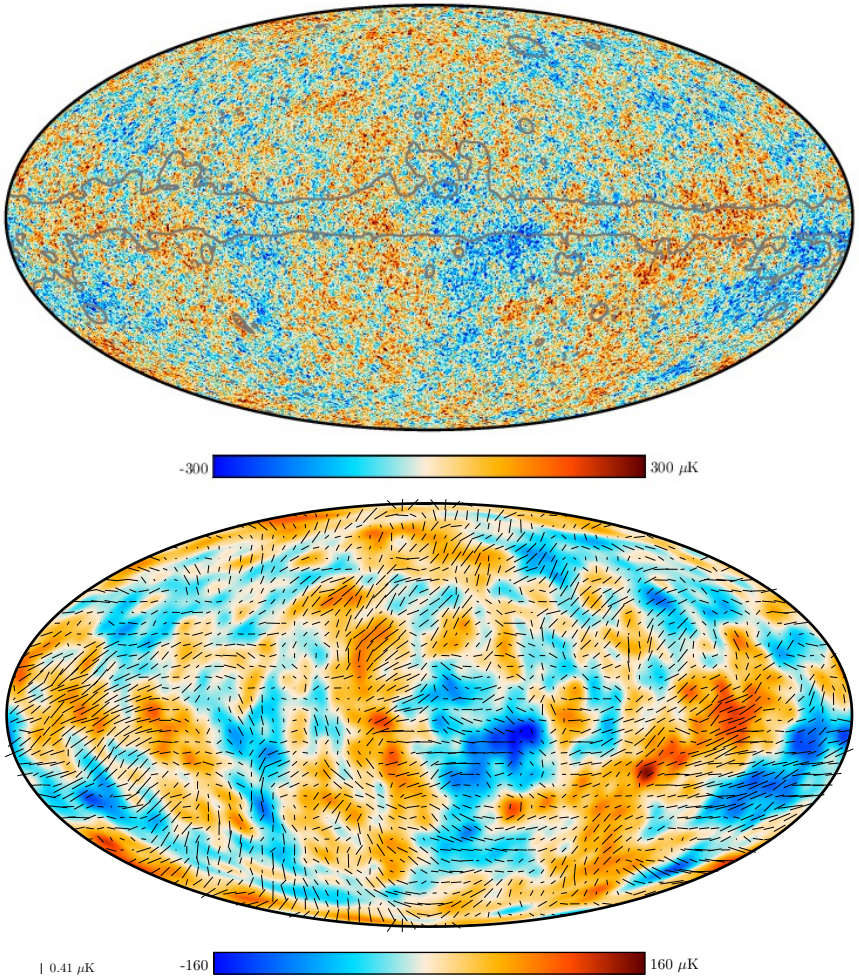


Angular resolution ~ 5 arcminutes

Planck 2015 Results. I. Overview of products and scientific results

The ALMOST ``smooth'' isotropic universe

Full-sky map of CMB temperature anisotropies and polarization from *Planck* satellite (2018)

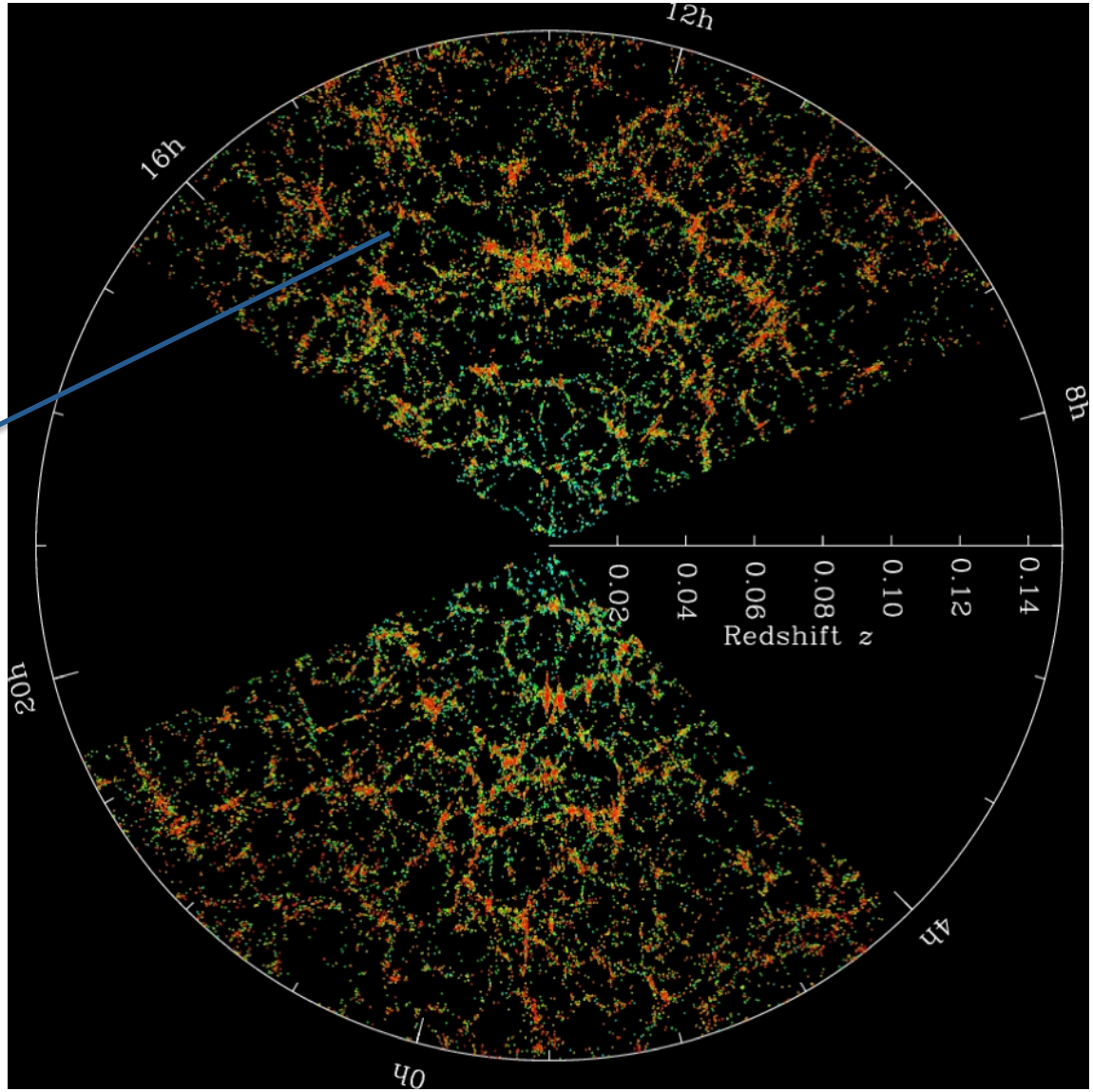


Angular resolution ~ 5 arcminutes

Planck 2018 Results. I. Overview and the cosmological legacy of Planck.

The ``smooth'' isotropic universe

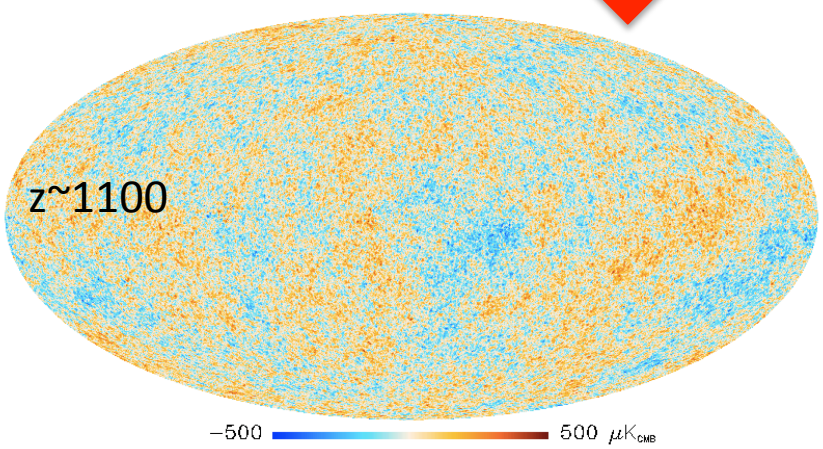
Sloan-Digital-Sky-Survey (SDSS)
more than 930,000 galaxies
and more than 120,000 quasars
on more than a quarter of the sky



THE SPATIAL DISTRIBUTION OF THE GALAXIES IS NOT RANDOM IT FOLLOWS A STATISTICAL PATTERN: WHY? THIS IS RELATED TO THE ORIGIN OF THESE OBJECTS THAT FORMED FROM VERY SMALL INHOMOGENIETIES

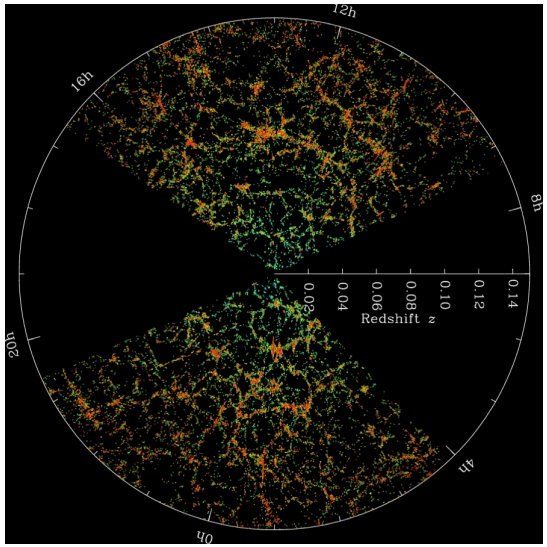
The ALMOST ``smooth'' isotropic universe

Quantum fluctuations of a scalar field, the inflaton, set the ***initial conditions*** for CMB anisotropies and Large-Scale-Structure formation



$$\frac{\Delta T}{T} \sim \frac{\delta\rho}{\rho}\Big|_{\text{initial}} \sim 10^{-5}$$

Gravitational instability



z~0.3

The NOT SO ``smooth'' homogeneous universe

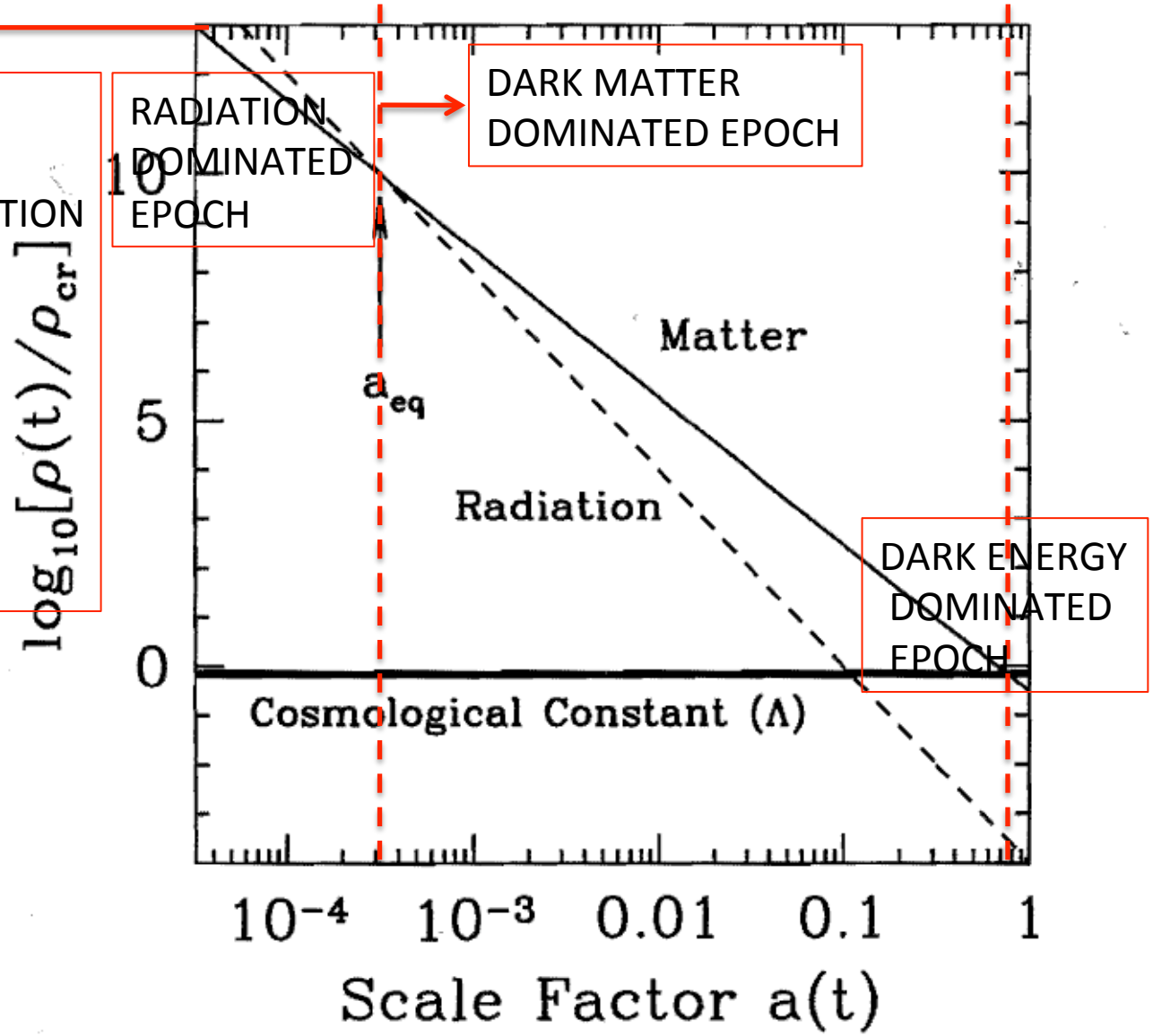
https://www.youtube.com/watch?v=FBkYIqtYb0I

Credit to Volker Springel and to Max-Planck-Institute for Astrophysics.

Follow the link http://www.mpa-garching.mpg.de/galform/data_vis/lcdm_color1_lowres_divx.avi (write the address by hand otherwise it does not work).

Beyond the standard Big-Bang model

WHAT ABOUT THE UNIVERSE BEFORE THE STANDARD RADIATION DOMINATED PHASE? (i.e., BEFORE WHAT WE CALL THE STANDARD BIG-BANG THEORY) → INFLATION



From Dodelson Scott, ``Modern Cosmology'' Academic press 2003.

Beyond the standard Big-Bang model

Developments – both theoretical and observational- in the last 2-3 decades point towards

- Existence of ***dark matter and dark energy*** (the latter driving the present acceleration of the universe)

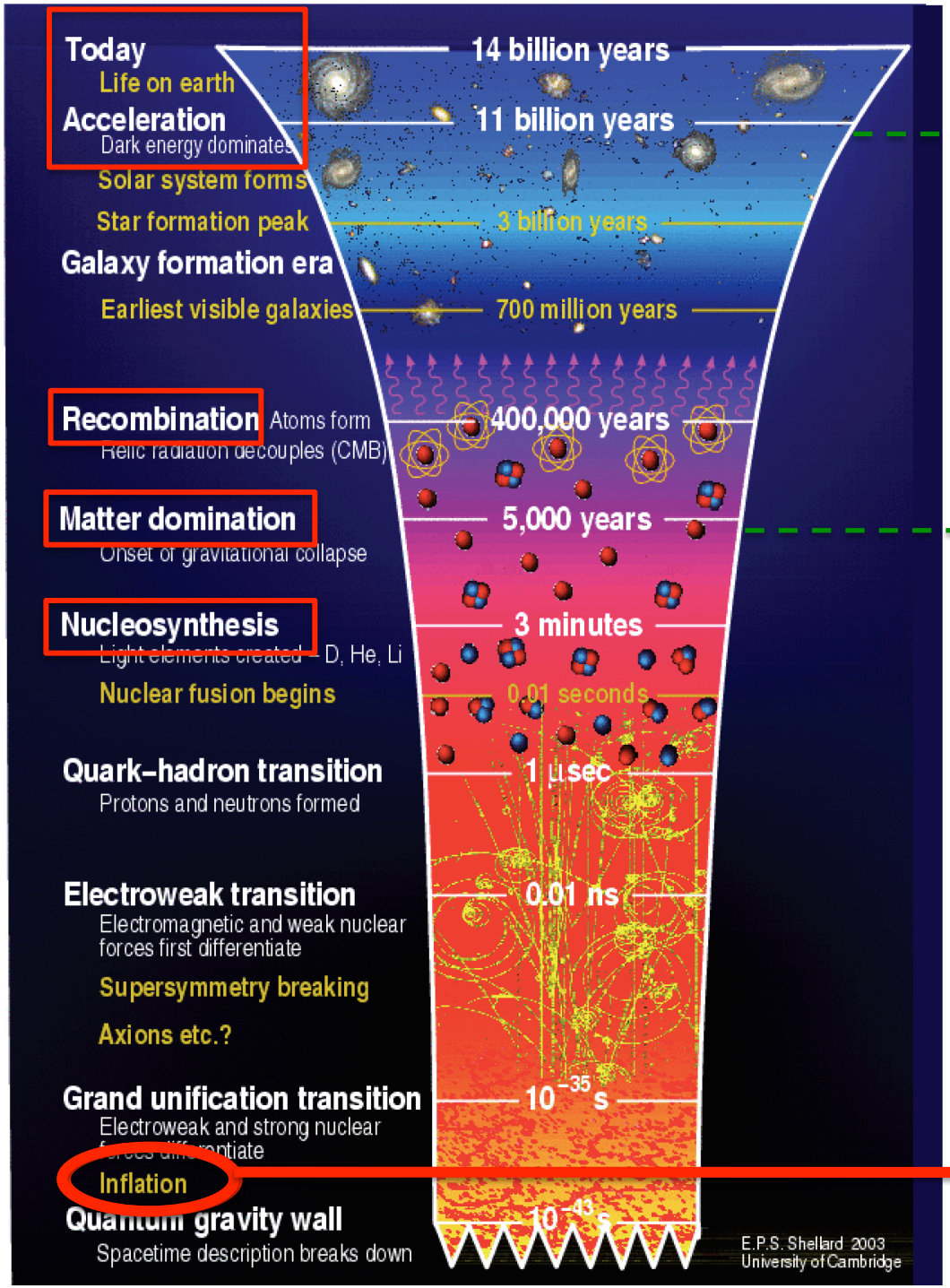
- ***Inflation*** as a fascinating and compelling theory to explain the origin of the first primordial density perturbations in the very early universe, i.e. the seeds from which all the structures we see in the universe today formed by gravitational instability (CMB anisotropies, Large-Scale-Structures like cluster of galaxies).

Going beyond the standard Big-Bang model also means:

- going beyond the zero-order approximation of a homogeneous and isotropic universe (and beyond equilibrium)

Baryogenesis/
dark matter particle production

How do we treat in a rigorous way the evolution ***of cosmological perturbations in General Relativity***?



We are here

Z~0 (REDSHIFT $1+z=a_0/a(t)$)

Dark Energy

Z_{rec}~1100 (T~0.25 eV): CMB

Z_{eq}~3500

Matter domination

Radiation domination

T~1 MeV

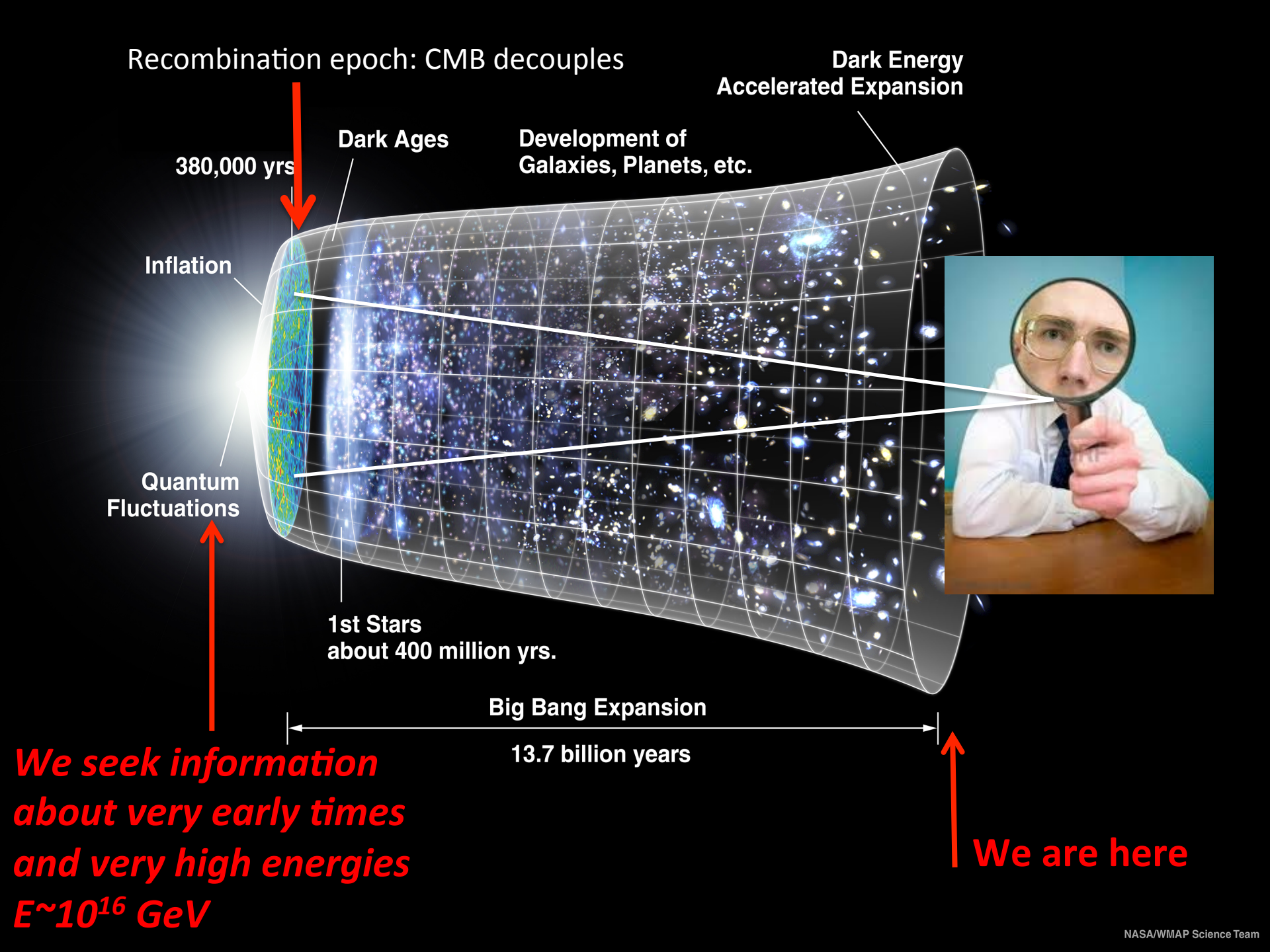
T~1 GeV

T~100 GeV (Dark Matter production??

T~10^{16} GeV

Baryogenesis?

We seek information about ***very early times and very high energies E up to 10^{16} GeV***


Recombination epoch: CMB decouples
Dark Energy
Accelerated Expansion
380,000 yrs
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
We seek information
about very early times
and very high energies
E~10^16 GeV
We are here
NASA/WMAP Science Team

THE MAIN COMPONENTS OF THE *PRESENT* UNIVERSE

Let be p=(isotropic) pressure of the fluid, ρ its energy density, w=p/ρ the equation of state, $c_s = (\partial p/\partial \rho)_S^{1/2}$ = (adiabatic) sound speed

- RADIATION (PHOTONS OF THE COSMIC MICROWAVE BACKGROUND)

$$p = (1/3)\rho; \quad \rho \propto a^{-4} \propto T^4 \Rightarrow T \propto a^{-1} \qquad\qquad w = 1/3 = c_s^2$$

- BARYONIC MATTER: electrons, protons etc: includes visible matter of the universe, non-relativistic, can be treated as pressureless as a first approximation

$$\rho(t) = mn(t); p = nT \ : \ T \ll m \, , (\text{i.e. } k_B T \ll mc^2) \Rightarrow p << \rho$$

$$\rho(t) \propto a(t)^{-3}$$

$$\Omega_b \, h^2 = 0.0224 \pm 0.0001$$ ***(from the satellite Planck 2018 measurements)***

- DARK MATTER: non-relativistic matter, decoupled from the rest of the world, non-gravitational interactions are negligible $\Rightarrow p \simeq 0 \ (w \simeq 0 \simeq c_s^2)$

$$\rho(t) \propto a(t)^{-3}$$

$$\Omega_{dm} \, h^2 = 0.120 \pm 0.001$$ ***(from the satellite Planck 2018 measurements)***

THE MAIN COMPONENTS OF THE *PRESENT* UNIVERSE

Let be p=pressure of the fluid, ρ its energy density, w=p/ρ the equation of state,
c_s = the (adiabatic) sound speed

- COSMOLOGICAL CONSTANT: an energy density that does not dilute with the expansion of the universe $\rho(t) = \text{const.}; \quad p = -\rho \ (w = -1);$

$$\Omega_\Lambda = 0.6847 \pm 0.0073$$

- NEUTRINOS:

$$\sum m_\nu < 0.12\,\text{eV} \quad (95\%\,\text{CL})$$

$$\Omega_\nu h^2 < 0.0012 \quad (95\%\,\text{CL})$$

(from *Planck 2018 temperature anisotropies and polarization* +lensing of CMB + BAO)

- SPATIAL CURVATURE:

$$\Omega_K = 1 - \Omega_0 = 0.001 \pm 0.002$$

(*Planck 2018 temperature anisotropies and polarization* +lensing of CMB + BAO)

THE MAIN COMPONENTS OF THE *PRESENT* UNIVERSE: THE ΛCDM MODEL

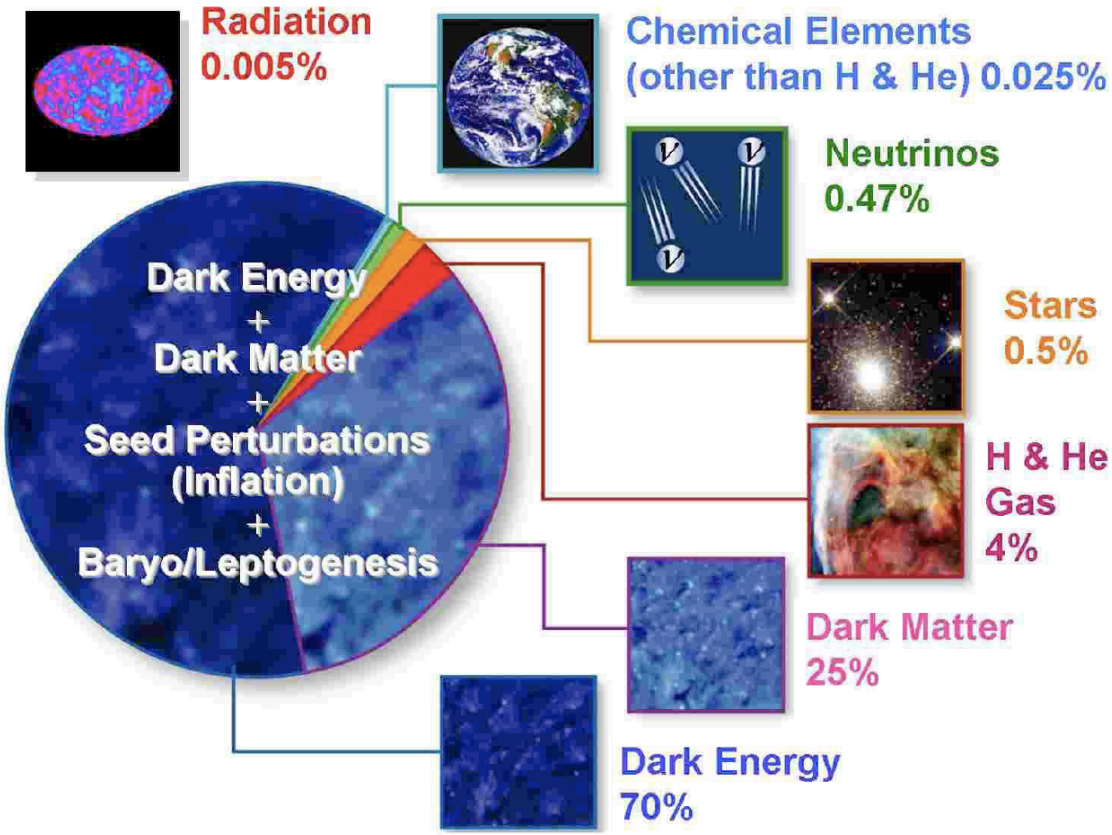
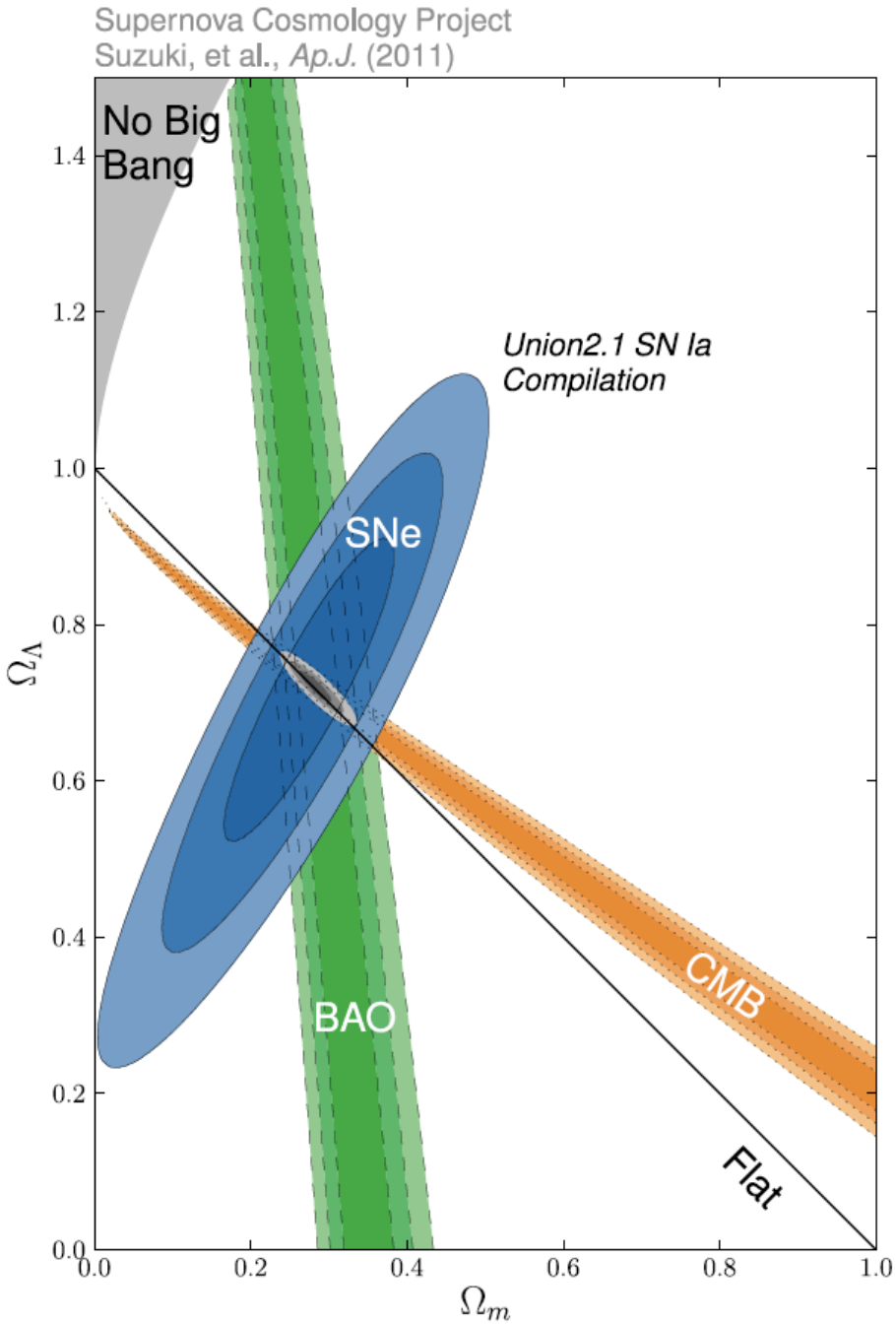


FIG. 1: Elements of the standard ΛCDM cosmological model. Illustrated by the pie is the fraction of the mass-energy of the Universe in various components. Also noted are some other necessary parts of the standard model, namely seed perturbations and baryo/leptogenesis.

From E.W. Kolb, arXiv:0709.3102 ``Cosmology and the unexpected''.

COSMIC CONCORDANCE



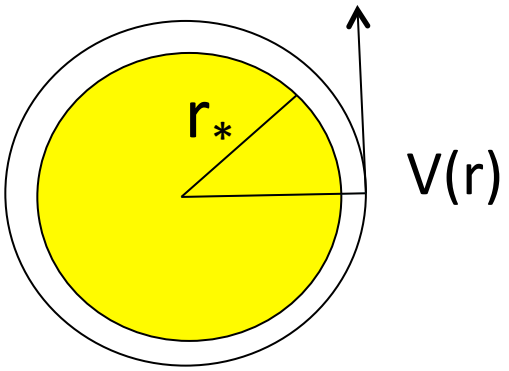
A reminder: Keplerian fall-off

Take rotational velocity in a spiral galaxy:

$$V^2(r) \propto \frac{M(r)}{r}$$

If only luminous matter: M(r) costant for r > r_*:

$$V(r) \propto \frac{1}{r^{1/2}}$$



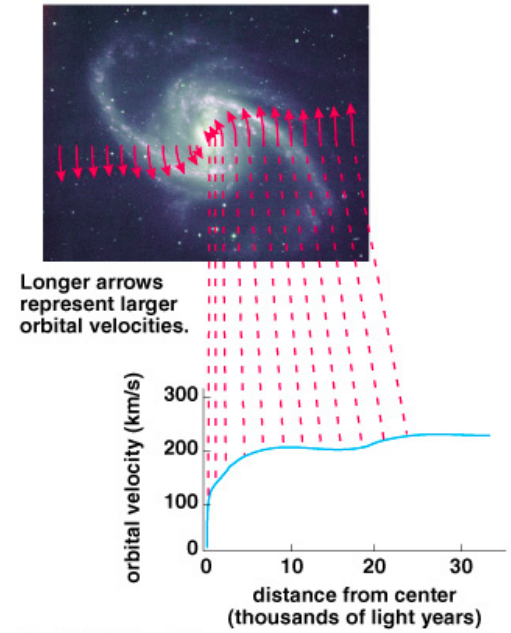
ROTATION CURVES (RC)

Symmetric circular rotation of a disk characterized by

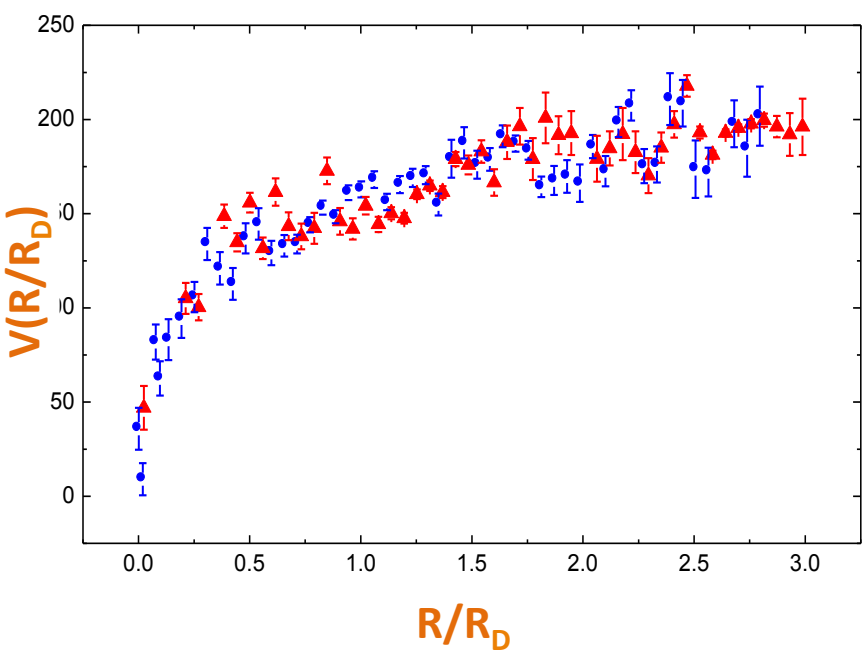
- Sky coordinates of the galaxy centre
- Systemic velocity V_{sys}
- Circular velocity $V(R)$
- Inclination angle

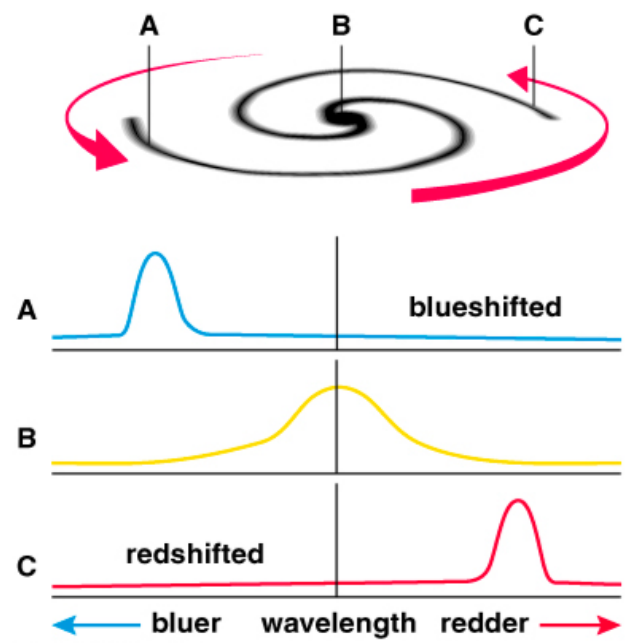
$$V_{obs}(\xi,\eta) = V_{sys} + V(R)\cos\theta\,\sin i$$

θ = azimuthal angle

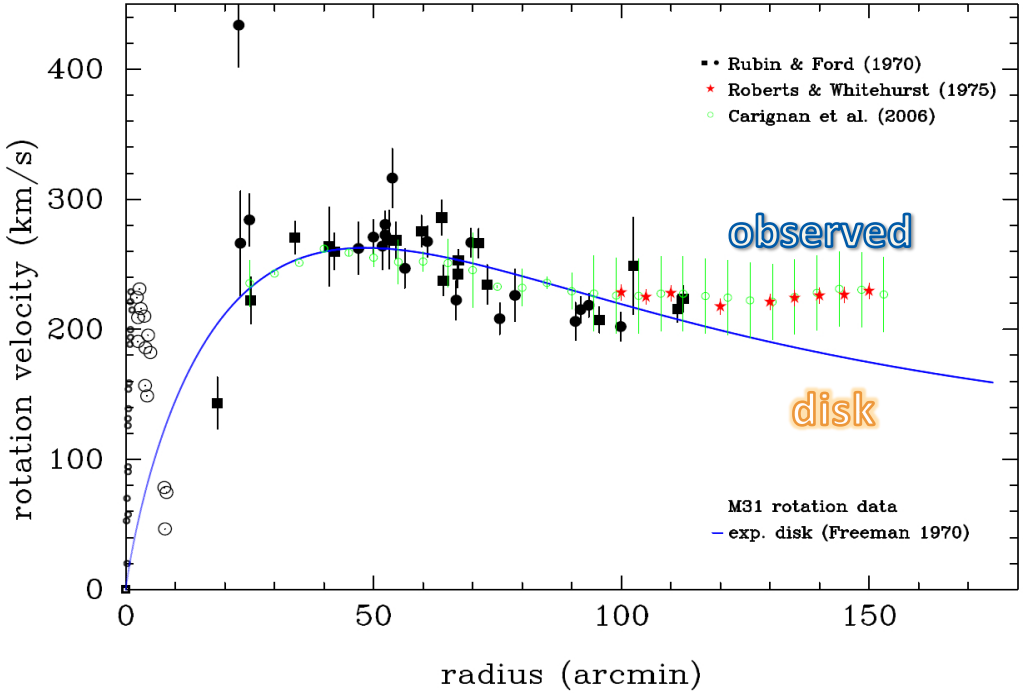


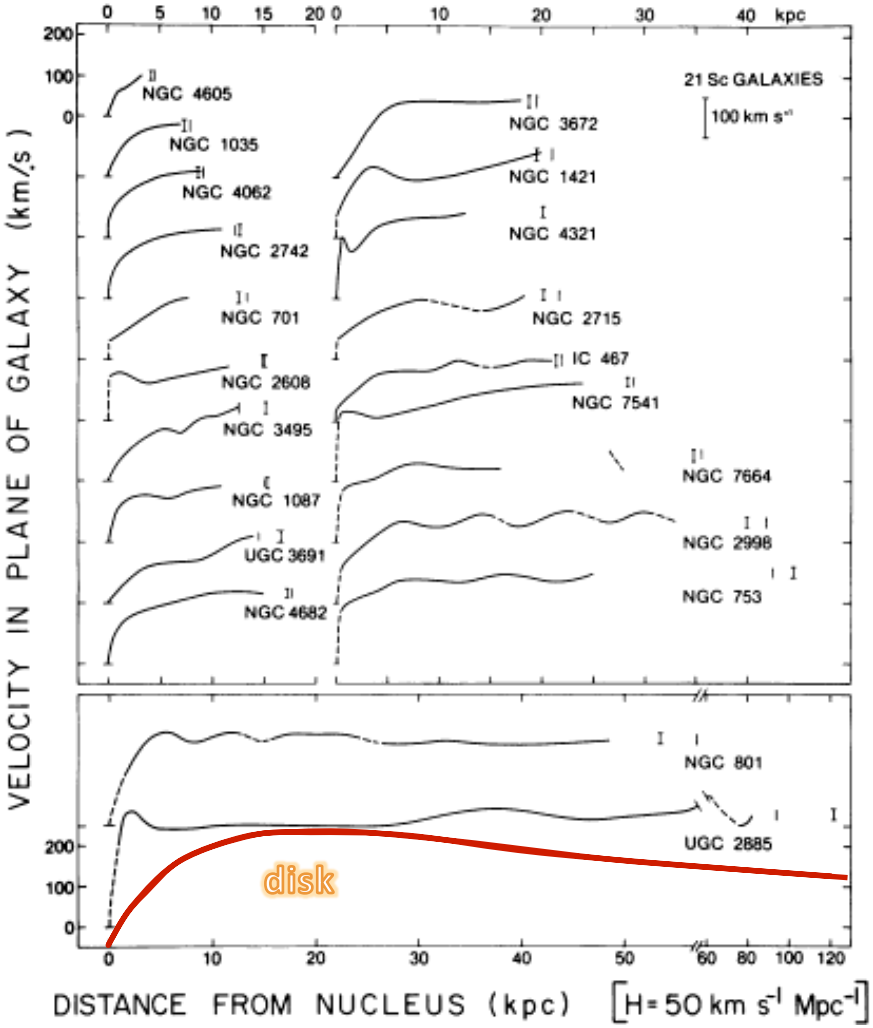
UGC2405 EXEMPLE OF HIGH QUALITY RC





Early discovery from optical and HI RCs



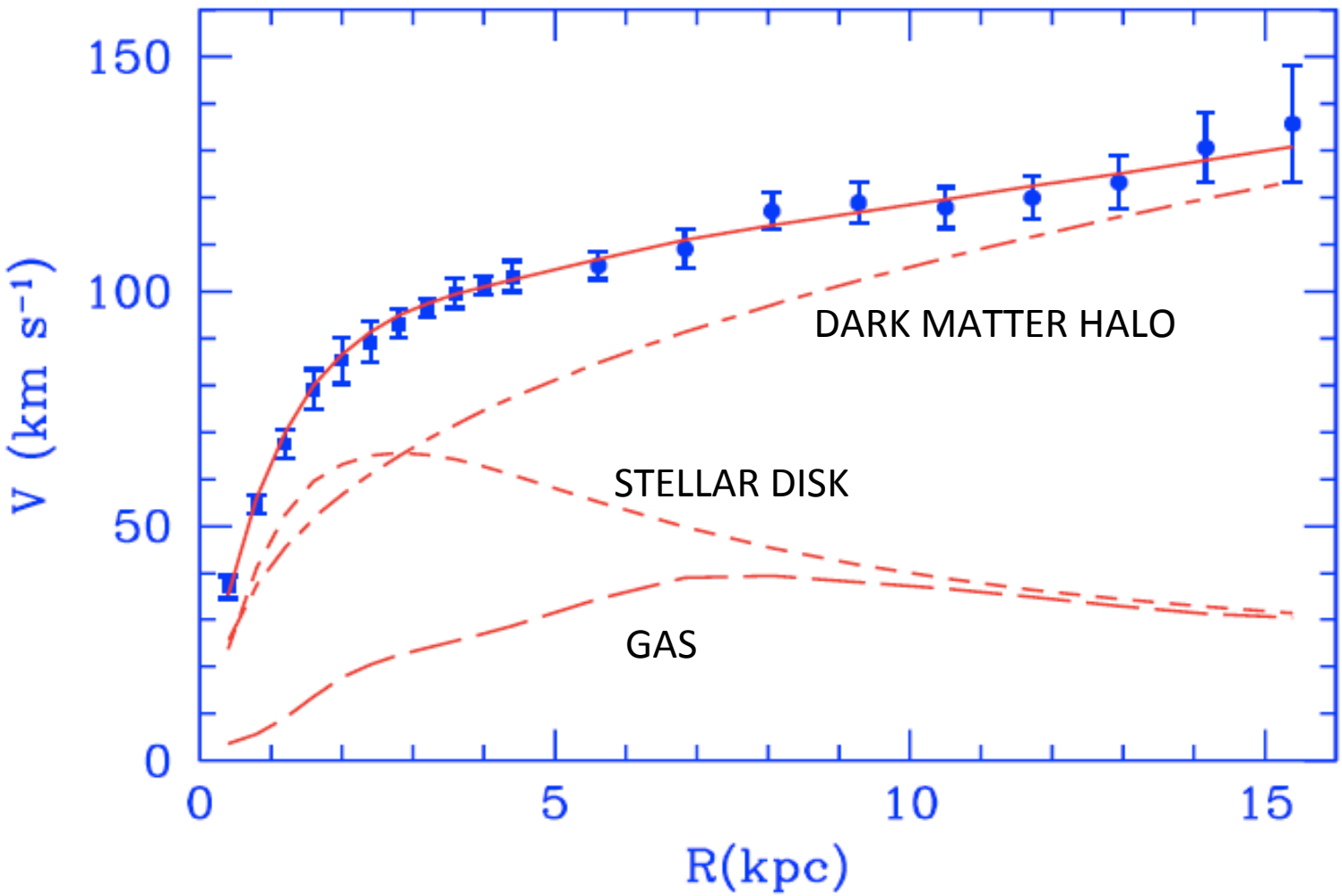


Rubin et al 1980

NO RC FOLLOWS THE STELLAR DISK VELOCITY PROFILE

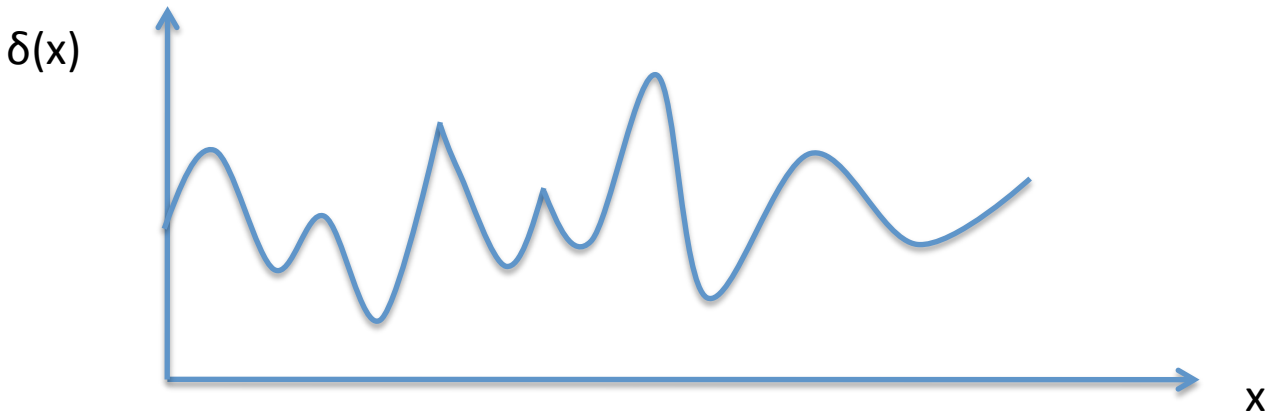
MASS DISCREPANCY AT OUTER RADII

EVIDENCE OF DARK MATTER from RC



From Corbelli & Salucci, MNRAS 311, 441 (2000)

DARK MATTER: POWER-SPECTRUM OF INHOMOGENEITIES



Fourier decomposition of density field

$$\delta(\mathbf{x}) = \frac{\delta\rho}{\bar{\rho}} = \sum_{\mathbf{k}} \delta_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}$$

Dimensionless power-spectrum

$$\Delta^2(k) \equiv \frac{\partial\sigma^2}{\partial \ln k} \propto k^3 |\delta_{\mathbf{k}}|^2 \propto k^{3+n}\, T^2(k)$$

Transfer function

Primordial power-law power-spectrum
(n is the scalar spectral index, n=0.96 from CMB at more than 5 sigma)

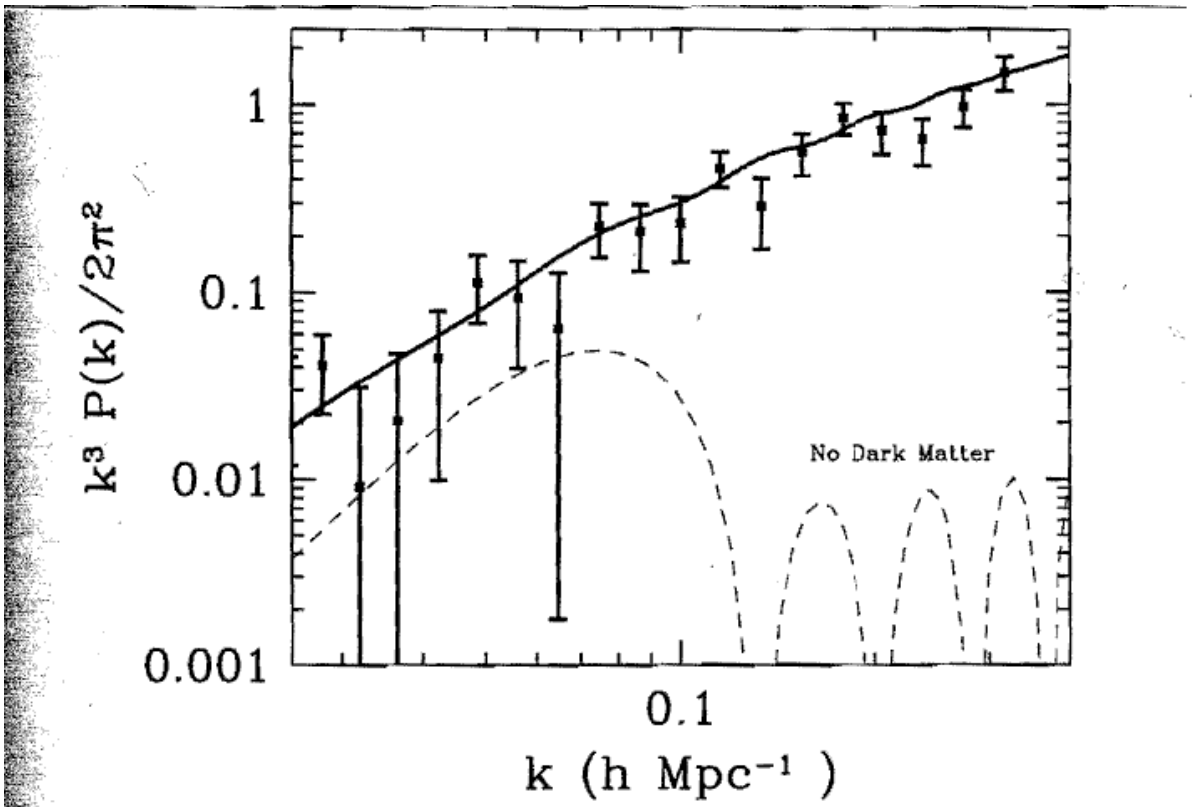
Use Poisson equation $4\pi G\bar{\rho}\,\delta = \nabla^2\Phi \;\Rightarrow\; \delta_{\mathbf{k}} \propto k^2 \Phi_{\mathbf{k}}$

$$|\Phi_{\mathbf{k}}^{\text{primordial}}|^2 \propto k^{n-4}$$

FROM INFLATION

$$\Phi_{\mathbf{k}} = \Phi_{\mathbf{k}}^{\text{primordial}} \times T(k) \times \{\text{Growth function}(a)\}$$

EVIDENCE OF DARK MATTER from a cosmological point of view

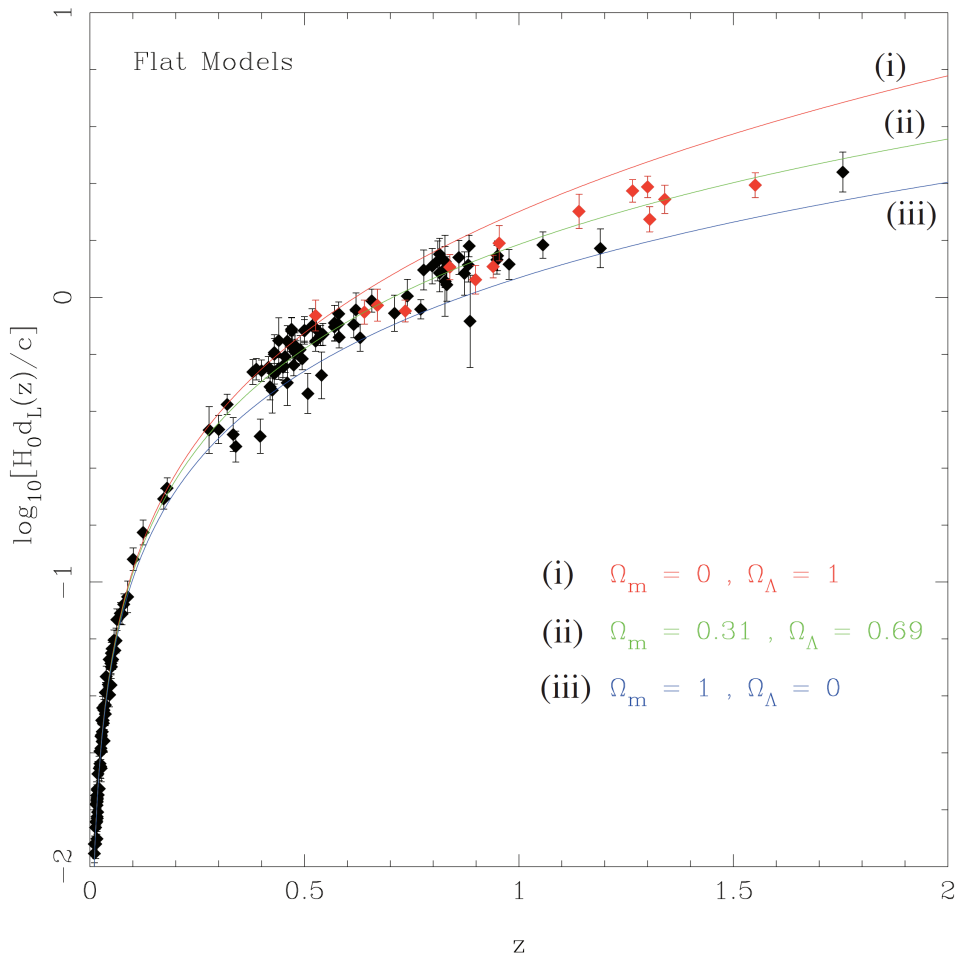


igure 1.13. The variance $\Delta^2 \equiv k^3 P(k)/2\pi^2$ of the Fourier transform of the galaxy dis-ibution as a function of scale. On large scales, the variance is smaller than unity, so the stribution is smooth. The solid line is the theoretical prediction from a model in which the niverse contains dark matter, a cosmological constant, with perturbations generated by infla-on. The dashed line is a theory with only baryons and no dark matter. Data come from the SCz survey (Saunders *et al.*, 2000) as analyzed by Hamilton and Tegmark (2001).

From Dodelson Scott, ``Modern Cosmology'' Academic press 2003.

DARK ENERGY

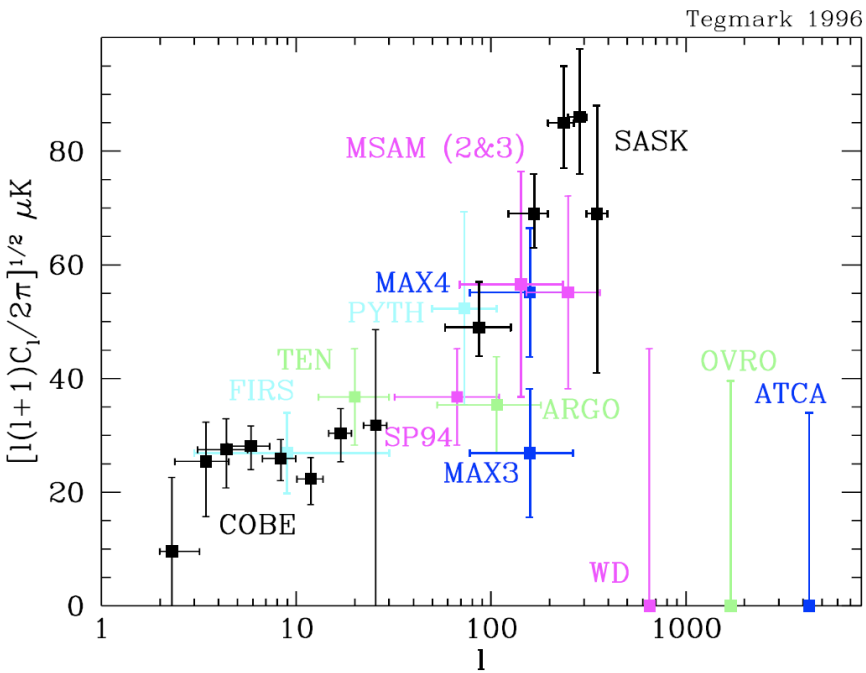
The dominant component in the universe (almost 70%) that is accelerating the universe today:
A cosmological constant Λ (with negative pressure)??

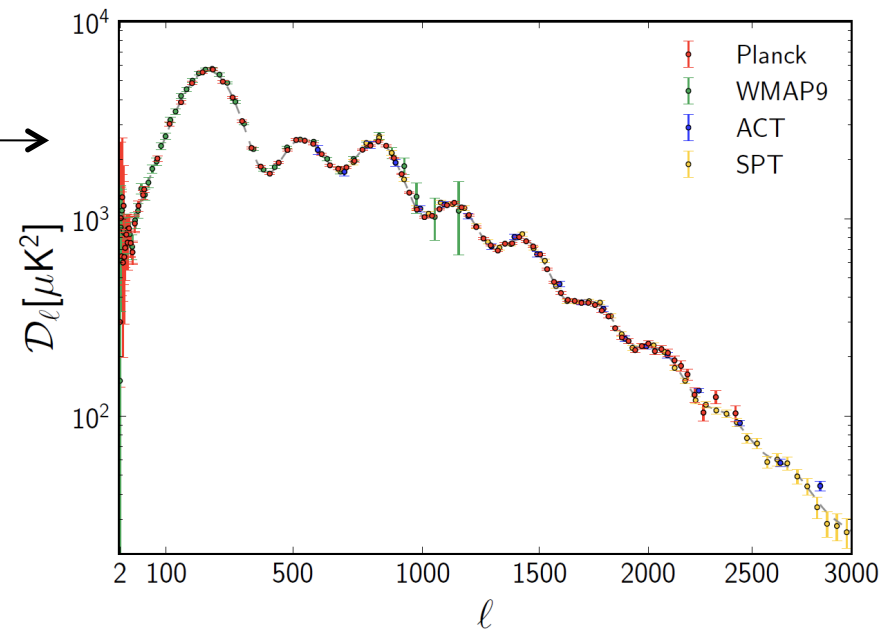


From Choudhury & Padmanabhan A&A 429, 807 (2005)

Enormous Progress!

Temperature CMB anisotropies: from the early 90s till the present





$$\theta \sim \frac{\pi}{\ell}$$

Summaryzing.....

Inflation+dark matter+study of cosmological perturbations

Tools to study cosmological perturbations during (and after) the early universe

How it is produced in the early universe?
(Also models of baryogenesis)

How the first primordial density perturbations have been generated in the early universe?
What are the models of the early universe and their predictions?

INFLATION

- A very early epoch in the history of the universe during which the expansion is accelerated

- a fascinating theory for the generation of the primordial density perturbations, i.e., the seeds of the CMB anisotropies and Large-Scale-Structures

- Inflationary models make definite theoretical predictions about the initial conditions, with specific observable effects today
  (e.g. the pattern of the CMB anisotropies)

- ***Over the last years these basic predictions have been confirmed by observations***

- gravitational waves from inflation are considered a smoking-gun of inflation (not detected yet; for the moment only upper limits from CMB data; interesting interplay with direct interferometric measurements).

References:

two good References are

The first 2 chapters of
- Andrew R Liddle and David H Lyth, Cosmological Inflation and Large-Scale Structure, Cambridge University Press, 2000.

The second chapter contains even more details than those sketched here

The 1st (and 2nd) chapter of
- Dodelson Scott, ``Modern Cosmology'' Academic press 2003.

As far as the recent discovery of the so called "dark baryons" which I mentioned in the first Lecture (when referring to the cosmic/pie, or energy budget of the Universe, see slide 21), an interesting reading can be found at
https://www.newscientist.com/article/2149742-half-the-universes-missing-matter-has-just-been-finally-found/