Geometric methods in Theoretical Physics – 2nd part

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1. Discuss the Energy levels for a quantum-mechanical system in the double well potential

$$V(x) = \begin{cases} V_m & x \in]-\infty, -a] \cup [a, +\infty[\\ V_0 & x \in [-b, b] \\ 0 & \text{otherwise} \end{cases}$$

   where $a > b > 0$ and $V_m > V_0 > 0$.

2. Given the potential $A = f(r)U^{-1}(x)dU(x)$, where $U(x) \in \mathrm{SU(2)}$, find what condition on f to fulfill the (anti-)selfduality equation $F = \pm \star F$.

3. Check the algebra of conformal transformation in 4 dimensions (for both Euclidean and Lorentzian signatures), where P_μ are the generators of Translations $x^\mu \to x^\mu + a^\mu$, D is the generator of Dilatations $x^\mu \to \lambda x^\mu$, K_μ are the generators of special conformal transformations $x^\mu \to \frac{x^\mu - b^\mu x^2}{1 - 2b_\sigma x^\sigma + b^2 x^2}$ and $M_{\mu\nu}$ are the usual generators of Lorentz transformations.

4. Prove that an arbitrary k-form on an almost complex manifold, one can define (p, q) tensors, with $k = p + q$ such that

$$\omega = \sum_{p+q=k} \omega^{(p,q)}.$$

   Show that one can decompose

$$d\left(\omega^{(p,q)}\right) = A^{(p-1,q+2)} + B^{(p,q+1)} + C^{(p+1,q)} + D^{(p+2,q)},$$

   for some $p + q + 1$ forms A, B, C and D.