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1. Consider a generic change of coordinates $x^\mu = x^\mu(\tilde{x})$. Expand it in power series about the point described by x^μ_P. Use this expansion to argue that one can always put an arbitrary metric $g_{\mu\nu}(\tilde{x})$ equal to $\eta_{\mu\nu}$ at the point P, up to corrections of the second order in x.

   Compare the number of conditions needed vs. the number of free parameters in the coordinate transformation at each order in the expansion.

2. Consider the following metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2,$$

   where $d\Omega^2$ is the metric on the 2-sphere $d\Omega^2 = d\theta^2 + \cos^2\theta\, d\phi^2$. Expand this metric close to the point $r^* = 2M$.

3. Discuss the motion of a free particle in Minkowski spacetime using Rindler coordinates, i.e.

$$ds^2 = e^{2a\xi}\left(-d\tau^2 + d\xi^2\right).$$

4. Consider an observer at a fixed value of the ξ coordinate in Rindler spacetime (with the metric given in the previous exercise).
   (a) Find the explicit form of the components of its velocity (u^τ, u^ξ) and of the acceleration $(\alpha^\tau, \alpha^\xi)$.
   (b) Write the equation for the trajectory of a light ray travelling from $\xi = 0$ at $\tau = 0$ to negative values of ξ.
   (c) Give the components of its momentum (p^τ, p^ξ), assuming that in the standard basis of Minkowski spacetime they are $p_0 = p_1 = -E/c$.
   (d) Determine the relation between the frequency of the signal emitted at $\xi = 0$ and the one received by another observer at $\xi = \xi_0 < 0$ (The energy of a particle of momentum p measured by an observer in motion with velocity u is $\mathcal{E} = -p^\mu u_\mu$).

5. Compute the area of a circle on a plane, on a sphere and on a hyperbolic space.
   (a) Write the metric of the plane in terms of angular coordinates and compute the integral giving the perimeter and the area of a circle of unit radius.
   (b) A 2-dimensional sphere can be described as the hypersurface solution to the equation $x^2 + y^2 + z^2 = R^2$ embedded in 3-dimensional flat Euclidean space: $ds_3^2 = dx^2 + dy^2 + dz^2$. Derive the induced metric on the sphere using spherical coordinates to solve the constraint equation *(Assume $R > 1$ from now on).*
   (c) Compute the perimeter of a circle centered at the point $z = R$, $x = y = 0$, as a function of the distance from the center.
   (d) Compute the area for the circle with perimeter 2π.

(e) Now consider the hypersurface described by $-x^2 - y^2 + z^2 = R^2$. Solve the constraint using spherical and hyperbolic coordinates and derive the metric on this 2-dimensional hyperbolic space induced from ds_3^2.

(f) Compute the perimeter of a circle centered at the point $z = R$, $x = y = 0$, as a function of the distance from the center.

(g) Compute the area for the circle with perimeter 2π.