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1. Show that the geodesic equation transforms covariantly under arbitrary change of coordinates.

2. Consider the metric discussed in class describing flat space rotating with fixed angular velocity ω about the z axis

$$ds^2 = -\left[1-\omega^2(x^2+y^2)\right] dt^2 + 2\omega\,(y\,dx - x\,dy)dt + dx^2 + dy^2 + dz^2. \tag{1}$$

   (a) Verify this by expressing the metric in cylindrical coordinates (ρ, ϕ, z) and transforming $\phi \to \phi + \omega t$.

   (b) Obtain the geodesic equations for x, y and z in the rotating frame.
   (You may assume that the only non-zero Christoffel symbols are Γ^x_{tt}, Γ^x_{yt}, Γ^y_{tt} and Γ^y_{tx}. Also note that the metric is not diagonal, so the inverse is not straightforward)

   (c) Show that in the nonrelativistic limit these reduce to the Newtonian equations of motion for a rotating free particle exhibiting the centrifugal and the Coriolis force, i.e.

$$\begin{aligned} \ddot{x} &= \omega^2 x - 2\,\omega\,\dot{y}, \\ \ddot{y} &= \omega^2 y + 2\,\omega\,\dot{x}, \end{aligned} \tag{2}$$

   where $\dot{} \equiv \frac{d}{dt}$ and $\ddot{} \equiv \frac{d^2}{dt^2}$.

3. The hyperbolic plane is defined by the metric

$$ds^2 = \frac{1}{z^2}(dx^2 + dz^2), \qquad z > 0. \tag{3}$$

   (a) Show that vertical lines (i.e. $x = const$) satisfy the geodesic equation and that $z = 0$ is at infinite distance from any point in the upper half plane;

   (b) Show that the other possible geodesics are semicircles centered on the x-axis.
   (Use the parametrization with $(\dot{x}^2 + \dot{z}^2)/z^2 = 1$ to simplify the problem)