---

1. * Consider the hypersurface in $\mathbb{R}^n$ described by $M = \{x \mid f(x) = 0\}$. Show that the kernel of the differential operator on f gives the tangent space to M, namely

$$\ker[df(p)] = T_pM.$$

2. In $\mathbb{R}^2$, let $x^a = (x, y)$ denote Cartesian and $x'^a = (r, \phi)$ plane polar coordinates. Consider the vector fields X, Y, Z of components $X^a = (1, 0)$, $Y^a = (0, -1)$ and $Z^a = (-y, x)$. Find their components in the x' basis.

3. In a coordinate basis show that

$$\Gamma^{\rho}_{\mu\rho} = \partial_\mu(\log\sqrt{-g}),$$

where $g \equiv \det g_{\mu\nu}$. Then show that

$$D_\mu j^\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, j^\mu\right).$$

Note that $\det(\mathbb{1} + \epsilon A) = 1 + \epsilon\, \mathrm{tr} A + O(\epsilon^2)$.

4. Compute the Riemann tensor for the metric of Assignment 2, exercise 2.

5. Consider a 2-sphere with metric $ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2$.
   (a) Show that lines with constant ϕ are geodesics, while lines with constant θ are geodesics only at the equator.
   (b) Write the equations for the parallel transport of a vector V along

$$\theta = \theta_0, \quad \phi = \phi_0 + \tau.$$

   Derive $V^\theta = V^\theta(\tau)$ and $V^\phi = V^\phi(\tau)$, for $V = (1, 0)$ as initial condition. In what case $V(2\pi) = V(0)$?