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1. Check that the metric describing the rotating frame of Assignment 2, can be recovered by means of the following vielbeins:

$$e^0 = dt, \quad e^1 = dx + \omega\, y\, dt, \quad e^2 = dy - \omega\, x\, dt, \quad e^3 = dz.$$

Compute the spin connection and the curvature.

2. Fix the coefficient in front of the expression

$$M_P^2 \int R^{ab} \wedge e^c \wedge e^d\, \epsilon_{abcd}$$

in order to match the Einstein–Hilbert action, with the normalization given in class.

3. The following surface immersed in 5-dimensional Minkowski spacetime, with H constant, describes a hyperboloid:

$$-(X^0)^2 + (X^1)^2 + (X^2)^2 + (X^3)^2 + (X^4)^2 = H^{-2}.$$

Find the metric induced on this hypersurface, if we solve the constraint by

$$\begin{aligned} X^i &= e^{Ht}\, x^i, \qquad \text{for } i = 1,2,3, \\ X^0 - X^4 &= \tfrac{2}{H} e^{Ht}. \end{aligned}$$

This describes de Sitter spacetime in Lemaitre coordinates

(a) Do these coordinates cover the whole spacetime?

(b) Using Cartan's formalism, compute the Ricci tensor and scalar for this spacetime.

(c) Repeat the exercise with the following coordinates:

$$\begin{aligned} X^0 &= H^{-1} \sinh(Ht)\sqrt{1 - H^2 r^2}, \\ X^i &= x^i \qquad \text{for } i = 1,2,3, \\ X^4 &= H^{-1} \cosh(Ht)\sqrt{1 - H^2 r^2}, \end{aligned}$$

and $r^2 = \sum_i (x^i)^2$.