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1. Consider a particle moving radially in the Schwarzschild metric. Compute $\frac{d^2r(t)}{dt^2}$, where $r$ and $t$ are standard Schwarzschild coordinates and compare the result with what one expects from Newton's law. Show in which limit the two coincide. Finally, compute $\frac{d^2r(t)}{dt^2}$ for a photon and show that even in the limit $r \gg 2G_N M$ the result is different from what one expects for a non-relativistic particle.

2. An observer in circular motion in the Schwarzschild metric has velocity described by the vector
$$u^\mu = A\,(\delta_0^\mu + \Omega\,\delta_\phi^\mu).$$
   (a) Fix $A$ and discuss the result.
   (b) Compute the acceleration and the value of $\Omega$ that makes the trajectory a geodesic.

3. A static observer at $r = R_1$, $\theta = \pi/2$, $\phi = \phi_1$ in Schwarzschild spacetime sends a light signal with wavelength $\lambda$ towards another static observer at $r = R_2 > R_1$, $\theta = \pi/2$, $\phi = \phi_2$. Compute the wavelength measured by the second observer (using energy conservation along geodesic motion). Does this depend on the angles $\phi_1$ and $\phi_2$?

4. Draw the surface
$$x^2 + y^2 = \left(\frac{z^2}{8m} + 2m\right)^2$$
embedded in flat Euclidean $\mathbb{R}^3$. Compute the metric induced on the same surface, using polar coordinates on the $x, y$ plane, and compare it with sections of the Schwarzschild metric at fixed time.

5. Consider the motion of a massive particle in the Schwarzschild background.
   (a) Draw the potential as a function of the angular momentum $l$ and compare it with the Newtonian one. Check the difference of the allowed region for bound orbits and explain the difference.
   (b) When do you find an unstable circular orbit? What is the minimum distance from the horizon for this orbit?
   (c) Compute how long is one year for such orbits, from the point of view of the orbiting observer as well as from the point of view of a distant observer.