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1. Compute the stress-energy tensor for the electromagnetic field described by the action

$$-\frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu}$$

and specify its form for the configuration

$$F = p\,\sin\theta\,d\theta\wedge d\phi + \frac{q}{r^2}dr\wedge dt,$$

assuming that the metric has the following form:

$$ds^2 = -f(r)^2dt^2 + f(r)^{-2}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

2. Check Einstein's equations for the stress-energy tensor computed in the previous exercise.

3. * Take the Kerr–Newman metric, compute the area of the horizons and, using the expression for the angular momentum and the radius of the horizon, find a formula for the mass of the black hole in terms of the charges. By varying the resulting expression check the first law of black hole mechanics, where

$$\kappa = \frac{1}{2}\frac{r_+ - r_-}{r_+^2 + a^2}, \quad \phi = e\,\frac{r_+}{r_+^2 + a^2}, \quad \Omega = \frac{a}{2(r_+^2 + a^2)}.$$

4. * In the movie Interstellar there is a planet orbiting a massive black hole, such that 1 hour on the planet corresponds to 7 years on Earth. Assume that the black hole is described by the Kerr metric and that the planet is in geodesic motion in the innermost circular orbit. What are the allowed values of mass and the angular momentum compatible with the situation depicted in the movie?