PHASE TRANSITIONS IN NUCLEAR SHAPE

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This subject has received considerable attention in the last few years, both on the theoretical and on the experimental side. It has been first observed by F.Iachello [1] that a new class of dynamic symmetries can be associated with the critical point of the phase transition. The first example of it is the E(5) symmetry related to the phase transition between the harmonic oscillator and the γ-unstable limit (symmetries U(5) and O(6) in the IBA representation). Short later, the new symmetry X(5) has been found [2] to be associated with the phase transition between harmonic oscillator and axially symmetric rotor (U(5) and SU(3), respectively). It has also been noted (by Jolie, Casten, von Brentano and Werner) [3]) that the O(6) vertex of the Casten triangle can be interpreted as the critical point in the phase transition between the (axially symmetric) prolate and oblate deformations. Other aspects of phase transitions in nuclei are discussed in the ref.s [4, 5, 6, 7, 8, 9].

Several experimental works have provided evidence for the new symmetries in nuclei at the borderline between a spherical and a deformed region. The E(5) symmetry has been first observed in the ^{134}Ba nucleus by Casten and Zamfir [10], and later also in ^{94}Ru by Frank, Alonso and Arias [11] in their investigation of th Ru isotope chain.

Nuclei close to the X(5) symmetry in the region $A \approx 150$ have been investigated by Casten and Zamfir (^{152}Sm)[12] and by Krücken *et al.* (^{150}Nd) [13], while, in a different transitional region, a phenomenological study of ^{104}Mo has been reported by the Florence group (see below). New researches in this field are under way, and a few results of them are expected to be reported in the Erice Conference.

The contributions from italian groups to this field of research are discussed with more details in the following. They include a theoretical work by Fortunato and Vitturi [3] on a new analytical solution of the Bohr hamiltonian with two $\gamma-$independent potentials in the β variable, namely a Coulomb-like and a Kratzer-like potentials displaying a SO(2,1) algebraic structure, and two works by the group of Florence, the first [1] of which discusses the level scheme of ^{104}Mo as an example of the X(5) symmetry while the second, presented at the NS2002 Conference in Legnaro [17] gives some preliminary results of an investigation of possible phase transitions in the octupole mode. More results on this particular subject, including a new parameterization scheme for simultaneous quadrupole and octupole deformations close to the axial limit [3], will be presented at the Erice Conference.

In the following, this section will report a summary of the work of the group of Padova, the reprints of the two published papers of the group of Florence and a short "addendum" with a few results of the work under way, part of which will be the object of the talk to be presented at the Erice conference.

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New analytically solvable $\gamma-$unstable hamiltonians.

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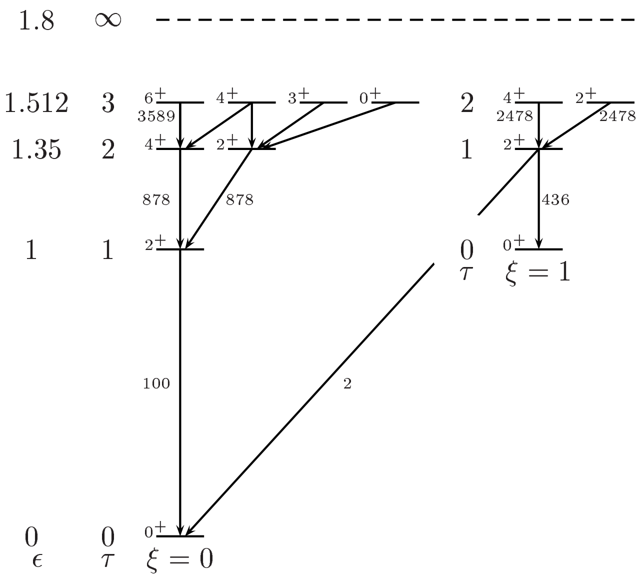
In the case of the phase transition from spherical to $\gamma-$unstable nuclei, the E(5) dynamical symmetry assumes, for the potential in the β variable, an infinite square-well potential [1]. This represents a critical point between an harmonic oscillator and a potential with a minimum for a finite value of β. It has the remarkable advantage of yielding an exactly solvable Bohr hamiltonian.
The purpose of this investigation [3] was to show that, still within the condition of $\gamma-$instability, there are other classes of analytically solvable potentials, whose underlying algebraic structure is associated to the group SO(2,1).
The Bohr hamiltonian for a $\gamma-$unstable potential (hence dependent only on β) may be separed and the second order differential equation in β may be recast in its standard form with a simple substitution. This equation, without discussing the details, reads:

$$\chi''(\beta) + \Big\{ \epsilon - u(\beta) - \frac{(\tau+3/2)^2}{\beta^2} + \frac{1}{4\beta^2} \Big\} \chi(\beta) \; = \; 0. \qquad (1)$$

This equation is transformed in the Whittaker differential equation when the reduced potential $u(\beta)$ takes two particular forms: a Coulomb-like form $u(\beta) = -A/\beta$ and a Kratzer-like one $u(\beta) = -A/\beta + B/\beta^2$. Eigensolutions of this particular cases are the Whittaker's functions (that may be further reduced to expressions involving Laguerre polynomials) and the spectra acquire simple regular patterns. In the case of Coulomb-like case the spectrum is akin to the well-know spectrum of the hydrogen atom, albeit in a space of dimesion 5 and with different meanings of quantum numbers and costants:

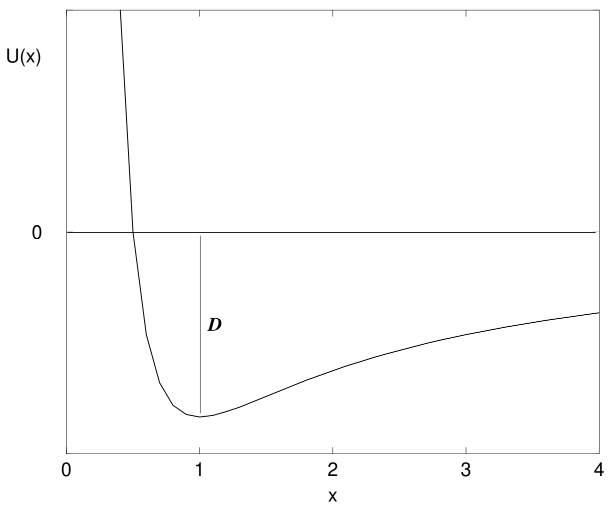
$$\varepsilon_{\tau,\xi} = \frac{A^2/4}{(\tau+\xi+2)^2} \qquad (2)$$

where the τ quantum number is associated with the SO(5) invariance and ξ labels different bands. The latter comes from the fact that the hypergeometric function hidden in the Whittaker's function should be a finite series and hence the first argument must be a negative integer. This condition fixes the spectrum that we show, in reduced units (independent on the overall constant A), in fig.1.



This spectrum displays very peculiar features: there is a threshold at 1.8 that corresponds to an infinite quantum number and there is degeneracy of the τ-th member of the ξ-th band with the $(\tau-1)$-th state of the $(\xi+1)$-th band.

This degeneracy is removed if one introduces in the Bohr hamiltonian the Kratzer-like potential. Playing with the parameters one can move from a spherical situation ($B \to 0$ or Coulomb-like limit) to a deformed one. An example of parameterization is shown in fig. 2 where the horizontal scale is expressed in the adimensional variable β_0/β, while the vertical scale is in units of the depth of the pocket ($\mathcal{D}$). The minimum corresponds to the value β_0, as one can easily see rewriting the Kratzer potential as:



$$u_K(\beta) = -2\mathcal{D}\Big(\frac{\beta_0}{\beta} - \frac{1}{2}\frac{\beta_0^2}{\beta^2}\Big) \qquad (3)$$

Whenever one of the two parameters $\{\beta_0, \mathcal{D}\}$ is fixed two limits may be evidenced: when the second parameter is very small we regain the Coulomb-like spectrum, while when the parameter is very big we obtain a typical $\beta-$rigid, $\gamma-$unstable O(6) spectrum that displays only one band with a $\tau(\tau+3)$ spacing rule. We have also calculated electromagnetic transition rates for the most important transitions in the various case. It is worth noticing that in the Coulomb-like limit of the Kratzer case we regain not only the energy levels, but also the B(E2) values.

From an algebraic point of view, the hamiltonian may be recast as a linear combination of the infinitesimal generators of the SU(1,1) group, which have the commutation relations:

$$[\hat{Z}_1, \hat{Z}_2] = -4i\hat{Z}_3 \quad [\hat{Z}_3, \hat{Z}_2] = -2i\hat{Z}_2 \quad [\hat{Z}_3, \hat{Z}_1] = 2i\hat{Z}_1. \qquad (4)$$

There are four non-compact isomorphic Lie algebras su(1,1) $\sim$ so(2,1) $\sim$ sl(2,R)$\sim$ sp(2,R) that can be thus associated with this problem. The chain of subalgebras that gives the labels of the set of orthonormal states $\{|\ \xi\tau\alpha LM\rangle\}$ is explicitly given as [4]:

$$\begin{array}{ccccccccc} SU(1,1) & \times & SO(5) & \supset & U(1) & \times & SO(3) & \supset & SO(2) \\ \lambda & & \tau & \alpha & \xi & & L & & M \end{array} \qquad (5)$$

where λ is an SU(1,1) lowest weight and α indexes the SO(3) multiplicity.

The possibility of analytic solutions in the presence of a Kratzer potential is not limited to the case of $\gamma-$instability. Following Iachello[2] one can assume a harmonic dependence on the γ variable, as an approximation to the true potential in the region around $\gamma \simeq 0^o$. This potential is expected to describe an axially symmetric deformed rotor. An approximate separation of variables for the Bohr hamiltonian can still be adopted for potentials of the type $u(\beta)+v(\gamma)$. The total spectrum is thus composed of two parts: $\epsilon \simeq \epsilon_\beta + \epsilon_\gamma$. Insering our choice of the potentials in the hamiltonian we find, for the $\beta-$part of the spectrum, a general structure that resembles the one we have presented for the $\gamma-$unstable case.

A preliminary account of this case will be presented at the Erice Conference by one of the authors (L.F.). The spectrum for the Coulomb-like potential in β plus an harmonic

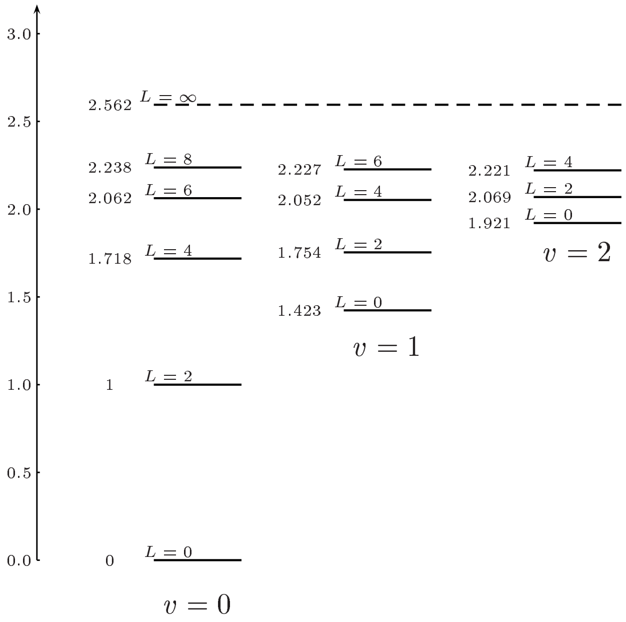


Figure 1: $\beta-$spectrum of the Coulomb-like rotor ($n_\gamma = 0$). The vertical scale is expressed in units of the energy of the ($v = 0, L = 1$) state.

oscillator in γ is easily found to be

$$\epsilon^{(\beta)}_{v,L} \quad = \quad \frac{A^2/4}{\left(\sqrt{\frac{9}{4}+\frac{L(L+1)}{3}}+\frac{1}{2}+v\right)^2}. \tag{6}$$

This spectrum (fig. 1) looks rather similar to the one studied in the case of $\gamma-$instability. Nevertheless it is worth mentioning that the ratio of the ($v = 0, L = 4$) to the ($v = 0, L = 2$) energies is 1.718, while the ratio of the ($v = 1, L = 0$) (beginning of the second band) to the ($v = 0, L = 2$) energies is 1.423. Both this values may be very interesting for a comparison with experimental data. Other peculiarities are the presence of the threshold, corresponding to an infinite L quantum number, that is located at 2.5616 and the absence of the deceneracy pattern found in the $\gamma-$unstable case.

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Further comments on the excited γ bands in ^{104}Mo.

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In our previous paper [1], the theoretical predictions [2] for the critical–point symmetry X(5) have been compared with the existing experimental data on the ^{104}Mo nucleus. A rather good agreement has been found, not only for the $K = 0$ ground–state band, but also for the excited bands with $n_\gamma = 1$, $K = 2$ and $n_\gamma = 2$, $K = 4$, when the value of $\langle\beta^2\rangle$, introduced in the eq. (4) of ref. [2], is kept constant. If, however, $\langle\beta^2\rangle$ is identified with the expectation value of β^2 over the proper eigenfunction of the β mode, one should expect a 30% increase in the value of this parameter when the angular momentum goes from 2 to 10 (see table II of ref. [1]).
Here we suggest an alternative form of the potential which, still leaving unchanged the results for the $K = 0$ band, leads to a simpler behaviour for the part of the Hamiltonian which depends on γ.
We can start from eq.s (4) of ref. [2]. Here, however, in the second equation of the system we reinsert the current value of β^2 in the place of its average value:

$$\left[-\frac{1}{\beta^4}\,\frac{\partial}{\partial\beta}\,\beta^4\frac{\partial}{\partial\beta}+\frac{1}{4\beta^2}\,\frac{4}{3}\,I(I+1)+u(\beta)\right]\xi_I(\beta)=(\epsilon-\epsilon_\gamma)\,\xi_I(\beta)\ , \tag{7}$$

$$\left[-\frac{1}{\beta^2}\,\frac{1}{\sin 3\gamma}\,\frac{\partial}{\partial\gamma}\sin 3\gamma\,\frac{\partial}{\partial\gamma}+\frac{1}{4\beta^2}\,K^2\left(\frac{1}{\sin^2\gamma}-\frac{4}{3}\right)+v(\gamma)\right]\eta_K(\gamma;\beta)=\epsilon_\gamma\;\eta_K(\gamma;\beta)\ . \tag{8}$$

Let us consider the limit of eq. 7 for $\gamma \ll 1$, with a potential v that is quadratic in the product $\beta\gamma$ (in this limit, $\beta\,\gamma \approx \sqrt{2}\;a_2$):

$$\left[\frac{1}{\beta^2}\,\frac{1}{\gamma}\,\frac{\partial}{\partial\gamma}\,\gamma\frac{\partial}{\partial\gamma}-\frac{K^2}{4\beta^2\gamma^2}+\frac{c}{2}\,(\beta\gamma)^2+\epsilon_\gamma+\frac{K^2}{4\beta^2}\right]\eta_K(\gamma;\beta)=0 \tag{9}$$

(here, we have used an expansion up to the second order for $1/\sin^2\gamma \approx (1/\gamma^2)+1/3$). If – as it is the case – the typical frequency of the γ vibrations is much higher than those of the β motion, we can use the adiabatic approximation and evaluate the eigenfunctions of the γ oscillator for each value of the (slowly varying) parameter β. We define $\tilde\epsilon_\gamma = \epsilon_\gamma + K^2/(4\beta^2)$ and we obtain

$$\left[\frac{1}{\gamma}\,\frac{\partial}{\partial\gamma}\,\gamma\frac{\partial}{\partial\gamma}-\frac{(K/2)^2}{\gamma^2}+\frac{c}{2}\,\beta^4\,\gamma^2+\beta^2\,\tilde\epsilon_\gamma\right]\eta_K(\gamma;\beta)=0 \tag{10}$$

and the eigenvalues of $\beta^2\tilde\epsilon_\gamma$ are given by $(n_\gamma+1)\sqrt{c}\,\beta^2$. In conclusion, we obtain

$$\tilde\epsilon_\gamma = (n_\gamma+1)\,\sqrt{c}\ , \tag{11}$$

independent of β. This is, indeed, only an approximation. The correct treatment of the change of variables from the $\beta-\gamma$ pair to the pair $x = \beta$, $y = \beta\gamma$ leads to additional terms in the coefficients of the derivatives and to a new mixed–derivative term. These new terms, however, would be small (at most, of the order of γ) and will be neglected.

Table 1: Ratios of excitation energies over the band head, $E(I,K) - E(I=K,K)$, to the energy of the first excited state, $E(2,0)$, for the original X(5) model and for the present model – indicated here as $\tilde{\mathrm{X}}(5)$ – compared with those of ^{104}Mo.

	$n_\gamma = 0,\ K = 0$			$n_\gamma = 1,\ K = 2$			$n_\gamma = 2,\ K = 4$		
I^π	X(5)	$\tilde{\mathrm{X}}(5)$	^{104}Mo	X(5)	$\tilde{\mathrm{X}}(5)$	^{104}Mo	X(5)	$\tilde{\mathrm{X}}(5)$	^{104}Mo
2^+	1.00	1.00	1.000	0.00	0.00	0.000			
3^+				0.86	0.90	1.124			
4^+	2.91	2.91	2.917	1.90	1.96	2.094	0.00	0.00	0.000
5^+				3.10	3.19	3.451	1.19	1.29	1.253
6^+	5.43	5.43	5.619	4.43	4.53	4.745	2.53	2.68	2.602
7^+				5.89	6.00	6.370	3.99	4.20	4.107
8^+	8.48	8.48	8.959	7.48	7.60	7.577			

Now, we introduce the value of ϵ_γ in the eq. 7 and we chose, as in ref [2], a square–well potential $u(\beta) = 0$ for $0 \leq \beta \leq \beta_W$ and $= +\infty$ elsewhere. We finally define, as in ref [2], $\tilde{\xi}(\beta) = \beta^{3/2}\ \xi(\beta)$, to obtain

$$\frac{\partial^2 \tilde{\xi}}{\partial \beta^2} + \frac{1}{\beta}\frac{\partial \tilde{\xi}}{\partial \beta} + \left[\epsilon - (n_\gamma + 1)\sqrt{c} - \frac{1}{\beta^2}\left(\frac{I(I+1)}{3} + \frac{9}{4} - \frac{K^2}{6}\right)\right]\tilde{\xi}_{I,K}(\beta) = 0\ . \qquad (12)$$

If we define $\epsilon_\beta \equiv k_\beta^2 = \epsilon - (n_\gamma + 1)\sqrt{c}$, $z = \beta k_\beta$, and $\nu = \sqrt{I(I+1)/3 + 9/4 - K^2/6}$ we finally obtain, as in the ref. [2], that $\tilde{\xi}(z)$ is solution of a Bessel equation of irrational order ν, with the boundary condition $\tilde{\xi}(\beta_W k_\beta) = 0$. In conclusion, the present approach gives the same predictions of ref. [2] for the $K = 0$ bands, while for those with $K \neq 0$ our predictions deviates slightly from those of ref. [2]. Namely, we find again

$$E(s, I, n_\gamma, K, M) = E_0 + B\ (x_{s,\nu})^2 + A n_\gamma + C K^2 \qquad (13)$$

with $x_{s,\nu}$ the s^{th} zero of the Bessel function $J_\nu(x)$, but the value of the index ν turns out to depend on K, while the parameters E_0, A, B and C are now predicted to be strictly constant. The comparison of the predicted excitation energies with those of the original X(5) model and with the empirical values for ^{104}Mo are given in the Table 1.
More details will be reported elsewhere [3]. Here, we only add a final comment: two different (and both sensible) approximations of the quantum Hamiltonian are obtained with a different choice of the restoring force that keeps the nuclear surface close to the axial shape. Their predictions differ slightly (as one could expect) for the non–axial oscillations, but are exactly equal in the case of axial shape. This result can encourage us in treating with similar approximations the axial limit of the combined quadrupole + octupole modes, as it can be found in the next paper of this section.

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