

# SBL Electron Neutrino Disappearance



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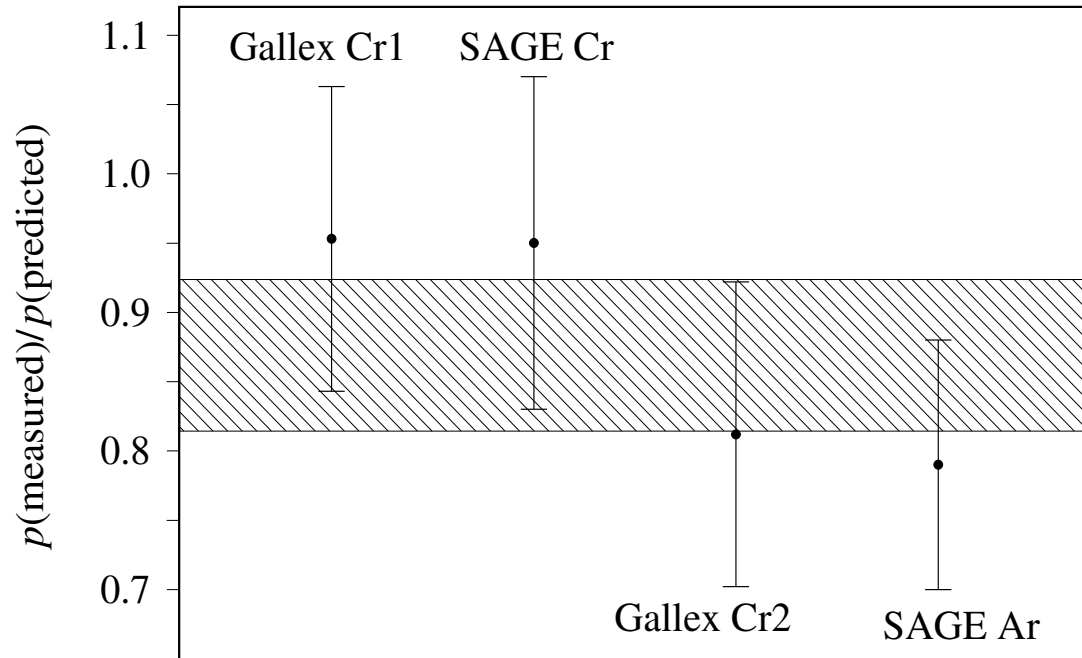
EURO $\nu$  Workshop - 24 March 2009

work in collaboration with Carlo Giunti

## Active-Sterile $\nu$ mixing ?

- Charged massive spin 1/2 particles can have only a Dirac mass term
- According to Majorana neutral massive spin 1/2 particles can coincide with their antiparticles having a Majorana mass term.
- Non-SM right-handed neutral particles can have both Dirac and Majorana mass terms.
- If these non-SM right-handed neutral particles are light (sterile neutrinos  $\nu_s$ ) can mix with ordinary active neutrinos.
- The observable effect is a disappearance of active neutrinos.
- We focus on SBL Gallium, Miniboone and Reactors experiments.

# $\nu_e$ Disappearance in Gallium radioactive source experiments



$R \equiv$  wheighted average value of the ratio of measured and predicted  $^{71}\text{Ge}$  production rates (p) :

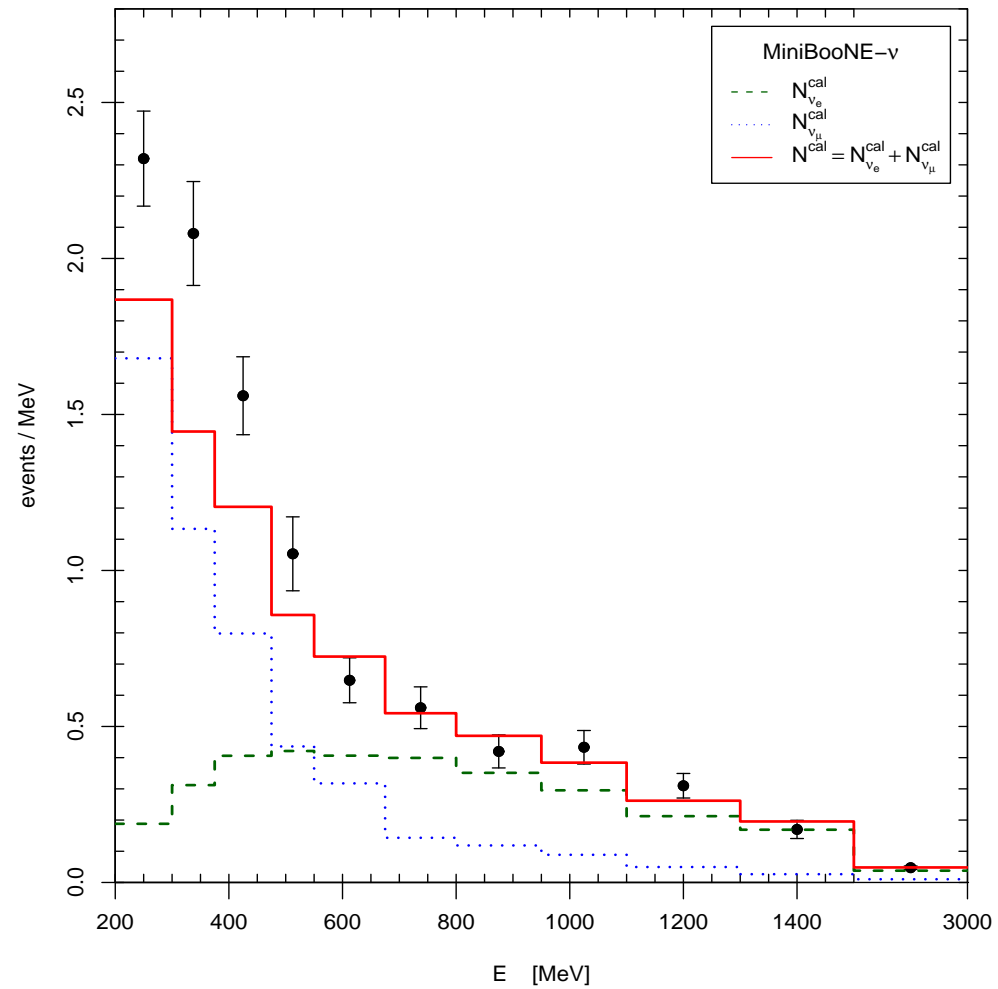
$$R \equiv \frac{p(\text{measured})}{p(\text{predicted})} = 0.87 \pm 0.05$$

[arXiv:0901.2200\[nucl-ex\]](https://arxiv.org/abs/0901.2200)

Ga radioactive source exp. results may be interpreted as an indication of the disappearance of  $\nu_e$  due to active-sterile oscillations!

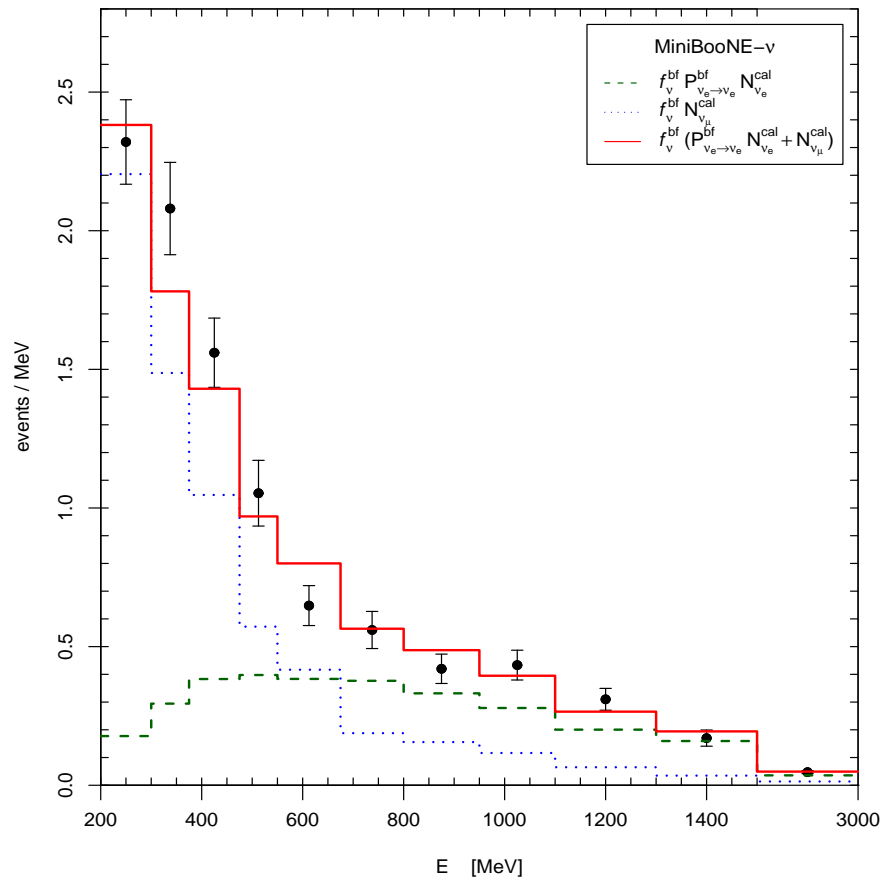
[hep-ph/0610352](https://arxiv.org/abs/hep-ph/0610352) Carlo Giunti & ML

# Miniboone- $\nu$ data : Low Energy Excess or ...



arXiv:0812.2243

# $\nu_e$ Disappearance in Miniboone- $\nu$ data



arXiv:0902.1992v1 and Phys. Rev. D 77, 093002 (2008) C.Giunti & ML

A renormalization of the absolute event rate by a constant factor  $f_{\nu}$  ( $\Delta f_{\nu} = 0.15$ ) with a simultaneous disappearance of the  $\nu_e$  in the beam .

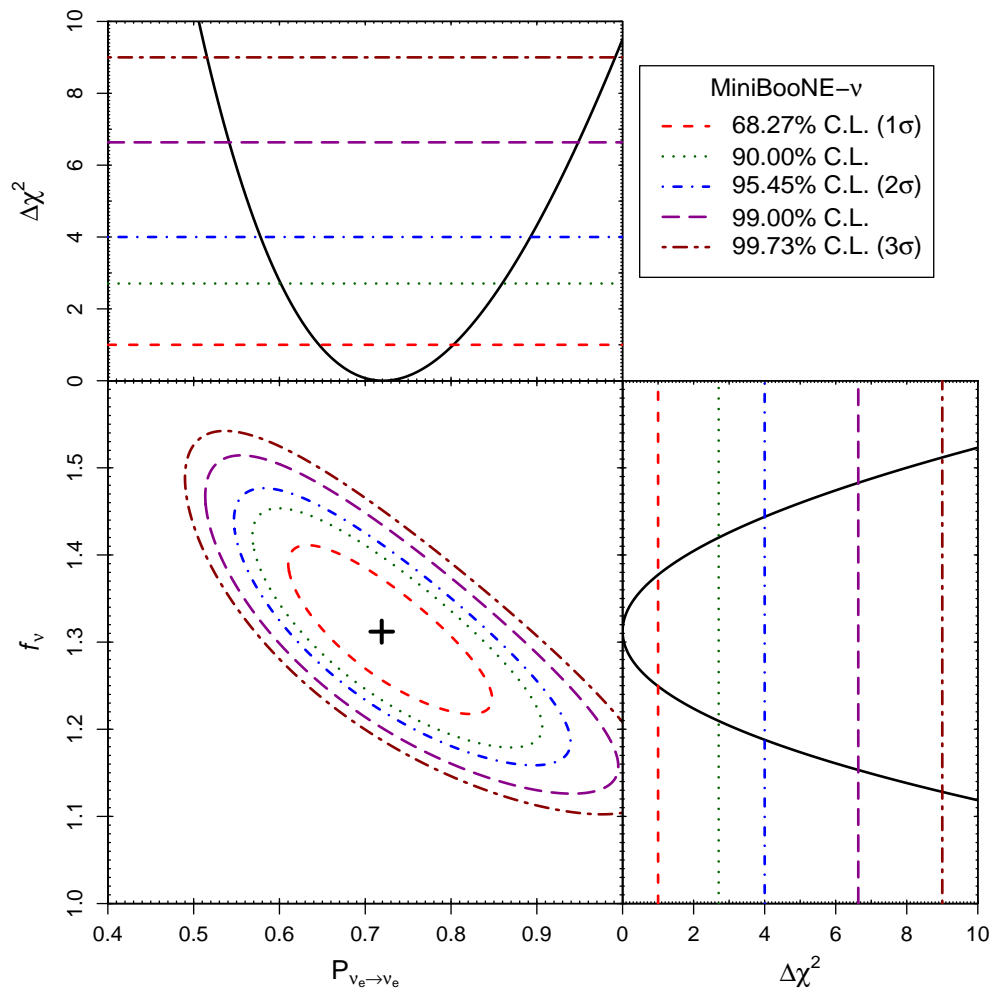
A constant  $P_{\nu_e \rightarrow \nu_e} \leftrightarrow \Delta m^2 \gtrsim 20 \text{ eV}^2$  .

| $j$ | Energy Range [MeV] | $N_{\nu_e,j}^{\text{cal}}$ | $N_{\nu_\mu,j}^{\text{cal}}$ | $N_{\nu,j}^{\text{cal}}$ | $N_{\nu,j}^{\text{exp}}$ |
|-----|--------------------|----------------------------|------------------------------|--------------------------|--------------------------|
| 1   | 200 – 300          | 24.2                       | 162.4                        | 186.7                    | 232                      |
| 2   | 300 – 375          | 21.8                       | 86.4                         | 108.2                    | 156                      |
| 3   | 375 – 475          | 39.4                       | 81.2                         | 120.6                    | 156                      |
| 4   | 475 – 550          | 29.5                       | 34.5                         | 64.1                     | 79                       |
| 5   | 550 – 675          | 47.0                       | 42.4                         | 89.4                     | 82                       |
| 6   | 675 – 800          | 47.0                       | 19.7                         | 66.7                     | 70                       |
| 7   | 800 – 950          | 49.1                       | 20.0                         | 69.1                     | 64                       |
| 8   | 950 – 1100         | 41.8                       | 16.4                         | 58.2                     | 65                       |
| 9   | 1100 – 1300        | 41.2                       | 12.1                         | 53.3                     | 63                       |
| 10  | 1300 – 1500        | 29.1                       | 9.7                          | 38.8                     | 34                       |
| 11  | 1500 – 3000        | 54.5                       | 18.2                         | 72.7                     | 73                       |

We calculate the best fit values of the parameters  $P_{\nu_e \rightarrow \nu_e}$  and  $f_\nu$  by minimizing the least-square function :

$$\chi_{\text{MB-}\nu}^2 = \sum_{j=1}^{11} \frac{(N_{\nu,j}^{\text{the}} - N_{\nu,j}^{\text{exp}})^2}{N_{\nu,j}^{\text{the}}} + \left( \frac{f_\nu - 1}{\Delta f_\nu} \right)^2, \quad N_{\nu,j}^{\text{the}} = f_\nu \left( P_{\nu_e \rightarrow \nu_e} N_{\nu_e,j}^{\text{cal}} + N_{\nu_\mu,j}^{\text{cal}} \right)$$

# Fit to Miniboone- $\nu$ data



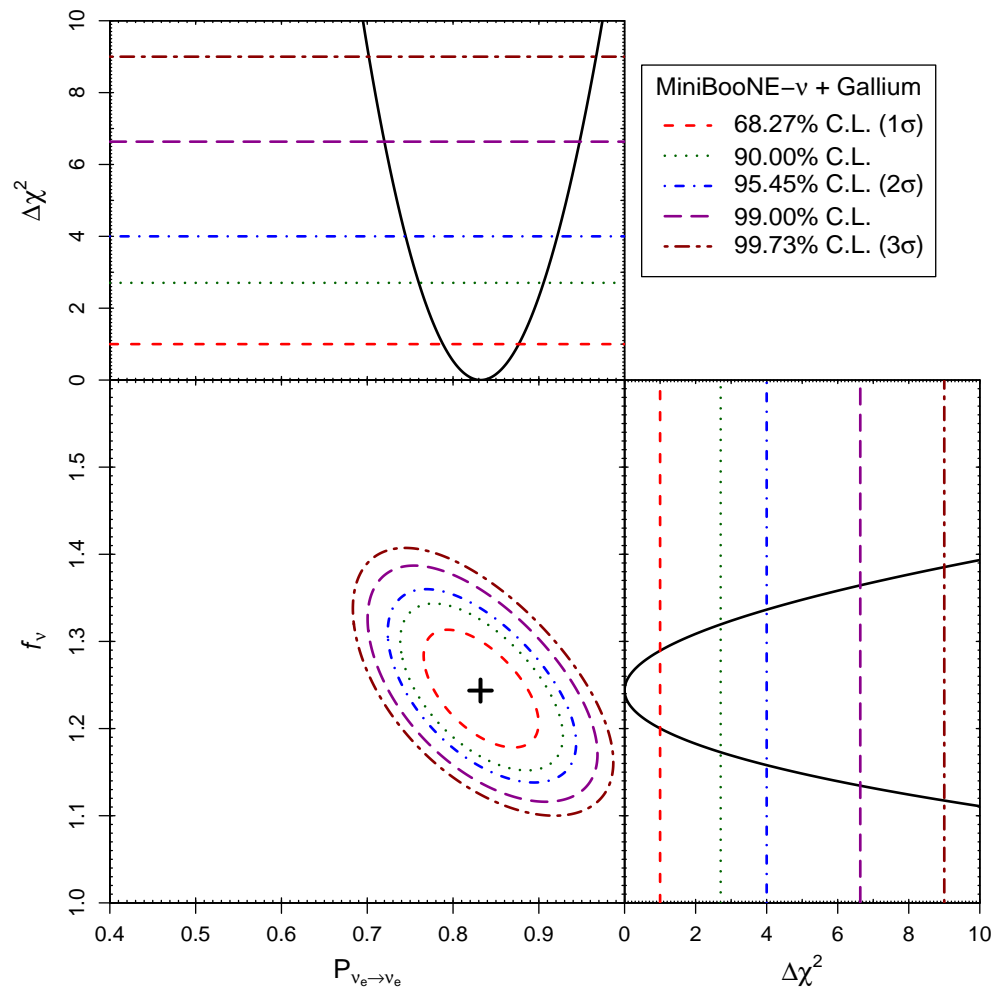
$$\chi^2_{min} = 17.7 / (9 \text{ dof})$$

$$GoF = 3.8\%$$

$$P_{\nu_e \rightarrow \nu_e}^{bf} = 0.72^{+0.08}_{-0.07}$$

$$f_\nu^{bf} = 1.31^{+0.07}_{-0.06}$$

# Fit to Miniboone- $\nu$ & Gallium data



$$\chi_{min}^2 = 20.1/(10 \text{ dof}) \quad GoF = 2.8\% \quad P_{\nu_e \rightarrow \nu_e}^{bf} = 0.83 \pm 0.04 \quad f_{\nu}^{bf} = 1.24_{-0.04}^{+0.05}$$



# Miniboone- $\nu$ & Gallium : Osc vs No Osc

|         |   | MB- $\nu$ | MB- $\nu$ +Ga |
|---------|---|-----------|---------------|
| No Osc. | $\chi_{\min}^2$                           | 27.2      | 34.0          |
|         | NDF                                       | 10        | 11            |
|         | GoF                                       | 0.2%      | 0.04%         |
|         | $f_{\nu}^{\text{bf}}$                     | 1.15      | 1.15          |
|         |   |           |               |
| Osc.    | $\chi_{\min}^2$                           | 17.7      | 20.1          |
|         | NDF                                       | 9         | 10            |
|         | GoF                                       | 3.8%      | 2.8%          |
|         | $P_{\nu_e \rightarrow \nu_e}^{\text{bf}}$ | 0.72      | 0.83          |
|         | $f_{\nu}^{\text{bf}}$                     | 1.31      | 1.24          |
|         |   |           |               |
| PG      | $\Delta\chi_{\min}^2$                     |           | 2.4           |
|         | NDF                                       |           | 1             |
|         | GoF                                       |           | 12.4%         |

## Active-Sterile $\nu_e$ mixing !

- The parameter goodness-of-fit of 12.4% implies that the results of the MiniBooNE neutrino and the Gallium radioactive source experiments are compatible in the framework of the  $\nu_e$  disappearance hypothesis.
- The goodness of fit of 2.8% is acceptable and much better than the 0.04% obtained without  $\nu_e$  disappearance.
- $P_{\nu_e \rightarrow \nu_e} = 1$  is disfavored at more than  $3\sigma$  (the precise value is 99.98% CL).

The large disappearance of  $\nu_e$  found in Gallium and in Miniboone- $\nu$  data, may be due to oscillations into sterile neutrinos  $\nu_e \rightarrow \nu_s$  since

- $\nu_e \rightarrow \nu_\mu$  transitions are restricted by the results of CCFR , KARMEN , NOMAD and MINIBOONE ;
- $\nu_e \rightarrow \nu_\tau$  transitions are limited by the results of CHORUS and NOMAD .

## SBL Reactor $\bar{\nu}_e$ experiments

$R^{(d)}$  denotes the ratio of measured and predicted event rates at the source-detector distance  $d$ :

Gosgen :

$$R_{\text{Gosgen}}^{(37.9 \text{ m})} = 1.018 \pm 0.019(\text{stat}) \pm 0.015(\text{uncorr}) \pm 0.060(\text{corr}) , \quad (1)$$

$$R_{\text{Gosgen}}^{(45.9 \text{ m})} = 1.045 \pm 0.019(\text{stat}) \pm 0.015(\text{uncorr}) \pm 0.060(\text{corr}) , \quad (2)$$

$$R_{\text{Gosgen}}^{(64.7 \text{ m})} = 0.975 \pm 0.036(\text{stat}) \pm 0.030(\text{uncorr}) \pm 0.060(\text{corr}) , \quad (3)$$

Bugey :

$$R_{\text{Bugey}}^{(15 \text{ m})} = 0.988 \pm 0.004(\text{stat}) \pm 0.05(\text{syst}) , \quad (4)$$

$$R_{\text{Bugey}}^{(40 \text{ m})} = 0.994 \pm 0.010(\text{stat}) \pm 0.05(\text{syst}) , \quad (5)$$

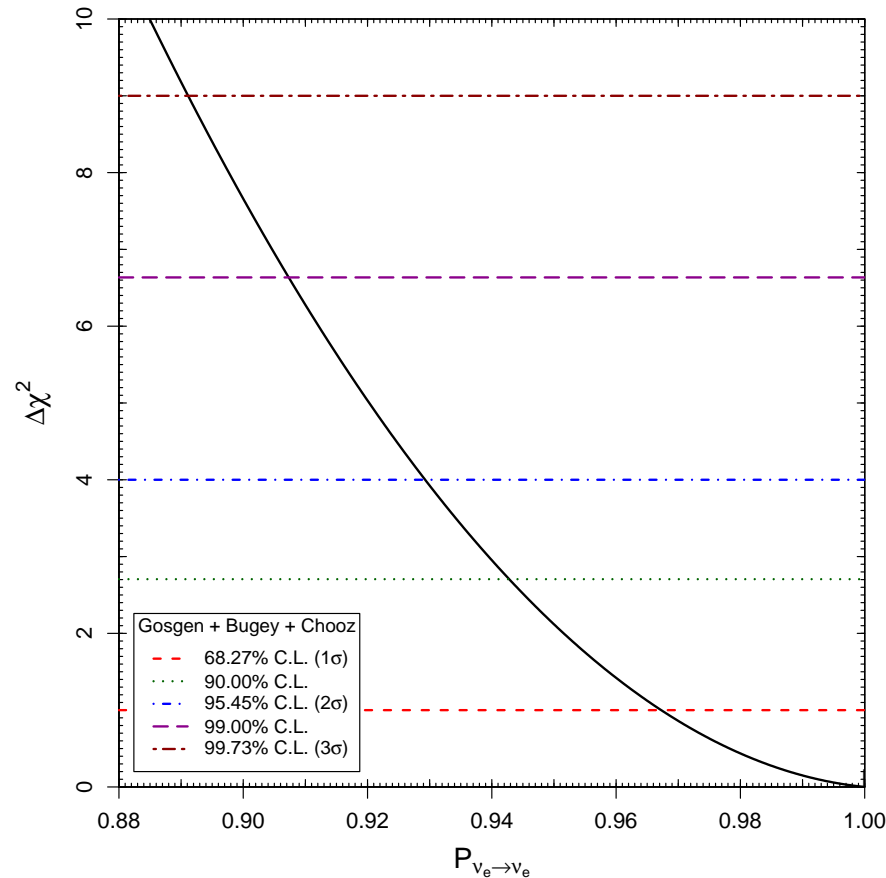
$$R_{\text{Bugey}}^{(95 \text{ m})} = 0.915 \pm 0.132(\text{stat}) \pm 0.05(\text{syst}) . \quad (6)$$

Chooz :

$$R_{\text{Chooz}}^{(1 \text{ km})} = 1.01 \pm 0.028(\text{stat}) \pm 0.036(\text{syst}) . \quad (7)$$

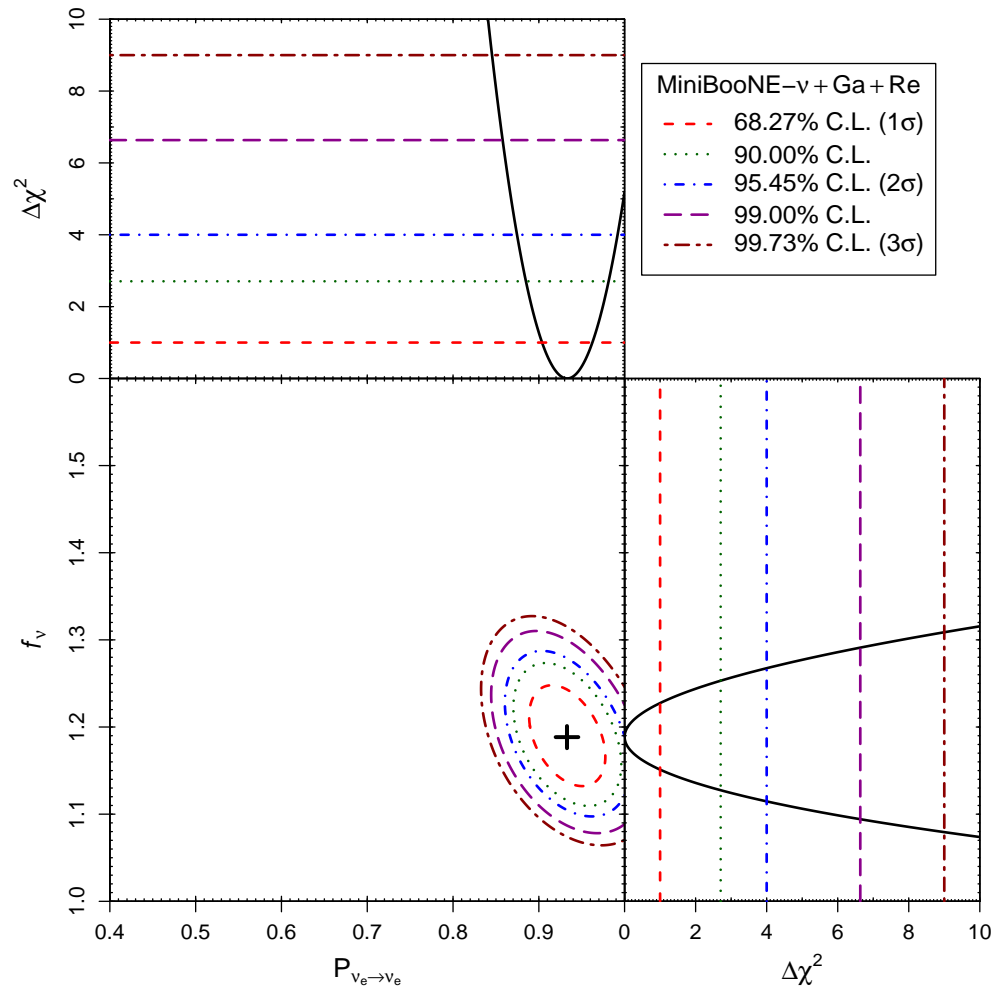
N.B. The Chooz systematic uncertainty of the reactor neutrino flux has the same value as that of Gosgen and Bugey, i.e. approximately 3% (ILL value).

## Fit to SBL Reactor $\bar{\nu}_e$ data



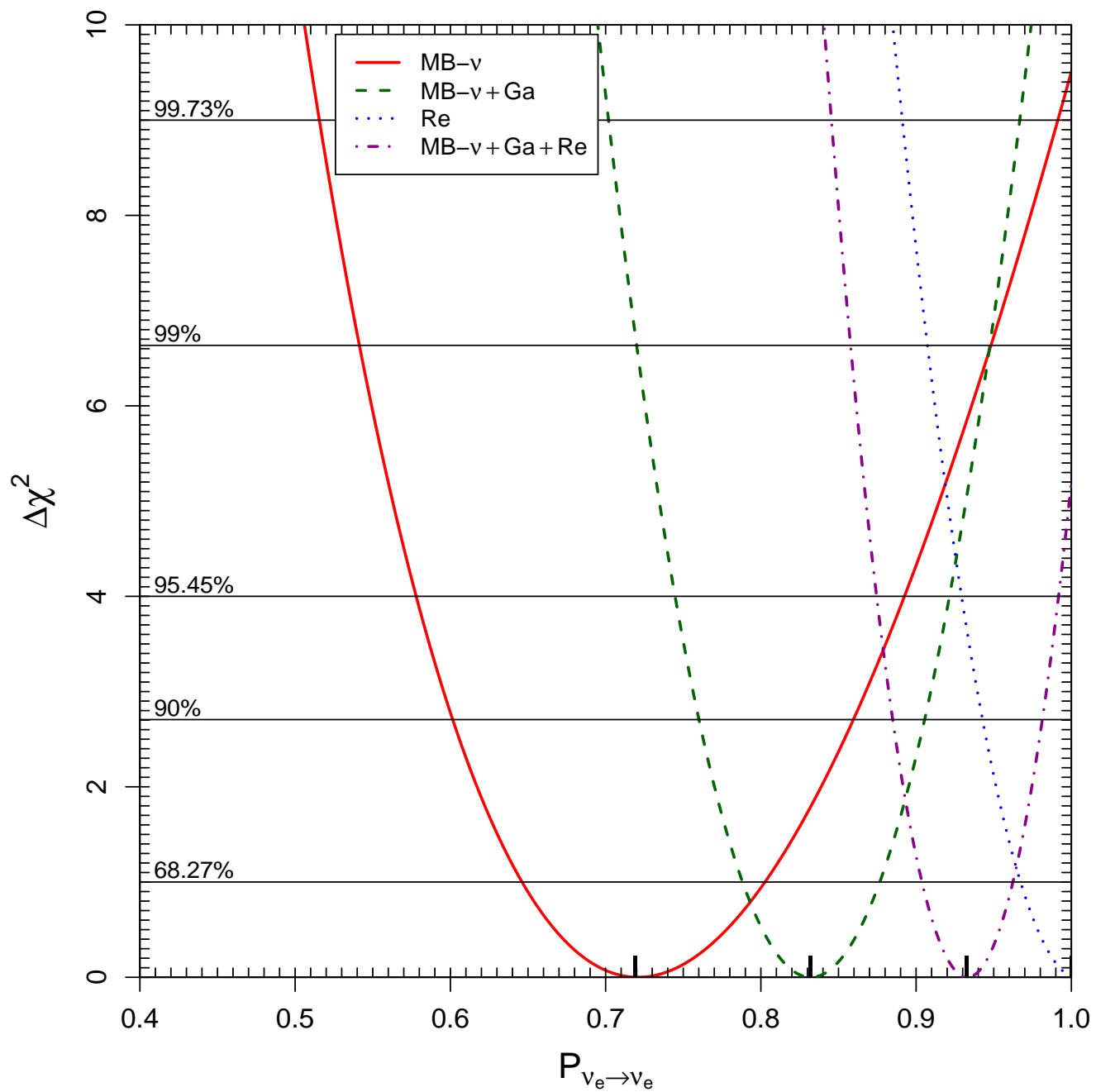
The lower limits for  $P_{\nu_e \rightarrow \nu_e}$  indicate that reactor data allow a small  $\bar{\nu}_e$  disappearance. Therefore, we tried a combined analysis of MiniBooNE neutrino, Gallium and reactor data under the hypothesis of  $\nu_e$  disappearance with  $P_{\nu_e \rightarrow \nu_e} = P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ .

# Combined fit of Miniboone- $\nu$ & Gallium & Reactor (1)



$$\chi_{min}^2 = 31.7 / (17 \text{ dof}) \quad GoF = 1.7\% \quad P_{\nu_e \rightarrow \nu_e}^{bf} = 0.93 \pm 0.03 \quad f_{\nu}^{bf} = 1.19 \pm 0.04$$

# Combined fit of Miniboone- $\nu$ & Gallium & Reactor (2)



# Miniboone- $\nu$ & Gallium & Reactor : Osc vs No Osc

|         |   | MB- $\nu$ | MB- $\nu$ +Ga | MB- $\nu$ +Ga+Re |
|---------|---|-----------|---------------|------------------|
| No Osc. | $\chi_{\min}^2$                           | 27.2      | 34.0          | 36.9             |
|         | NDF                                       | 10        | 11            | 18               |
|         | GoF                                       | 0.2%      | 0.04%         | 0.5%             |
|         | $f_{\nu}^{\text{bf}}$                     | 1.15      | 1.15          | 1.15             |
| Osc.    | $\chi_{\min}^2$                           | 17.7      | 20.1          | 31.7             |
|         | NDF                                       | 9         | 10            | 17               |
|         | GoF                                       | 3.8%      | 2.8%          | 1.7%             |
|         | $P_{\nu_e \rightarrow \nu_e}^{\text{bf}}$ | 0.72      | 0.83          | 0.93             |
|         | $f_{\nu}^{\text{bf}}$                     | 1.31      | 1.24          | 1.19             |
| PG      | $\Delta\chi_{\min}^2$                     |           | 2.4           | 11.1             |
|         | NDF                                       |           | 1             | 2                |
|         | GoF                                       |           | 12.4%         | 0.4%             |

## Tension between $\nu_e$ and $\bar{\nu}_e$ data

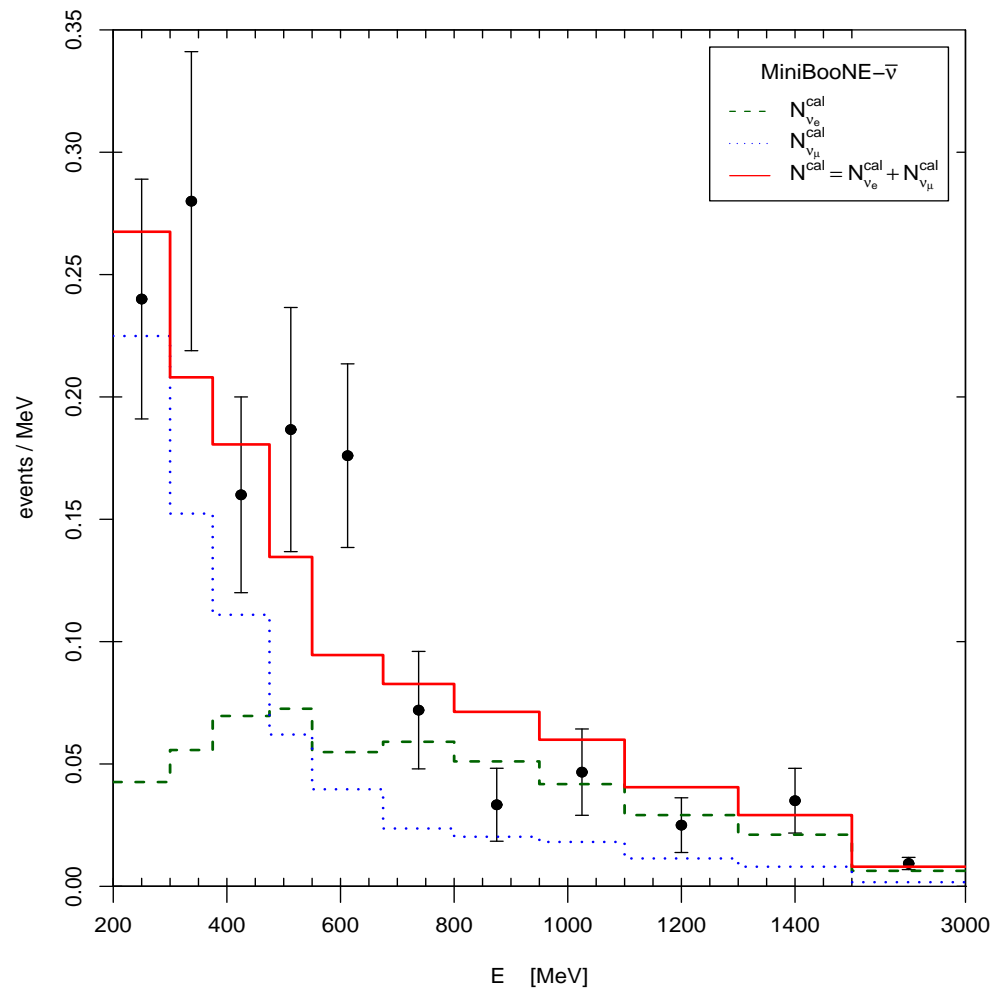
- The rather low parameter goodness-of-fit, 0.4% , shows that there is tension between MiniBooNE and Gallium neutrino data on one side and reactor antineutrino data on the other side.
- The goodness of fit of 1.7% is acceptable and it is better than the 0.5% obtained without  $\nu_e$  disappearance.
- $P_{\nu_e \rightarrow \nu_e} = 1$  is disfavored at more than  $2\sigma$  (97.74% CL).

Possible explanations of this tension could be:

1. Statistical fluctuations.
2. Systematic uncertainties have been underestimated.
3. Our hypothesis of  $\nu_e$  disappearance is excluded.
4. There is a violation of CPT symmetry leading to  $P_{\nu_e \rightarrow \nu_e} \neq P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ .

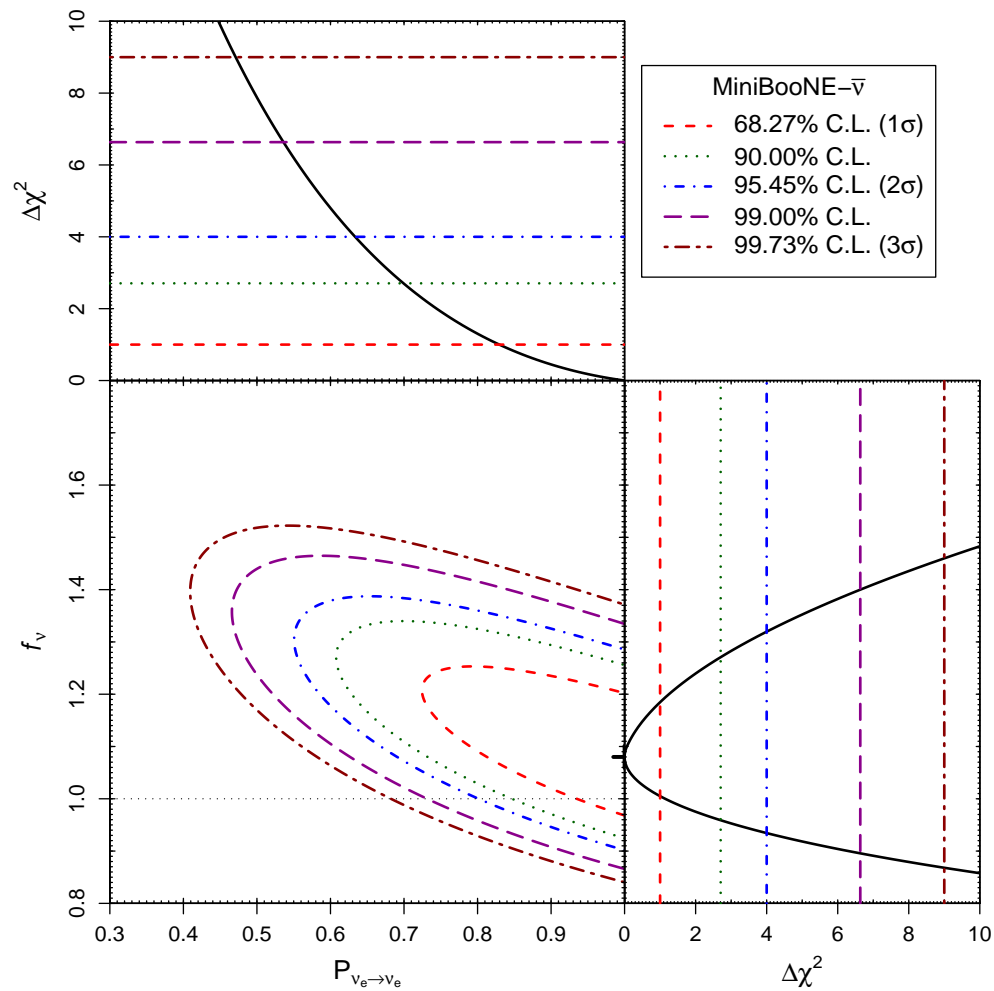


# More data : Miniboone- $\bar{\nu}$



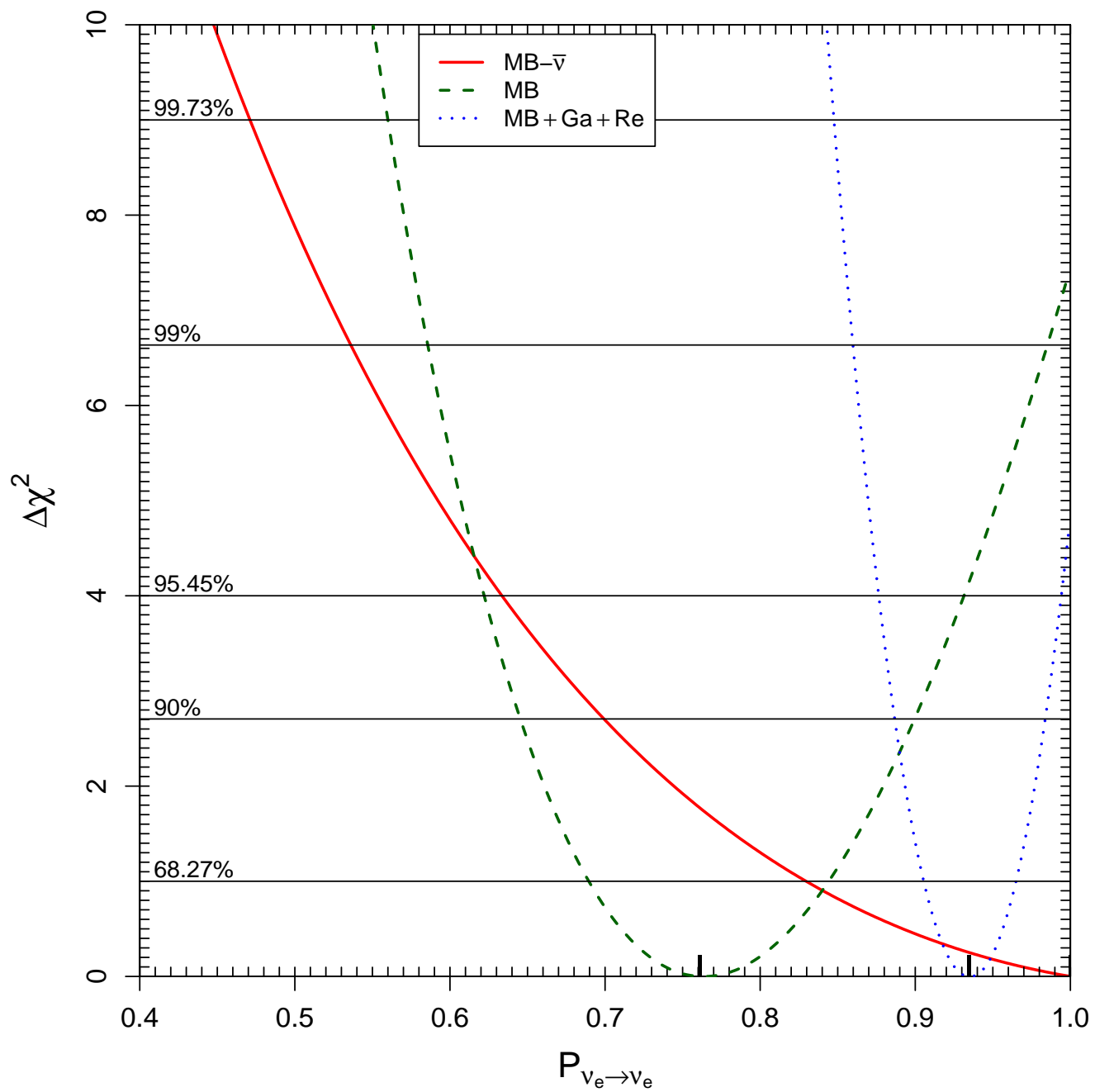
Karagiorgi talk @ FNAL dec 2008

# Fit to Miniboone- $\bar{\nu}$ data



$$\chi_{min}^2 = 16.9/(9 \text{ dof}) \quad GoF = 5.0\% \quad P_{\nu_e \rightarrow \nu_e}^{bf} = 1.00^{+0.00}_{-0.17} \quad f_{\bar{\nu}}^{bf} = 1.08^{+0.10}_{-0.08}$$

# Combined fit of Miniboone & Gallium & Reactor



## Miniboone & Gallium & Reactor : Osc vs No Osc

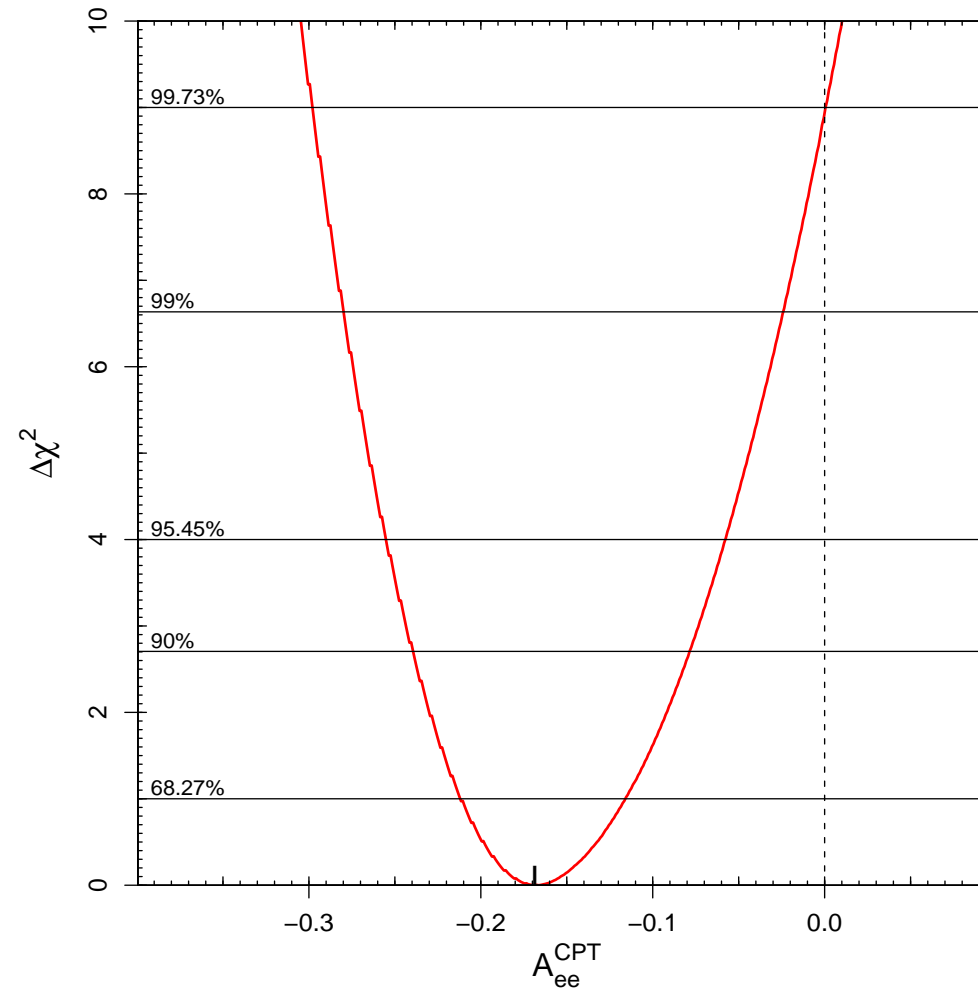
|         |   | MB- $\nu$ | MB- $\nu$ +Ga | MB- $\nu$ +Ga+Re | MB+Ga+Re |
|---------|---|-----------|---------------|------------------|----------|
| No Osc. | $\chi_{\min}^2$                           | 27.2      | 34.0          | 36.9             | 53.8     |
|         | NDF                                       | 10        | 11            | 18               | 29       |
|         | GoF                                       | 0.2%      | 0.04%         | 0.5%             | 0.3%     |
|         | $f_{\nu}^{\text{bf}}$                     | 1.15      | 1.15          | 1.15             | 1.15     |
|         | $f_{\bar{\nu}}^{\text{bf}}$               |           |               |                  | 1.08     |
| Osc.    | $\chi_{\min}^2$                           | 17.7      | 20.1          | 31.7             | 48.9     |
|         | NDF                                       | 9         | 10            | 17               | 27       |
|         | GoF                                       | 3.8%      | 2.8%          | 1.7%             | 0.6%     |
|         | $P_{\nu_e \rightarrow \nu_e}^{\text{bf}}$ | 0.72      | 0.83          | 0.93             | 0.93     |
|         | $f_{\nu}^{\text{bf}}$                     | 1.31      | 1.24          | 1.19             | 1.19     |
|         | $f_{\bar{\nu}}^{\text{bf}}$               |           |               |                  | 1.10     |
| PG      | $\Delta\chi_{\min}^2$                     |           | 2.4           | 11.1             | 8.3      |
|         | NDF                                       |           | 1             | 2                | 3        |
|         | GoF                                       |           | 12.4%         | 0.4%             | 4.1%     |

## Tension between $\nu_e$ and $\bar{\nu}_e$ data

- The parameter goodness-of-fit of 4.1% do not allow us to reject the compatibility of the data under the hypothesis of  $\nu_e$  disappearance. This results indicate that the possibility that the tension between MiniBooNE neutrino and Gallium data on one side and reactor data on the other side is due to statistical fluctuations may be correct.
- The goodness of fit of 0.6% is rather low and it is better than the 0.3% obtained without  $\nu_e$  disappearance.
- $P_{\nu_e \rightarrow \nu_e} = 1$  is disfavored at more than  $2\sigma$  (97.04% CL).
- Next we consider a possible violation of the CPT equality  $P_{\nu_e \rightarrow \nu_e} = P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$  as a possible explanation of the tension between MiniBooNE and Gallium neutrino data on one side and reactor antineutrino data on the other side under the hypothesis of  $\nu_e$  disappearance.
- We quantify the amount of CPT violation through the asymmetry

$$A_{ee}^{\text{CPT}} \equiv P_{\nu_e \rightarrow \nu_e} - P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} .$$

# CPTV fit of Miniboone & Gallium & Reactor



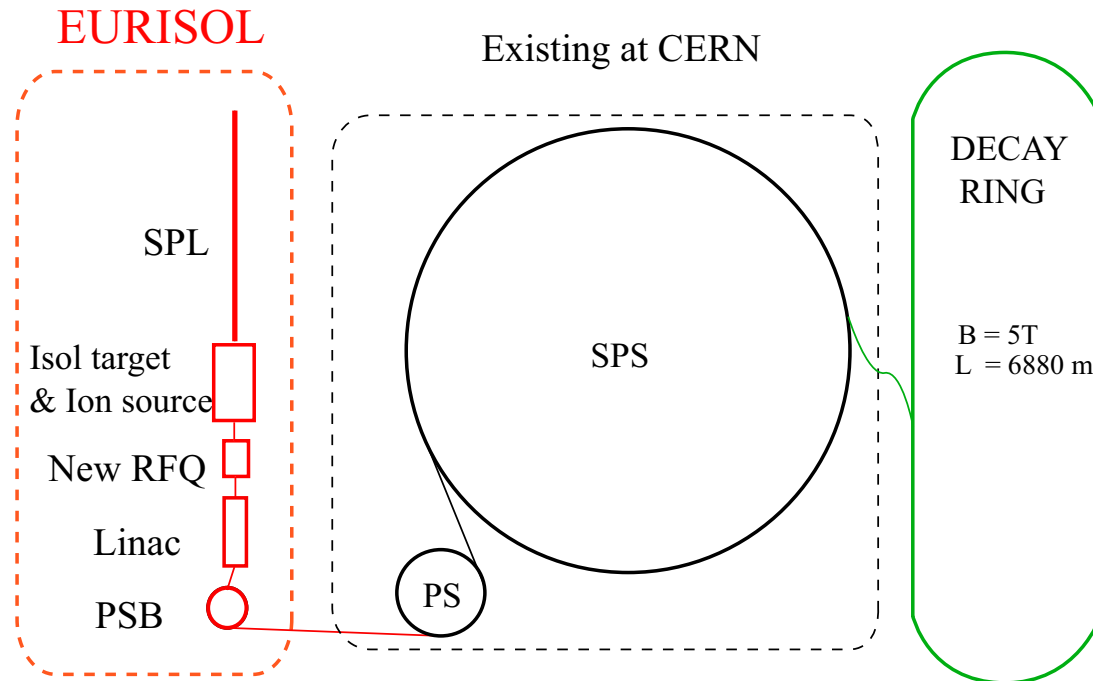
$$\chi_{min}^2 = 38.3/(26 \text{ dof}) \quad GoF = 5.7\% \quad A_{ee}^{\text{CPT}} = -0.165_{-0.04}^{+0.05}$$

## Interesting results !

- The relatively low goodness of fit of 5.7% is due to the relatively low goodness of fit of the MiniBooNE neutrino (3.8%) and antineutrino (5.0%) data.
- There is an indication of CPT violation ( $A_{ee}^{\text{CPT}} < 0$ ) at 99.71% CL.
- Since the indication of CPT violation that we have found has been obtained under the hypothesis of  $\nu_e$  disappearance into sterile neutrinos, it could be due to very exotic CPT-violating properties of the sterile neutrinos.
- Let us emphasize that the possibility of CPT violation is extremely interesting and should be explored in future experiments by measuring the CPT asymmetries:

$$A_{\alpha\beta}^{\text{CPT}} \equiv P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha} \quad \alpha, \beta = e, \mu.$$

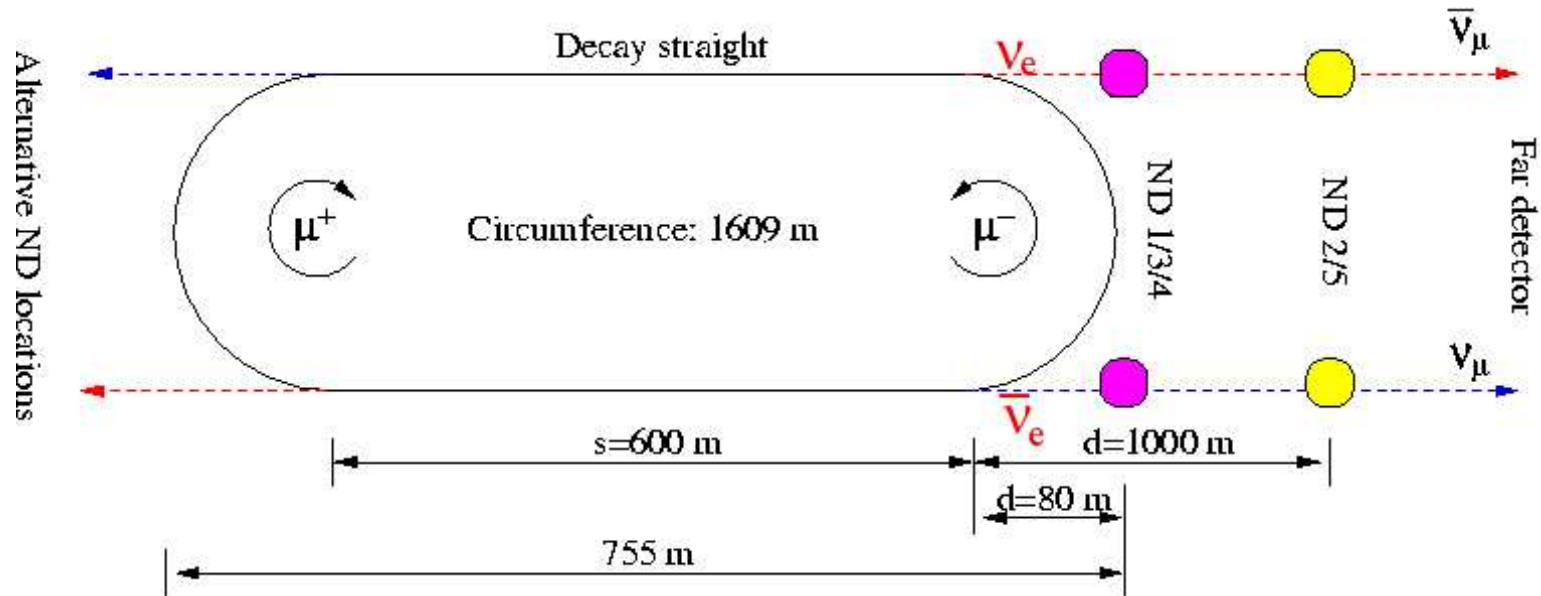
## Future SBL experimental CPT tests with a $\beta$ beam



Future SBL Beta-Beam experiments [ P.Zucchelli PLB 532 (2002) 166] with a pure  $\nu_e$  or  $\bar{\nu}_e$  beam from nuclear decay of accelerated ions have the potentiality to check the possible SBL disappearance of  $\nu_e$  and  $\bar{\nu}_e$  with high accuracy .



## Future SBL experimental CPT tests with a $\nu$ factory



Future SBL  $\nu$  factory experiments [CERN-Yellow-Report hep-ph/0210192] with pure  $\nu_e$  AND  $\bar{\nu}_e$  beams from muon decay of accelerated muons have the capability to check the possible SBL disappearance of  $\nu_e$  and  $\bar{\nu}_e$  again with high accuracy . Non standard interactions studied from [ A.Rubbia, ML et al. JHEP06(2001)032]  $\rightarrow$  [J.Tang and W.Winter arXiv:0903.3039].

... if they are roses they'll flower...



... A BRIGHT FUTURE for Majorana  $\nu$  physics with  $\beta$  &  $\mu$  beams !!!

Backup slides

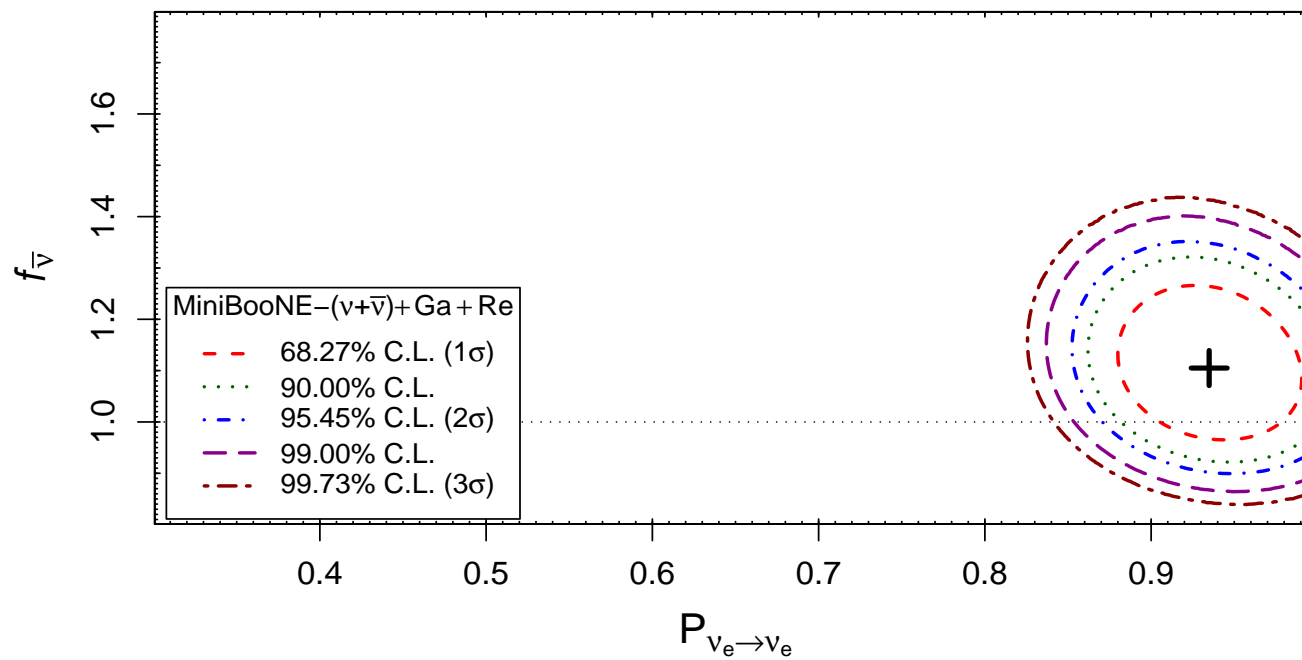
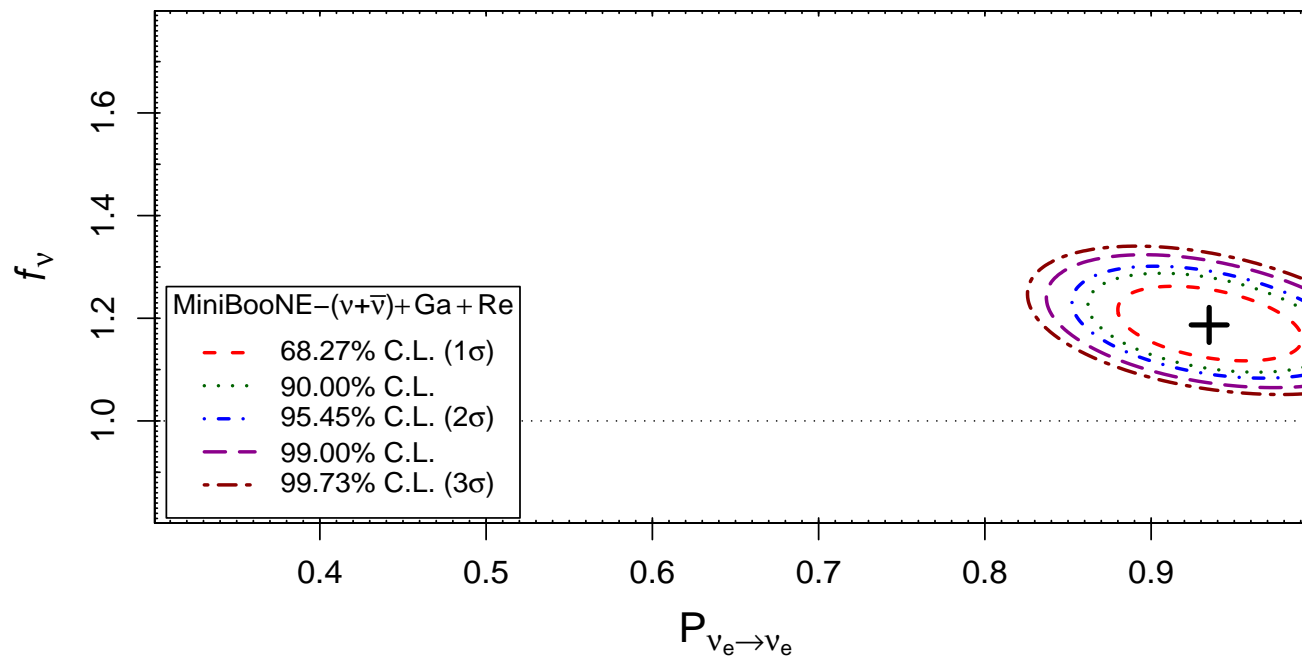
# Background systematic uncertainties: Many errors are similar between neutrino and antineutrino mode

| Source                        | $\bar{\nu}$ mode uincer. (%) |             | $\nu$ mode uincer. (%) |             |
|-------------------------------|------------------------------|-------------|------------------------|-------------|
|                               | 200-475                      | 475-1250    | 200-475                | 475-1250    |
| $E_{\nu}^{QE}$ range (MeV)    |                              |             |                        |             |
| Flux from $\pi^+/\mu^+$ decay | 0.4                          | 0.7         | 1.8                    | 2.2         |
| Flux from $\pi^-/\mu^-$ decay | 3.3                          | 2.2         | 0.1                    | 0.2         |
| Flux from $K^+$ decay         | 2.3                          | 4.9         | 1.4                    | 5.7         |
| Flux from $K^-$ decay         | 0.5                          | 1.1         | -                      | -           |
| Flux from $K^0$ decay         | 1.5                          | 5.7         | 0.5                    | 1.5         |
| Target and beam models        | 1.9                          | 3.0         | 1.3                    | 2.5         |
| $\nu$ cross section           | 6.4                          | 12.9        | 5.9                    | 11.9        |
| NC $\pi^0$ yield              | 1.7                          | 1.6         | 1.4                    | 1.9         |
| Hadronic interactions         | 0.5                          | 0.6         | 0.8                    | 0.3         |
| External interactions (dirt)  | 2.4                          | 1.2         | 0.8                    | 0.4         |
| Optical model                 | 9.8                          | 2.8         | 8.9                    | 2.3         |
| Electronics & DAQ model       | 9.7                          | 3.0         | 5.0                    | 1.7         |
| <b>Total (unconstrained)</b>  | <b>16.3</b>                  | <b>16.2</b> | <b>12.3</b>            | <b>14.2</b> |
| <b>Statistical Error</b>      | <b>12.8</b>                  | <b>13.2</b> | <b>4.9</b>             | <b>5.1</b>  |

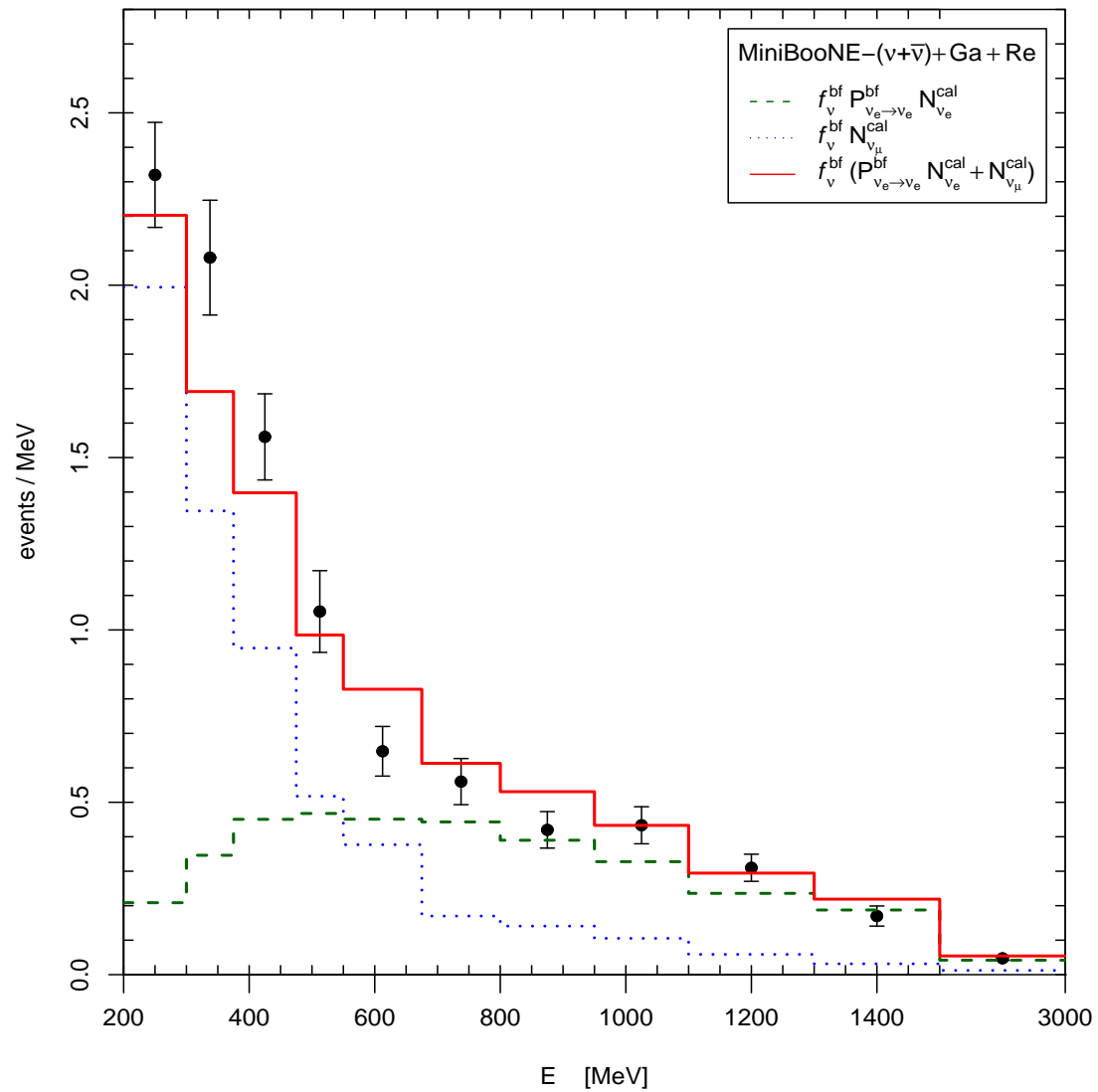
Expect large cancellation of systematic errors in ratio analysis.

Antineutrino statistical error will dominate.

# Combined fit of Miniboone & Gallium & Reactor



# Best fit of MiniBooNE & Gallium & Reactor vs MiniBooNE- $\nu$ data



|                     | BF   | 68.27%      | 90%         | 95.45%      | 99%         | 99.73%      |
|---------------------|------|-------------|-------------|-------------|-------------|-------------|
| MB- $\nu$           | 0.72 | 0.65 - 0.80 | 0.60 - 0.86 | 0.58 - 0.89 | 0.54 - 0.95 | 0.52 - 0.99 |
| MB- $\nu$ +Ga       | 0.83 | 0.79 - 0.88 | 0.76 - 0.91 | 0.75 - 0.92 | 0.72 - 0.95 | 0.70 - 0.97 |
| Re                  | 1.00 | 0.97 - 1.00 | 0.94 - 1.00 | 0.93 - 1.00 | 0.91 - 1.00 | 0.89 - 1.00 |
| MB- $\nu$ +Ga+Re    | 0.93 | 0.90 - 0.96 | 0.89 - 0.98 | 0.88 - 0.99 | 0.86 - 1.00 | 0.85 - 1.00 |
| MB- $\bar{\nu}$     | 1.00 | 0.83 - 1.00 | 0.70 - 1.00 | 0.63 - 1.00 | 0.54 - 1.00 | 0.47 - 1.00 |
| MB- $\bar{\nu}$ +Re | 1.00 | 0.97 - 1.00 | 0.95 - 1.00 | 0.93 - 1.00 | 0.91 - 1.00 | 0.89 - 1.00 |
| MB                  | 0.76 | 0.69 - 0.84 | 0.65 - 0.90 | 0.62 - 0.93 | 0.59 - 0.98 | 0.56 - 1.00 |
| MB+Ga+Re            | 0.93 | 0.91 - 0.96 | 0.89 - 0.98 | 0.88 - 0.99 | 0.86 - 1.00 | 0.85 - 1.00 |

Table 1: Best-fit values (BF) and allowed ranges of  $P_{\nu_e \rightarrow \nu_e}$  at the indicated value of confidence level.

|         |   | MB- $\bar{\nu}$ | MB- $\bar{\nu}$ +Re | MB    | MB+Ga+Re |
|---------|---|-----------------|---------------------|-------|----------|
| No Osc. | $\chi_{\min}^2$                           | 16.9            | 19.8                | 44.1  | 53.8     |
|         | NDF                                       | 10              | 17                  | 21    | 29       |
|         | GoF                                       | 7.6%            | 28.5%               | 0.2%  | 0.3%     |
|         | $f_{\bar{\nu}}^{\text{bf}}$               | 1.08            | 1.08                | 1.08  | 1.08     |
| Osc.    | $\chi_{\min}^2$                           | 16.9            | 19.8                | 36.7  | 48.9     |
|         | NDF                                       | 9               | 16                  | 19    | 27       |
|         | GoF                                       | 5.0%            | 23.0%               | 0.9%  | 0.6%     |
|         | $P_{\nu_e \rightarrow \nu_e}^{\text{bf}}$ | 1.00            | 1.00                | 0.76  | 0.93     |
|         | $f_{\bar{\nu}}^{\text{bf}}$               | 1.08            | 1.08                | 1.19  | 1.10     |
|         | $f_{\nu}^{\text{bf}}$                     |                 |                     | 1.28  | 1.19     |
| PG      | $\Delta\chi_{\min}^2$                     |                 | 0.0                 | 2.1   | 8.3      |
|         | NDF                                       |                 | 1                   | 1     | 3        |
|         | GoF                                       |                 | 100.0%              | 14.8% | 4.1%     |

Table 2: Values of  $\chi^2$ , number of degrees of freedom (NDF) and goodness-of-fit (GoF) for the fit of MiniBooNE antineutrino (MB- $\bar{\nu}$ ), MiniBooNE antineutrino and reactor (MB- $\bar{\nu}$ +Re), MiniBooNE neutrino and antineutrino (MB) and MiniBooNE neutrino and antineutrino, Gallium and reactor (MB+Ga+Re) data. The first four lines correspond to the case of no oscillations (No Osc.). The following six lines correspond to the case of oscillations (Osc.). The last three lines give the parameter goodness-of-fit (PG).



## Parameter Goodness-of-fit (PG)

- The goodness-of-fit is the probability to obtain a worse fit under the assumption that the model under consideration is correct. It is the standard statistic used for the estimation of the quality of a fit obtained with the least-squares method, assuming the validity of the approximation in which  $\chi_{\min}^2$  has a  $\chi^2$  distribution with  $\text{NDF} = N_D - N_P$  degrees of freedom, where  $N_D$  is the number of data points and  $N_P$  is the number of fitted parameters. The fit is usually considered to be acceptable if the goodness-of-fit is larger than about 1%.
- The value of  $(\Delta\chi_{\min}^2)_{A+B}$  corresponding to the Parameter Goodness-of-fit (PG) of two experiments A and B is given by  $(\chi_{\min}^2)_{A+B} - [(\chi_{\min}^2)_A + (\chi_{\min}^2)_B]$ . It has a  $\chi^2$  distribution with number of degrees of freedom  $\text{NDF} = P_A + P_B - P_{A+B}$ , where  $P_A$ ,  $P_B$  and  $P_{A+B}$  are, respectively, the number of parameters in the fits of A, B and A+B data.

[M. Maltoni and T. Schwetz, Phys. Rev. D68 (2003) 033020 (hep-ph/0304176).]