Neutrino Majorana

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Ettore Majorana. Short biography

Majorana paper “Symmetrical theory of the electron and the positron”

Present status of neutrino Majorana

Ettore Majorana was born in Catania on 5.08.1906 There were five children in the family. Ettore father was an engineer, specialist in telephone communications

Unusual abilities to numerical calculations Ettore reveal when he was very young. At the age of four he could multiply two three-digit numbers and got correct result in seconds

In 1921 the family moved to Rome

Ettore finished Liceo in 1923 with high marks. In autumn of 1923 he entered Engineer faculty of the Rome University

Among his fellow-students and friends was E. Segre.

Segre and then Amaldi in 1927 switched to physics and started to work with E. Fermi who was appointed in 1926 as a Professor of theoretical physics of Rome University

E. Segre often talked about extraordinary mathematical abilities of Majorana in Fermi group and he tried to convince Majorana to follow his example

Majorana switched to physics in the beginning of 1928 after a talk with Fermi

Fermi was developing at that time the statistical model which became known as Thomas-Fermi model. He explained Majorana the model and showed him the table with numerical values of the potential

Next day Majorana returned back to the Institute with his own table for the potential. He transformed second order nonlinear Thomas-Fermi equation into Riccati equation and solved it numerically. Majorana and Fermi results coincided

A few days later Majorana transferred to Physics faculty. He impressed everybody by his lively mind and broad interests. For his criticism he was called the "Great Inquisitor" Majorana received doctorate in 1929. He got 110/110 with distinction. Thesis were devoted to the investigation of the structure of nuclei and to the theory of the alpha-decay. His supervisor was Fermi

After doctorate Majorana visited the Institute for a few hours every day. He spend most of his time in library studying papers of Dirac, Heisenberg, Pauli, Weil and Wigner

He had a great interest to the group theory and even wanted to write a book on the subject

At that time Fermi group was involved in investigation of atomic and molecular physics

Majorana wrote six papers on the subject

The splitting of levels of electrons in different heavy atoms due to electron spin. The calculation of levels of electrons in helium. Detailed calculation of the atomic spectrum of calcium. The theory of molecular ion He_2^+. The theory of H_2 molecule.

In the last paper of this period Majorana developed the theory of non adiabatic spin-flip transitions of polarized particles in rapidly varying magnetic field. Majorana considered the $J = 1/2$ case. His result was generalized by Bloch and Rabi for arbitrary J. Majorana result is well known and is widely used in polirized neutron spectrometers.

These papers demonstrate profound ability of Majorana of using symmetry properties of the states in order to simplify the problem and to choose the suitable approximation (which was not usual at that time) They demonstrated detailed knowledge by Majorana of experimental data. In 1932 Majorana received teaching diploma ("libero docente") Committee (Fermi, Lo Surdo, Persico) concluded " the candidate has a complete mastery in theoretical physics"

In the end of 1931-beginning of 1932 all members of the Fermi group started to concentrate their efforts on nuclear physics Amaldi returned from Leipzig and made a series of seminars on Retherford, Chadwick and Ellis papers. Fermi, Majorana, Rasetti, Segre and a few others attended the seminars. They were often interrupted by questions which gave Fermi opportunity for detailed blackboard explanations. Majorana was usually silent but sometimes he made very keen observations, always to point

F. Jolliot and I.Curies were the first who observed recoil protons with energy about 5 MeV from "penetrating radiation" produced by bombardment of Be by α particles. They interpreted their results as a new type of interaction of γ and proton

After reading F. Jolliot papers Majorana noticed:" They did not understood what they observed. Proton recoil is probably produced by a heavy neutral particle"

A few days later "Nature" was received in Rome with Chadwick paper on the discovery of neutron (1932)

Many models of nuclei in which n, p and also e^{-}, α were considered as constituents of nuclei

Majorana was one of the first who came to an idea that only p and n are constituents of nuclei

Majorana started to develop the theory of nuclear forces. He developed the theory of space exchange forces between p and n (Majorana potential)

Fermi was very interested in the idea and pushed him to publish his results. Majorana refused and even did not allow Fermi to mention them in his talk on atomic nuclei at the conference in Paris

Fermi managed, however, to persuade Majorana to go to Leipzig (Heisenberg) and Copenhagen (Bohr)

Majorana was abroad during seven months starting from January 1933

Majorana had difficulties in establishing new relations with people. Abroad he was practically alone. However, he became friendly with Heisenberg to whom he had great respect. Heisenberg convinced Majorana to publish his paper on nuclear theory which appeared in 1933

After returning from Germany Majorana started to come to the Institute at via Panisperna rather rare and after some months did not come at all.

He was at home and became interested in political economy, philosophy, construction of ships, medicine... He even wrote a paper on statistical laws in physics and social sciences which was discovered and published after his disappearence

Meanwhile new talanted physicists appeared in Italy: Wick, Racah, Giovanni Gentili Jr., Pincherle , Watagin. It was time to create a new chair in theoretical physics. The competition for the chair was announced at the beginning of 1937 by the University of Palermo.

It was a problem to convince Majorana to take part in the competition

Finally Fermi and his group managed to convince Majorana to take part in the competition

Majorana had no publications during several years. He sent to "Nuovo Cimento" his most important paper "Symmetrical theory of the electron and the positron"

Probably without the competition the paper would never appear

Then happened the following. By the request of Senator Giovanni Gentili Majorana for his extraordinary abilities was appointed without competition as professor at Napoli University (The commitee made ther following short list: Wick, Racah, Giovanni Gentili Jr.)

In January 1938 Majorana moved to Napoli

In Napoli he had lonely life and lived in different hotels

He went to the University only when he had lectures (on Quantum mechanics). After lectures he visited Carrelli, professor of Experimental Physics, with whom he became friendly, and discuss with him different problems in physics.

Never mentioned what he was doing. Discussed his neutrino theory. Carrelli had impression that Majorana considered this theory as his most important contribution to physics

On March 23 1938 Majorana decided to go to Palermo.

On March 25 Carrelli received a telegram from Majorana from Palermo. He asked Carrelli do not worry about a letter which he would receive

In the letter which came later Majorana wrote that he found life useless and decided to commit suicide.

Carrelli called to Fermi and Fermi informed Luciano, Ettore brother

Luciano immediately went to Napoli. He understood that on the evening of March 25 Ettore took boat to Napoli. He was seen sleeping in his cabin when the boat was entering the bay of Napoli. He did not arrive to Napoli. His body was never found.

Fermi once noticed that if a person with such intelligence decided to disappear he will succeed in that

During several months there was investigation conducted by family and police. Vatican tried to find out whether he entered some monastery. No traces were found

Mussolini took personal interest in the investigation. Ettore mother wrote to him with a letter from Fermi

”I have no hesitation in saying and it is no way of exaggeration, that of all the Italian and foreign scholars whom I have had the opportunity of knowing, Majorana is the one whose depth most impressed me. Capable of developing baldly hypothesis and, at the same time criticizing his own work and that of others, highly skilled in calculations and a mathematician of a great depth, who never lost sight of the true nature of the physics problems behind the vail of figures and mathematical techniques, Ettore Majorana was highly endowed with that rare combination of gifts which go to make a typical theoretician of the first rank”

Fermi (From Cocconi memories)

"There are various kind of scientists in the world. The second and third-rate ones do their best but do not get very far. There are also first-rate people who make very important discoveries which are of capital importance for the development of the science. Then there are genius like Galillo and Newton. Ettore Majorana was one of these. Majorana had greater gifts that anyone else in the world; unfortunately he lacked one quality which other men generally have: plain common sense"

From Pontecorvo memories

E. Majorana was very critical to himself and other people. He was permanently unhappy with himself. He was a pessimist but had very accute sense of humor. He was conditioned by complicated and absolutely nontrivial living rules

E. Majorana was quite rich and I (Pontecorvo) can not avoid thinking that his life might not have finished so tragically should he have been obliged to work for a living. For that reason and also because he did not like to publish the results of all investigations he had made, Majorana contribution to physics is much less than it could be

SYMMETRICAL THEORY OF THE ELECTRON AND POSITRON

The Dirac sea leads to the symmetrical theory of electrons and positrons.

Majorana was not satified by the methods how the symmertry was reached

In fact the symmetry is formally lost when vacuum is considered as the state with all levels of electrons with negative energies occupied

“As for electron and positrons are concerned, we should expect from the theory only a formal improvement. However, in our oppinion it is important (for possible extension of the theory) that the very notion of negative states disappears.”

Consider complex spinor field $\psi(x)$. Dirac equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\psi(x) = 0$$

For the conjugated field $\psi^{c}(x) = C\bar{\psi}^{T}(x)$

$$(C\gamma_{\alpha}^{T}C^{-1} = -\gamma_{\alpha}; \quad C^{T} = -C)$$

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\psi^{c}(x) = 0$$

Majorana presented the field $\psi(x)$ in the form

$$\psi(x) = \frac{1}{\sqrt{2}}\chi_1 + i\frac{1}{\sqrt{2}}\chi_2$$

$$\chi_1(x) = \frac{\psi(x)+\psi^{c}(x)}{\sqrt{2}}; \;\; \chi_2(x) = \frac{\psi(x)-\psi^{c}(x)}{\sqrt{2}i}$$

The fields $\chi_{1,2}(x)$ satisfy Dirac equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\chi_{1,2}(x) = 0$$

and additional conditions

$$\chi^{c}_{1,2}(x) = \chi_{1,2}(x)$$

Majorana used a representation in which γ^{α} are imaginary matrices and $\psi^{c}(x) = \psi^{*}(x)$. In this representation $\chi_1(x)$ and $\chi_2(x)$ are real and imaginary parts of $\psi(x)$

He consructed first quantum theory for the field $\chi_1(x)$ ($\chi_2(x)$)

The current for the field
$\chi_i(x) = \chi_i^c(x); \quad i = 1, 2$

$$j_\alpha^i(x) = \bar{\chi}_i(x)\gamma^\alpha\chi_i(x) = -\chi_i^T(x)(\gamma^\alpha)^T\bar{\chi}_i(x)^T = -\bar{\chi}_i(x)\gamma^\alpha\chi_i(x) = 0$$

Thus, $\chi_i(x)$ is the field of neutral particles without charge and magnetic moment.

Vector of the energy and momentum

$$P^\alpha = \int \sum_r p^\alpha a_r^\dagger(p) a_r(p) d^3p$$

Operators $a_r(p)$ and $a_r^\dagger(p)$ satisfy usual anticommutation relations.

Thus, $a_r(p)$ ($a_r^\dagger(p)$) is the operator of absorption (creation) of neutral particle with momentum p and helicity r.

$|p\rangle = a_r^\dagger(p)|0\rangle$ is state of the particle with momentum p and helicity r. No states with negative enegies, no antiparticles

Ihe case of the complex field $\psi(x) = \frac{1}{\sqrt{2}}\chi_1 + i\frac{1}{\sqrt{2}}\chi_2$ the current $j^i_\alpha(x) = \bar{\psi}_i(x)\gamma^\alpha\psi_i(x)$ different from zero.

After quantization Majorana came to symmetrical theory of particles and antiparticles with operators of total momentum and of charge

$$P^\alpha = \int \sum_r p^\alpha [c^\dagger_r(p)c_r(p) + d^\dagger_r(p)d_r(p)]d^3p$$

$$Q = e\int \sum_r [c^\dagger_r(p)c_r(p) - d^\dagger_r(p)d_r(p)]d^3p$$

The vacuum state satisfies $c_r(p)|0\rangle = 0$ and $d_r(p)|0\rangle = 0$

$$|p\rangle = c^\dagger_r(p)|0\rangle; \quad |p\rangle = d^\dagger_r(p)|0\rangle$$

are states of particle with charge e and mass m and antiparticle with charge $-e$ and same mass m

Majorana conclusion

“A generalization of Jordan-Wigner quantization method allows not only to give symmetrical form to the electron-positron theory but also to constract an essentially new theory for particles without electric charge (neutrons and hypothetical neutrinos)”

and

“Although it is perhaps not possible now to ask experiment to choose between the new theory and that in which the Dirac equations are simply extended to neutral particles, one should keep in mind that the new theory is introducing in the unexplored field a smaller number of hypothetical entities”

Soon after Majorana paper appeared Racah (1937) proposed a possibled test of the Majorana hypothesis

The chain

$$(A,Z) \to (A,Z+1) + e^- + \nu; \quad \nu + (A',Z') \to (A',Z'+1) + e^-$$

is possible if neutrino is Majorana particle and is forbidden if neutrino is Dirac particle.

At that time (and many years later) there was no possibility to realize this idea

In 1938 Furry suggested that Racah chain can induce neutrinoless double β-decay of nuclei

$$(A,Z) \to (A,Z+2) + e + e$$

in the case if neutrino is Majorana particle

MAJORANA NEUTRINO; PRESENT STATUS

Are massive neutrinos and antineutrinos identical or different ?

This problem which had been put forward by Majorana about 70 years ago is the most fundamental problem of todays neutrino physics

Basics

Neutrino interaction is perfectly described by the Standard Model Lagrangian with CC and NC currents

$$j_{\alpha}^{\mathrm{CC}} = 2\sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_{\alpha} l_L$$

$$j_{\alpha}^{\mathrm{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_{\alpha}\nu_{lL}$$

After the discovery of neutrino oscillations we know that neutrino masses are different from zero and neutrino mixing takes place

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}$$

Nature of neutrinos with definite masses ν_i is determined by the type of neutrino mass term.

Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \bar{n}_L \, M^{\mathrm{M}} (n_L)^c + \mathrm{h.c.}$$

M^{M} is a symmetrical matrix

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \end{pmatrix}$$

$\nu_{s_i L}$ are sterile fields

No global gauge invariance

There is no conserved total lepton number L which could allow to distinguish neutrinos and antineutrinos

$$\mathcal{L}^{\mathrm{M}} == -\frac{1}{2}\sum_i m_i\,\bar{\nu}_i\nu_i$$

ν_i is Majorana field

$$\nu_i^c(x) = \nu_i(x)$$

Origin of the violation of L ?

The most plausible (and popular) is see-saw

Neutrino masses are many orders of magnitude smaller than masses of quarks and leptons

The see-saw connect smallness of neutrino masses with violation of the total lepton number by right- handed Majorana mass term at a large scale

Sterile fields

$$\nu_{s_1 L} = (\nu_{eR})^c; \nu_{s_2 L} = (\nu_{\mu R})^c; \nu_{s_3 L} = (\nu_{\tau R})^c$$

$$M^{\mathrm{M+D}} = \begin{pmatrix} 0 & m_{\mathrm{D}}^T \\ m_{\mathrm{D}} & M_R \end{pmatrix},$$

Assume that eigenvalues of M_R are much larger than m_{D}. Can be presented in block-diagonal form

$$U^T M^{\mathrm{M+D}} U = \begin{pmatrix} m_\nu & 0 \\ 0 & M_R \end{pmatrix},$$

Neutrino mass matrix $m_\nu = -m_{\mathrm{D}}^T M_R^{-1} m_{\mathrm{D}}^T$

Neutrino masses are much smaller than masses of quarks or leptons.

Heavy Majorana particles, see-saw partners of light Majorana neutrinos, must exist. CP-violating decays of these particles in the early Universe is a possible source of the barion asymmetry of the Universe .

Dirac mass term

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l'l} \bar{\nu}_{l'L} M^{\mathrm{D}}_{l'l} \nu_{lL} + \mathrm{h.c.}. = -\sum_i m_i \bar{\nu}_i \nu_i$$

Neutrino mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\ \nu_{iL}$$

Invariance under global gauge transformation

$$\nu'_{lL}(x) = e^{i\alpha}\ \nu_{lL}(x); \nu'_{lR}(x) = e^{i\alpha}\ \nu_{lR}(x) \quad l'(x) = e^{i\alpha}\ l(x); \quad q'(x) = q(x)$$

$L = L_e + L_\mu + L_\tau$ is conserved

$\nu_i(x)$ is complex 4-component field

$$L(\nu_i) = -L(\bar{\nu}_i) = 1$$

Smallness of neutrino masses?

In principle can be explained by models with large extra dimensions

For the SM weak interaction, theories with massless Dirac and Majorana neutrinos are equivalent

Effects which allow to reveal the Majorana nature of neutrinos are proportional to $\frac{m_i^2}{Q^2} \ll 1$ and very strongly suppressed

The neutrinoless double β-decay is an exceptional process. Large targets , small backgrounds, high energy resolutions make experiments on the search for this decay an unique source of information about the nature of ν_i

The half-life of $0\nu\beta\beta$-decay

$$\frac{1}{T^{0\,\nu}_{1/2}(A,Z)} = |m_{\beta\beta}|^2\,|M(A,Z)|^2\,G^{0\,\nu}(E_0,Z)$$

$m_{\beta\beta} = \sum_i U_{ei}^2\,m_i$ effective Majorana mass

$M(A,Z)$ is nuclear matrix element

The most stringent lower bounds were obtained in the germanium Heidelberg-Moscow experiment

$$T^{0\,\nu}_{1/2}(^{76}\text{Ge}) \geq 1.9 \cdot 10^{25}\text{years} \quad (90\%\text{CL})$$

and cryogenic CUORICINO experiment

$$T^{0\,\nu}_{1/2}(^{130}\text{Te}) \geq 1.8 \cdot 10^{24}\,\text{years} \quad (90\%\text{CL})$$

Taking into account different calculations of the nuclear matrix elements

$$|m_{\beta\beta}| \leq (0.3 - 1.2)\ \text{eV}$$

The aim of future experiments (CUORE, MAJORANA, EXO, SUPER NEMO, and others)

$$|m_{\beta\beta}| \simeq \text{a few } 10^{-2}\ \text{eV}$$

The effective Majorana mass strongly depends on the neutrino mass spectrum and the lightest neutrino mass

Three standard neutrino mass spectra

I. Hierarchy of neutrino masses

$$m_1 \ll m_2 \ll m_3$$

$$|m_{\beta\beta}| \simeq \left| \sin^2\theta_{12}\sqrt{\Delta m^2_{12}} + e^{2i\,\alpha_{23}}\sin^2\theta_{13}\sqrt{\Delta m^2_{23}} \right|$$

$\alpha_{23} = \alpha_3 - \alpha_2$ is the difference of the Majorana CP phases.

Both terms are small Compensation is possible

Upper bound

$$|m_{\beta\beta}| \leq 6.6 \cdot 10^{-3}\ \text{eV}$$

is smaller that the expected future sensitivity

II. Inverted hierarchy of neutrino masses

$$m_3 \ll m_1 < m_2$$

$$|m_{\beta\beta}| \simeq \sqrt{|\Delta m^2_{13}|}\;(1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha_{12})^{\frac{1}{2}}$$

The only unknown parameter is $\sin^2\alpha_{12}$.

$$0.9 \cdot 10^{-2} \leq |m_{\beta\beta}| \leq 5.8 \cdot 10^{-2}\ \mathrm{eV}$$

in the range of the anticipated sensitivities of the future experiments

III. Quasi-degenerate neutrino mass spectrum

If the lightest neutrino mass $m_0 \gg \sqrt{|\Delta m^2_{23}|}$

$$m_1 \simeq m_2 \simeq m_3$$

$$|m_{\beta\beta}| \simeq m_0\,(1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha_{12})^{\frac{1}{2}}$$

Large $|m_{\beta\beta}|$ are expected

From the observation of the $0\nu\beta\beta$-decay

$$|m_{\beta\beta}| \leq m_0 \leq 4.4\,|m_{\beta\beta}|$$

In order to determine $|m_{\beta\beta}|$ from experimental data nuclear matrix elements must be known.

Complicated nuclear problem

Different calculations of NME for the same nuclear transition differ by factor 2-3 and more

If $0\nu\beta\beta$-decay of *different nuclei* is observed ratio of NME can be measured

This can be used as a model independent test of different calculations

Are massive neutrinos and antineutrinos identical or different ? This problem, which has been put forward by E. Majorana about 70 years ago, is the most fundamental problem of the modern neutrino physics.

Without its solution the origin of the small neutrino masses and neutrino mixing probably can not be revealed.

It is still waiting for its (experimental) solution.