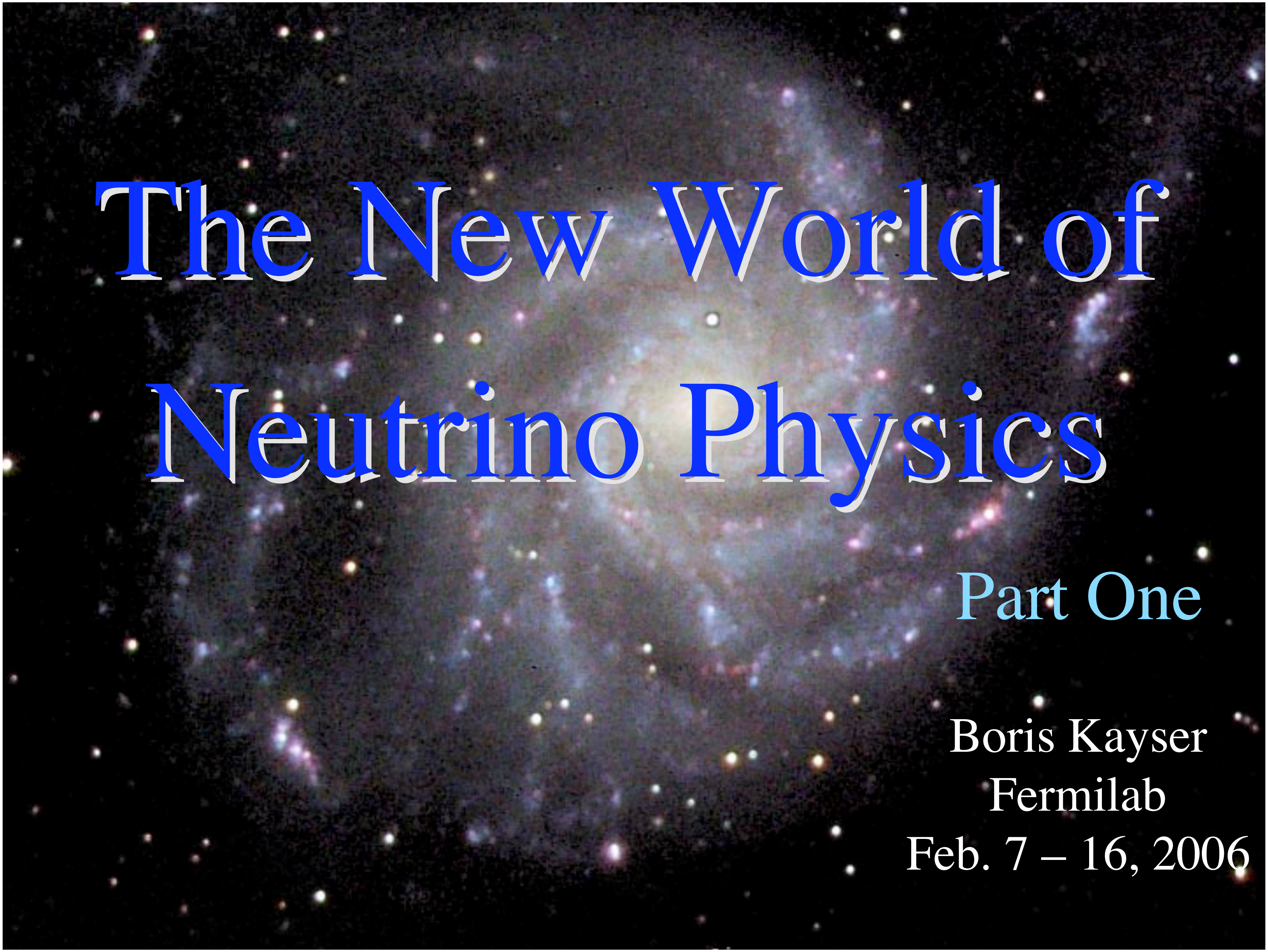

The New World of Neutrino Physics

Part One

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Fermilab
Feb. 7 – 16, 2006

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

Neutrinos are Special

The constituents of matter:

quarks, charged leptons, neutrinos.

Apart from the neutrinos, the lightest of these constituents is the electron.

But —

Neutrino masses ~ $10^{-(6-7)}$ x Electron mass

Neutrino masses, while nonzero, are *very tiny*.

Quark mixing angles are *small*.

But Leptonic mixing angles are **large**.

The quarks and charged leptons, being electrically charged, cannot be their own antiparticles.

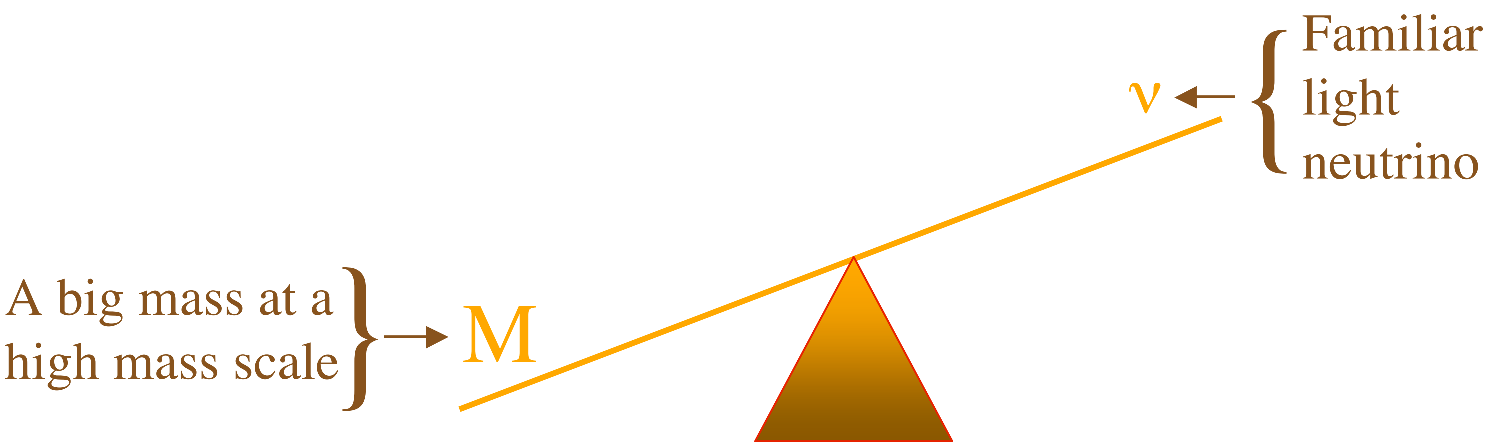
Neutrinos might be their own antiparticles: $\bar{\nu} = \nu$.

Neutrino mass probably has a different origin than the masses of the other constituents of matter.

Neutrino Mass is Physics Beyond the Standard Model

The most popular theory of why neutrinos are so light is the —

See-Saw Mechanism



The see-saw mechanism suggests that the big mass M, and the physics behind neutrino mass, are at $\sim 10^{15}$ GeV.

This puts the physics of neutrino mass ***way*** beyond the domain of the Standard Model.

Neutrinos and the Universe

* Neutrinos and photons are far and away the most abundant particles in the universe.

  If we wish to understand the universe, we must understand neutrinos.

* Neutrinos have played a role in shaping the large-scale structure of the universe.

  Observations of that structure have yielded information on neutrino mass.

* The see-saw mechanism predicts ***heavy*** neutrino “see-saw partners” to the light neutrinos.

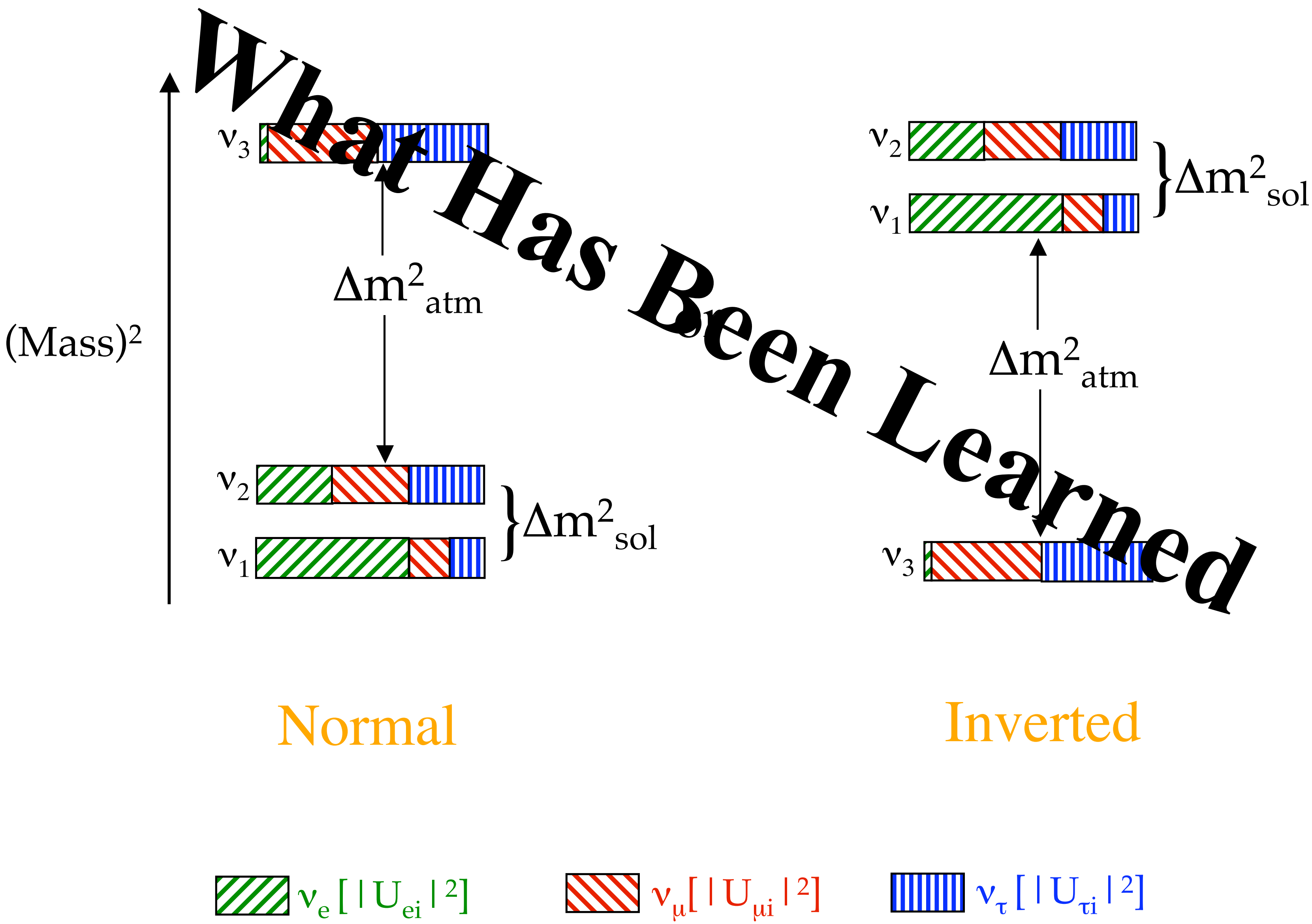
  Decays of these heavy neutrinos in the early universe may have been the origin of the excess of matter over antimatter in the universe.

The Plan for Part One

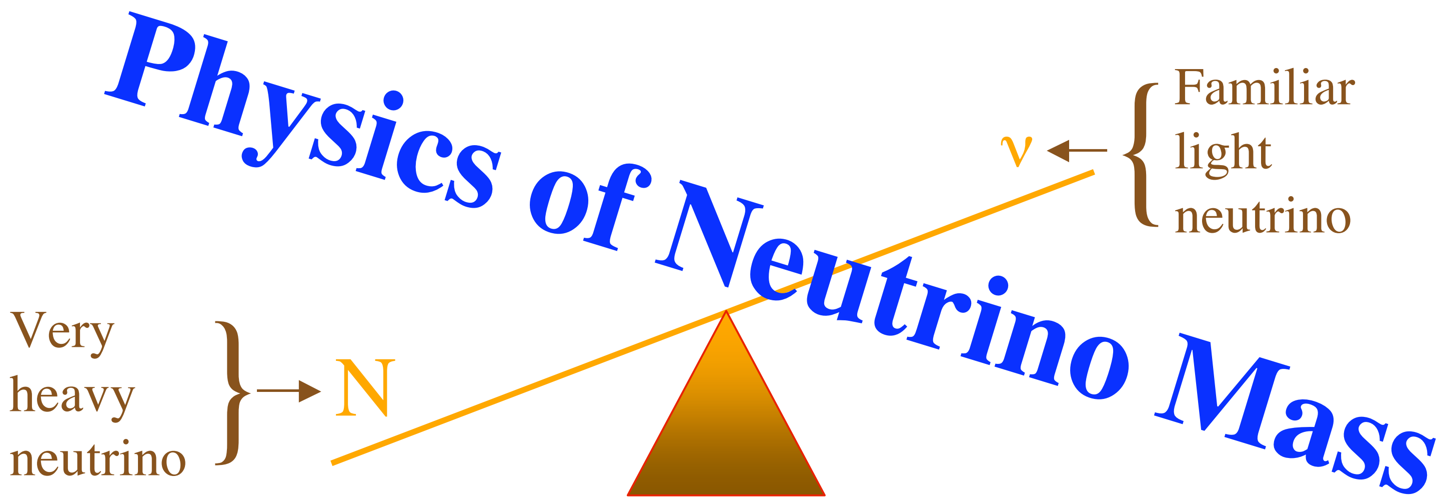
Neutrino Oscillation in Vacuum


What Has
Been Seen

The Neutrino Mass Spectrum



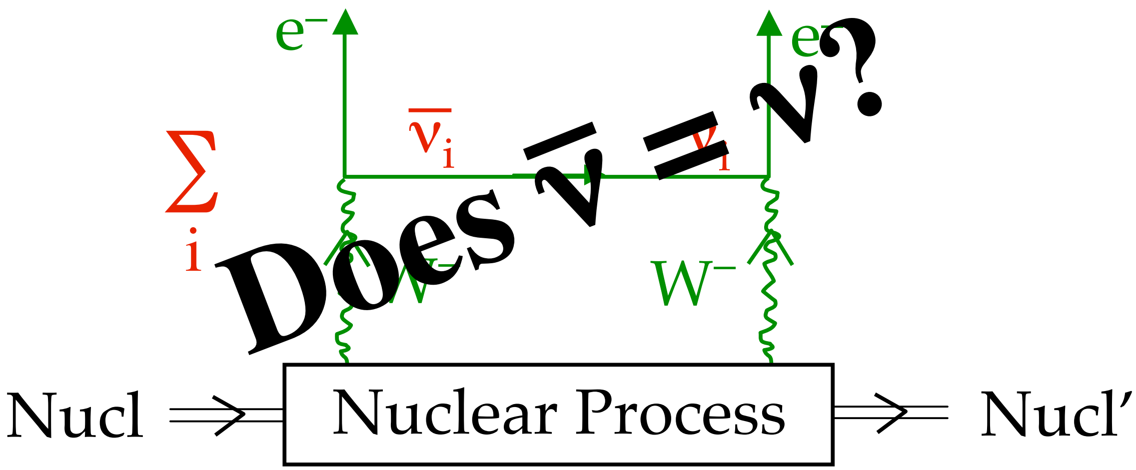
See-Saw Mechanism



The Open Questions

To Demonstrate That $\bar{\nu} = \nu$:

Neutrinoless Double Beta Decay [$0\nu\beta\beta$]



Leptogenesis

Probability [$N \rightarrow e^-$ + …] $\neq$ Probability [$N \rightarrow e^+$ + …]

MATTER   antimatter

in the early universe.

Are Neutrinos the Origin of the Matter–Antimatter Asymmetry of the Universe?

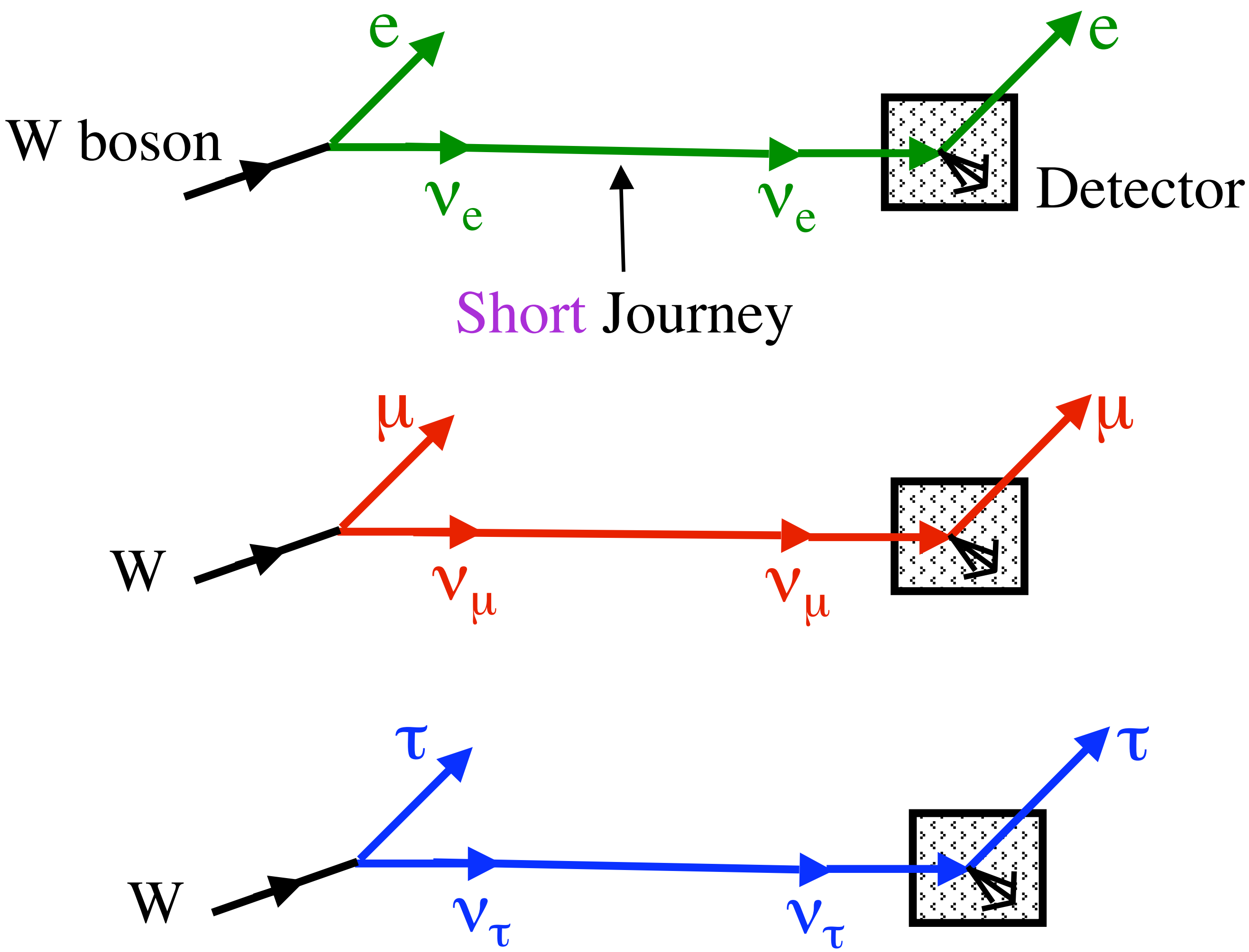
Neutrino Oscillation in Vacuum

Neutrinos Come in at Least Three Flavors

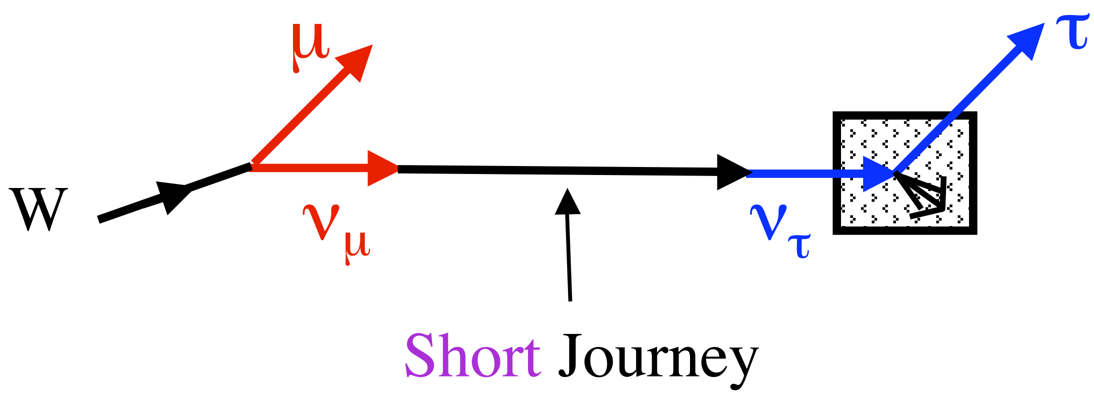
The known neutrino flavors: ν_e , ν_μ , ν_τ

Each of these is associated with the corresponding charged-lepton flavor: e , μ , τ

The Meaning of this Association



Over short distances, neutrinos do not change flavor.



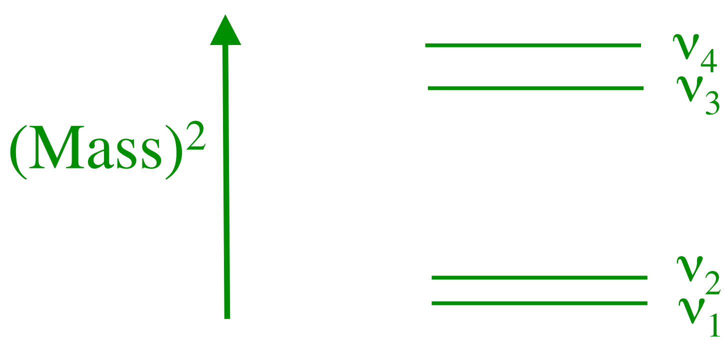
Does Not Occur

But if neutrinos have masses, and leptons mix, neutrino flavor changes do occur during ***long*** journeys.

Let Us Assume Neutrino Masses and Leptonic Mixing

Neutrino mass —

There is some spectrum of 3 or more neutrino mass eigenstates ν_i:



Mass (ν_i) $\equiv$ m_i

Neutrino mixing —

$\ell_e \equiv e,\ \ell_\mu \equiv \mu,\ \ell_\tau \equiv \tau$

When $W^+ \rightarrow \ell_\alpha^{\ +} + \nu_\alpha$,

e, μ, or τ

the produced neutrino state $|\nu_\alpha>$ is

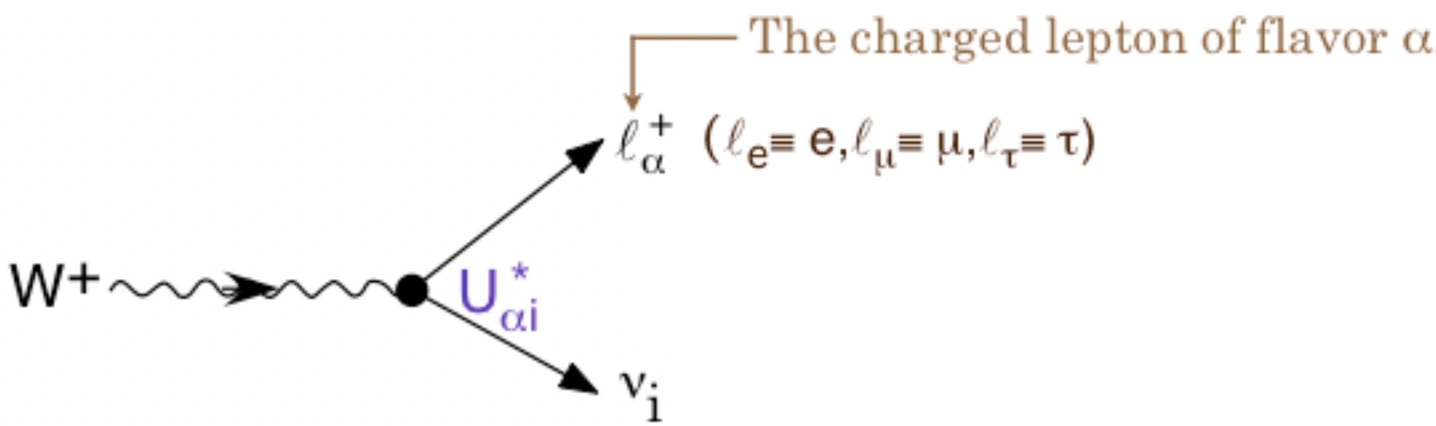
$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\ |\nu_i> \quad .$$

Neutrino of flavor α

Neutrino of definite mass m_i

Leptonic Mixing Matrix

Another way to look at W decay:



A given ℓ_α^+ can be accompanied by *any* ν_i.

$$\mathrm{Amp}(W^+ \rightarrow \ell_\alpha^+ + \nu_i) = U^*_{\alpha i}$$

The neutrino state $|\nu_\alpha>$ produced together with ℓ_α^+

is $|\nu_\alpha> = \sum_i U^*_{\alpha i} |\nu_i>$.

According to the Standard Model, extended to include neutrino mass and leptonic mixing —

- The number of different ν_i is the same as the number of different ℓ_α (3).
- The mixing matrix U is 3 x 3 and unitary:

$$UU^\dagger = U^\dagger U = 1.$$

Some models include "sterile" neutrinos — neutrinos that experience none of the known forces of nature except gravity.

In such models, there are $N > 3$ ν_i, and U is N x N, but still unitary.

Just as each neutrino of definite flavor ν_α is a superposition of mass eigenstates ν_i, so each mass eigenstate is a superposition of flavors .

From $|\nu_\alpha> = \Sigma_i\, U^*_{\alpha i}\, |\nu_i>$ and the unitarity of U,

$$|\nu_i> = \Sigma_\alpha\, U_{\alpha i}\, |\, \nu_\alpha> .$$

The flavor-α fraction of ν_i is —

$$|<\nu_\alpha|\nu_i>|^2 = |U_{\alpha i}|^2 .$$

The Standard Model (SM) description of neutrino *interactions* (not masses or leptonic mixing) is well-confirmed.

We will assume it is true, and extend it to include mixing.

For the lepton couplings to the W boson, we then have —

Left-handed

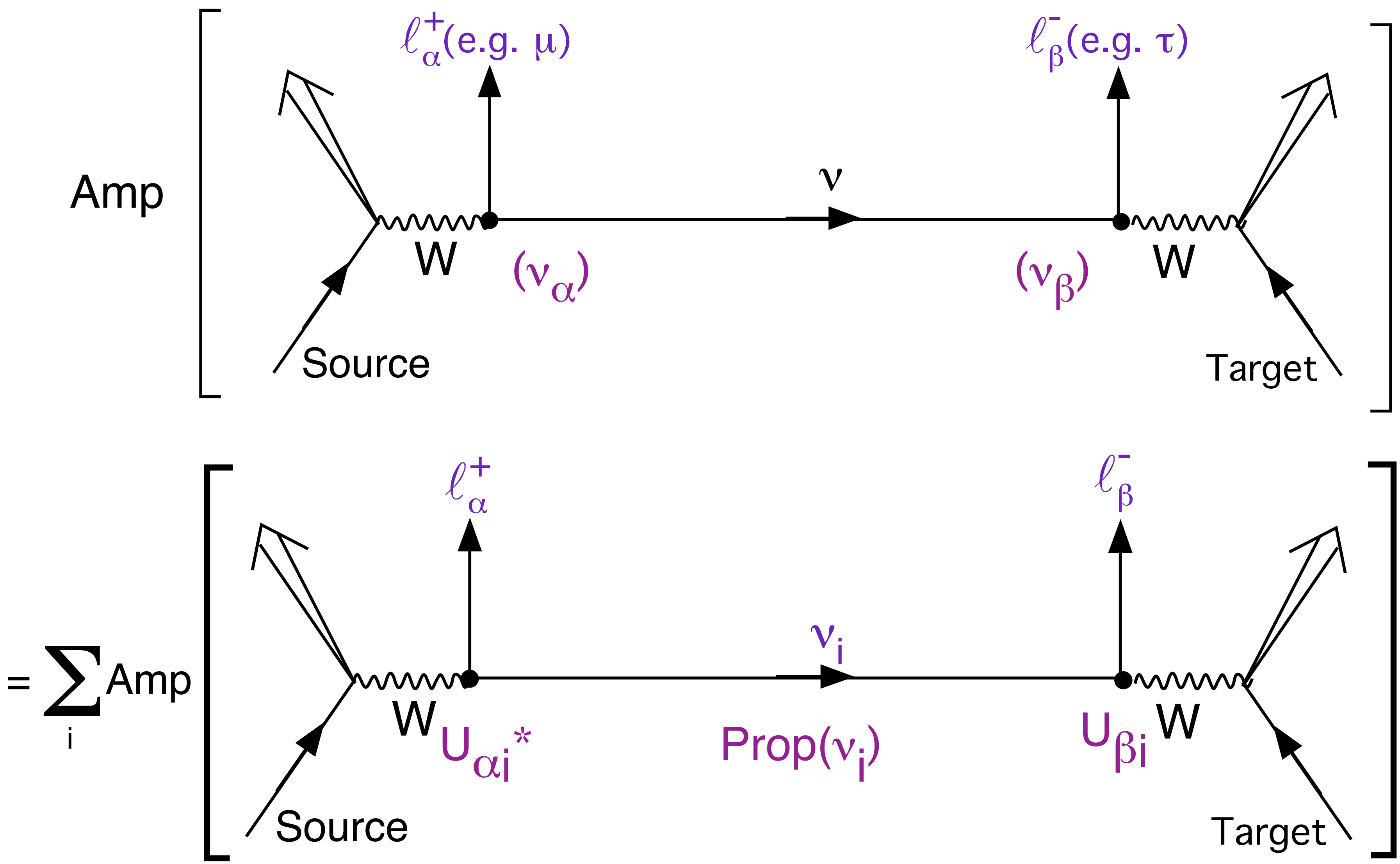
$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell_{L\alpha}} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \overline{\nu_{L\alpha}} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell_{L\alpha}} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu_{Li}} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

Neutrino Flavor Change (Oscillation) in Vacuum

(Approach of B.K. & Stodolsky)



$$\text{Amp}\,[\nu_\alpha \rightarrow \nu_\beta] = \Sigma U_{\alpha i}{}^{*}\ \text{Prop}(\nu_i) U_{\beta i}$$

What is Propagator (ν_i) ≡ Prop(ν_i)?

In the ν_i rest frame, where the proper time is τ_i,

$$i\frac{\partial}{\partial \tau_i}|\nu_i(\tau_i)>= m_i|\nu_i(\tau_i)> \quad .$$

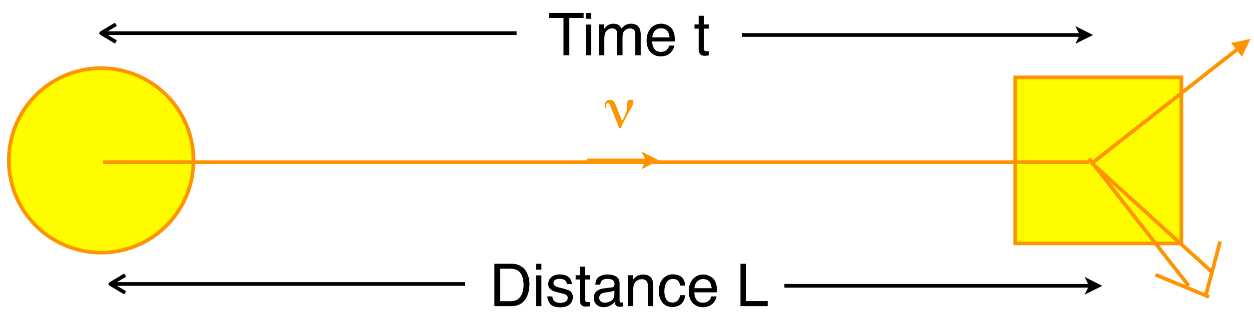
Thus,

$$|\nu_i(\tau_i)>= e^{-im_i\tau_i}|\nu_i(0)> \quad .$$

Then, the amplitude for propagation for time τ_i is —

$$\text{Prop}(\nu_i) \equiv < \nu_i(0)|\nu_i(\tau_i)>= e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t.

They are common to all components of the beam.

For each ν_i, by Lorentz invariance,

$$m_i \tau_i = E_i t - p_i L \ .$$

Neutrino sources are ~ constant in time.

Averaged over time, the

$$e^{-iE_1t} - e^{-iE_2t} \qquad \text{interference}$$

is —

$$< e^{-i(E_1-E_2)t} >_t \;=\; 0$$

$$\text{unless } E_2 = E_1 \;.$$

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad .$$

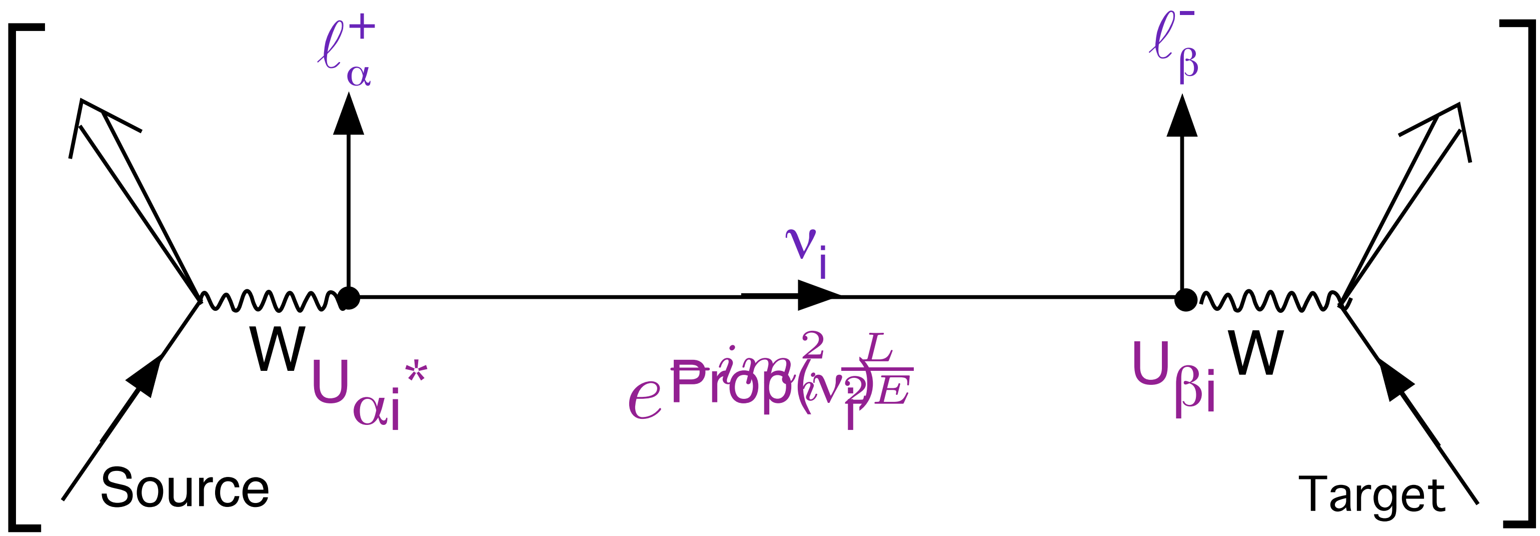
Then the phase in the ν_i propagator $exp[-im_i\tau_i]$ is —

$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= \underbrace{E(t - L)}_{\text{Irrelevant overall phase}} + m_i^2 L/2E \quad .$$

Amp [$\nu_\alpha \rightarrow \nu_\beta$]

$= \sum_i$ Amp



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \overset{CPT}{=} P(\nu_\beta \to \nu_\alpha) = P(\nu_\alpha \to \nu_\beta; U \to U^*)$$

Thus,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\underset{(-)}{+} 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

A complex U would lead to the CP violation
$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \neq P(\nu_\alpha \to \nu_\beta)$.

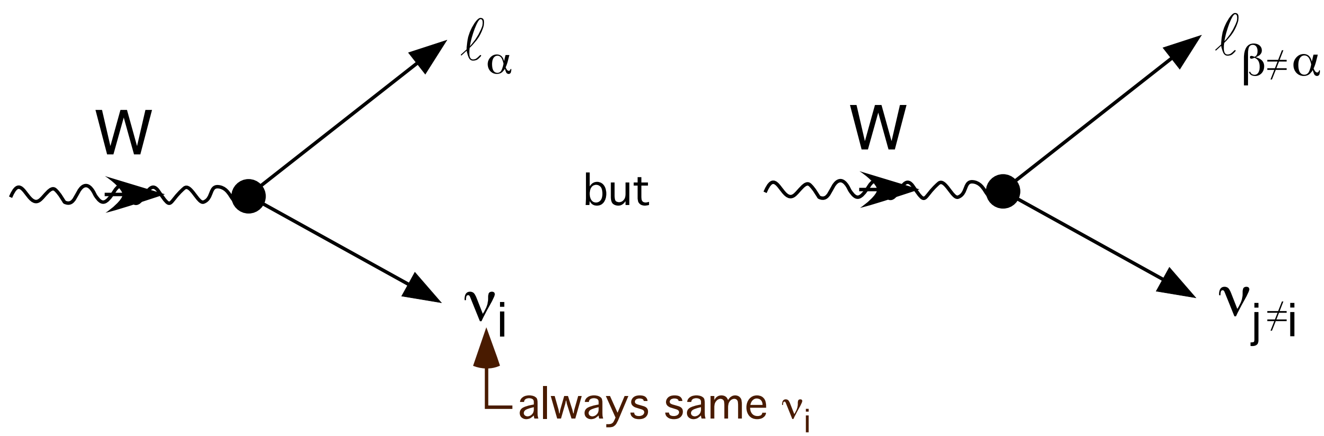
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P\left(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta\right) = \delta_{\alpha\beta}$$

Flavor *change* $\Rightarrow$ ν Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha, i} = 0, \text{ so that } P\left(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta\right) = \delta_{\alpha\beta}.$$

Flavor *change* $\Rightarrow$ Mixing

3. One can detect ($\nu_\alpha \to \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

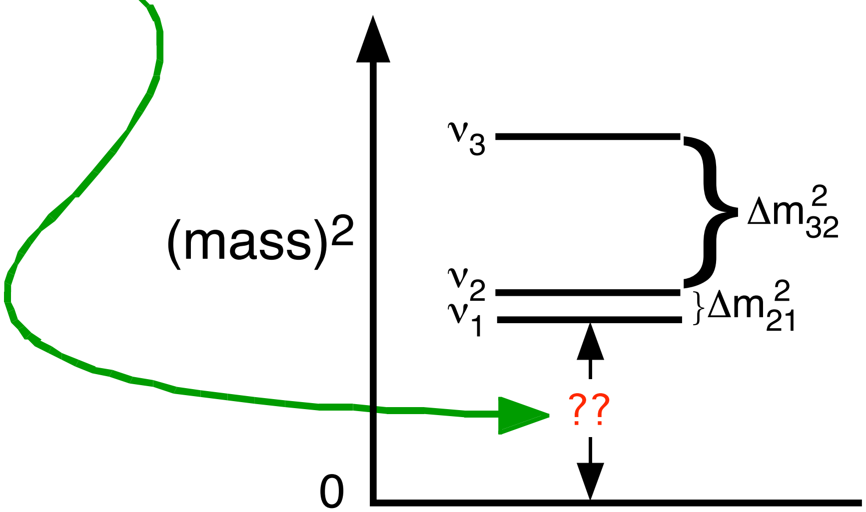
$\sin^2[1.27\Delta m^2(\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2(\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. P $(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors β≠α could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

8. Assuming all coherent ν_i in a beam have a common momentum p, rather than a common energy E, is a harmless error.

This assumption leads to the same $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$.

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i} U^*_{\beta i} e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}}$$

$$= U_{\alpha 3} U^*_{\beta 3} e^{2i\Delta_{31}} + U_{\alpha 2} U^*_{\beta 2} e^{2i\Delta_{21}} \underbrace{+ U_{\alpha 1} U^*_{\beta 1} \quad -(U_{\alpha 3} U^*_{\beta 3} + U_{\alpha 2} U^*_{\beta 2})}_{\text{Unitarity}}$$

$$= 2i[U_{\alpha 3} U^*_{\beta 3} e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U^*_{\beta 2} e^{i\Delta_{21}} \sin \Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m^2_{ij} \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^* (\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \right|^2$$

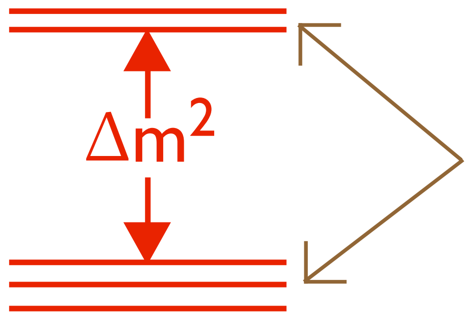
$$= 4[|U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2} U_{\beta 2}|^2 \sin^2 \Delta_{21}$$

$$+2|U_{\alpha 3} U_{\beta 3} U_{\alpha 2} U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \underset{(-)}{+} \delta_{32})] \quad .$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3} U^*_{\beta 3} U^*_{\alpha 2} U_{\beta 2})$, a CP $-$ violating phase.

Two waves of different frequencies,
and their ~~CP~~ interference.

When One Big Δm^2 Dominates



These splittings are invisible if $\Delta m^2 \dfrac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

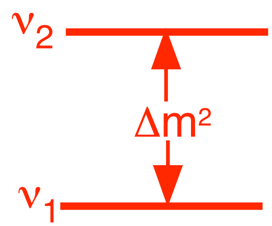
$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \cong S_{\alpha\beta} \sin^2(\Delta m^2 \frac{L}{4E}) \; ; \; S_{\alpha\beta} \equiv 4 \left| \sum_{i \text{ Clump}} U^*_{\alpha i} U_{\beta i} \right|^2 .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) \cong 1 - 4T_\alpha(1 - T_\alpha) \sin^2(\Delta m^2 \frac{L}{4E}) \; ; \; T_\alpha \equiv \sum_{i \text{ Clump}} |U^*_{\alpha i}|^2 .$$

"i Clump" is a sum over only the mass eigenstates on one end of the big gap Δm^2.

When There are Only Two Flavors and Two Mass Eigenstates



$$U = \begin{array}{cc} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} & \left[\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right] \end{array} \quad ; \quad S_{\alpha\beta} = 4T_\alpha(1 - T_\alpha) = \sin^2 2\theta$$

Mixing angle

For $\beta \neq \alpha$, $\quad P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.

For no flavor change, $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.