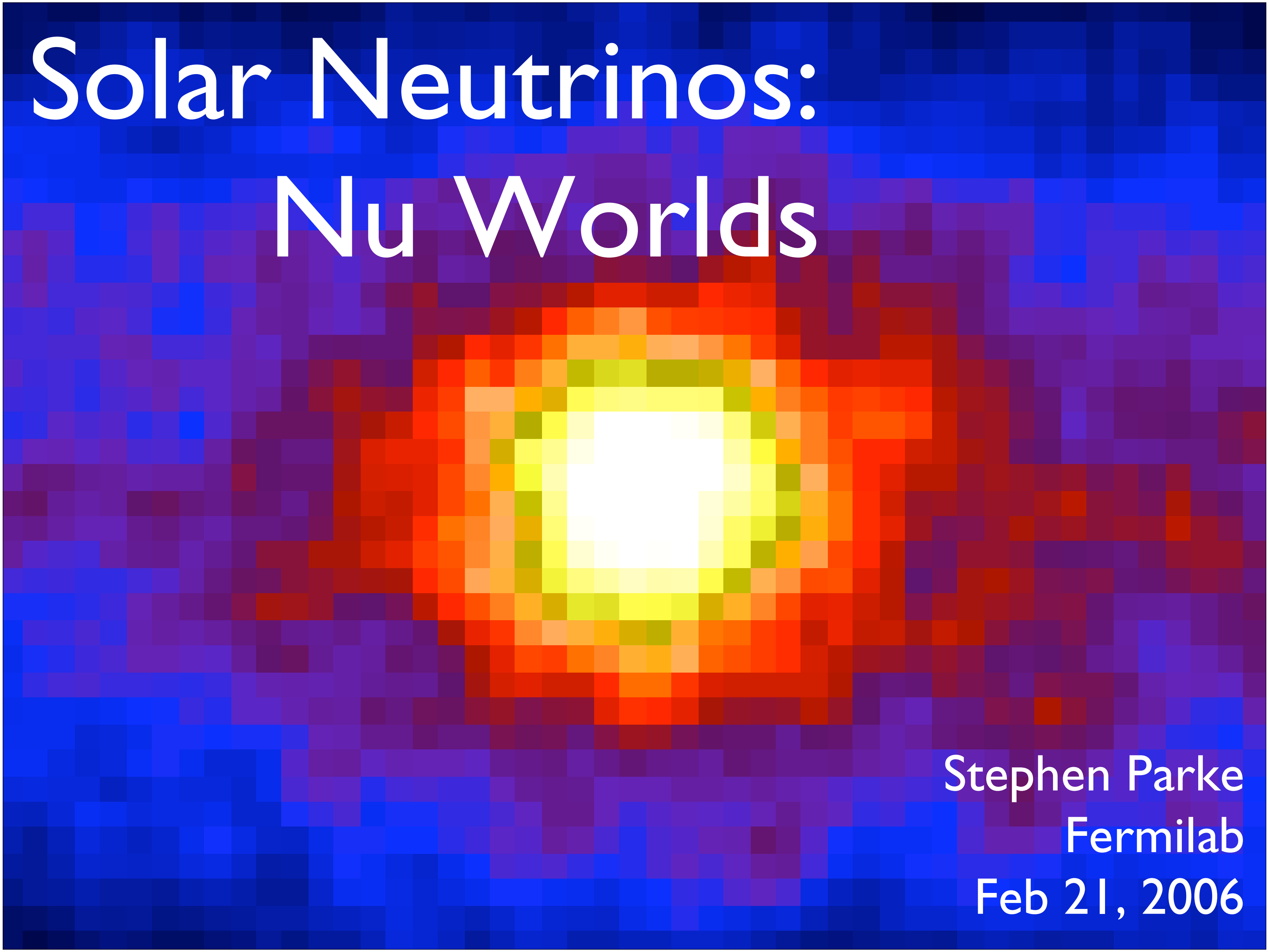
# Solar Neutrinos: Nu Worlds



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Fermilab
Feb 21, 2006

## Solar Engine:

$$4p + 2e^- \rightarrow {}^4He + 2\nu_e + 26.7MeV$$

$$E = mc^2$$

1 $\nu_e$ for every 13.4 MeV (=2.1 $\times 10^{-12}$ J)

$\mathcal{L}_\odot$ at earth's surface 0.13 watts/cm$^2$

$$\phi_\nu = \frac{0.13}{2.1 \times 10^{-12}} = 6 \times 10^{10}/cm^2/sec$$

http://theory.fnal.gov/people/parke/TALKS/2006

# Solar Spectrum:

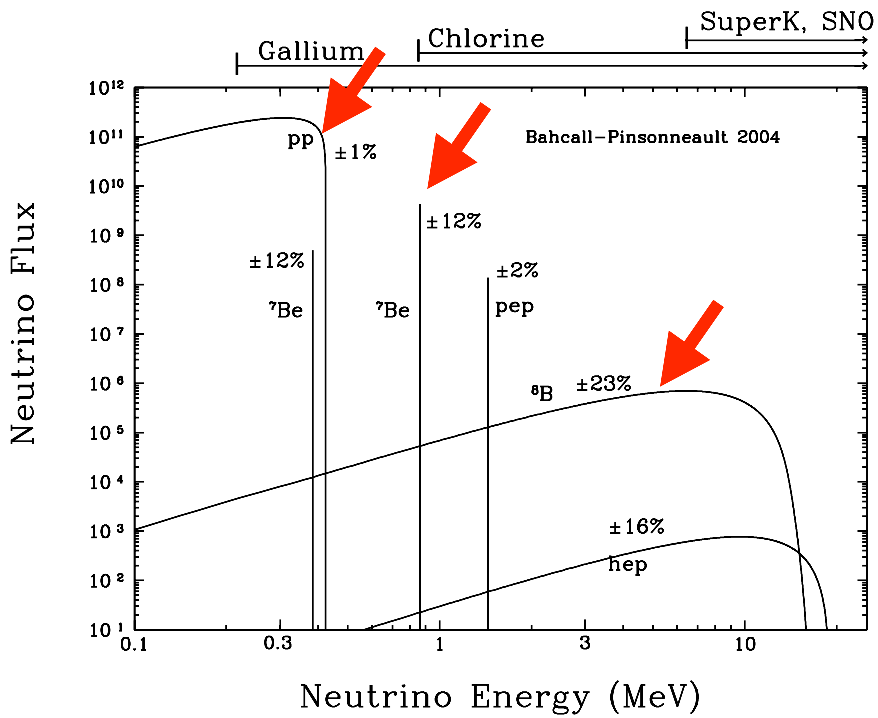



**Figure 1.** The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos $cm^{-2}\,s^{-1}\,MeV^{-1}$ at the Earth's surface. For line sources, the units are number of neutrinos $cm^{-2}\,s^{-1}$. Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).

$$p + p \longrightarrow ^2 H + e^+ + \nu_e$$

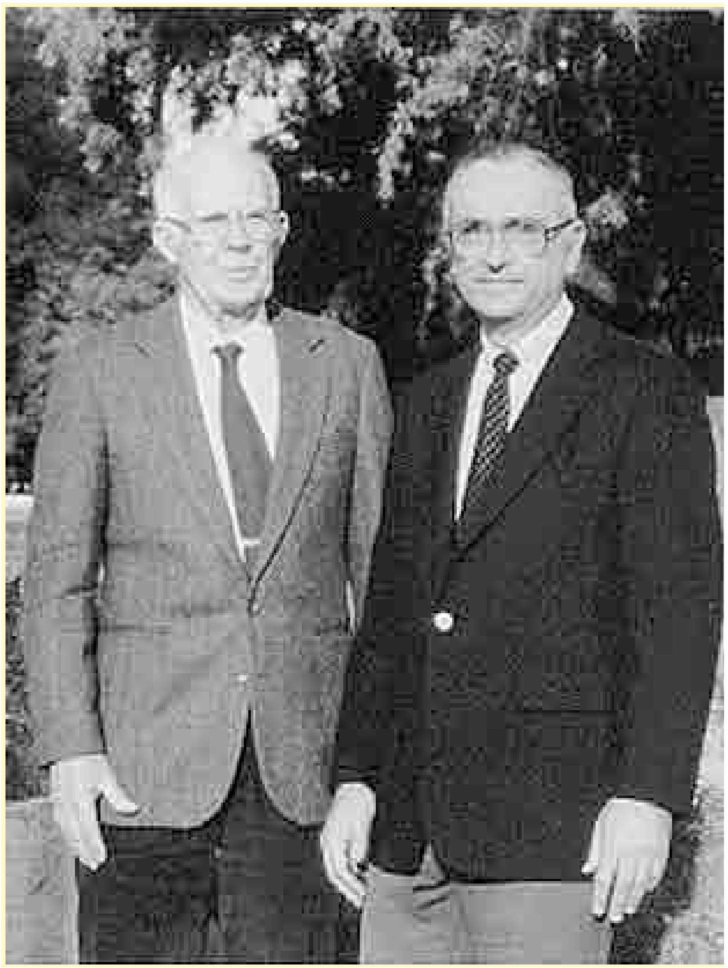
$$\phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} cm^{-2} sec^{-1}$$

$$^7Be + e^- \longrightarrow ^7 Li + \nu_e$$

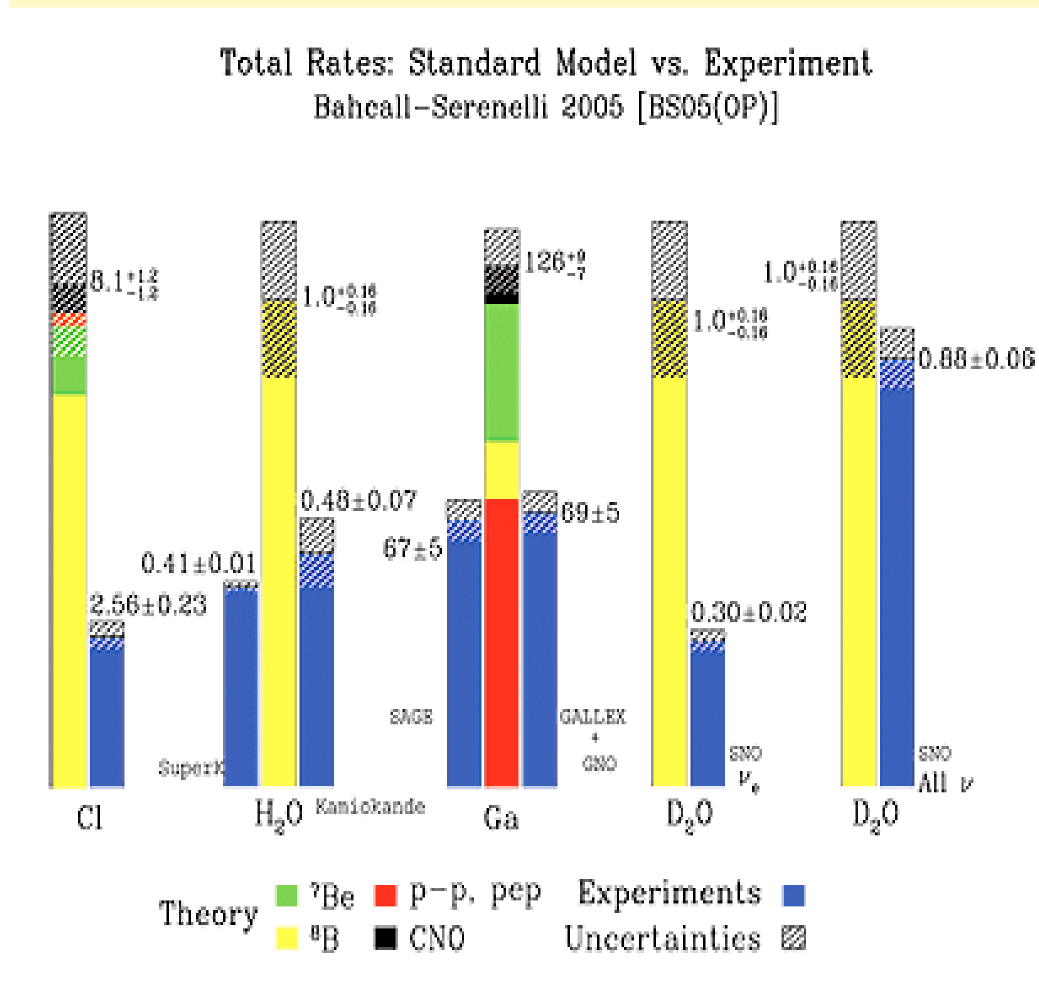
$$\phi_{^7Be} = 4.86(1 \pm 0.12) \times 10^{9} cm^{-2} sec^{-1}$$

$$^7Be + p \longrightarrow ^8 B \longrightarrow ^8 Be^* + e^- + \nu_e$$

$$\phi_{^8B} = 5.82(1 \pm 0.23) \times 10^{6} cm^{-2} sec^{-1}$$

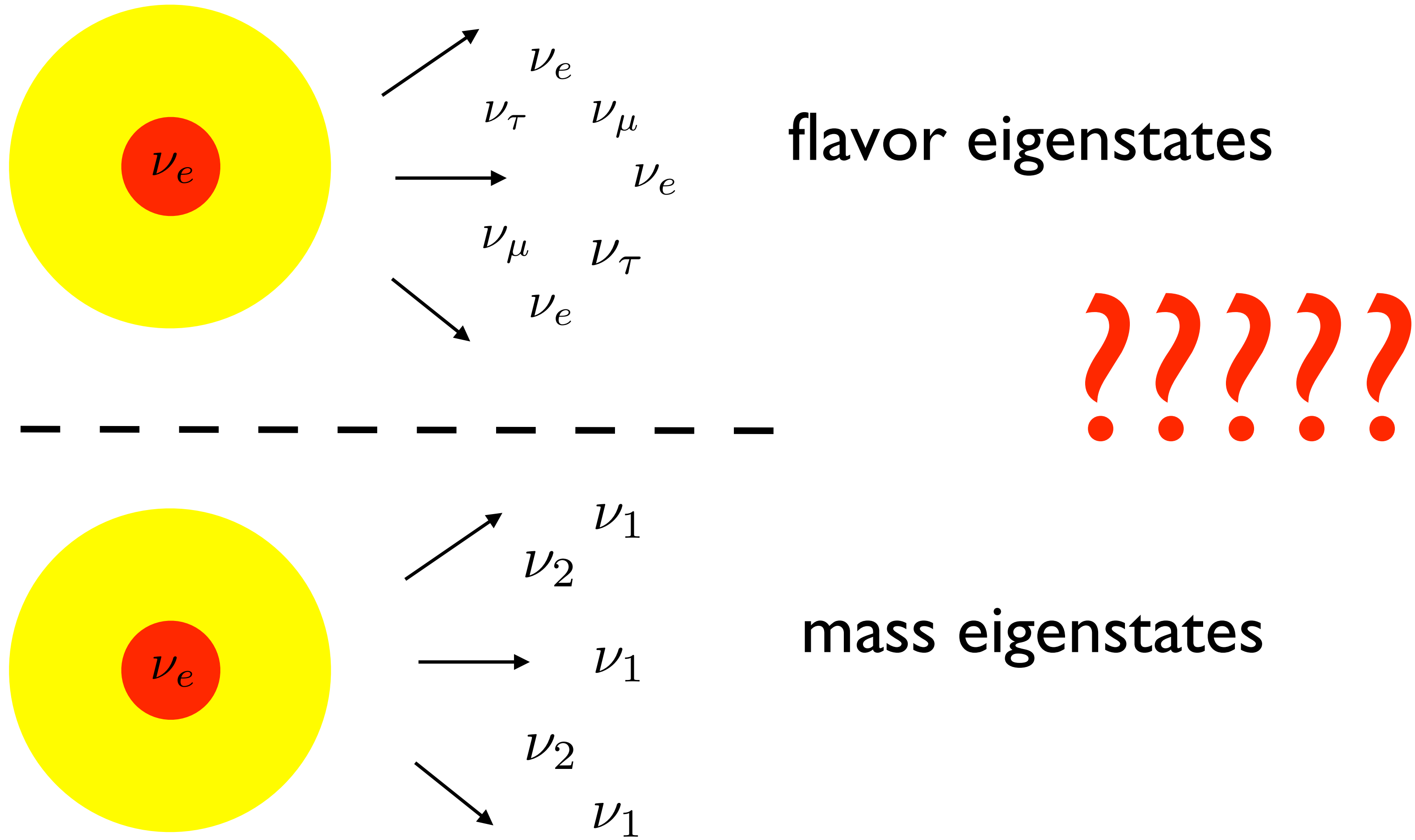


Ray Davis & John Bahcall




Theory v Exp.

# Identical Solar Twins:



Kinematical Phase:

$$\delta m^2_\odot = 8.0 \times 10^{-5} eV^2$$

$$\sin^2\theta_\odot = 0.31$$

$$\Delta_\odot = \frac{\delta m^2_\odot L}{4E} = 1.27 \quad \frac{8{\times}10^{-5}\ eV^2 \cdot 1.5{\times}10^{11}\ m}{0.1{-}10\ MeV}$$

???

$$\Delta_\odot \approx 10^{7\pm1}$$

Effectively Incoherent !!!

## Vacuum $\nu_e$ Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

where $f_1$ and $f_2$ are the fraction of $\nu_1$ and $\nu_2$ at production.

In vacuum $f_1 = \cos^2\theta_\odot$ and $f_2 = \sin^2\theta_\odot$.

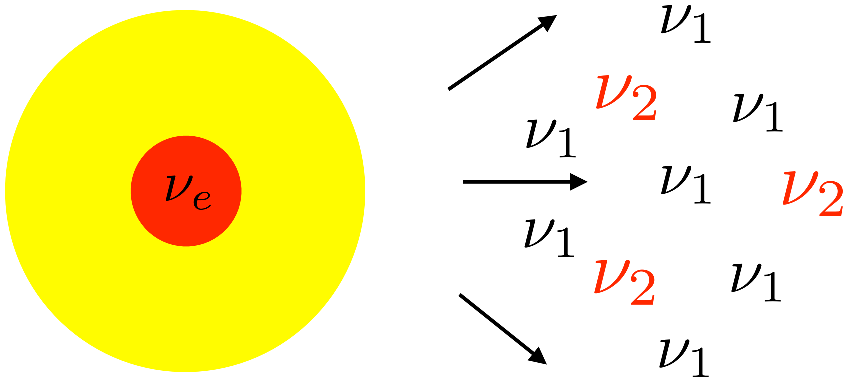
Note energy independence.

$$\langle P_{ee} \rangle = \cos^4\theta_\odot + \sin^4\theta_\odot = 1 - \frac{1}{2}\sin^2 2\theta_\odot$$

for pp and $^7$Be this is approximately THE ANSWER.

$f_1 \sim 69\%$ and $f_2 \sim 31\%$ and $\langle P_{ee} \rangle \approx 0.6$

# pp and $^7$Be



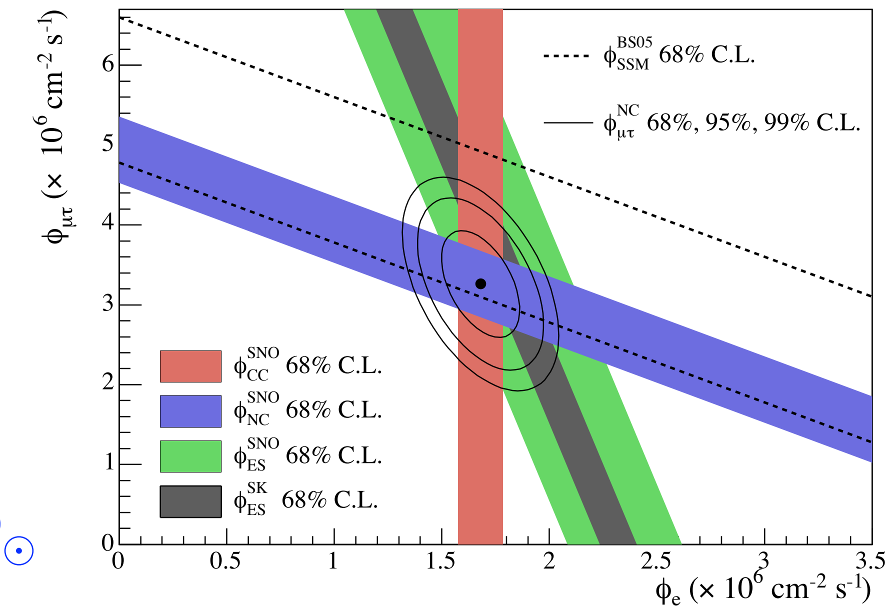

$f_1 \sim 69\%$

$f_2 \sim 31\%$

$$\langle P_{ee} \rangle \approx 0.6$$

$f_3 = \sin^2 \theta_{13} < 4\%$

# What about $^8B$ ?



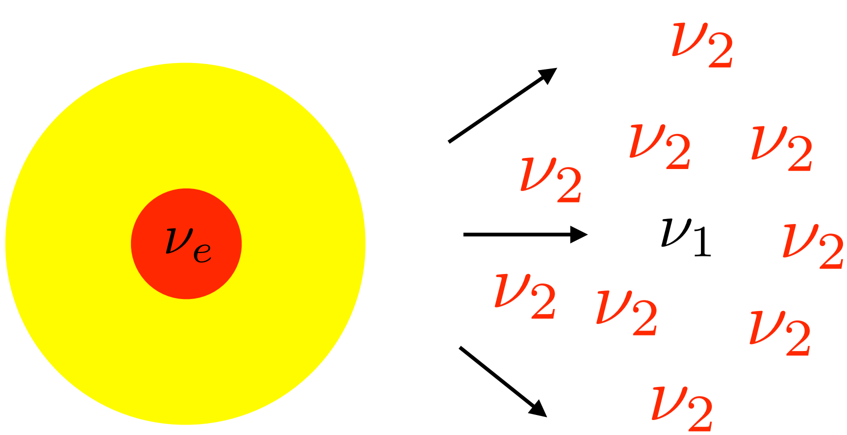

## SNO's CC/NC

$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

$$f_1 = \left(\frac{CC}{NC} - \sin^2\theta_\odot\right) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31)/0.4 \approx 10 \pm ???\%$$

# $^8B$




$f_2 \sim 90\%$

$f_1 \sim 10\%$

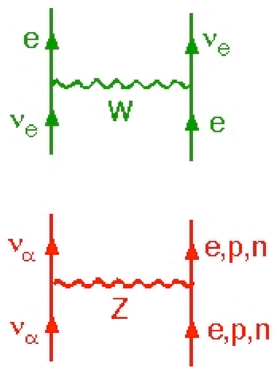
$$\langle P_{ee} \rangle = \sin^2\theta + f_1 \cos 2\theta_\odot \approx \sin^2\theta_\odot = 0.31$$

Wow!!! How did that happen???

energy dependence!!!

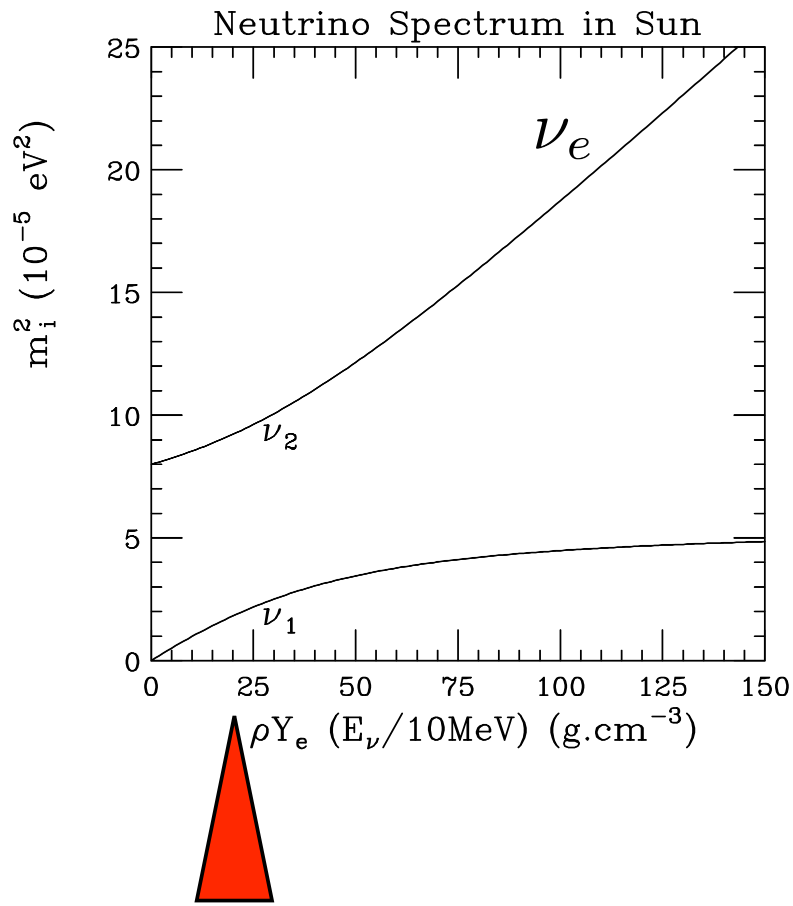
# MSW

Coherent Forward Scattering:




Wolfenstein '78

MATTER EFFECTS CHANGE THE NEUTRINO MASSES AND MIXINGS




Mikheyev + Smirnov Resonance WIN '85

## Neutrino Evolution:

$$-i\frac{\partial}{\partial t}\nu = H\nu$$

in the mass eigenstate basis

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } H = \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix}$$

$$E = \sqrt{p^2+m^2}$$

$$H = (p + \tfrac{m_1^2+m_2^2}{4p})I + \tfrac{1}{4E}\begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}$$

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

## in the flavor basis

$$\nu \to U\nu \text{ and } H \to UHU^\dagger$$

where $\nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix}$ and $U = \begin{pmatrix} \cos\theta_\odot & \sin\theta_\odot \\ -\sin\theta_\odot & \cos\theta_\odot \end{pmatrix}$

$$0 < \theta_\odot < \frac{\pi}{2}$$

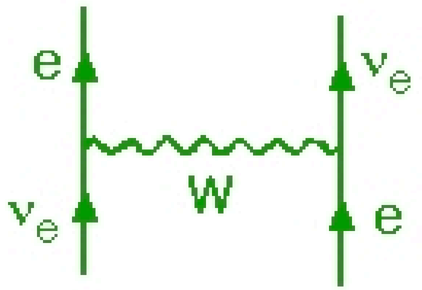
and therefore in flavor basis

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

i.e. $\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{mass} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{flavor}$

# Coherent Forward Scattering:

dimensions $[G_F N_e] = M^{-2} L^{-3} = M$




$$\pm\sqrt{2} G_F N_e \; \delta_{ee}$$

$N_e$ is number density of electrons

$+$(-) for neutrinos (anti-neutrinos)

Wolfenstein '78


Same for all active flavors, therefore overall phases

$$\begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \frac{G_F N_e}{\sqrt{2}} I_2 + \frac{1}{2} \begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & -\sqrt{2} G_F N_e \end{pmatrix}$$

Including Matter Effects in the Flavor Basis:

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m^2 \cos 2\theta_\odot + 2\sqrt{2} G_F N_e E_\nu & \delta m^2 \sin 2\theta_\odot \\ \delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu \end{pmatrix}$$

Diagonalize by identifying with

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\ \delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N \end{pmatrix}$$

## Masses and Mixings in MATTER: $\delta m_N^2$ and $\theta_\odot^N$

$$\begin{aligned} \delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu \\ \delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot \end{aligned}$$

Notice:
(1) Possible zero when $\delta m^2 \cos 2\theta_\odot = 2\sqrt{2} G_F N_e E_\nu$
(2) the invariance of the product $\delta m^2 \sin 2\theta_\odot$

$\nu_e$ disappearance in Loooong Block of Lead:

$$1 - P(\nu_e \to \nu_e) = \sin^2 2\theta_\odot^N \sin^2 \Delta_N$$

$$\Delta_N = \frac{\delta m_N^2 L}{4E}$$

same form as vacuum

The Solution:

$$\delta m_N^2 = \sqrt{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu)^2 + (\delta m^2 \sin 2\theta_\odot)^2}$$

$$\sin^2 \theta_\odot^N = \frac{1}{2}\left(1 - \frac{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu)}{\delta m_N^2}\right)$$

$$\theta_\odot^N > \theta_\odot$$

Quasi-Vacuum: $2\sqrt{2}G_F N_e E_\nu \ll \delta m^2 \cos 2\theta_\odot$

pp and $^7$Be

$$\delta m_N^2 = \delta m^2 \text{ and } \theta_\odot^N = \theta_\odot$$

Resonance (Mikheyev + Smirnov '85): $2\sqrt{2}G_F N_e E_\nu = \delta m^2 \cos 2\theta_\odot$

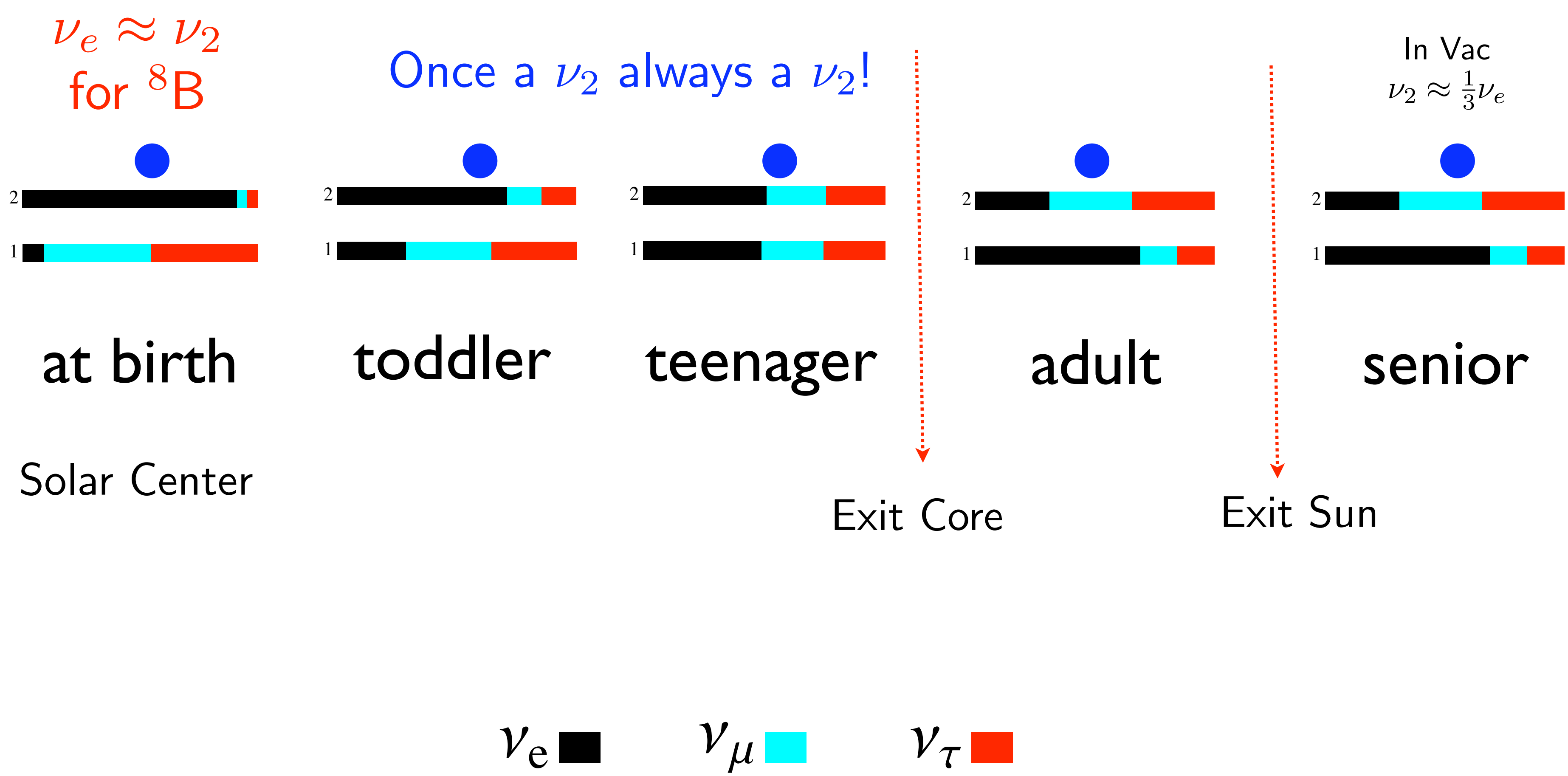
$$\delta m_N^2 = \delta m^2 \sin 2\theta_\odot \text{ and } \theta_\odot^N = \pi/4$$

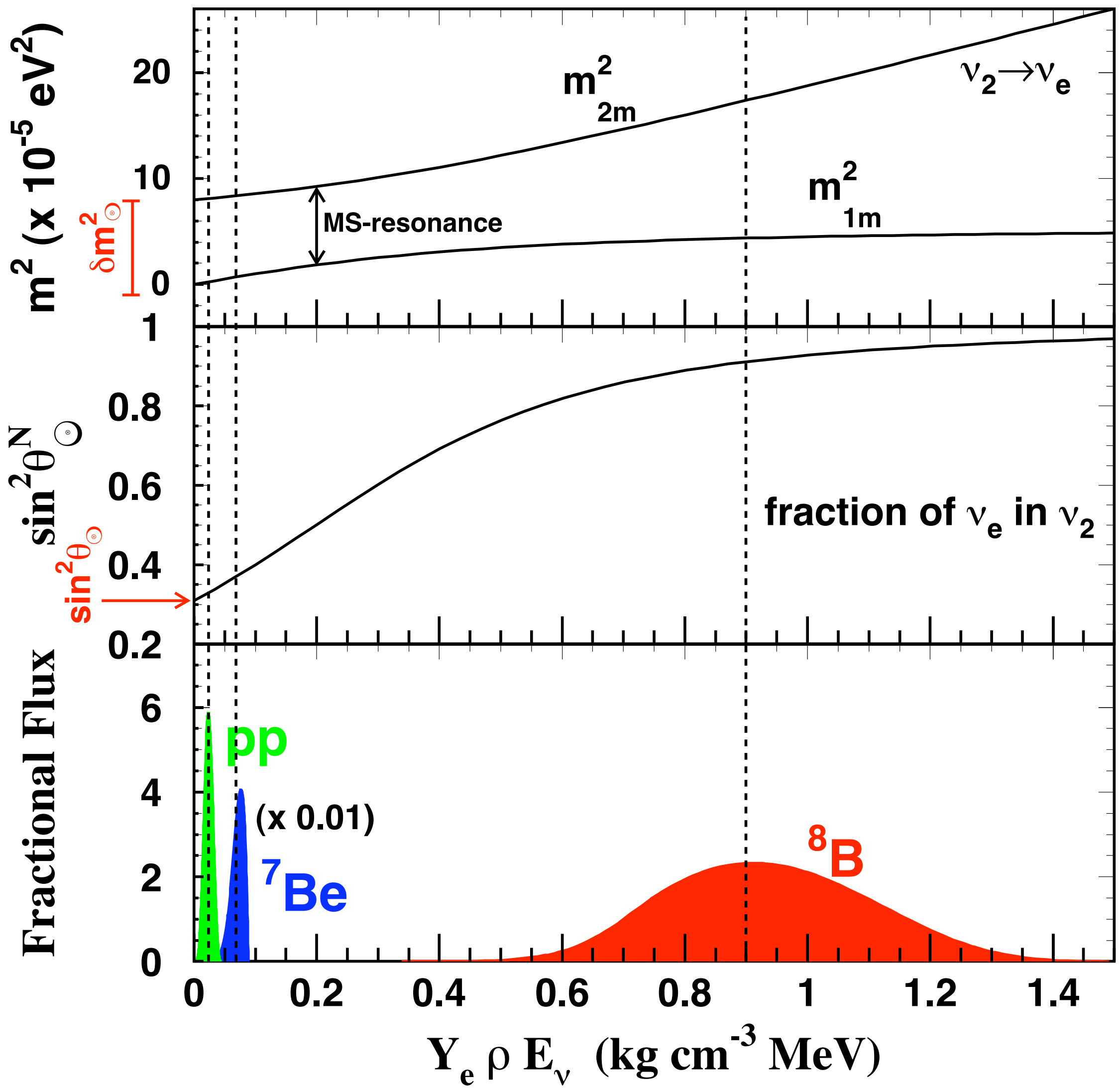
Matter Dominated: $2\sqrt{2}G_F N_e E_\nu \gg \delta m^2 \cos 2\theta_\odot$

$^8B$

$$\delta m_N^2 \to 2\sqrt{2}G_F N_e E_\nu \text{ and } \theta_\odot^N \to \pi/2$$

# Life of a Boron-8 Solar Neutrino:






In Vacuum

$$\delta m^2_\odot = 8.0 \pm 0.4 \times 10^{-5}\ eV^2$$

$$\sin^2\theta_\odot = 0.31 \pm 0.03$$

Whereas for $^8$B at center of Sun

$$\delta m^2_N = 14 \times 10^{-5}\ eV^2$$

$$\sin^2\theta^N_\odot = 0.91$$

hep-ph/0601198

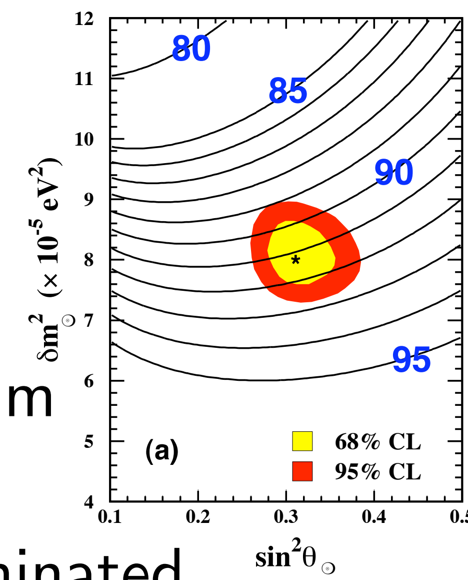
# Mass Eigenstate Purity:

| | $\langle f_1 \rangle$ (%) | $\langle f_2 \rangle$ (%) |
|---|---|---|
| vac | $69 \pm 3$ | $31 \mp 3$ |
| pp | $67 \pm 4$ | $33 \mp 4$ |
| $^7$Be | $63 \pm 4$ | $37 \mp 4$ |
| $^8$B | $9 \mp 2$ | $91 \pm 2$ |

quasi-vacuum

matter dominated

$^8$B: $\nu_2$ fraction (%)




$f_1 = \cos^2\theta_\odot^N$ and $f_2 = \sin^2\theta_\odot^N$

$\langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$

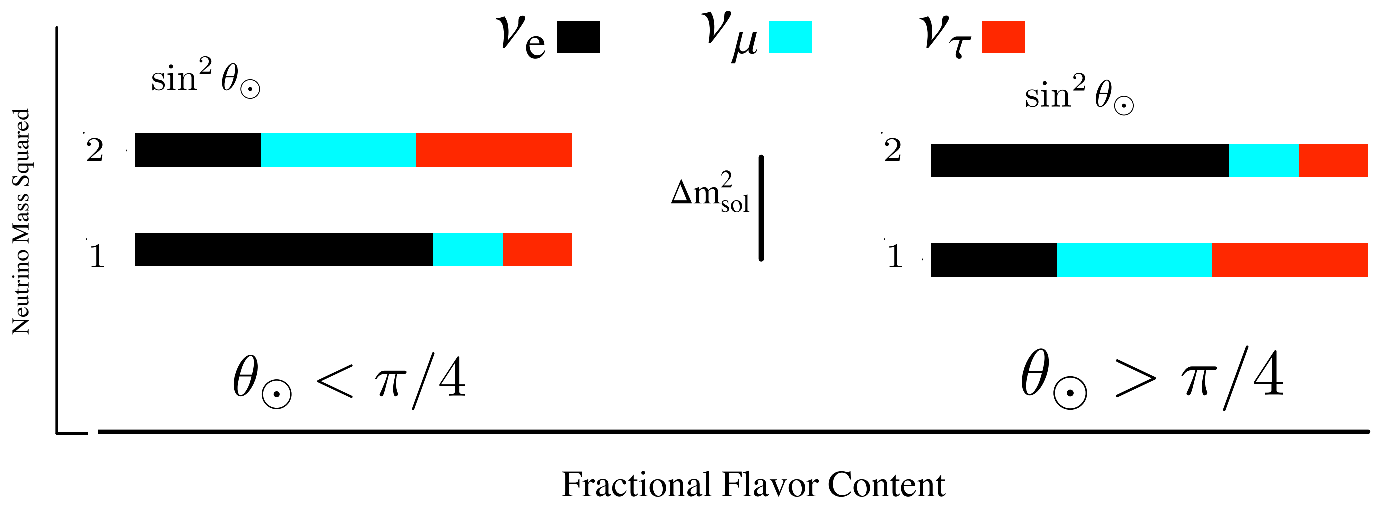
vac pp $^7$Be $^8$B

$N_e E_\nu$

$\langle P_{ee} \rangle = \cos^4\theta_\odot + \sin^4\theta_\odot$

$\Rightarrow \sin^2\theta_\odot$

# Solar Pair Mass Hierarchy:




KamLAND doesn't care since $\sin^2 2\theta_\odot$ same for $\theta_\odot$ and $\frac{\pi}{2} - \theta_\odot$

but SNO does !!!

for neutrino in matter
$\theta_\odot^N > \theta_\odot$

$$\langle P_{ee}\rangle = \cos^2\theta_\odot^N \cos^2\theta_\odot + \sin^2\theta_\odot^N \sin^2\theta_\odot = \frac{1}{2} + \frac{1}{2}\cos 2\theta_\odot^N \cos 2\theta_\odot$$

if $\theta_\odot < \pi/4$
$\langle P_{ee}\rangle \geq \sin^2\theta_\odot$

if $\theta_\odot > \pi/4$
$\langle P_{ee}\rangle \geq \frac{1}{2}(1 + \cos^2 2\theta_\odot) \geq \frac{1}{2}$

SNO: $\langle P_{ee}\rangle_{day} = 0.347 \pm 0.038$

Solar Hierarchy Determined !!!

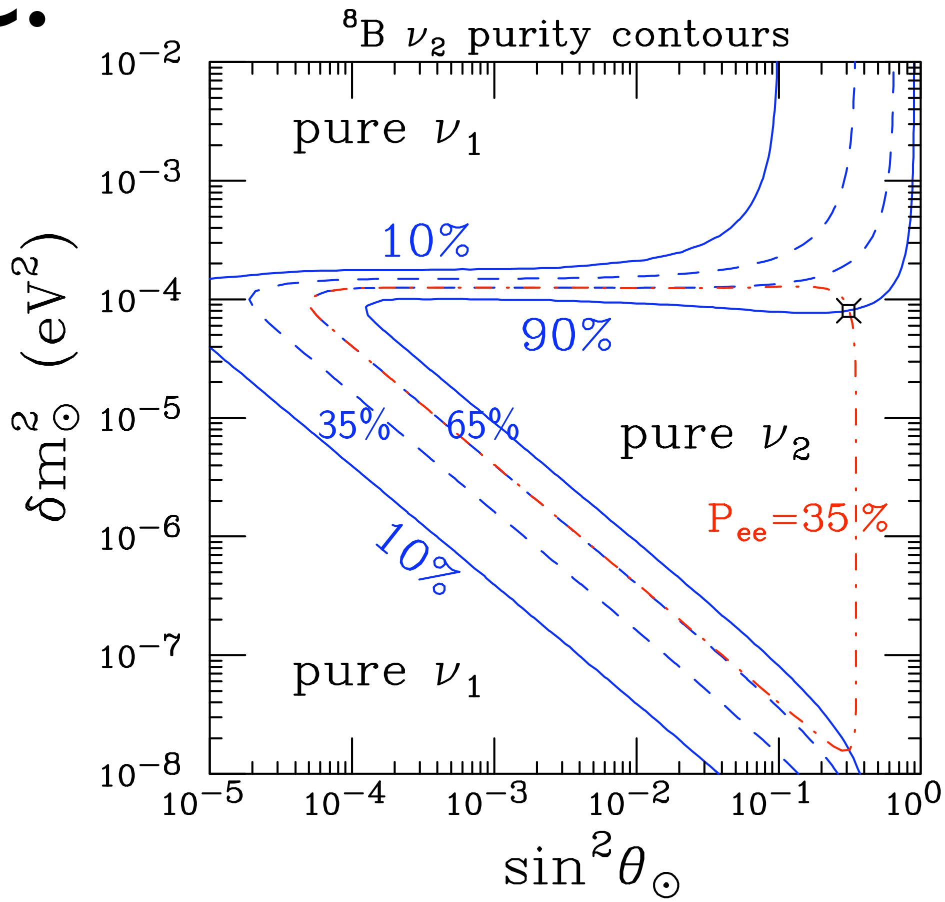
# The Big Picture:

$$P_{ee} = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

$$f_1 = (1 - P_x)\cos^2\theta_\odot^N + P_x \sin^2\theta_\odot^N$$

$$f_2 = (1 - P_x)\sin^2\theta_\odot^N + P_x \cos^2\theta_\odot^N$$

$P_x$ is the probability to jump from $\nu_2$ to $\nu_1$ (or $\nu_1$ to $\nu_2$) during MS-resonance crossing.




$$P_{ee} = \frac{1}{2} + \left(\frac{1}{2} - P_x\right)\cos 2\theta_\odot^N \cos 2\theta_\odot$$

Jump Probability:

$$P_x \approx \exp\left(-\pi \, \frac{Width\ of\ Resonance}{Oscillation\ Length}\right)$$

SP PRL, 57, 1275 and 2322 (1986)

## Day/Night Asymmetry:

$\nu_2 \rightarrow (1-\alpha)\nu_2 + \alpha\nu_1$ passing thru the earth.

A=2(D-N)/(D+N) expected to be few %

SK:

$$A_{ES} = -1.8 \pm 1.6(\mathrm{stat})^{+1.3}_{-1.2}(\mathrm{syst})\%$$

## Spectral Distortion:

A characteristic of matter effects is that the Fraction of $\nu_2$ is energy dependent .

Smaller at smaller E.

Implies an increase in $P_{ee}$ near threshold.

Maybe in SK with 4.5MeV threshold!!!

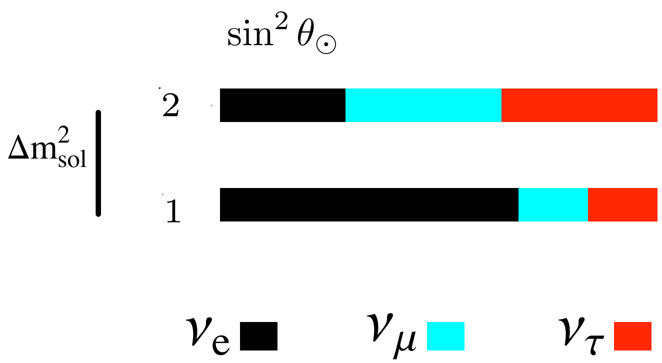
# Summary:

The low energy pp and $^7$Be Solar Neutrinos exit the sun as two thirds $\nu_1$ and one third $\nu_2$ due to (quasi-) vacuum oscillations.

$f_1 = 65 \pm 2\%$, $f_2 = 35 \mp 2\%$ with $P_{ee} \approx 0.56$

The high energy $^8$B Solar Neutrinos exit the sun as "PURE" $\nu_2$ mass eigenstates due to matter effects.

$f_2 = 91 \pm 2\%$ and $f_1 = 9 \mp 2\%$ with $P_{ee} \approx 0.35$.

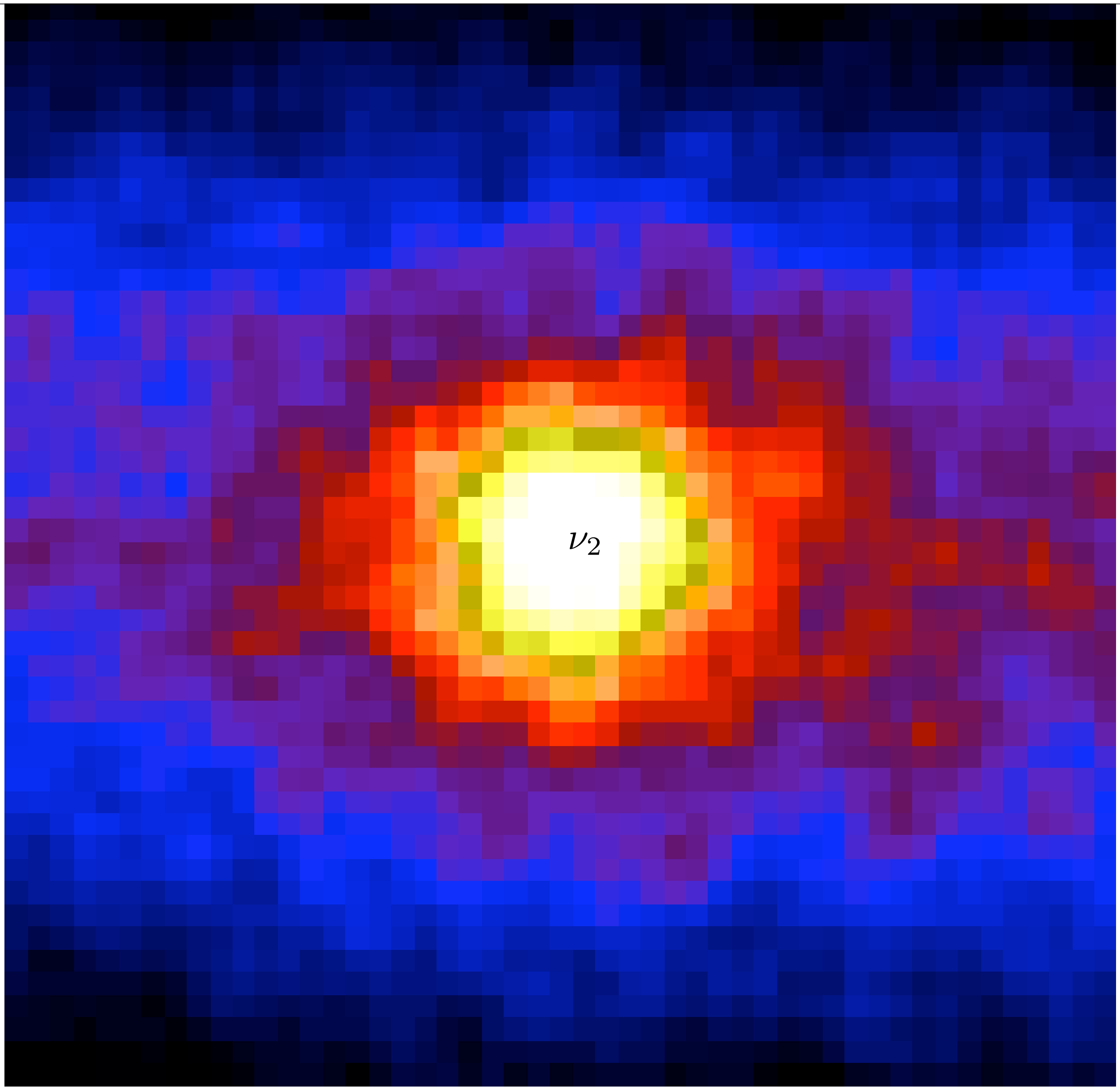



$$\delta m^2_\odot = 8.0 \pm 0.4 \times 10^{-5} eV^2$$

$$\sin^2\theta_\odot = 0.310 \pm 0.026$$

at 68% CL

SNO, KamLAND, SK/K, GNO/Gallex, SAGE, Cl




These are $\nu_2$ Neutrinos !!!

What happens to the neutrino oscillation length in the semi-classical limit, $\hbar \to 0$ ?

- $L_{osc} \longrightarrow \infty$
- $L_{osc} \longrightarrow 0$
- Other