Recent developments in Neutrino physics

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Glimpses at the field

50 anniversary of the neutrino discovery

F. Reines and C.L. Cowan, publications 1953 - 1956

From several 10th of events (lowest order in G_F)

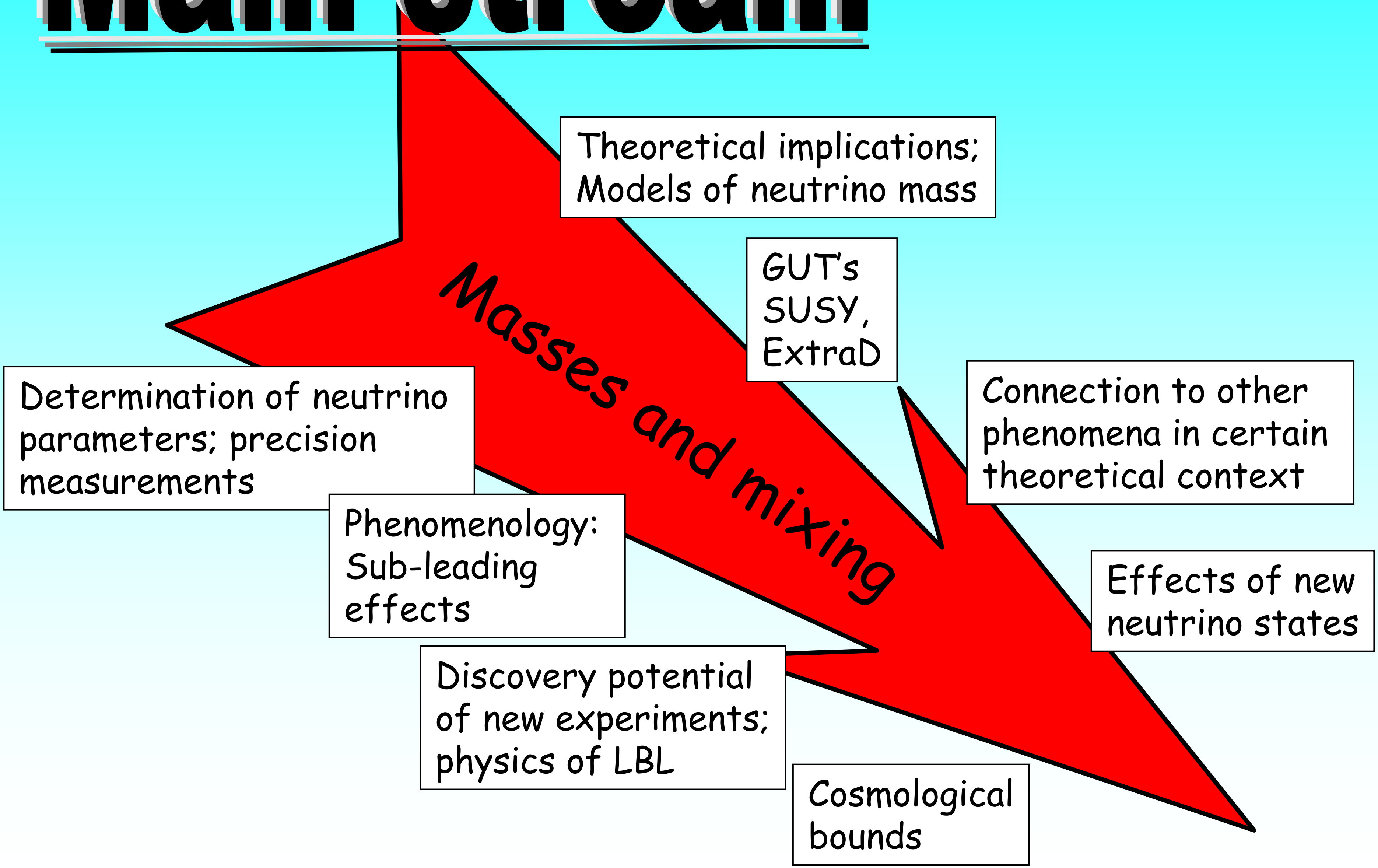
To 300 000 of the double beta decay events (second order in G_F)

25 years from idea to discovery

about 70 years to measure mass

Proton decay?
30 year from GUT formulation

Main stream


Masses and mixing
Theoretical implications; Models of neutrino mass
GUT's SUSY, ExtraD
Connection to other phenomena in certain theoretical context
Effects of new neutrino states
Determination of neutrino parameters; precision measurements
Phenomenology: Sub-leading effects
Discovery potential of new experiments; physics of LBL
Cosmological bounds

Neutrino map

Masses and mixing

Non-standard interactions

- Light scalars
-Magnetic moments

Neutrino astrophysics

High energy cosmic neutrinos: sources, fluxes, detection

Supernova neutrinos

Neutrinos in Cosmology

Neutrinos and LSS
Bounds on neutrino parameters

Neutrinos and Dark energy

Leptogenesis

New neutrino states

Phenomenology, LSND, Astrophysical consequences

The last neutrino anomaly?

Neutrino anomalies: driving force of developments of the field for many years

What is left?

LSND?

What is hot?

MiniBooNE

`` Neutrino Standard Model"

Searches for physics beyond ``neutrinoSM"

Main challenge

What is behind the observed pattern of neutrino masses and mixing?
(as well as masses and mixings of other fermions)

What is the underlying physics?

How far we can go in this understanding using usual notions
of the field theory (or effective field theory) and
in terms of symmetries, various mechanisms of symmetry breaking, etc. ?

Do we need something more beyond this?

Is something important missed in our approaches, ideas, principles?

Are we on right track?

Phenomenology of neutrino mass and mixing

1. Masses, Mixing, Effects

2. Phenomenology,
   determination of parameters,
   reconstructing neutrino spectrum

3. Beyond "standard picture"

Bottom-up: Implications

Part I.

Masses, Mixing, Effects

Flavors, Masses, Mixing

Flavors and masses

Flavor neutrino states:

ν_e ν_μ ν_τ

e μ τ

correspond to certain charged leptons

interact in pairs

Eigenstates of the CC weak interactions

ν_s Sterile neutrinos?

Mass eigenstates

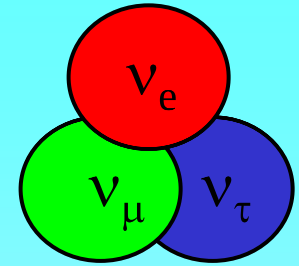
ν_1 ν_2 ν_3

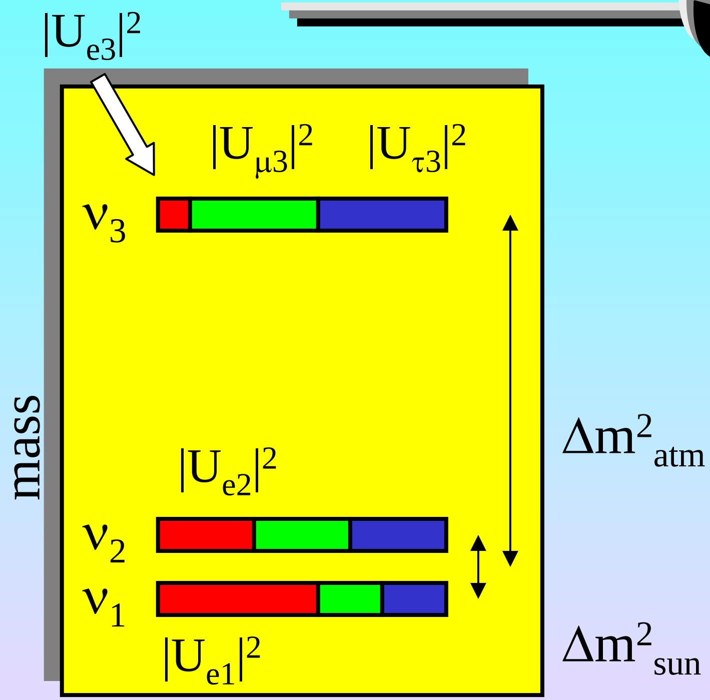
m_1 m_2 m_3

Mixing

Flavor states $\neq$ Mass eigenstates

Mixing angles





Normal mass hierarchy

$$\Delta m^2_{atm} = \Delta m^2_{32} = m^2_3 - m^2_2$$

$$\Delta m^2_{sun} = \Delta m^2_{21} = m^2_2 - m^2_1$$

Moduli of mixing elements are paremeterization independent

$$\tan^2\theta_{12} = |U_{e2}|^2 \; / \; |U_{e1}|^2$$

$$\sin^2\theta_{13} = |U_{e3}|^2$$

$$\tan^2\theta_{23} = |U_{\mu 3}|^2 \; / \; |U_{\tau 3}|^2$$

Mass eigenstates can be marked by the e-flavor (in parameterization independent way):

ν_1 is the state with maximal amount of the e-flavor

ν_3 is the state with minimal amount of the e-flavor

Mixing matrix

$$\nu_f = U_{PMNS}\,\nu_{mass}$$

where $\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$ $\nu_{mass} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$

the matrix is unitary:

$$U_{PMNS}^{+}\,U_{PMNS} = I$$

Pontecorvo-Maki-Nakagawa-Sakata mixing matrix

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$U_{\alpha\iota} = |U_{\alpha i}|\, e^{i\phi_{\alpha i}}$$

Due to unitarity and possibility to renormalize wave functions of neutrinos and charge leptons only one phase is physical

Parameterization

$U_{PMNS} = U_{23}\ I_\delta\ U_{13}\ U_{12}$

$I_\delta = \mathrm{diag}\,(1,\ e^{i\delta},\ e^{-i\delta})$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{12} = \cos\theta_{12}$, etc.

δ is the Dirac CP violating phase

θ_{12} is the ``solar'' mixing angle

θ_{23} is the ``atmospheric'' mixing angle

θ_{13} is the mixing angle restricted by CHOOZ/PaloVerde experiments

Two aspects of mixing

$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

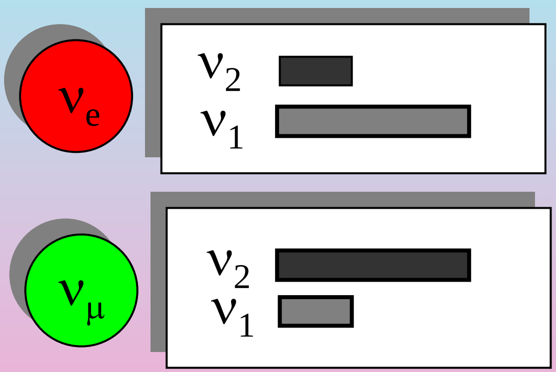
coherent mixtures of mass eigenstates

inversely

$$\nu_2 = \sin\theta\, \nu_e + \cos\theta\, \nu_\mu$$
$$\nu_1 = \cos\theta\, \nu_e - \sin\theta\, \nu_\mu$$

vacuum mixing angle

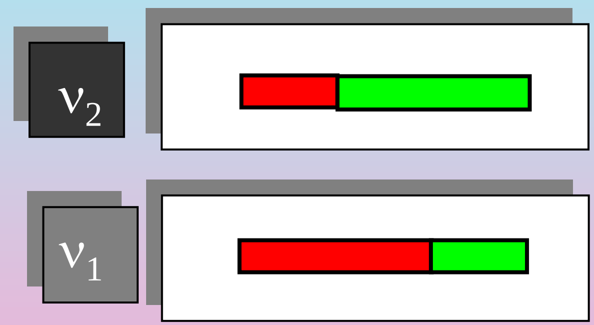
flavor composition of the mass eigenstates



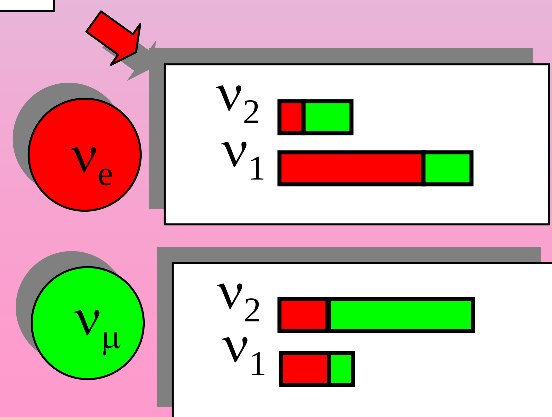
amplitudes

wave packets

inserting



Flavors of eigenstates



The relative phases of the mass states in ν_e and ν_μ are opposite

Interference of the parts of wave packets with the same flavor depends on the phase difference $\Delta\phi$ between ν_1 and ν_2

Who mixes neutrinos?

Non-trivial interplay of

Charged current weak interactions

Kinematics of specific reactions

Difference of the charged lepton masses

ν_e → $\beta-$ decays, energy conservation

ν_μ → $\pi-$ decays, chirality suppression

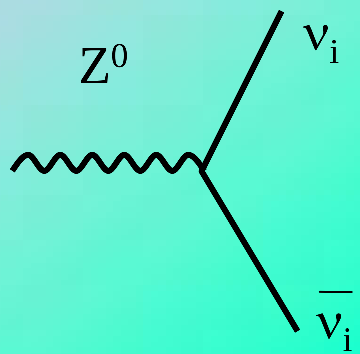
ν_τ → Beam dump, D - decay

What about neutral currents?

Can NC interactions prepare mixed state?

Z is flavor blind

What is the neutrino state produced in the Z-decay in the presence of mixing?

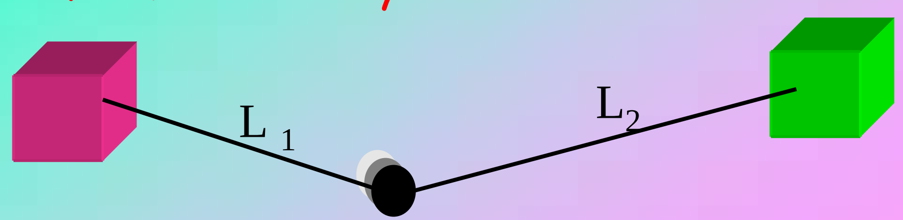


$$|f\rangle = \frac{1}{\sqrt{3}} \left[|\bar{\nu}_1 \nu_1\rangle + |\bar{\nu}_2 \nu_2\rangle + |\bar{\nu}_3 \nu_3\rangle \right]$$

$$|\langle f | H | Z\rangle|^2 = 3 |\langle \bar{\nu}_1 \nu_1 | H | Z\rangle|^2$$

Do neutrinos from Z^0- decay oscillate?

Two detectors experiment: detection of both neutrinos



If the flavor of one of the neutrino is fixed, another neutrino oscillates

$$P = \sin^2 2\theta \ \sin^2 [\pi (L_1 + L_2)/ l_\nu]$$

Determining oscillation parameters

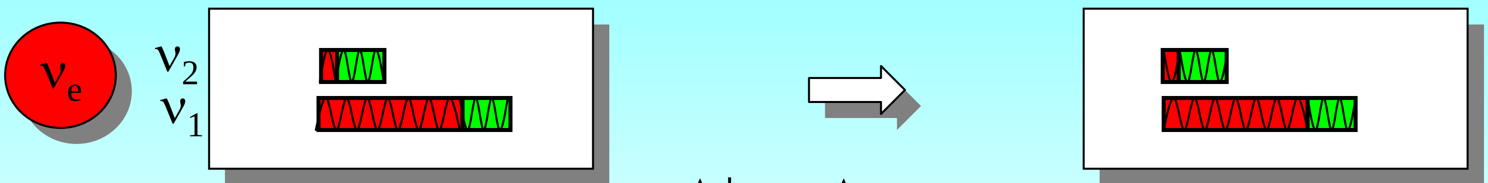
Parameters	Source of info	Effects involved
$\Delta m^2_{12}, \theta_{12}$	Solar neutrinos KamLAND	Adiabatic conversion, MSW Averaged oscillations Vacuum oscillations
$\Delta m^2_{23}, \theta_{23}$	Atmospheric neutrinos, K2K, MINOS	Vacuum oscillations
θ_{13}	CHOOZ, Atmospheric neutrinos + …	Vacuum oscillations Oscillations in matter

Oscillations

Vacuum Oscillations

- Flavors of mass eigenstates do not change
- Admixtures of mass eigenstates do not change: no ν_1 <-> ν_2 transitions

} Determined by θ



$\Delta\phi = 0$ $\qquad$ $\Delta\phi = \Delta v_{phase}\, t$

- Due to difference of masses ν_1 and ν_2 have different phase velocities

oscillations: effects of the phase difference increase which changes the interference pattern

Oscillation phase

- Phases should be calculated in the same space time point: x,t

$$\phi_i = E_i\, t - k_i\, x \;\sim\; E_i\,(t - x) + \frac{m_i^2}{2E_i}\, x$$

- $\Delta\phi = \Delta E t - \Delta p x$ $\qquad p = \sqrt{E^2 - m^2}$

$$\Delta p = (dp/dE)\,\Delta E + (dp/dm^2)\,\Delta m^2 = 1/v_g\,\Delta E + (1/2p)\,\Delta m^2$$

group velocity

- $$\Delta\phi = \Delta E\,(t - x/v_g) + \frac{\Delta m^2}{2E}\, x$$

In general (depending on conditions of production and detection) both quantities are non-zero

$\Delta E = 0$? ... v_g) = 0

AVERAGING OVER ΔE

L. O... al.
H. L...pkin

C. Giunti,
Center of the wave packet ...

Standard results are reproduced if both quantities are small

Oscillation effects should disappear in the limit $\Delta m^2 \rightarrow 0$

Parameters of oscillations

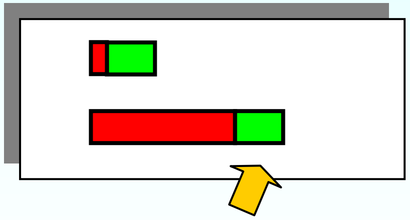
- Oscillation length: the distance at which the neutrino systems returns to the initial state

$$\Delta\phi = 2\pi \;\Rightarrow\; \Delta v_{phase}\, l_\nu = 2\pi \;\Rightarrow\; \Delta v_{phase} = \frac{\Delta m^2}{2E}$$

$$l_\nu = 2\pi/\Delta v_{phase} = 4\pi E/\Delta m^2$$

- Depth of oscillations: is given by maximal probability to find ν_μ in the originally produced ν_e states.

Muonic parts in the the wave packet sum up:



$\sin\theta \cos\theta$

$$\Rightarrow\; A_P = (2 \sin\theta \cos\theta)^2 = \sin^2 2\theta$$

Oscillation probability

$$P(\nu_\mu) = \frac{A_p}{2}\left(1 - \cos\frac{2\pi x}{l_\nu}\right) = \sin^2 2\theta \sin^2 \frac{\pi x}{l_\nu}$$

Features of neutrino oscillations in vacuum:

Oscillations -- effect of the phase difference increase between mass eigenstates

Admixtures of the mass eigenstates ν_i in a given neutrino state do not change during propagation

Flavors (flavor composition) of the eigenstates are fixed by the vacuum mixing angle

Paradoxes of Neutrino Oscillations

Still under discussion:

What interferes:
``equal energies or momenta"?

Steady source approximation?

Coherence length

Phase of oscillations?
Correct way to calculate

Relevance of wave packets

Meaning/relevance of the ``flavor states";

Limitations of the plane wave description

Role of the uncertainty principle;

Effect of particles accompanying neutrino production (interaction)
effect of ``recoil"

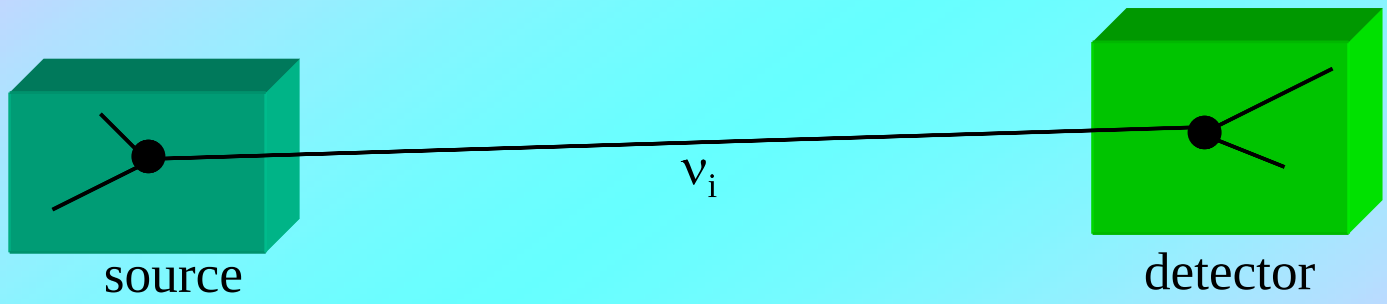
Standard formula and beyond

Plane wave:	V. Gribov, B. Pontecorvo, S. Bilenky, B. Pontecorvo, H. Fritzsch, P. Minkowski
Wave packet picture:	B. Kayser R. G. Winter
Field theory:	C. Giunti, C. W. Kim, U. W. Lee W. Grimus and P. Stockinger M. Blasone, G Vitiello ...

Consistent description

Whole process of the oscillations experiment includes

neutrino production
propagation in between the source and detector
detection



- Production, propagation and detection as a unique process neutrinos ν_1 and ν_2 are virtual particles propagating between the production x_P and detection x_D points
- Neutrinos ν_i are described by propagators $S_i(x_P - x_D)$
- Integration should be performed over finite production and detection regions (integration over x_P , x_D)
- Finite accuracy of ``measurements'' of the energy and momenta of external particles

Truncating the process

For x_P - x_D >> $1/\Delta p$ neutrinos can be considered as real (on shell) particles with negligible corrections due to virtuality

Whole the process can be truncated in three parts:

Production

Propagation of neutrinos as wave packets

Detection

Correct boundary (initial and final) conditions should be imposed

Wave packet description

Oscillations are essentially finite space - finite time phenomenon that is all the components; production, propagation, detection should be considered (occur) in the finite time intervals and finite region of space.

K2K - test

Evolution equation

'Physical derivation'

Input
- neutrinos are ultrarelativistic $E \sim p + m^2/2E$
- no spin-flip, no change of the spinor structure
- lowest order in m/E

In vacuum the mass states are the eigenstates of Hamiltonian

$$i \frac{d\nu_{mass}}{dt} = \left(p\, I + \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \right) \nu_{mass}$$

$$\nu_{mass} = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Using relation $\nu_{mass} = U^{+} \nu_f$ find equation for the flavors

$$i \frac{d\nu_f}{dt} = \frac{M^2}{2E} \nu_f$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$M^2 = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^{+}$$

mass matrix in flavor basis

the term pI proportional to unit matrix is omitted

Graphic representation

Graphic representation

$\vec{\nu} = (\mathrm{Re}\ \nu_e^+\nu_\mu,\ \ \mathrm{Im}\ \nu_e^+\nu_\mu,\ \ \nu_e^+\nu_e - 1/2)$

elements of density matrix

$\vec{B} = \frac{2\pi}{l_m}(\sin 2\theta_m,\ 0,\ \cos 2\theta_m)$

$l_m = 2\pi/\Delta H$ oscillation length

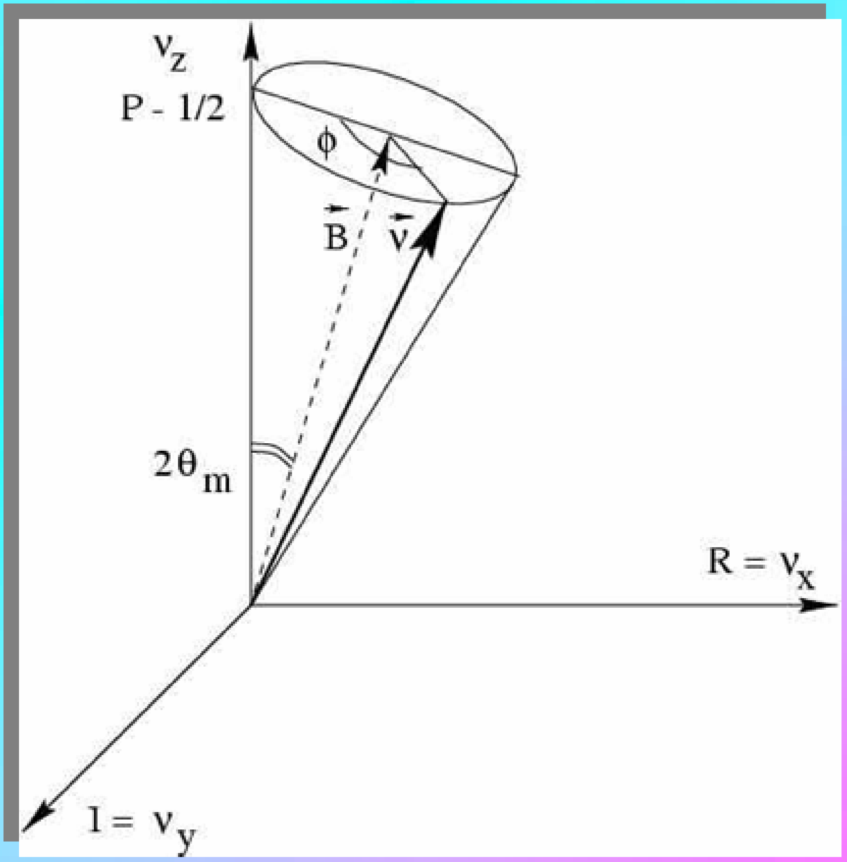
Evolution equation

$$\frac{d\vec{\nu}}{dt} = (\vec{B} \times \vec{\nu})$$

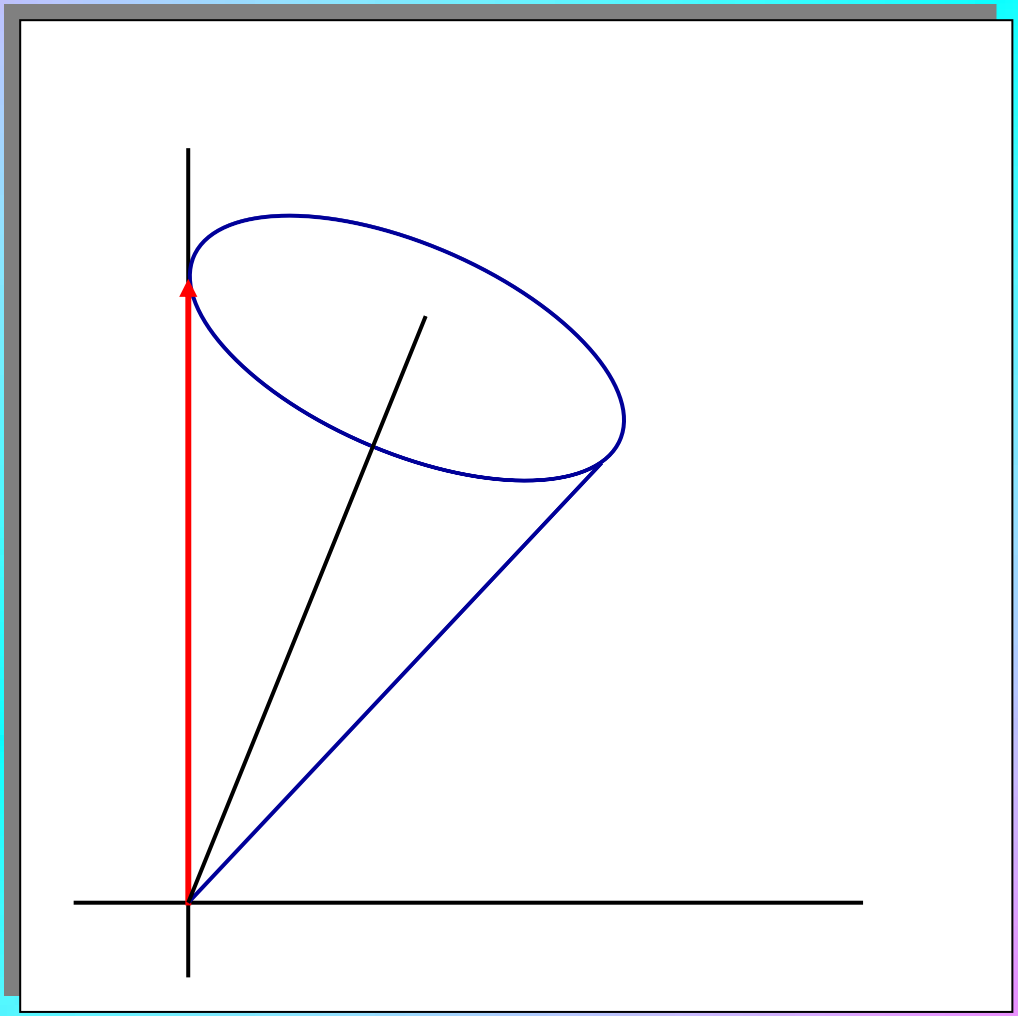
Coincides with equation for the electron spin precession in the magnetic field

$\phi = 2\pi t/\ l_m$ - phase of oscillations

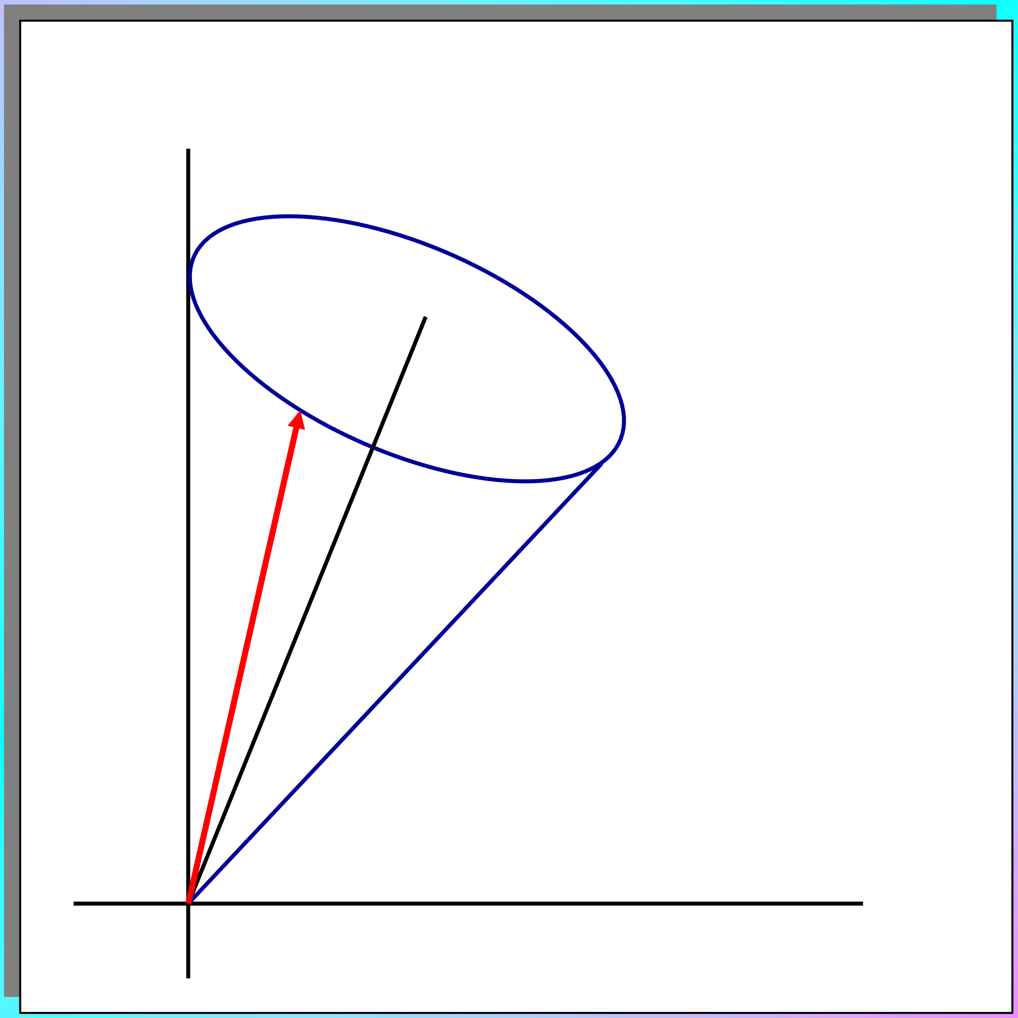
$P = \nu_e^+\nu_e = \nu_Z + 1/2 = \cos^2\theta_Z/2$ probability to find ν_e



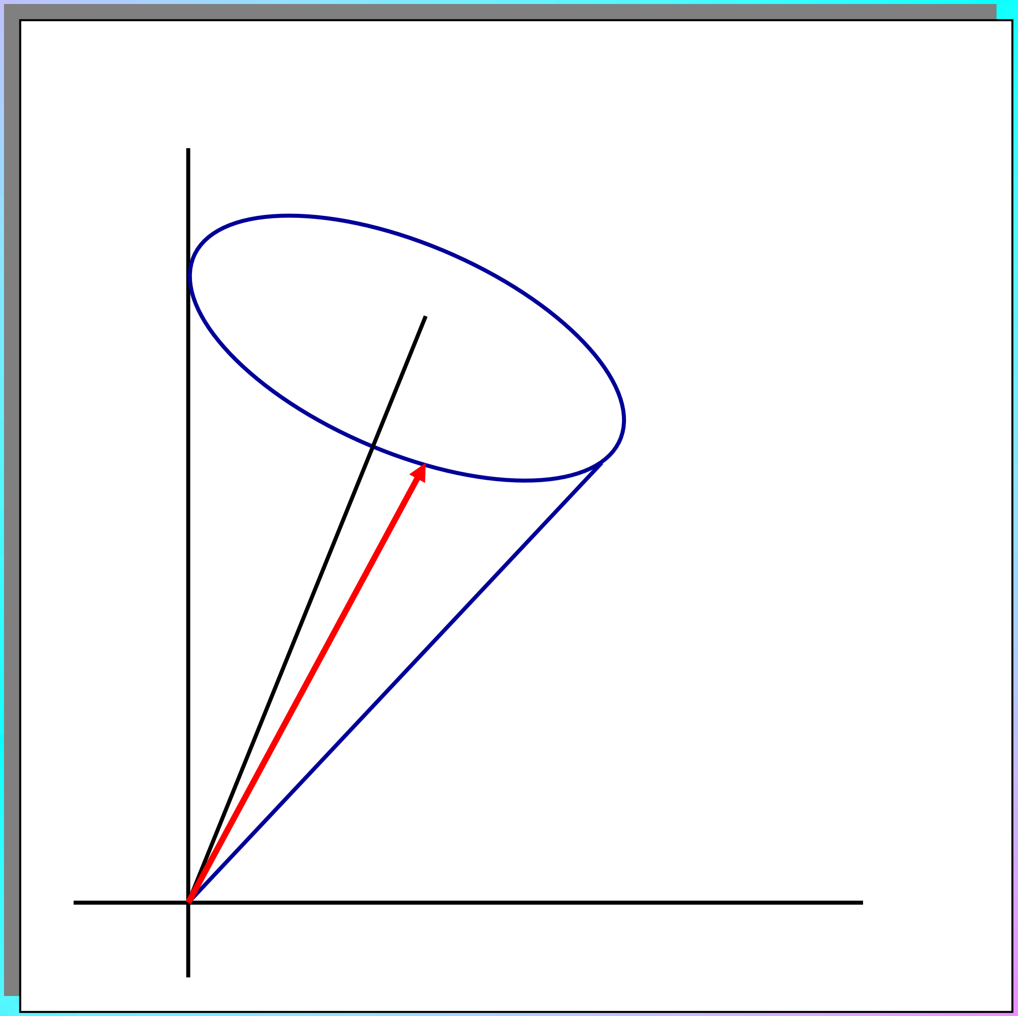
Oscillations



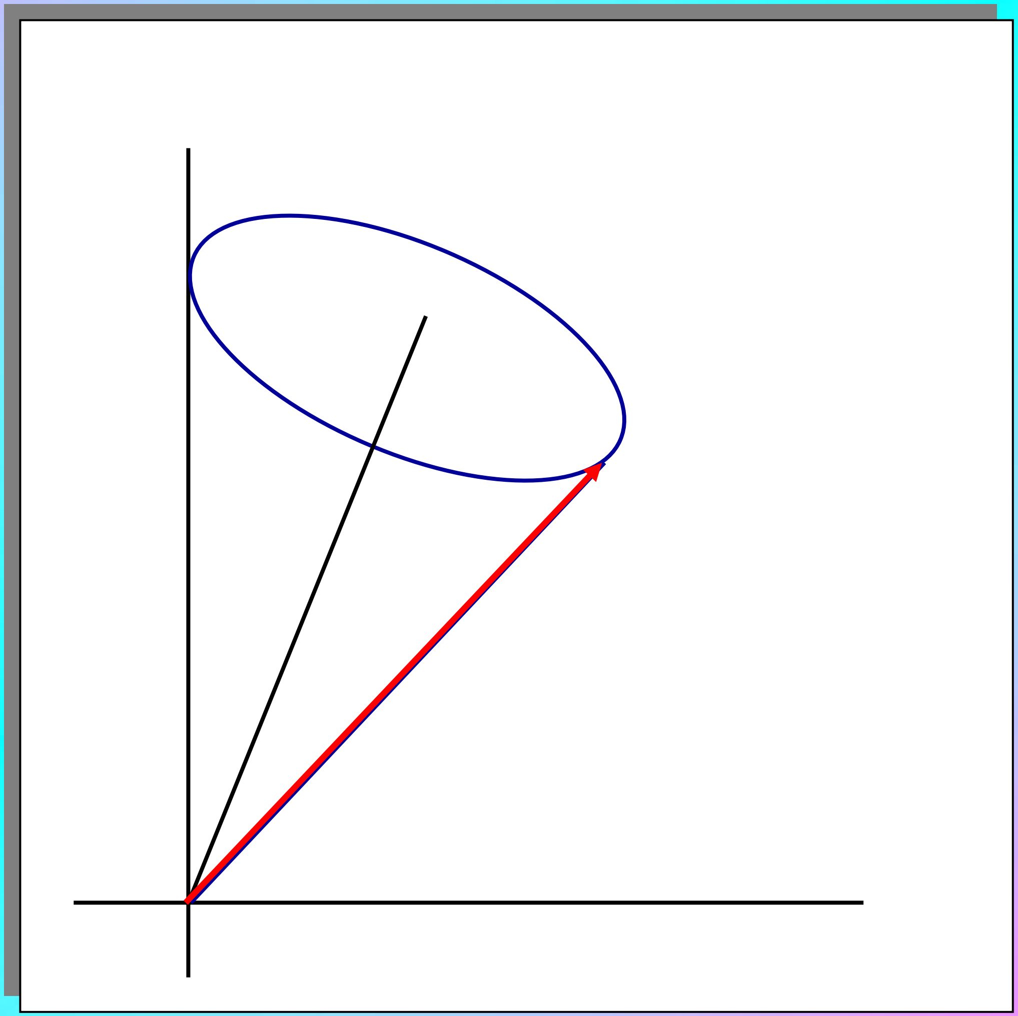
Oscillations



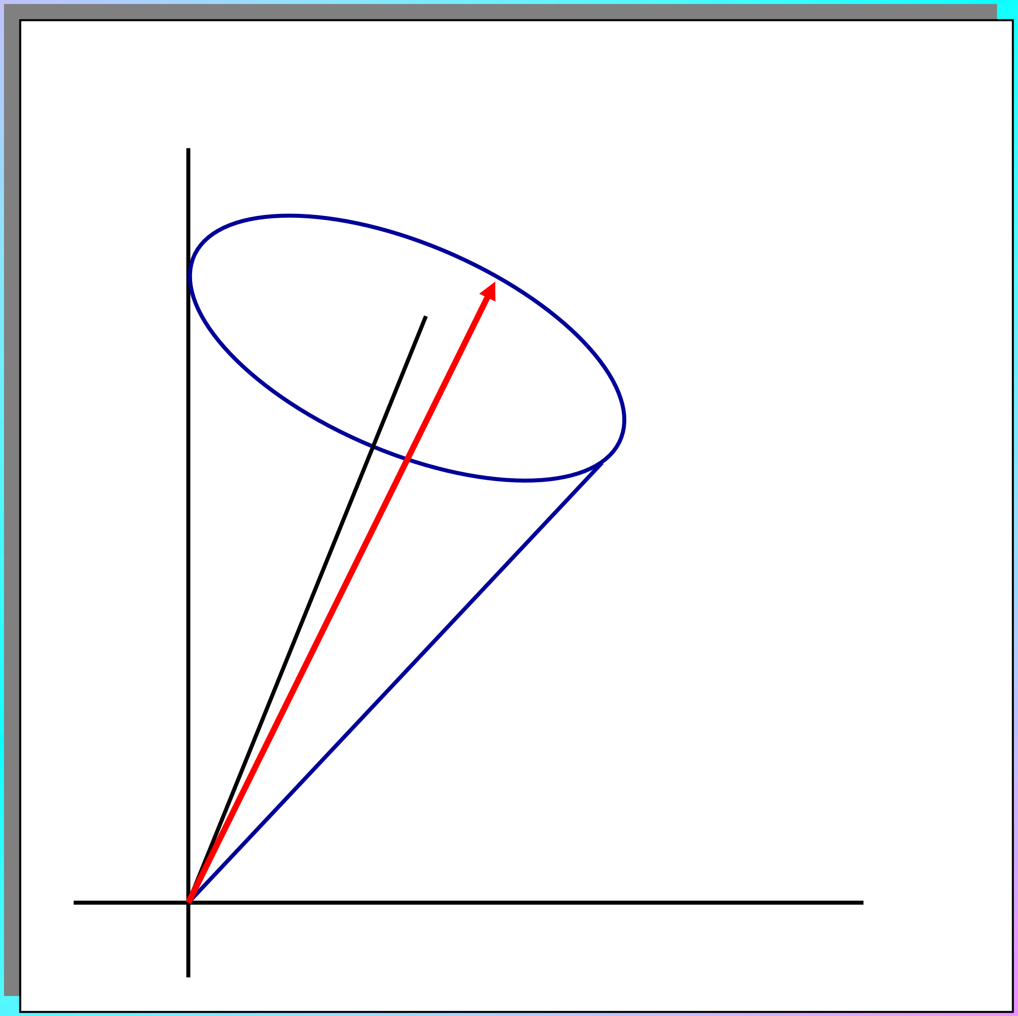
Oscillations



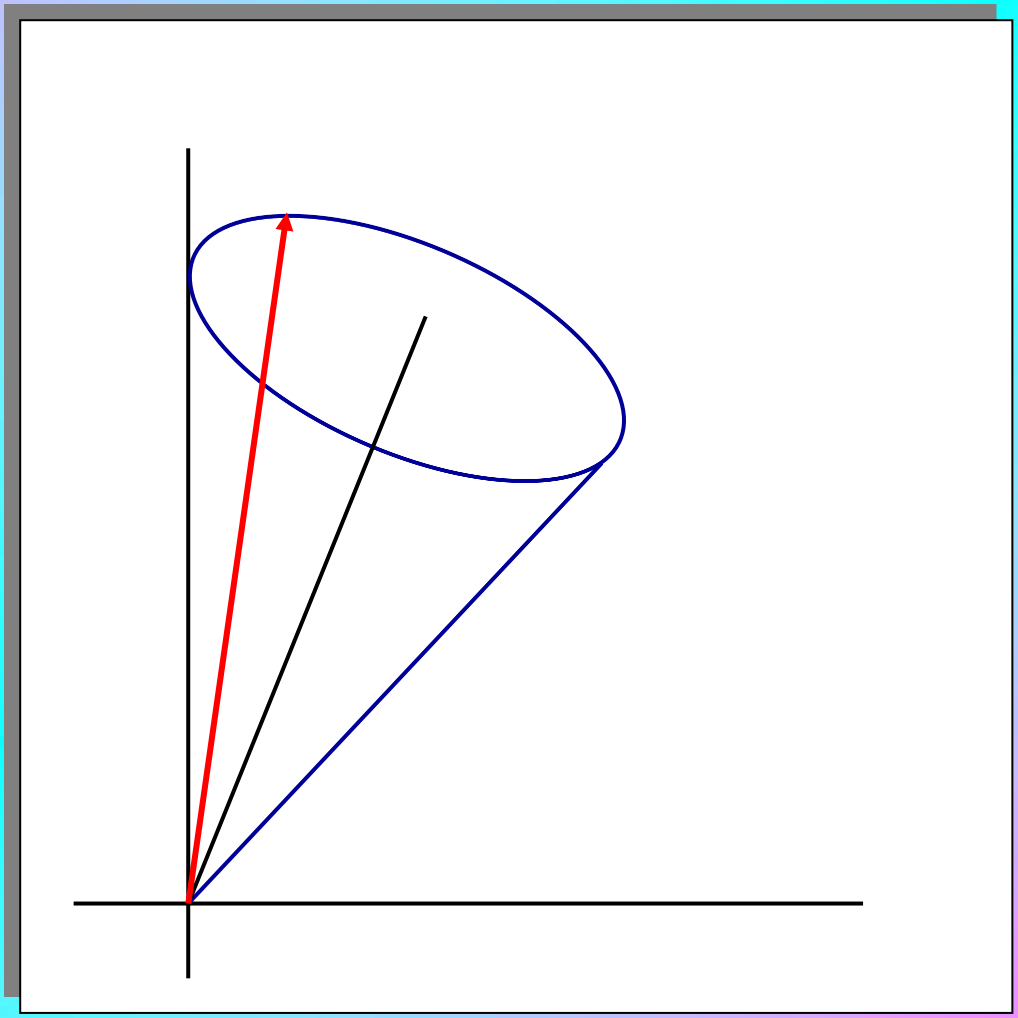
Oscillations



Oscillations



Oscillations

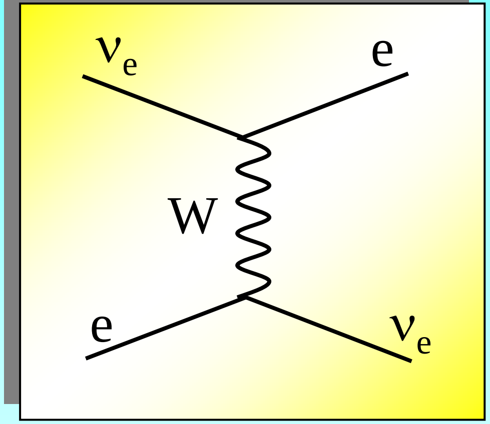


Matter effects

Refraction

L. Wolfenstein, 1978

Elastic forward scattering → Potentials V_e, V_μ



- $V \sim 10^{-13}$ eV inside the Earth for E = 10 MeV
- Difference of potentials is important → for ν_e ν_μ :

$$V_e - V_\mu = \sqrt{2}\, G_F n_e$$

- Refraction index:

$$n - 1 = V / p$$

- Refraction length:

$$l_0 = 2\pi / (V_e - V_\mu) = \sqrt{2}\, \pi / G_F n_e$$

- $n - 1$ $\begin{cases} \sim 10^{-20} & \text{inside the Earth} \\ < 10^{-18} & \text{inside the Sun} \\ \sim 10^{-6} & \text{inside the neutron star} \end{cases}$

- Neutrino optics → focusing of neutrinos fluxes by stars
  complete internal reflection, etc

derivation

Matter potential

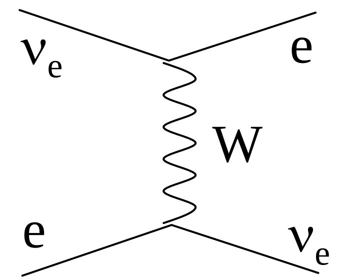
At low energies: neglect the inelastic scattering and absorption effect is reduced to the elastic forward scattering (refraction) described by the potential V:

$$H_{int}(\nu) = \langle \psi \,|\, H_{int} \,|\, \psi \rangle = V \, \bar{\nu} \nu$$

ψ is the wave function of the medium

CC interactions with electrons

$$H_{int} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^{\mu}(1 - \gamma_5) \nu \; \bar{e} \gamma_{\mu}(1 - \gamma_5) e$$



$\langle \bar{e} \gamma_0 e \rangle = \langle e^+ e \rangle = n_e$ - is the electron number density

$\langle \bar{e} \vec{\gamma} e \rangle = n_e \vec{v}$

$\langle \bar{e} \vec{\gamma} \gamma_5 e \rangle = n_e \vec{\lambda}_e$ - averaged polarization vector of e

For unpolarized medium at rest:

$$V = \sqrt{2} \, G_F n_e$$

Evolution equation in matter

$$i \frac{d\nu_f}{dt} = H_{tot}\, \nu_f \qquad \nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$H_{tot} = H_{vac} + V$ is the total Hamiltonian

$H_{vac} = \frac{M^2}{2E}$ is the vacuum (kinetic) part

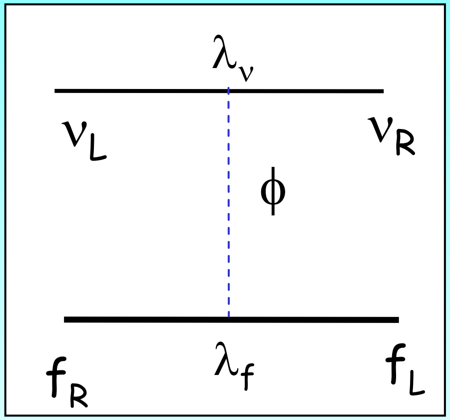
$V = \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$ matter part $\qquad V_e = \sqrt{2}\, G_F n_e$

H_{tot}

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{2E} \cos 2\theta + V_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

"Soft" neutrino mass

Exchange by very light scalar



$m_\phi \sim 10^{-8} - 10^{-6}$ eV

$f = e, u, d, \nu$

Recently: in the context of MaVaN scenario

D B Kalplan, E. Nelson,
N. Weiner , K. M. Zurek
M. Cirelli, M.C. Gonzalez-Garcia,
C. Pena-Garay
V. Barger, P Huber, D. Marfatia

chirality flip – true mass:

$m_{soft} = \lambda_\nu \lambda_f n_f / m_\phi$

$\lambda_f \sim \phi / M_{Pl}$

In the evolution equation:

$m_{vac} \rightarrow m_{vac} + m_{soft}$

generated by some short range physics (interactions) EW scale VEV

medium dependent mass

Eigenstates and mixing in matter

	in vacuum:		*in matter:*
Effective Hamiltonian	H_0	→	$H = H_0 + V$
Eigenstates	ν_1, ν_2	→	ν_{1m}, ν_{2m}
Eigenvalues	$m_1^2/2E, m_2^2/2E$	→	H_{1m}, H_{2m}

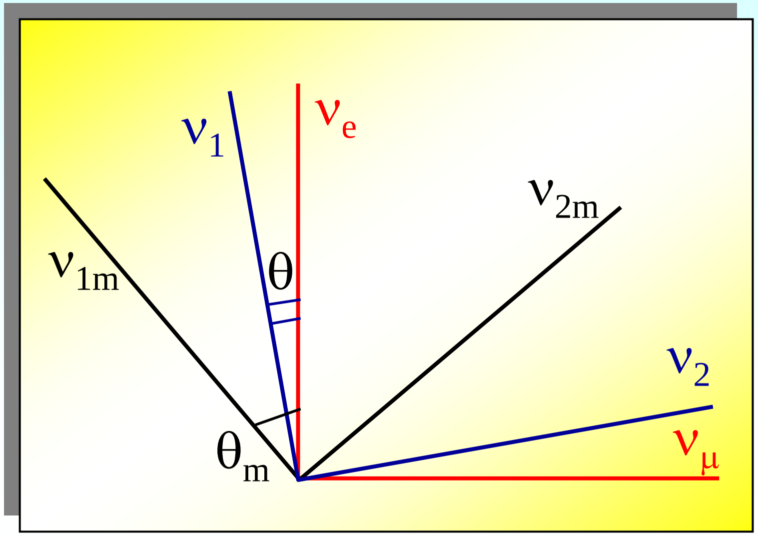
depend on n_e, E

instantaneous

ν_f ⇔ θ ⇔ ν_{mass}

ν_f ⇔ θ_m ⇔ ν_H

Mixing angle determines flavors
(flavor composition) of the eigenstates



Mixing angle in matter Resonance

Diagonalization of the Hamiltonian:

- $$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2\sqrt{2} G_F n_e E/\Delta m^2)^2 + \sin^2 2\theta}$$

- Mixing is maximal for

$$\sqrt{2} G_F n_e = \frac{\Delta m^2}{2E} \cos 2\theta$$

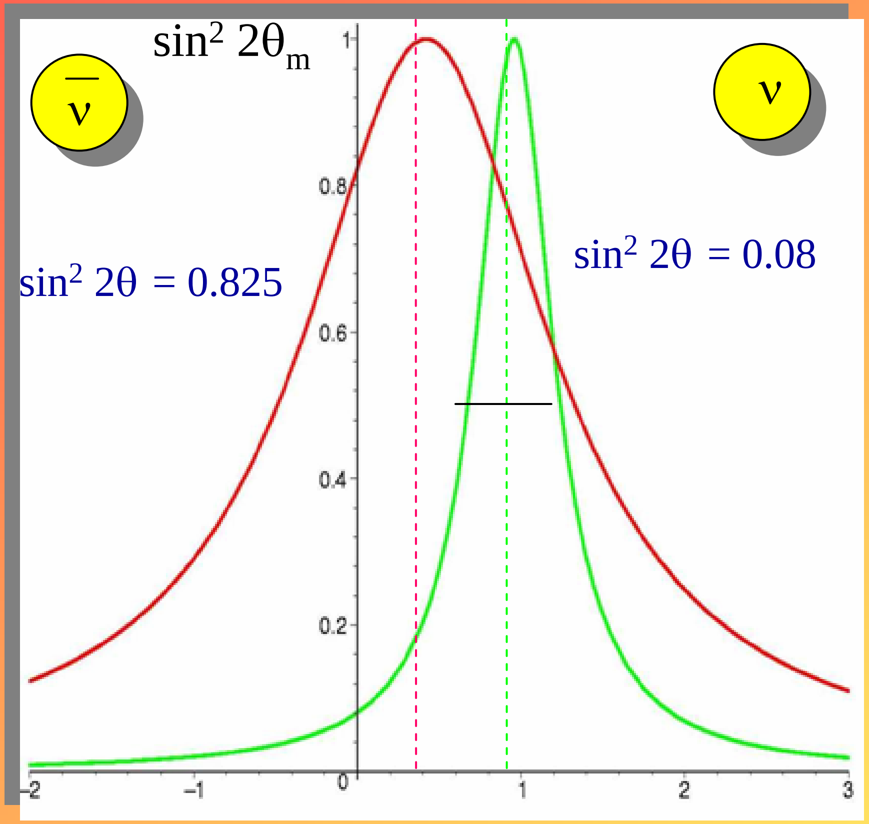
Resonance condition

$\sin^2 2\theta_m = 1$ $\qquad H_e = H_\mu$ $\qquad$ level crossing

- Difference of the eigenvalues

$$H_2 - H_1 = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - 2\sqrt{2} G_F n_e E/\Delta m^2)^2 + \sin^2 2\theta}$$

Resonance



l_ν / l_0 $\sim n E$

- Resonance width:
- Resonance layer:

$\Delta n_R = 2 n_R \tan 2\theta$

$n = n_R + \Delta n_R$

In resonance:

$\sin^2 2\theta_m = 1$

- Flavor mixing is maximal
- Level split is minimal

$l_\nu = l_0 \cos 2\theta$

Vacuum oscillation length $\approx$ Refraction length

For large mixing: $\cos 2\theta \sim 0.4$ the equality is broken: strongly coupled system → shift of frequencies.

- Manifestations depend on density profile
- Determines scale of ρ and E of strong flavor transition occurs

Level crossing

Dependence of the neutrino eigenvalues on the matter potential (density)

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

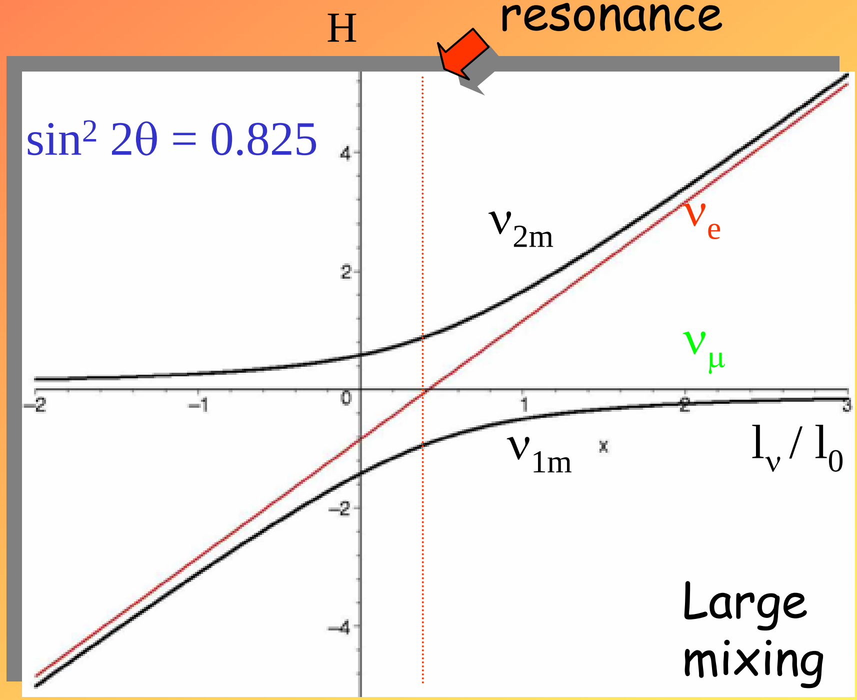
V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

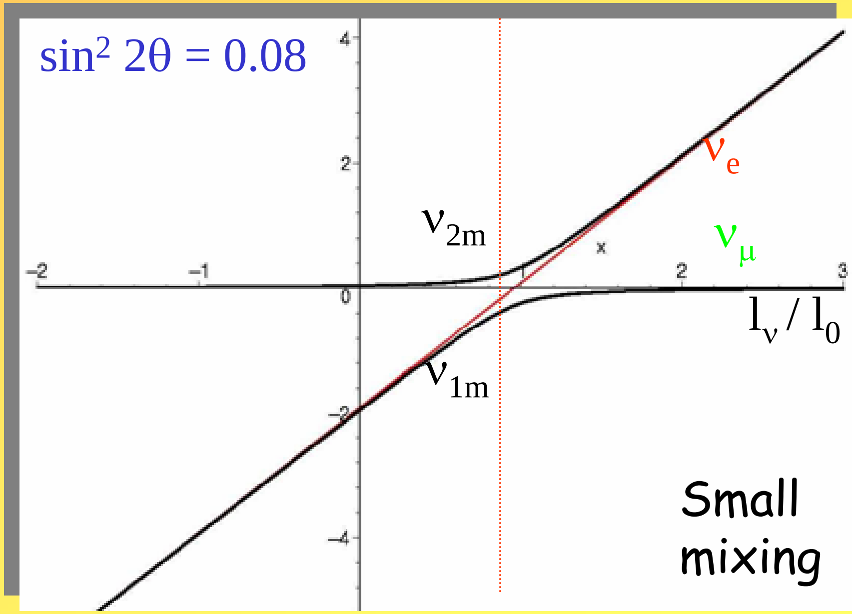
$$\frac{l_\nu}{l_0} = \cos 2\theta$$

Crossing point - resonance

- the level split is minimal
- the oscillation length is maximal

For maximal mixing: at zero density





Oscillation length and refraction length

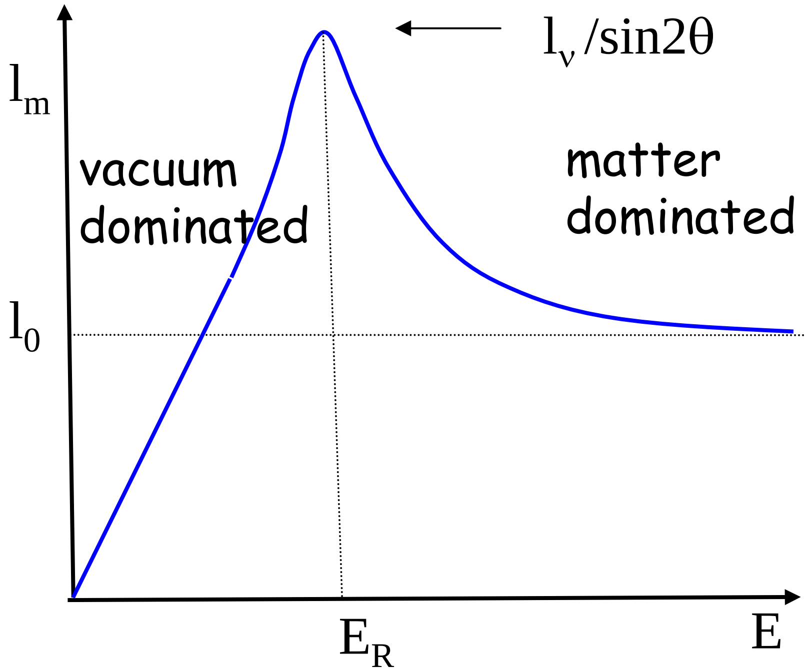
Oscillation length in matter

$$l_m = \frac{2\pi}{H_2 - H_1}$$

Refraction length

$$l_0 = \frac{2\pi}{\sqrt{2}\, G_F n_e}$$

Determines the phase produced by interaction with matter



Resonance condition:

$$l_\nu = l_0 \cos 2\theta$$

Oscillations in matter

Propagation of neutrinos in the matter of the Earth

- solar neutrinos
- supernova neutrinos
- accelerator neutrinos, LBL

Oscillations in matter

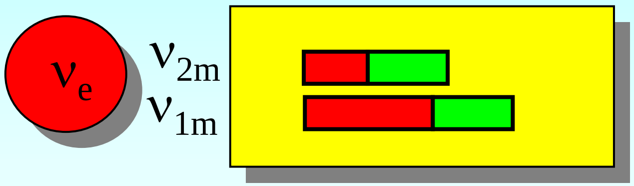
Physical picture

In uniform matter (constant density)
mixing is constant

$\theta_m(E, n)$ = constant

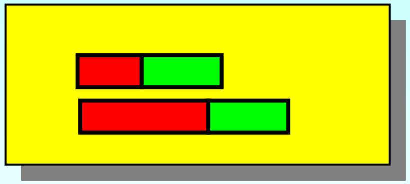
- Flavors of the eigenstates do not change
- Admixtures of matter eigenstates do not change: no $\nu_{1m} <-> \nu_{2m}$ transitions
- Monotonous increase of the phase difference between the eigenstates $\Delta\phi_m$

→ **Oscillations** as in vacuum



$\Delta\phi_m = 0$

→



$\Delta\phi_m = (H_2 - H_1) L$

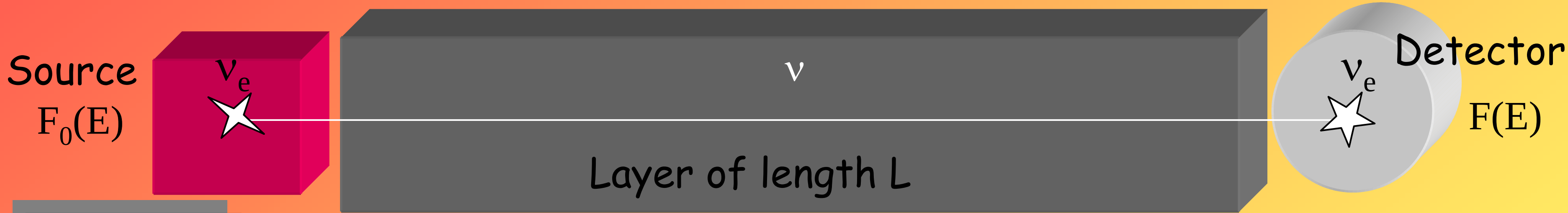
Parameters of oscillations (depth and length)
are determined by mixing in matter
and by effective energy split in matter

$\sin^2 2\theta$, l_ν → $\sin^2 2\theta_m$, l_m

Resonance enhancement of oscillations

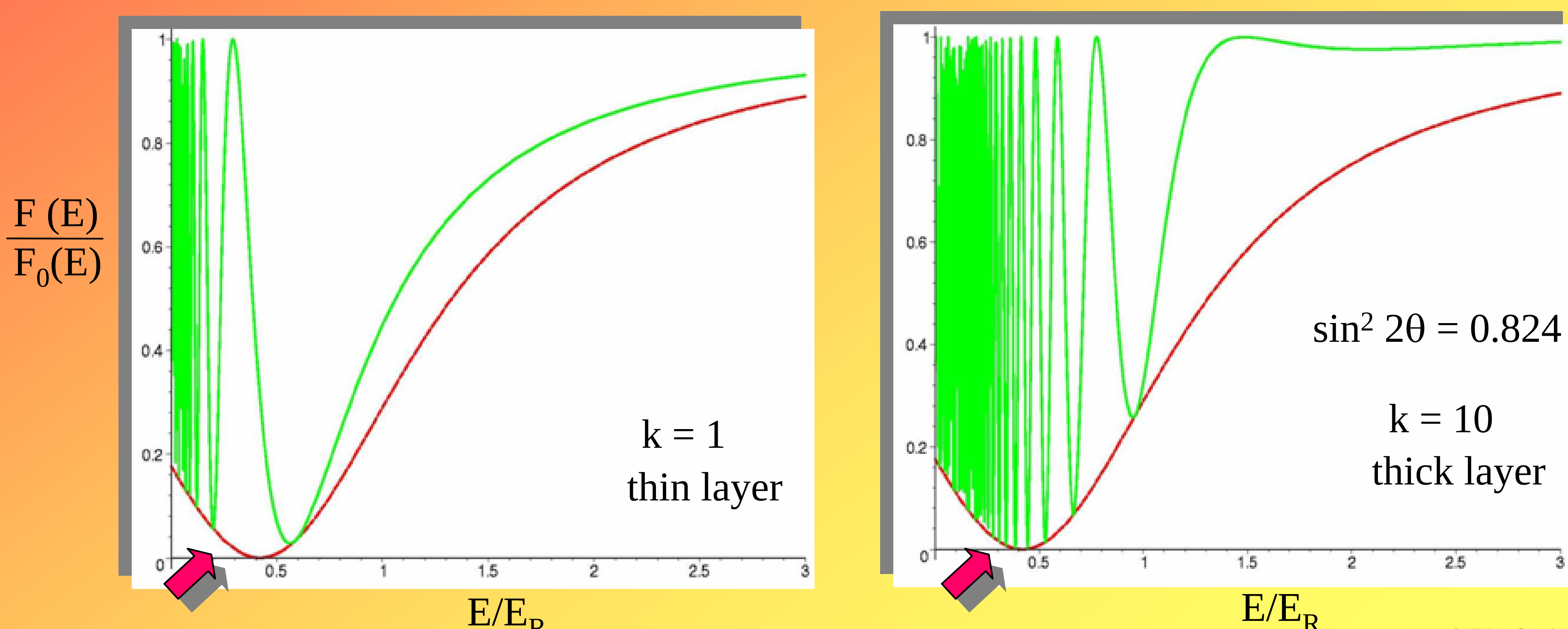
Constant density

oscillations determined by θ_m and l_m (Δ H)



$k = \pi L / l_0$

$\sin^2 2\theta = 0.824$



A Yu Smirnov

High energy neutrinos in the mantle of the Earth
(constant density is a good first approximation)

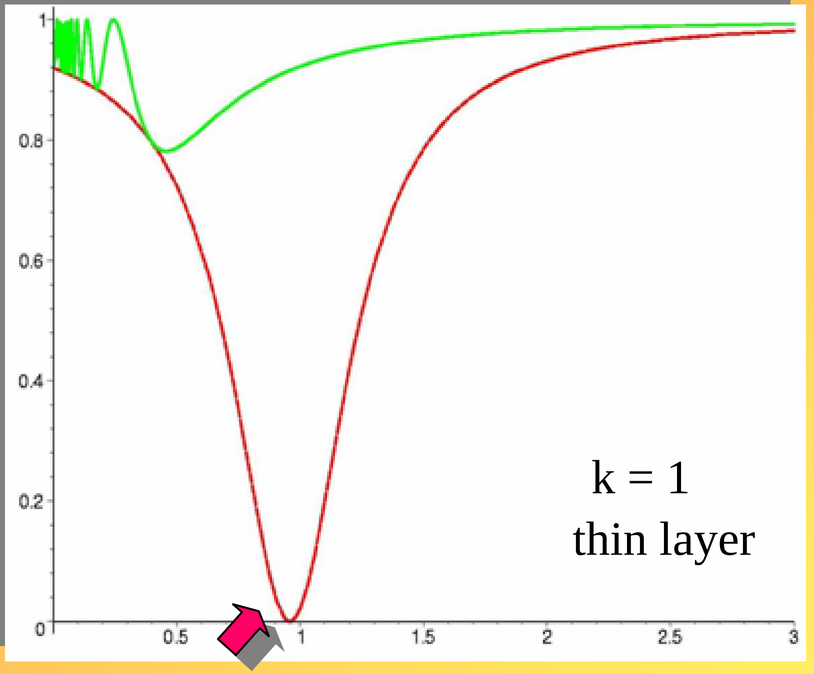
Applications:

Atmospheric neutrinos

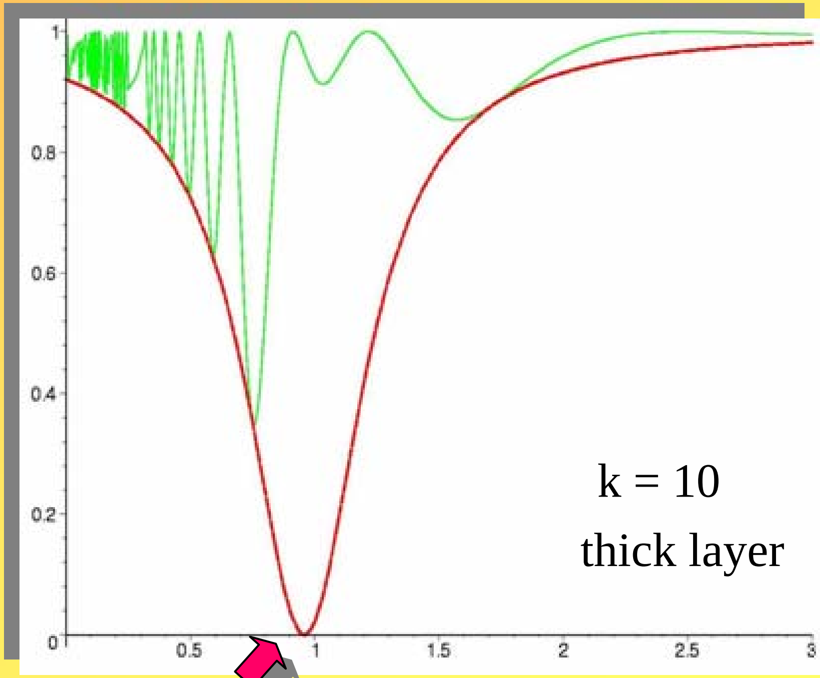
Accelerator neutrinos,
LBL experiments

$\sin^2 2\theta = 0.08$

$\frac{F(E)}{F_0(E)}$



E/E_R



E/E_R

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Degrees of freedom

Arbitrary state: $\nu(t) = \cos\theta_a \, \nu_{1m} + \sin\theta_a \, \nu_{2m} \, e^{-i\phi(t)}$

Effects associated to different degrees of freedom

- $\theta_a = \theta_a(t)$ - determines the admixtures of the eigenstates

Adiabaticity violation

- $\phi(t)$ is the phase difference between the two eigenstates

$$\phi(t) = \int_0^t H \, dt'$$

Oscillations

- Flavors (flavor composition) of the eigenstates are determined by the mixing angle in matter

Adiabatic conversion

$$\langle \nu_e | \nu_{1m} \rangle = \cos\theta_m \qquad \langle \nu_\mu | \nu_{1m} \rangle = - \sin\theta_m$$

- Combination of effects

$\theta_a(t) + \phi(t)$ -> parametric effects, etc.

$\theta_m(t) + \phi(t)$ -> ad. conv. + oscillations

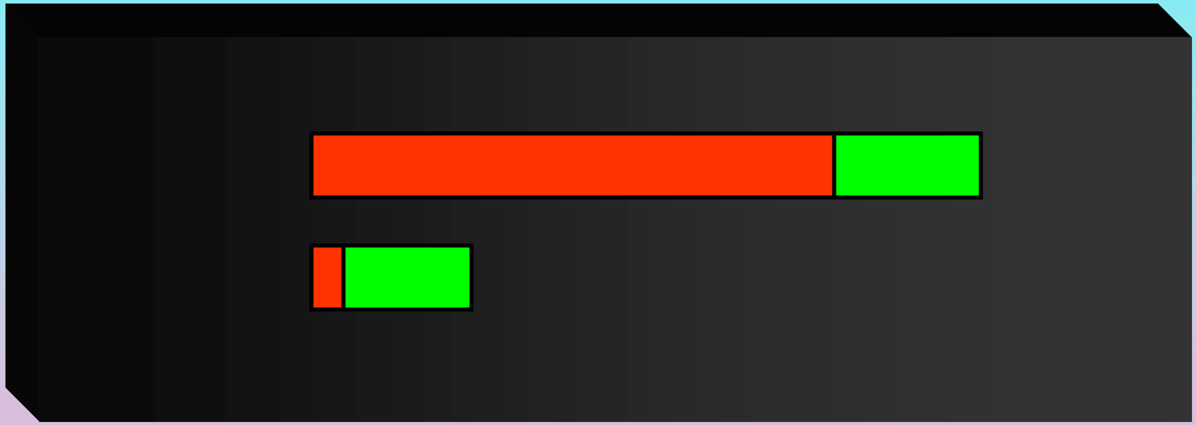
The MSW - effect

Adiabatic or partially adiabatic flavor conversion of neutrinos in medium with varying density

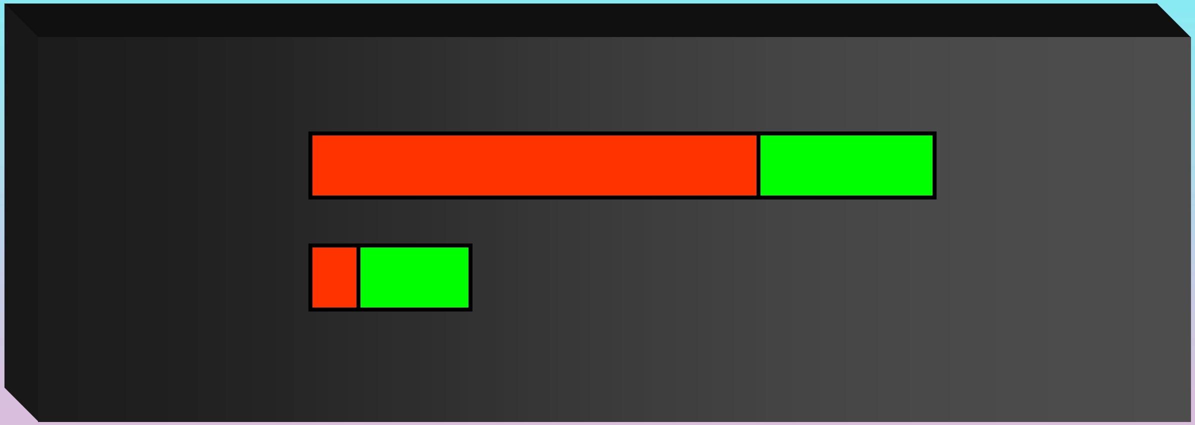
→ Flavor of the neutrino state follows density change

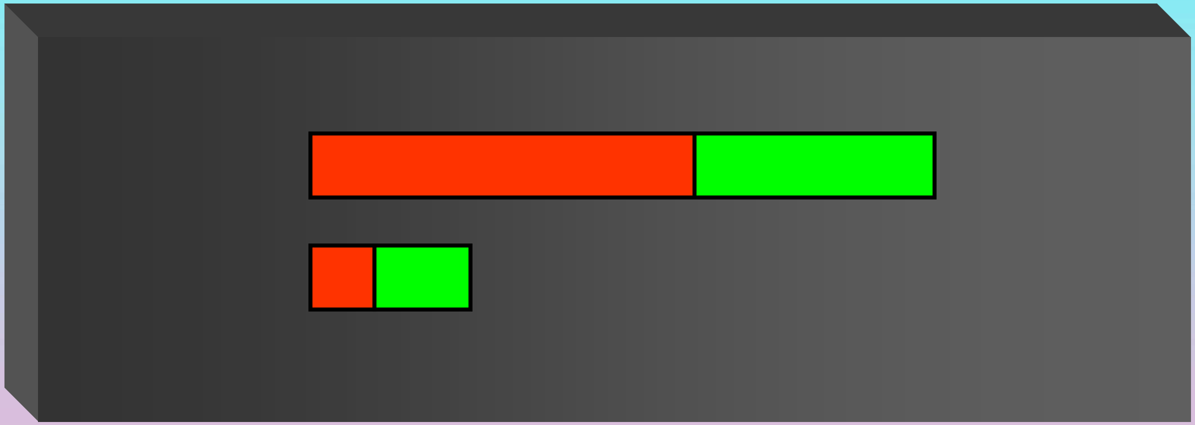
Adiabatic case

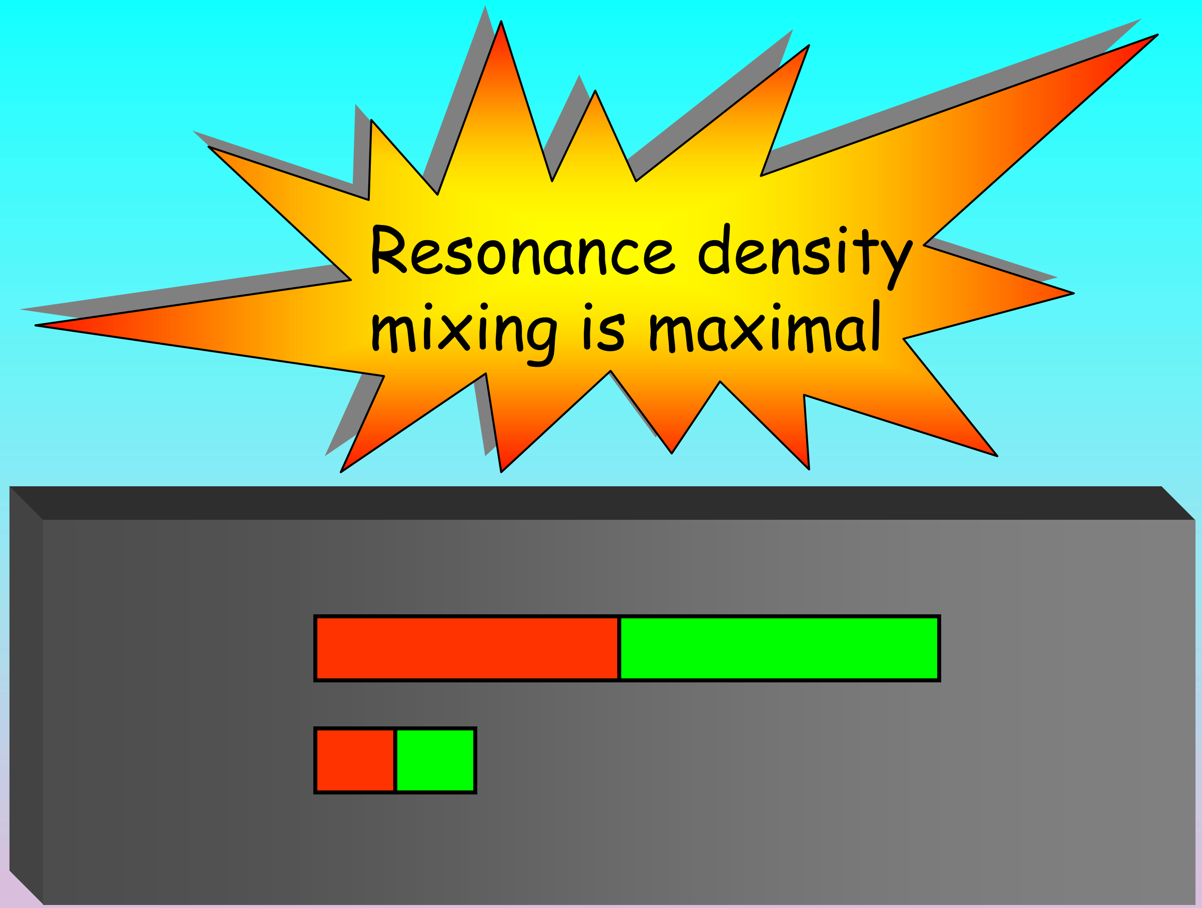
Physical picture

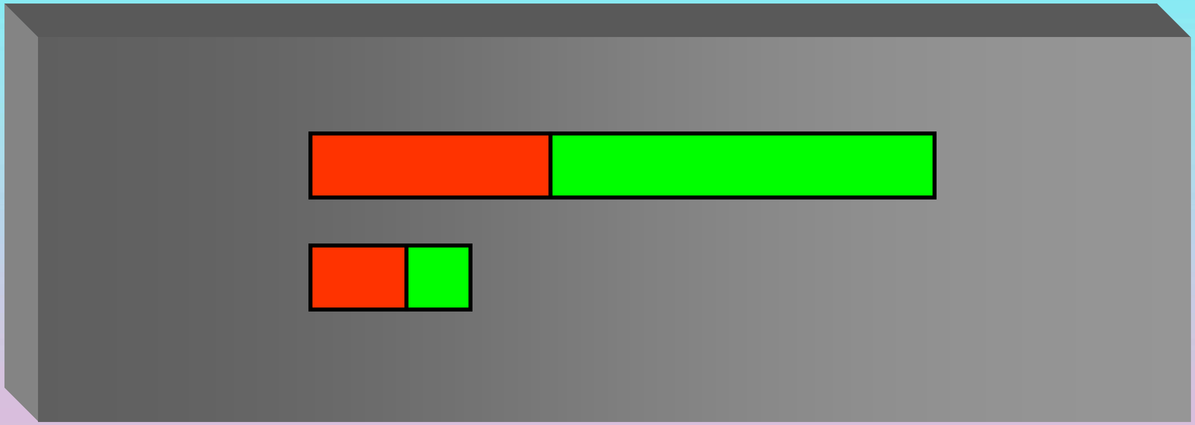


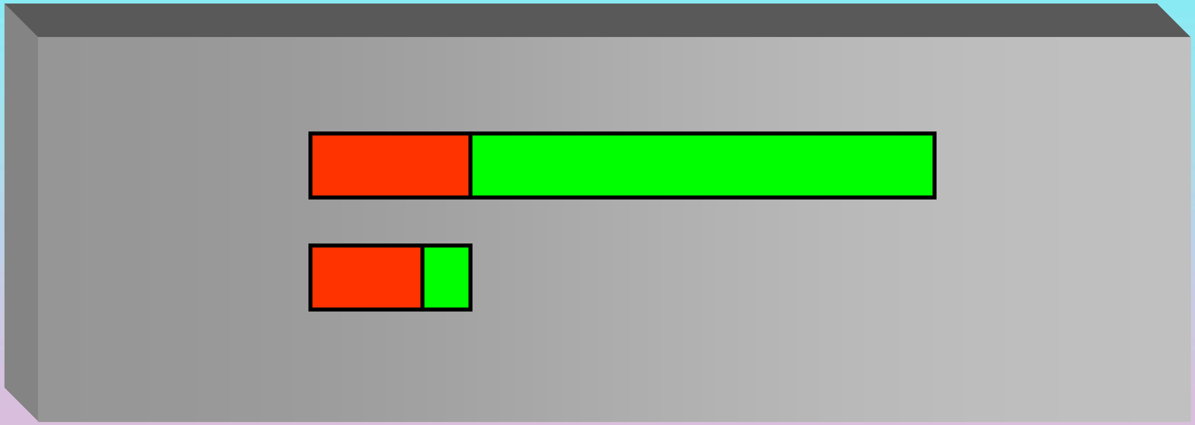
- Admixtures of the eigenstates do not change (adiabaticity) → Determined by mixing θ_m^0 in the production point
- Flavors of the eigenstates follow the density change → Flavor: $\theta_m = \theta_m(\rho(t))$
- Phase difference of the eigenstates changes leading to oscillations → $\phi = (H_1 - H_2)\, t$






Resonance density
mixing is maximal





Evolution of eigenstates in matter

In non-uniform medium the Hamiltonian depends on time: $H_{tot} = H_{tot}(n_e(t))$

➡ Its eigenstates, ν_{matter}, do not split the equations of motion

$$i \frac{d\nu_f}{dt} = H_{tot}\, \nu_f \qquad \nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Inserting $\nu_f = U(\theta_m)\, \nu_{matter}$ we get evolution equation for

$$\nu_{matter} = \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} 0 & i \dfrac{d\theta_m}{dt} \\ -i \dfrac{d\theta_m}{dt} & H_2 - H_1 \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} \qquad \theta_m = \theta_m(n_e(t))$$

The Hamiltonian is non-diagonal
no split of equations ➡ transitions $\nu_{1m} \Leftrightarrow \nu_{2m}$

Adiabaticity

- Adiabaticity condition

$$\frac{\left|\frac{d\theta_m}{dt}\right|}{H_2 - H_1} \ll 1$$

External conditions (density) change slowly the system has time to adjust them

Essence:
transitions between the neutrino eigenstates can be neglected

$$\nu_{1m} \not\leftrightarrow \nu_{2m}$$

⇒ The eigenstates propagate independently

Some more details

Adiabatic conversion formula

Initial state: $\nu(0) = \nu_e = \cos\theta_m^0\, \nu_{1m}(0) + \sin\theta_m^0\, \nu_{2m}(0)$

Adiabatic evolution to the surface of the Sun (zero density):

$\nu_{1m}(0) \;\text{-->}\; \nu_1$

$\nu_{2m}(0) \;\text{-->}\; \nu_2$

Final state: $\nu(f) = \cos\theta_m^0\, \nu_1 + \sin\theta_m^0\, \nu_2\, e^{-i\phi}$

Probability to find ν_e averaged over oscillations

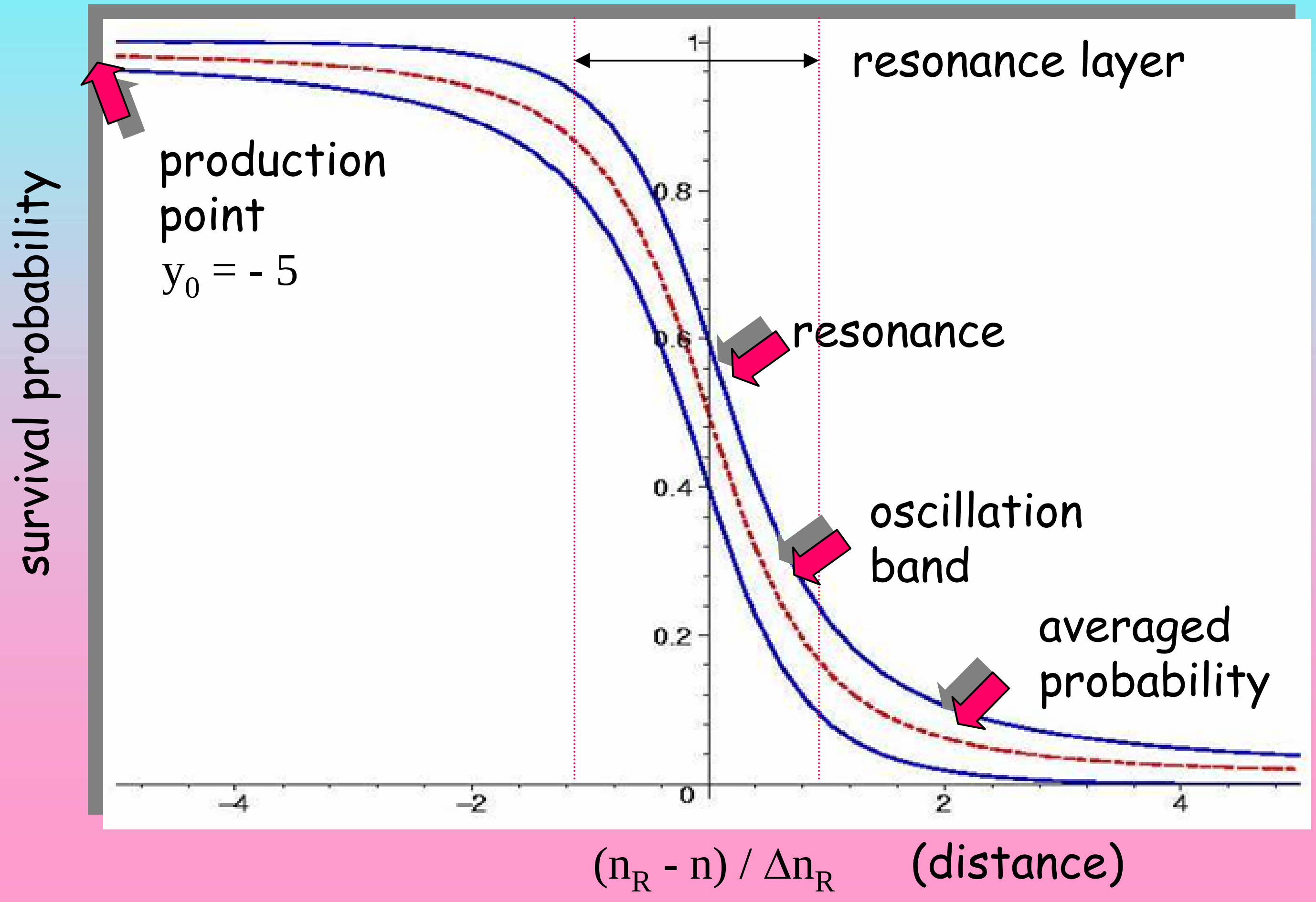
$$P = |\langle \nu_e | \nu(f) \rangle|^2 = (\cos\theta \cos\theta_m^0)^2 + (\sin\theta \sin\theta_m^0)^2$$

$$= 0.5[\, 1 + \cos 2\theta_m^0 \cos 2\theta\,]$$

$$P = \sin^2\theta + \cos 2\theta \cos^2\theta_m^0$$

Spatial picture

The picture is universal in terms of variable $y = (n_R - n) / \Delta n_R$
no explicit dependence on oscillation parameters, density distribution, etc.
only initial value y_0 matters



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Adiabaticity violation

Physical picture

- Transitions ν_{1m} <-> ν_{2m} occur
  admixtures of the eigenstates change
- Flavors of the eigenstates
  follow the density change → Flavor: $\theta_m = \theta_m(\rho(t))$
- Phase difference of the eigenstates
  changes leading to oscillations → $\phi = (H_1 - H_2)\, t$

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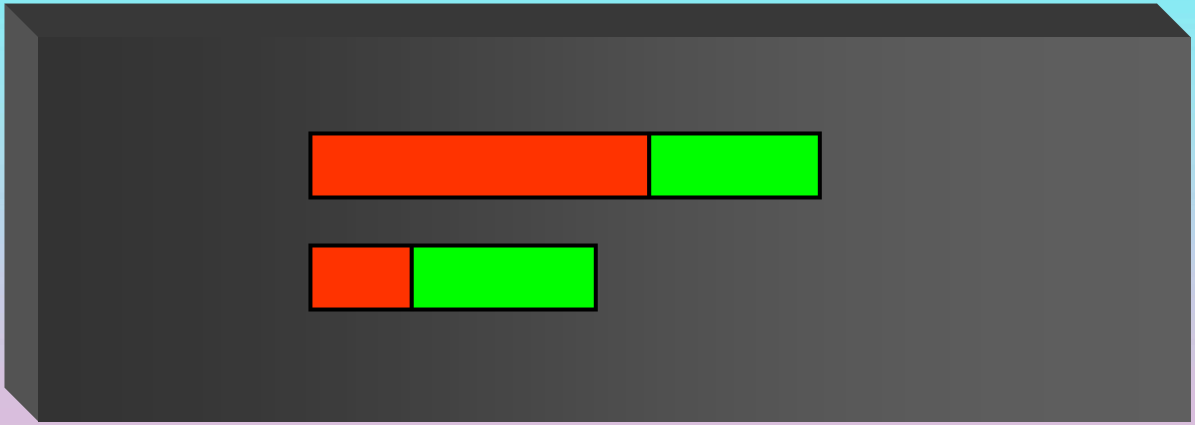

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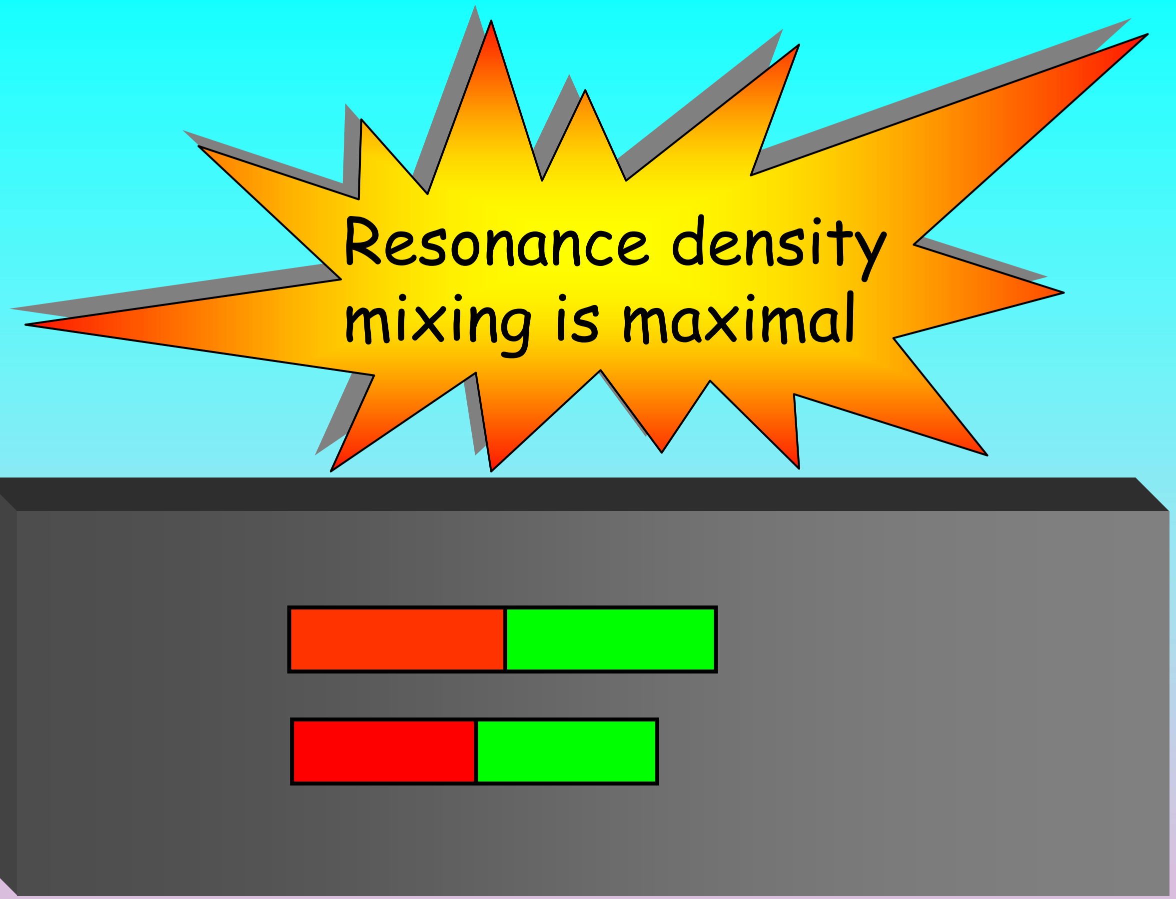
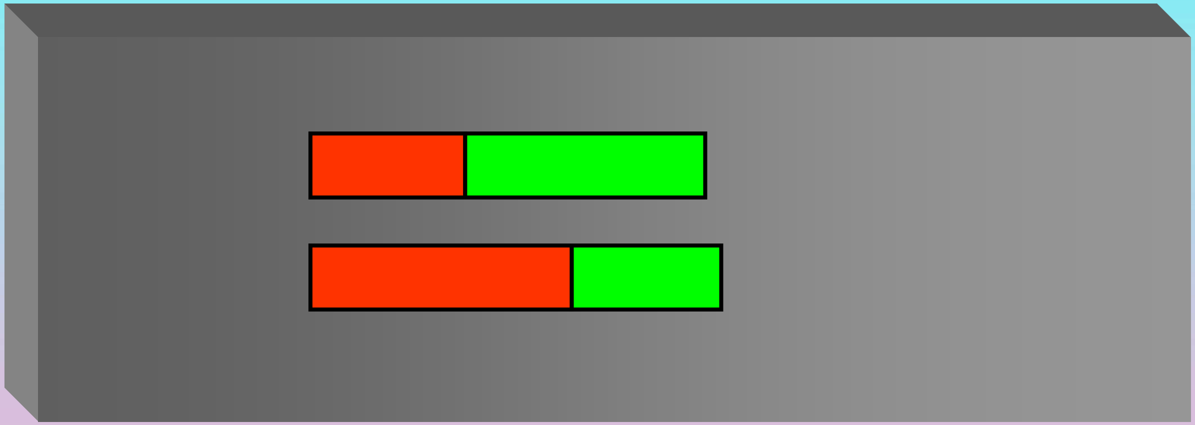

Resonance density
mixing is maximal



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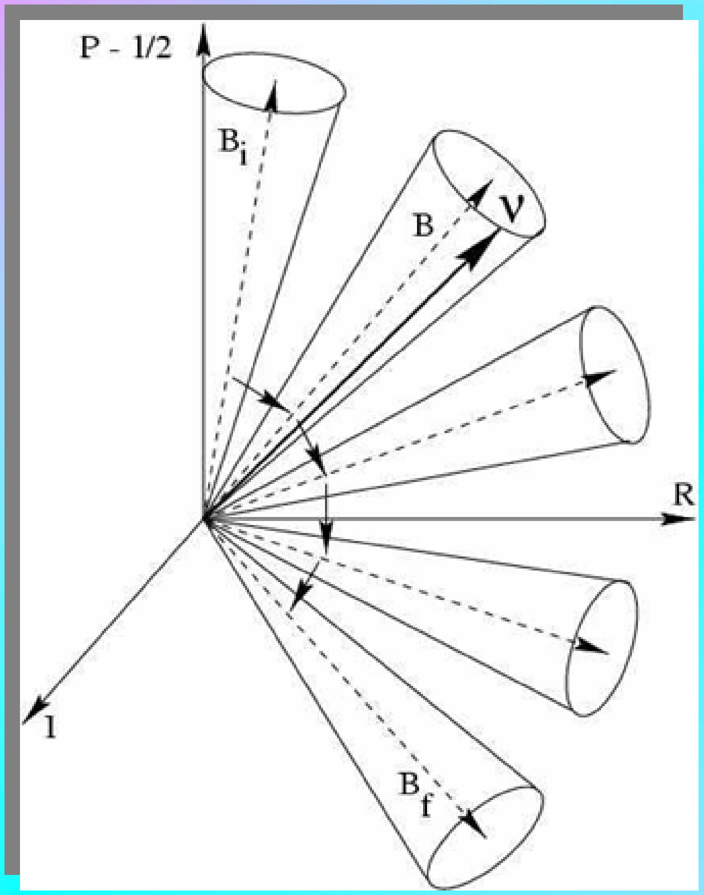
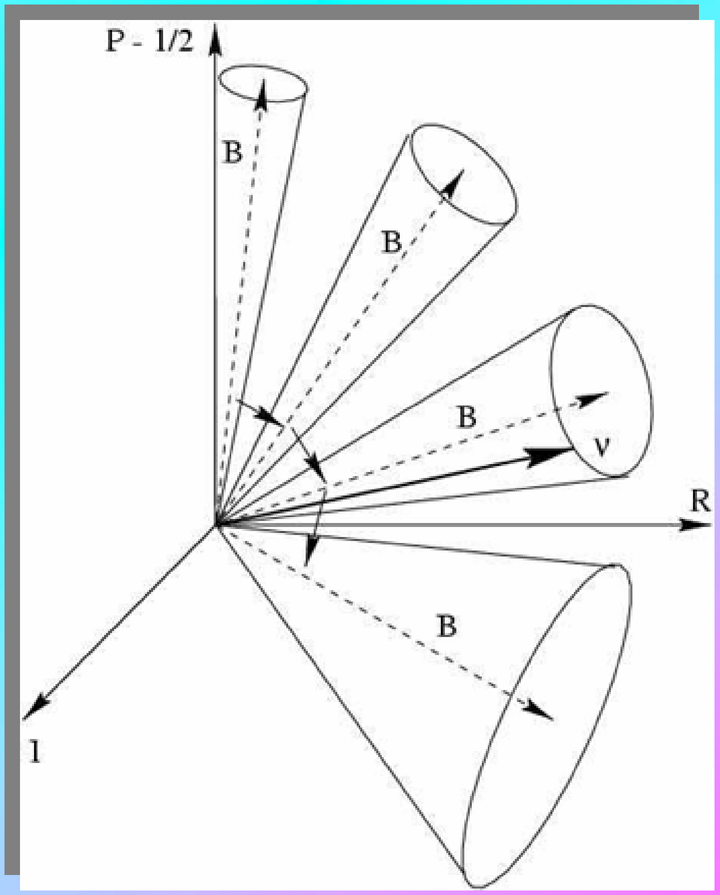
Graphic representation

Pure adiabatic conversion

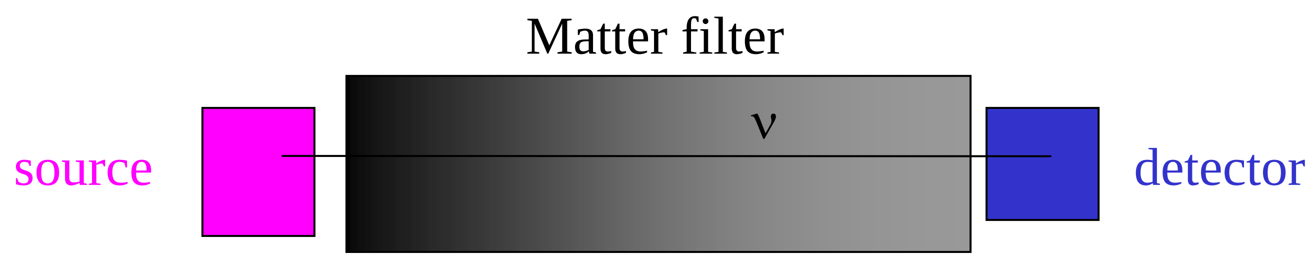


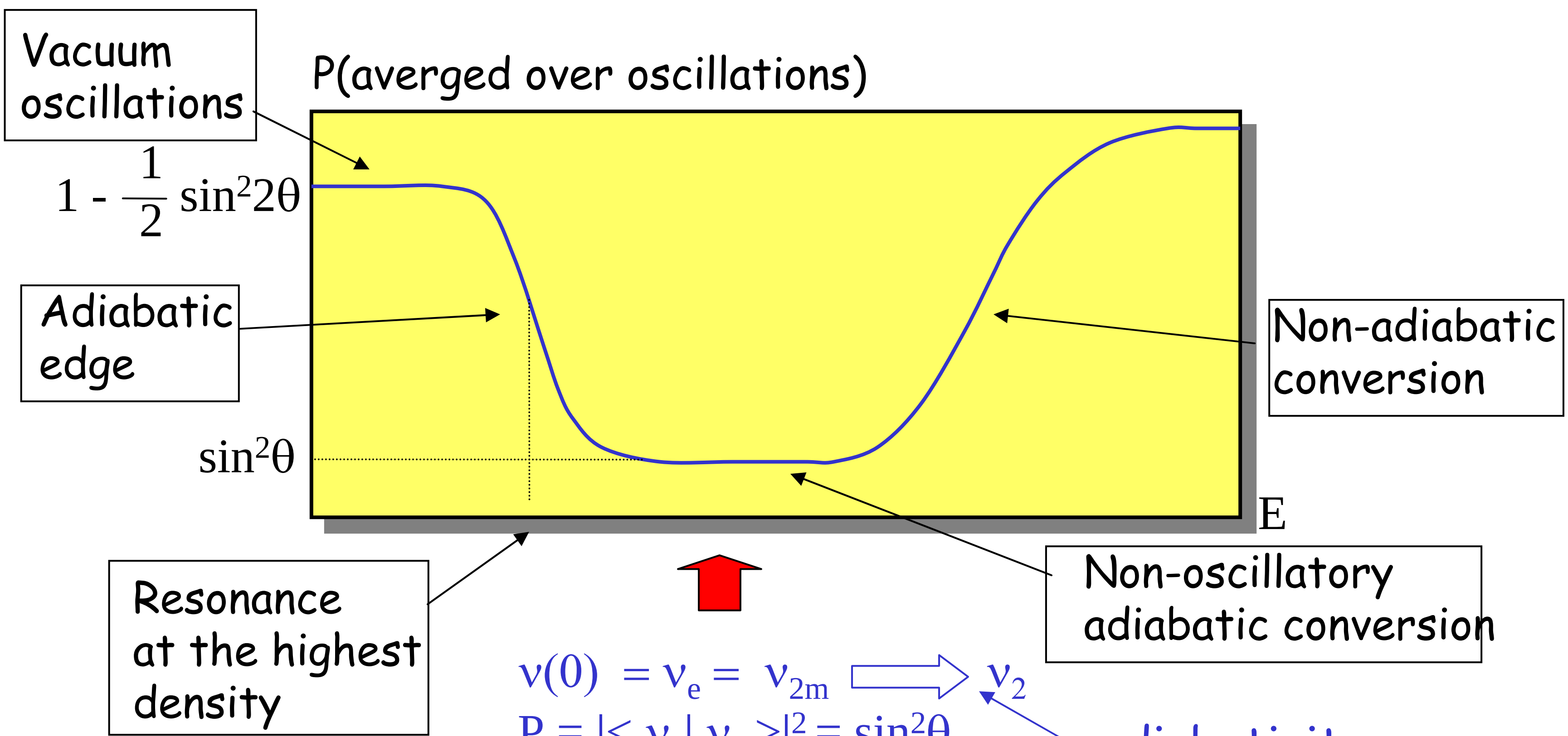
Partialy adiabatic conversion



Survival Probability

Non-uniform medium





$\nu(0) = \nu_e = \nu_{2m} \Rightarrow \nu_2$

$P = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2\theta$

adiabaticity

Oscillations versus MSW

What is essential difference between oscillations and the MSW effect?

Both require mixing,
MSW is usually accompanying by oscillations

Oscillations

- Vacuum or uniform medium with constant parameters
- Phase difference increase between the eigenstates

ϕ

Different degrees of freedom

Adiabatic conversion

Non-uniform medium or/and medium with varying in time parameters

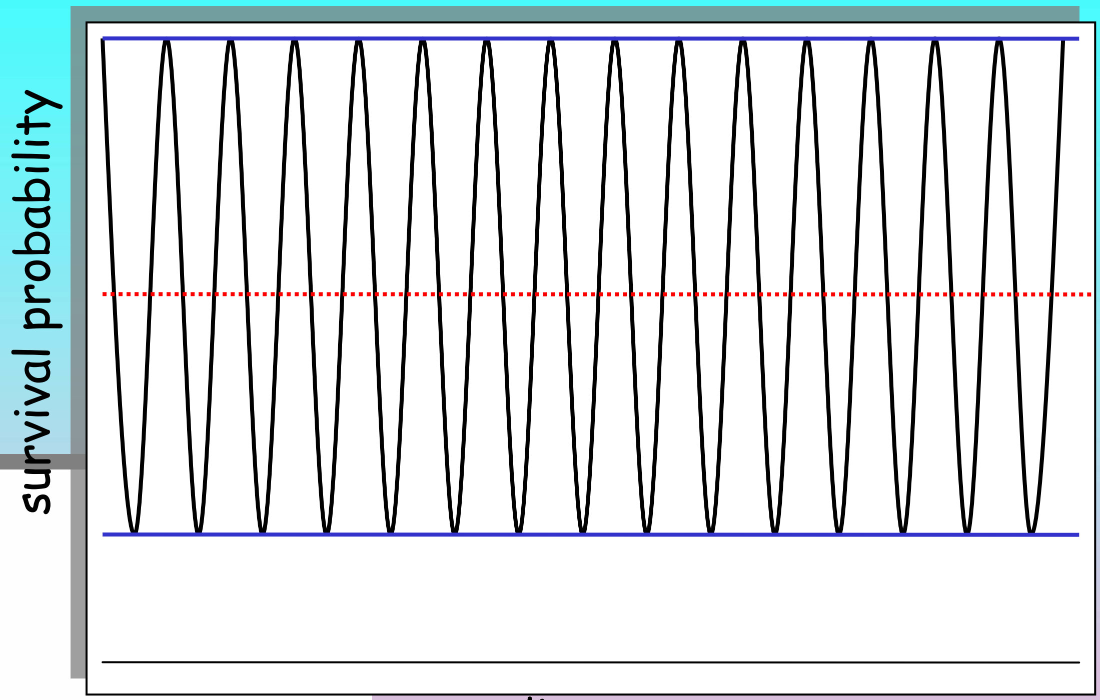
Change of mixing in medium = change of flavor of the eigenstates

θ_m

In non-uniform medium: interplay of both processes

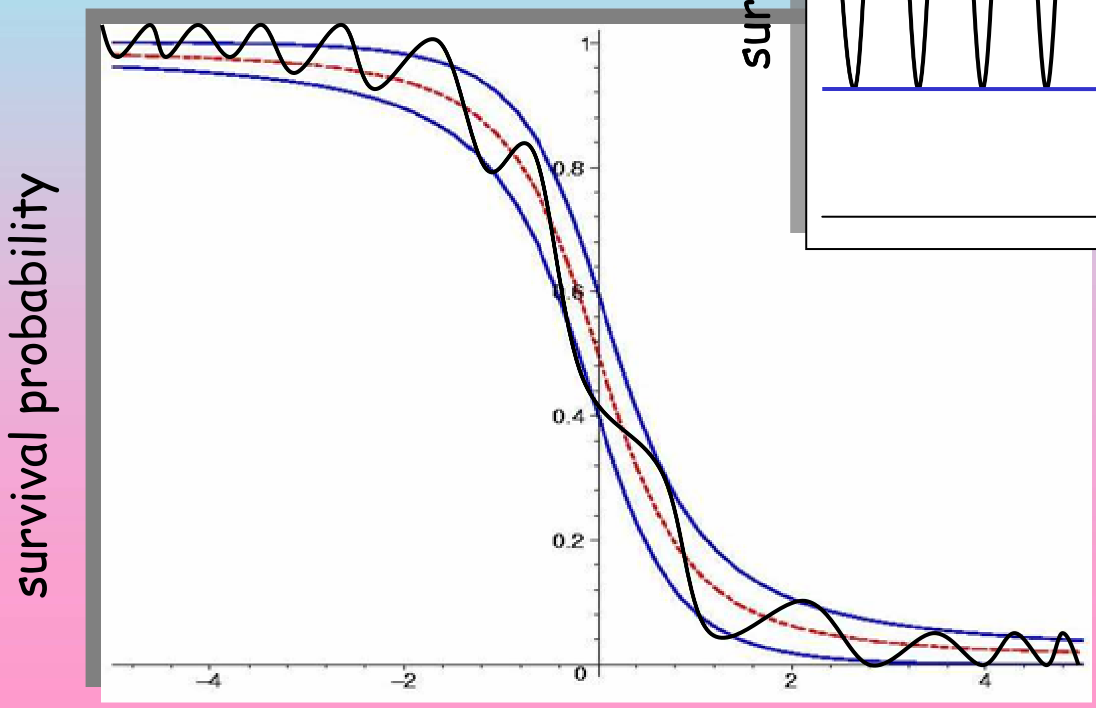
Spatial picture

Oscillations



distance

Adiabatic conversion



distance

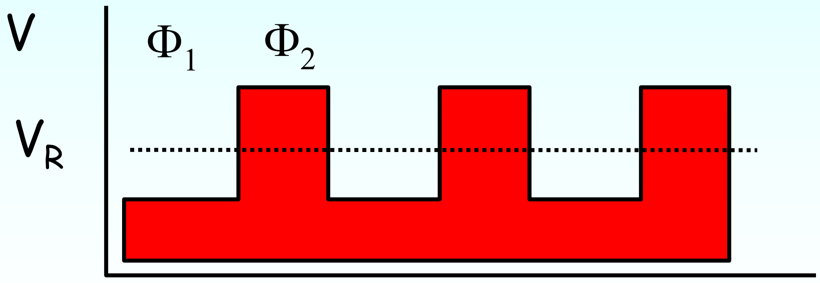
Parametric enhancement of oscillations

Enhancement associated to certain conditions for the phase of oscillations

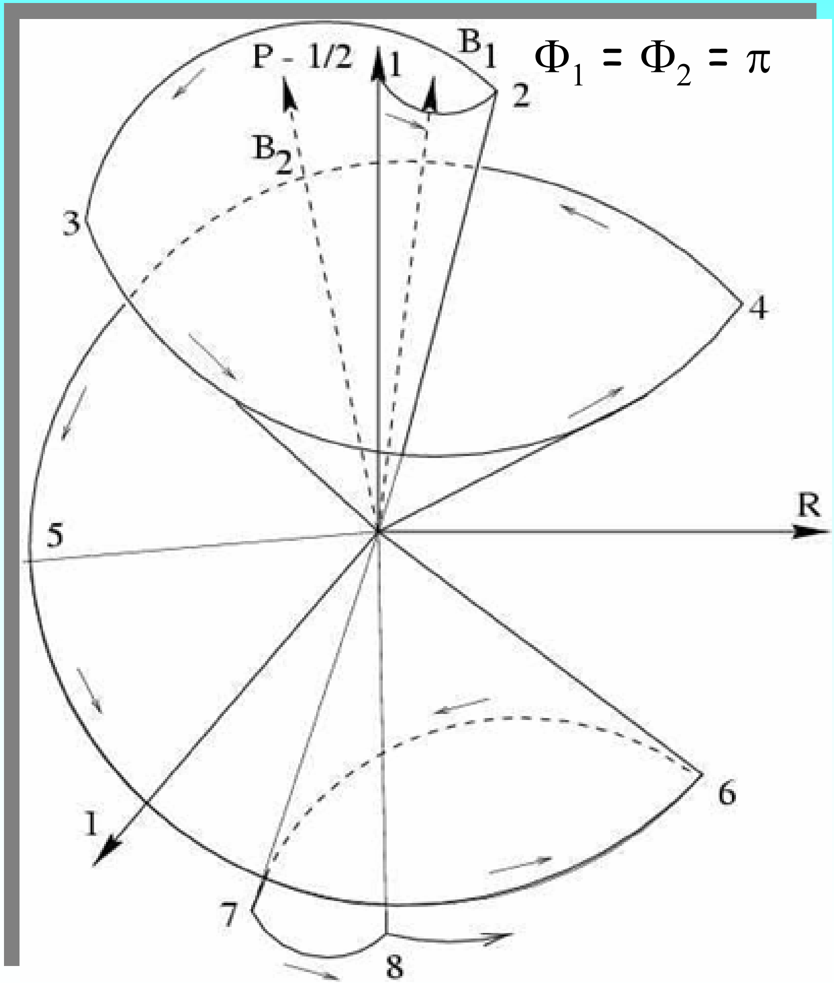
Another way to get strong transition

No large vacuum mixing and no matter enhancement of mixing or resonance conversion

V. Ermilova V. Tsarev, V. Chechin
E. Akhmedov
P. Krastev, A.S., Q. Y. Liu,
S.T. Petcov, M. Chizhov

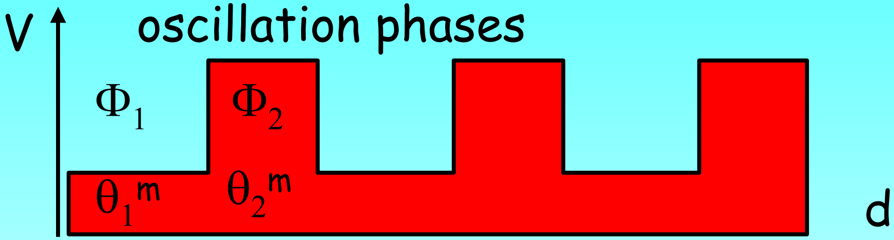


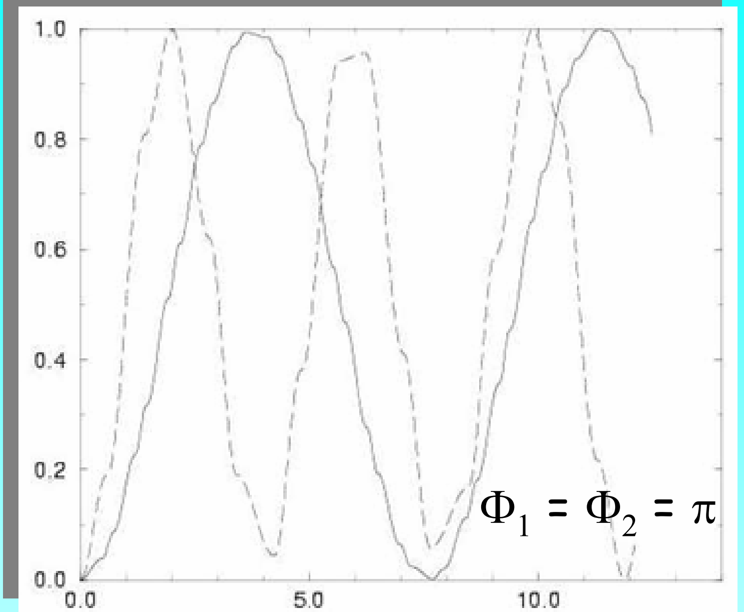
``Castle wall profile"



Parametric resonance

``Castle wall profile"





Resonance condition: E. Kh. Akhmedov

$$s_1 c_2 \cos 2\theta_1{}^m + s_2 c_1 \cos 2\theta_2{}^m = 0$$

$$s_i = \sin\Phi_i/2, \quad c_i = \cos\Phi_i/2, \quad (i = 1,2)$$

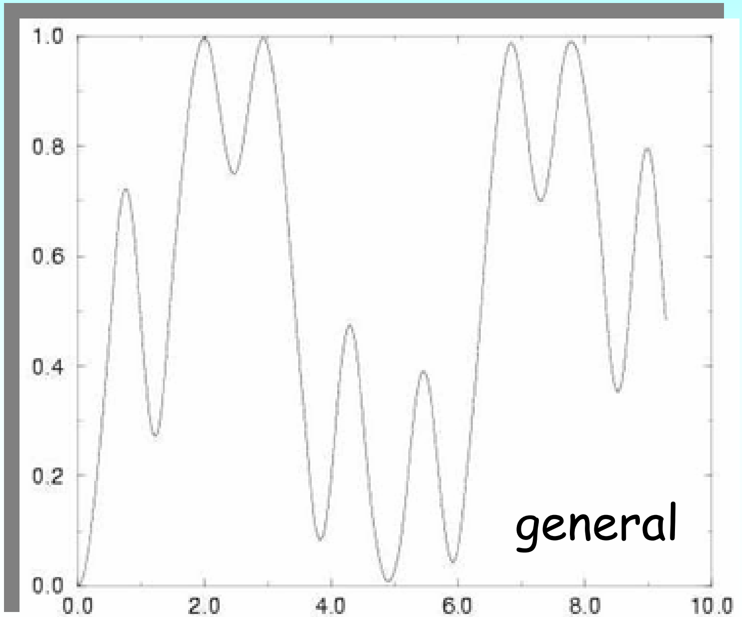
= maximal depth of oscillations

also S. Petcov M. Chizhov

Simplest realization:

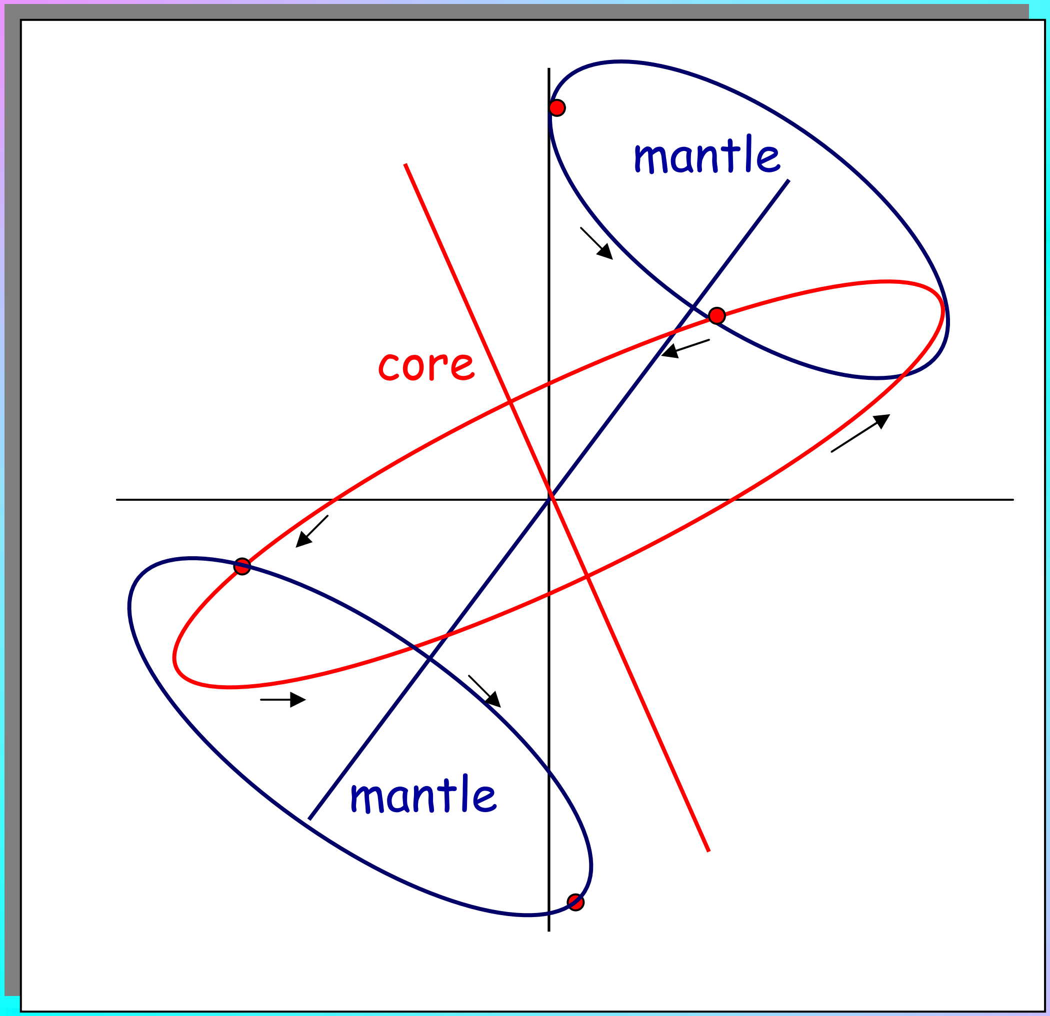
$c_1 = c_2 = 0$ $\Phi_1 = \Phi_2 = \pi$

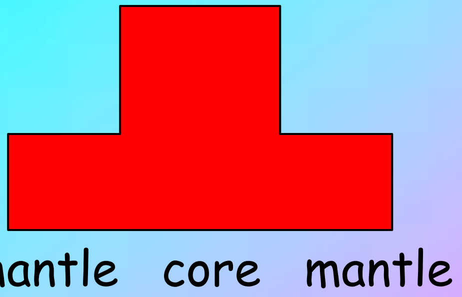
In general, certain correlation between the phases and mixing angles



distance

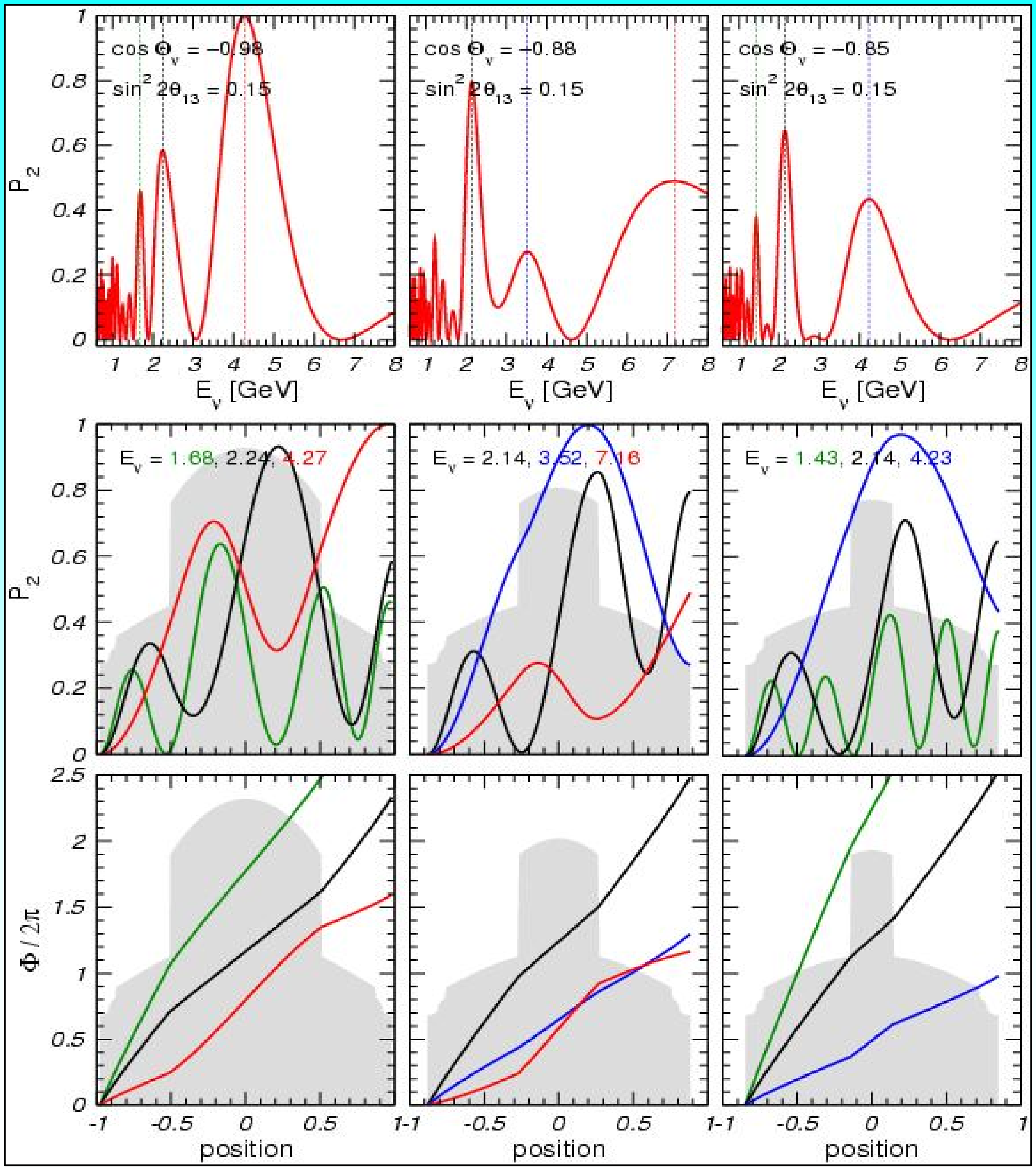
Parametric enhancement in the Earth





For the atmospheric neutrinos in multi GeV range

Parametric effects



$P_2 = P(\nu_e - \nu_\alpha)$

ν_α = combination of ν_μ and ν_τ in ν_3

Main challenge

Simple vs. complex

toward the underlying physics

fractal

Bottom-up

Where end meet?

Regularities

Symmetries

Anarchy, randomness

Landscape:
accidental…

To large extend masses and mixings are ``accidental'' parameters with rather complicated physics behind determined e.g. by vacuum of complex (many components) system of scalar fields