CHARGE CONJUGATION

The charge conjugation operator $\mathcal{C}$ is defined such that:

$$\hat{\mathcal{C}} \,|\, \nu(\vec{p}, s) \rangle = \underset{\text{phase} \uparrow = 1 \text{ usually}}{\eta_c} \,|\, \bar{\nu}(\vec{p}, s) \rangle$$

In the quantized theory $\mathcal{C}$ is a unitary operator

$$\mathcal{C}^\dagger \mathcal{C} = \mathbb{1}$$

Maybe it is better to indicate it as U_c.

We can see how $\mathcal{C}$ acts on a and b: $\qquad C|0\rangle = |0\rangle$

$$\mathcal{C} \,|\, \nu(\vec{p}, s) \rangle = \mathcal{C} a^\dagger(\vec{p}, s) |0\rangle = \mathcal{C} a^\dagger \mathcal{C}^\dagger \mathcal{C} |0\rangle$$
$$= b^\dagger(\vec{p}, s) |0\rangle$$

So $\mathcal{C} a \mathcal{C}^\dagger = \underset{\uparrow \text{phase} = +1}{b}$

The charge conjugate neutrino field, ν^c:

$$\nu^c \equiv \mathcal{C} \nu \mathcal{C}^\dagger$$

It can be shown that, taking $\qquad$ pg. 19 Kumi & Pensure

matrix not operator $\rightarrow C \equiv i\gamma^2\gamma^0$

and

$$\boxed{\nu^c = C \bar{\nu}^T}$$

Properties of C:

$C^2 = -1$

$C^\dagger = C^{-1} = C^T = -C$

$C \gamma_\mu C^\dagger = -\gamma_\mu^T$

CP conjugation

Parity: $\overset{U_P}{\swarrow}$

$$P\, \nu(t,\vec{x})\, P^{+} = \eta_P \gamma^0 \nu(t,-\vec{x})$$

L→R and viceversa. ?

CP:

$$\Sigma = CP$$

$$\Sigma | \nu(\vec{p},s) \rangle = \eta_\Sigma | \bar{\nu}(-\vec{p},s) \rangle$$

For fields we get

$$U_{CP}\, \nu(t,\vec{x})\, U_{CP}^{+} = \eta'_{CP}\, i\gamma^0\gamma^2 \nu^*(t,-\vec{x})$$

It can be shown that $\eta'_{CP} = -\eta^*_\Sigma$ pg. 21 K&P

Exercise:

Show that $(\nu_L)^c = (\nu^c)_R$?

MAJORANA NEUTRINO (Majorana 1937)

$$\boxed{\begin{aligned} \nu_M = \nu_M^c &\equiv \mathcal{C}\, \nu_M\, \mathcal{C}^{+} \\ &= C\, \bar{\nu}^T \end{aligned}}$$

Majorana condition !

In terms of creation and destruction operators

$$\nu_M = \int \frac{d\vec{p}}{(2\pi)^{3/2}} \sum_s \Big[u(\vec{p}, s)\, a(\vec{p}, s)\, e^{-ip\cdot x} + v(\vec{p}, s)\, \eta_c^*\, a^+\, e^{ip\cdot x} \Big]$$

Exercise

There are only one set of creation and annihilation operators

$\Rightarrow$ ν_M has only 2 degrees of freedom, corresponding to two spin states.

A Majorana particle cannot possess any quantum number as the Majorana condition is not invariant:

$$\nu \xrightarrow{U(1)_{Lep}} \nu\, e^{i\alpha_\nu \varphi}$$

$$\nu_M = C\, \bar{\nu}^T$$

$$\downarrow$$

$$\nu e^{i\alpha_\nu \varphi} = C\, e^{-i\alpha \varphi}\, \bar{\nu}^T$$

$$\nu = e^{-2i\alpha \varphi}\, C\, \bar{\nu}^T$$

$\neq$

If Majorana $\Rightarrow$ lepton number is broken.

Using the chiral representation,

$$\nu_L = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}$$

$$(\nu_L)^c = C\overline{\nu_L}^T = i\gamma^2 \nu_L^* = \begin{pmatrix} 0 \\ -i\sigma^2\varphi^* \end{pmatrix}$$

Notice $(\nu_L)^c = (\nu^c)_R$ ∇

$$\nu_M = \nu_L + \nu_L^c = \begin{pmatrix} \varphi \\ -i\sigma^2\varphi^* \end{pmatrix}$$

Only one two-component field φ.

For comparison ν_{Dirac}

$$\nu_D = \begin{pmatrix} \varphi \\ -i\sigma^2\chi^* \end{pmatrix}$$

$$\nu_D^c = \begin{pmatrix} \chi \\ -i\sigma^2\varphi^* \end{pmatrix}$$

It takes 2 Majorana ν (with opposite CP-parities) to make a Dirac neutrino.

— * —

CP properties of a Majorana ν:

Majorana condition

$$U_{CP}\,\nu_M\,U_{CP}^\dagger = \eta_{CP}\, i\gamma^0\gamma^2\nu^*(t,-\bar{x})$$
$$= \eta_{CP}\,\gamma^0 C\,\overline{\nu(t,-\bar{x})}^T = \eta_{CP}\,\gamma^0\,\nu(t,-\bar{x})$$

$\|$

$$U_{CP}\,C\bar{\nu}^T U_{CP}^\dagger = C\left(U_{CP}\,\bar{\nu}\,U_{CP}^\dagger\right)^T = C\left(U_{CP}\,\nu^\dagger\gamma^0\,U_{CP}^\dagger\right)^T$$

because T does not act on U_{CP}

$$= C\, \gamma_0^T \left(U_{CP}\, \bar{\nu}^\dagger\, U_{CP}^\dagger\right)^T$$

$$= C\, \gamma_0^T \left(U_{CP}\, \bar{\nu}\, U_{CP}^\dagger\right)^*$$

$$= C\, \cancel{\gamma_0^T}\, \eta_{CP}^*\, \cancel{\gamma_0^*}\, C^*\, \bar{\nu}^\dagger \quad \left(= \; C\, \eta_{CP}^*\, C^*\, \gamma_0\, \nu \right)$$

$$= C\, \eta_{CP}^*\, C^*\, \gamma_0\, \nu(t, -\bar{x})$$

$$= -\eta_{CP}^*\, \cancel{C^T C^*}\, \gamma_0\, \nu(t, -\bar{x})$$

$$C = -C^T \qquad = -\eta_{CP}^*\, \gamma_0\, \nu(t, -\vec{x})$$

$$C C^\dagger = \mathbb{1}$$

$$C^\dagger C = \mathbb{1}$$

$$C^T C^* = \mathbb{1}$$

$$\Longrightarrow \quad \eta_{CP} = -\eta_{CP}^*$$

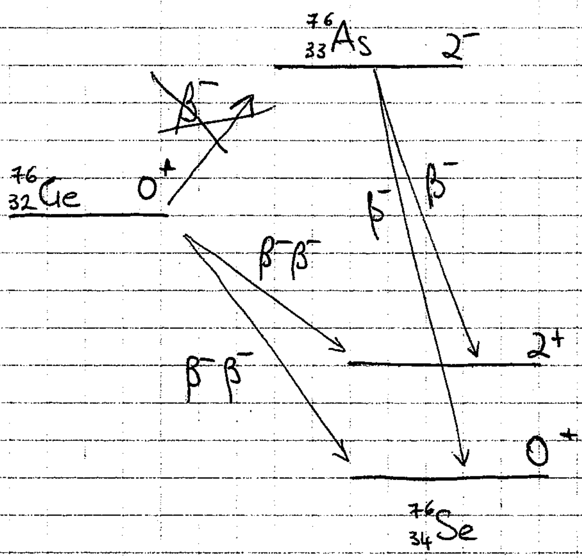
$$\Longrightarrow \quad \eta_{CP} = \pm i$$ Majorana CP phase

DOUBLE BETA DECAY

- $(\beta\beta)_{2\nu}$
- $(\beta\beta)_{0\nu}$

$(\beta\beta)_{2\nu}$

It is a rare process which takes place between nuclei.



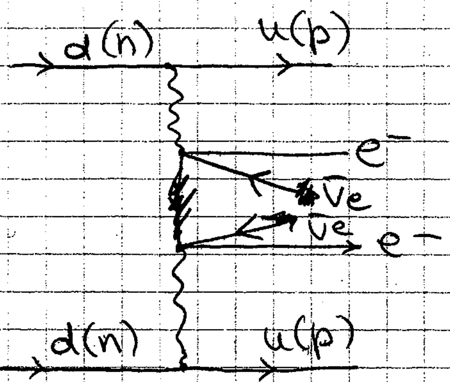
energy levels

Kinematically allowed

First proposed by Goeppert-Mayer (1935).

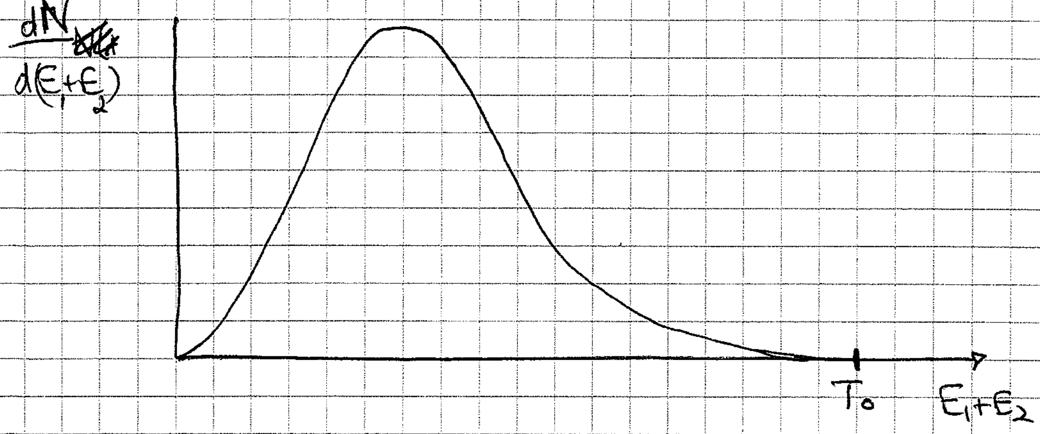
$$(A,Z) \longrightarrow (A,Z+2) + 2e^- + 2\bar{\nu}_e$$

$$\left(2n \longrightarrow 2p + 2e^- + 2\bar{\nu}_e\right)$$



$$\Gamma \sim G_F^4 \, T^{11}$$

What is measured is $E_1(e_1) + E_2(e_2)$



Typical lifetimes:

$$\tau\left[(\beta\beta)_{2\nu};\ ^{76}Ge\right] \sim 10^{21}\ yrs$$

$$\tau\left[(\beta\beta)_{2\nu};\ ^{100}Mo\right] \sim 10^{18}\ yrs$$

$$\tau\left[(\beta\beta)_{2\nu};\ ^{130}Te\right] \sim 10^{21}\ yrs$$

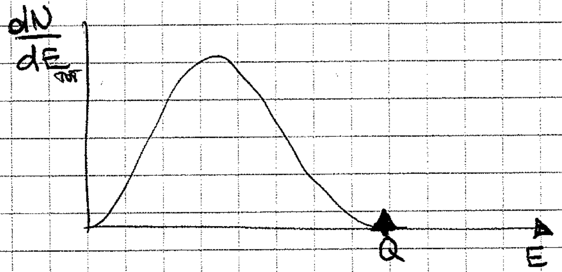
The main interest on $(\beta\beta)_{2\nu}$ is because it offers the opportunity to study the nuclear structure and test the predictions of different models of nuclear matrix ~~matri~~ elements calculation.

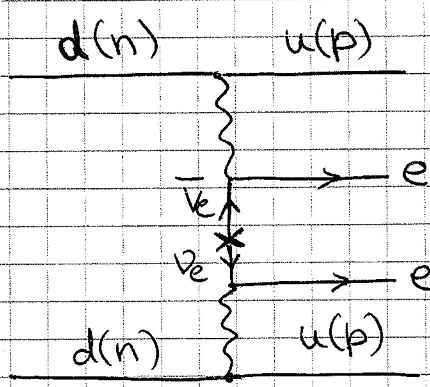
$(\beta\beta)_{0\nu}$

Motivated by Majorana neutrinos, ~~to~~ it was proposed in 1939 by Furry.

$$(A,Z) \longrightarrow (A,Z+2) + 2e^-$$

The energy spectrum vs $T = E_1 + E_2$ is a δ-function at the end point.





It violates the lepton number by 2 units !

It is a process of 2^{nd} order in the Fermi constant G_F.
The ν propagator

$$\langle 0 | T\left(\nu_{eL}(x_1)\, \nu_{eL}^T(x_2)\right)|0\rangle$$

$$\simeq \langle m \rangle \frac{i}{(2\pi)^4} \int \frac{d^4p}{p^2}\, e^{-ip(x_1-x_2)}\, \frac{1-\gamma_5}{2}\, C$$

where $\langle m \rangle = \sum_i U_{ei}^2\, m_i$

The half life time $T_{0\nu}^{1/2}$ is then:

$$\left(T_{0\nu}^{1/2}\right)^{-1} \propto \text{phase space}\, \left(M_{GT}+M_F\right)^2 \, |\langle m\rangle|^2$$

$$|\langle m\rangle| = \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 \right|$$

$$= \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}'} \right|$$

Predictions for $|\langle m\rangle|$:

- type of spectrum: NH, IH, QD, interpolating cases
- CPC or CPV.

$$|U_{e1}|^2 = \cos^2\theta_\odot\, c_{13}^2$$

$$|U_{e2}|^2 = \sin^2\theta_\odot\, c_{13}^2$$

$$|U_{e3}|^2 = \sin^2\theta_{13}$$

NH

$$m_1 \simeq 0$$
$$m_2 \simeq \sqrt{\Delta m^2_\odot}$$
$$m_3 \simeq \sqrt{\Delta m^2_A}$$

$$|\langle m \rangle| = \left| \sqrt{\Delta m^2_\odot}\; s^2_\odot c^2_{13} + \sqrt{\Delta m^2_A}\; s^2_{13}\, e^{i\alpha'_{31}} \right|$$

Ex. $\alpha_{31} = 0$

$$|\langle m \rangle| = \left| \sqrt{\Delta m^2_\odot}\; s^2_\odot c^2_{13} + \sqrt{\Delta m^2_A}\; s^2_{13} \right|$$
$$\simeq \sqrt{8 \cdot 10^{-5}\, eV^2}\; 0.31 \cdot 1 + \sqrt{2 \cdot 10^{-3}\, eV^2} \cdot 0$$
$$\cdot 0.02$$
$$\simeq \text{few meV}$$

Ex. $\alpha_{31} = \pi$

$$|\langle m \rangle| = \left| \;\cdots\; - \;\; \right|$$
$$\simeq$$

For CPV, the value interpolate continously from the ones of $\alpha_{31} = \pi$ to $\alpha_{31} = 0$.

IH

$m_3 \simeq 0$

$m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_A}$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m^2_A} \left(c^2_\odot + s^2_\odot e^{i\alpha_{21}} \right) c^2_{13} \right|$$

$\alpha_{21} = 0$ $\qquad |\langle m \rangle|_+ = \ \dots$ $\qquad$ Ex.

$\alpha_{21} = \pi$ $\qquad |\langle m \rangle|_- = \ \dots$ $\qquad$ Ex.

Again, for CPV the values interpolate between $|\langle m \rangle|_+$ and $|\langle m \rangle|_-$.

QD

$m_1 \simeq m_2 \simeq m_3 \equiv m_0$

$$|\langle m \rangle| \simeq m_0 \left| c^2_\odot c^2_{13} + s^2_\odot c^2_{13} e^{i\alpha_{21}} + \cancel{s^2_{13} e^{i\alpha_{31}}} \right|$$

$$\simeq m_0 \left| c^2_\odot + s^2_\odot e^{i\alpha_{21}} \right|$$

$\alpha_{21} = 0$ $\qquad |\langle m \rangle| \simeq m_0$

$\alpha_{21} = \pi$ $\qquad |\langle m \rangle| \simeq m_0 \cos 2\theta_\odot$ $\qquad$ minimal value

$\qquad \simeq m_0 \cdot 0.24$

Because of $\cos 2\theta_\odot >> 0$, the lower limit on $|\langle m \rangle|$ is significant. Very important for future searches.

EXPERIMENTAL SITUATION

- Heidelberg - Moscow experiment:

$$|\langle m \rangle| < 0.35 - 1.05 \text{ eV} \qquad \text{at } 90\% \text{ CL}$$

It uses ^{76}Ge.

There are two values for the bound because the nuclear matrix elements ($M_{GT} + M_F$) are known with an uncertainty of a factor of 3. They cannot be measured but only computed theoretically and they are different for different nuclei.

- IGEX:

$$|\langle m \rangle| < 0.33 - 1.35 \text{ eV} \qquad \text{at } 90\% \text{ CL}$$

It uses ^{76}Ge.

- NEMO 3 is taking data at present:

$$|\langle m \rangle| \lesssim 0.7 - 1.2 \text{ eV} \qquad \text{at } 90\% \text{ CL}$$

using ^{100}Mo and ^{82}Se.

- CUORICINO is taking data at present:

$$|\langle m \rangle| < 0.19 - 0.68 \text{ eV} \qquad \text{at } 90\% \text{ CL}$$

using ^{130}Te.

There is a claim of detection of $(\beta\beta)_{0\nu}$-decay by a subgroup of the Heidelberg-Moscow collaboration:

$$|\langle m \rangle| = 0.1 - 0.9 \text{ eV}$$

H.V. Klapdor-Kleingrothaus et al., PLB 586, 198 (2004)
and subsequently:

$$|\langle m \rangle| = 0.32 \pm 0.03 \text{ eV}$$

H.V. Klapdor-Kleingrothaus and I.V. Krivosheina, MPLA 21, 1547 (2006).

This claim is not yet confirmed by other experiments.