In fact:

$$U^m(t) = \begin{pmatrix} \cos\theta^m & \sin\theta^m \\ -\sin\theta^m & \cos\theta^m \end{pmatrix}$$

$$\dot{U}^m(t) = \dot{\theta}^m \begin{pmatrix} -\sin\theta^m & \cos\theta^m \\ -\cos\theta^m & -\sin\theta^m \end{pmatrix}$$

$$\left(U^m(t)\right)^+ \dot{U}^m(t) = \dot{\theta}^m \begin{pmatrix} C_m & -S_m \\ S_m & C_m \end{pmatrix} \begin{pmatrix} -S_m & C_m \\ -C_m & -S_m \end{pmatrix}$$

$$= \dot{\theta}^m(t) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

and

$$\left(U^m(t)\right)^+ H \, U^m(t) = \begin{pmatrix} E_A & 0 \\ 0 & E_B \end{pmatrix}$$

by definition of θ^m.

The propagation matrix is non diagonal ($\dot{\theta}^m \neq 0$) and ν_1^m, ν_2^m are not the energy eigenstates of the Hamiltonian. Mixing between ν_1^m and ν_2^m is present and a transition between one and the other is possible.

ADIABATIC APPROXIMATION

If the adiabaticity condition is satisfied:

$$|E_A - E_B| \gg |\dot{\theta}^m(t)|$$

then $\nu_1^m(t)$ and $\nu_2^m(t)$ are $\sim$ energy eigenstates and they do not mix significantly. $P_{JUMP} \cong 0$ and the evolution is adiabatic.

To be quantitative:

$$\dot{\theta}^m(t) = \frac{d\theta^m}{d\,tg 2\theta^m} \cdot \frac{d\,tg 2\theta^m}{dt}$$

$$= 1/2 \cos^2 2\theta^m \cdot \frac{\frac{\Delta m^2}{2E} \sin 2\theta_0 \, \dot{V}}{\left(\frac{\Delta m^2}{2E}\cos 2\theta_0 - V(t)\right)^2}$$

$$= 1/2 \cos^2 2\theta^m \cdot tg^2 2\theta^m \cdot \frac{\dot{V}}{\frac{\Delta m^2}{2E}\sin 2\theta_0}$$

$$= 1/2 \sin^2 2\theta^m \frac{\dot{V}}{\frac{\Delta m^2}{2E}\sin 2\theta_0}$$

$$= 1/2 \sin^2 2\theta^m \frac{\dot{V}}{V} \frac{V}{\frac{\Delta m^2}{2E}\sin 2\theta_0}$$

The condition is more restrictive at resonance where $|E_A - E_B|$ is minimal and $\sin^2 2\theta^m$ is maximal ($\sin^2 2\theta^m = 1$).

At resonance, the adiabaticity condition becomes:

$$|E_A - E_B| \gg \dot{\theta}^m(t)$$

$$\frac{\Delta m^2}{2E} \sin 2\theta_0 \gg 1/2 \left|\frac{\dot{V}}{V}\right| \frac{\cos 2\theta_0}{\sin 2\theta_0}$$

$$\frac{\Delta m^2}{2E}\,\frac{\sin^2 2\theta_0}{\cos 2\theta_0}\left|\frac{V}{\dot V}\right| \gg 1$$

with $$\frac{\dot V}{V} = \frac{1}{V}\frac{dV}{dr} = \frac{d\ln V}{dr}$$

Notice that the adiabaticity condition depends on θ_0.

One can also understand this as follows:

$$\text{resonance width} \equiv \delta r_R = \frac{2\,\mathrm{tg}\,2\theta_0}{\left|\frac{1}{V}\frac{dV}{dr}\right|}$$

$$\underbrace{\frac{\Delta m^2}{2E}\sin 2\theta_0}_{(E_A - E_B)_R}\cdot\overbrace{2\,\mathrm{tg}\,2\theta_0\,\frac{1}{\left|\frac{1}{V}\frac{dV}{dr}\right|}}^{\delta r_R} \gg 1$$

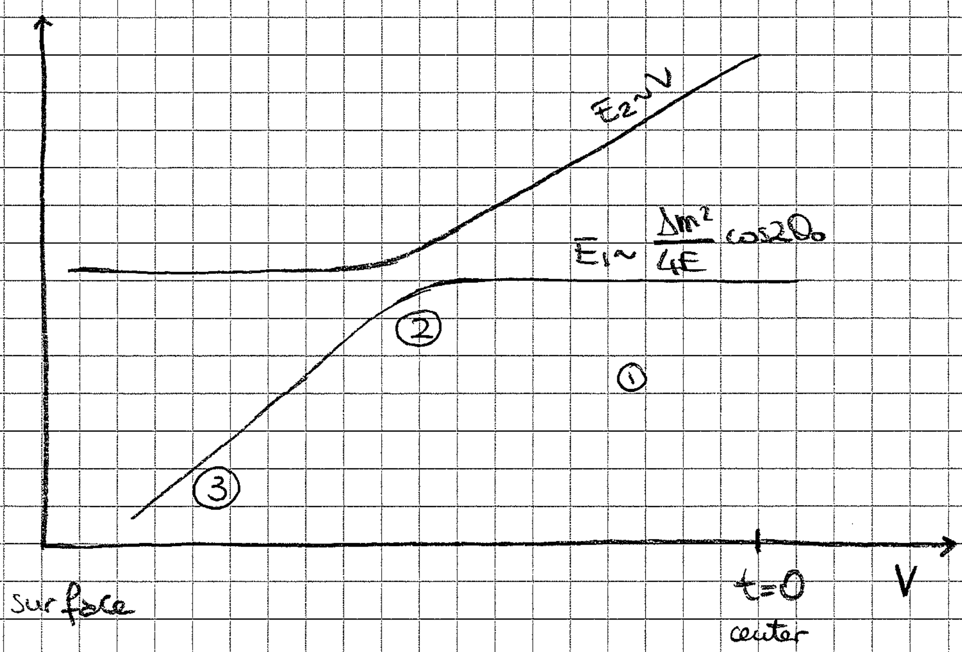
$$\frac{2\pi\,\delta r_R}{L_R} \gg 1$$

Many oscillations need to take place at resonance.

MSW effect — Mikheyev, Smirnov, Wolfenstein

Let's consider a neutrino produced in the Sun center $\left(V \gg \frac{\Delta m^2}{2E}\cos 2\theta_0\right)$ which travells towards the surface.

We plot the energy eigenvalues as a function of $V(t)$:



$$E_{1,2} = \frac{V \pm \sqrt{\left(V - \frac{\Delta m^2}{2E}\cos 2\theta_0\right)^2 + \left(\frac{\Delta m^2}{2E}\sin 2\theta_0\right)^2}}{2}$$

1 $\quad E_1 \sim \frac{\Delta m^2}{4E}\frac{\cos 2\theta_0}{\cancel{V}}\cdot\cancel{V} \sim \frac{\Delta m^2}{4E}\cos 2\theta_0$

$E_2 \sim V$

2 $\quad$ Resonance: $\quad E_1 \sim \frac{\Delta m^2}{2E}\,\frac{\cos 2\theta_0 - \sin 2\theta_0}{2}$

$$E_2 \sim \frac{\Delta m^2}{2E}\,\frac{\cos 2\theta_0 + \sin 2\theta_0}{2}$$

③ $$E_{1,2} = \pm \frac{\Delta m^2}{2E}$$

In the center of the Sun, the dominant term in the H^m is V $\Longrightarrow$ ν_e is mainly alligned with the heaviest eigenstate ν_2^m. ν_2^m evolves and (if adiabatic) becomes ν_2^0 at the surface. Then the probability of ν_e survival is:

~~$(\langle \nu_e | \nu_2 \rangle)$~~

$$P_{ee}(\text{Earth}) = \left| \langle \nu_e | \nu_2 \rangle \right|^2 = \sin^2\theta_0$$

For small values of θ_0, one can have a large depletion of the solar flux.

We can study this further. The $|\nu_e\rangle$ state in the Sun core is produced as an incoherent superposition of $|\nu_1^m\rangle$ and $|\nu_2^m\rangle$ as large E-difference.
So

$$|\nu_e\rangle \rightarrow |\nu_1^m\rangle \quad \text{with } P_1 = \cos^2\theta_m$$
$$|\nu_2^m\rangle \quad \text{with } P_2 = \sin^2\theta_m$$

We also introduce a jump probability for transitions between ν_1^m and ν_2^m, P_c.

$$|\nu_1^m\rangle \longrightarrow |\nu_1\rangle \quad \text{with } P = 1 - P_c$$
$$|\nu_2\rangle \quad \text{with } P = P_c$$

$$|\nu_2^m\rangle \longrightarrow |\nu_1\rangle \quad \text{with } P = P_c$$
$$|\nu_2\rangle \quad \text{with } P = 1 - P_c$$

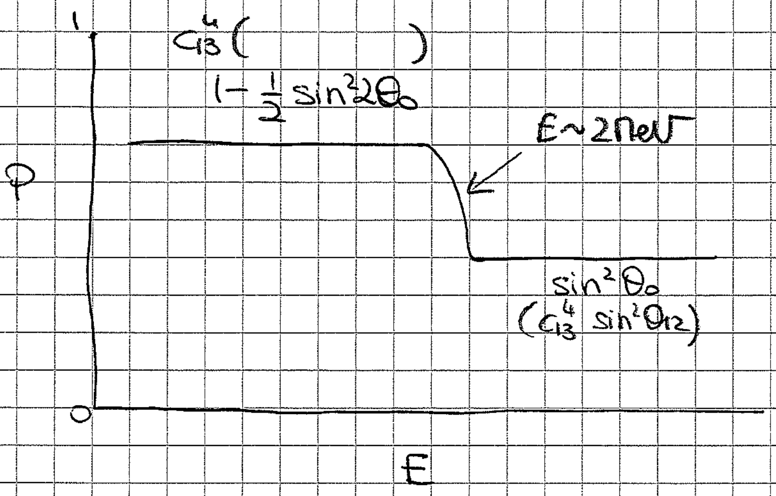
Finally, $P_{1e} = |\langle \nu_e | \nu_1 \rangle|^2 = \cos^2\theta_0$

$P_{2e} = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2\theta_0$

Then

$$P_{ee}^{Sun} = \cos^2\theta_{13}\left((1-P_c)\cos^2\theta_0 + P_c\sin^2\theta_0\right) + \sin^2\theta_{13}\left(P_c\cos^2\theta_0 + (1-P_c)\sin^2\theta_0\right)$$

For the simplified case $P_c = 0$, one has



Fits very well the spectral data with $\sin^2\theta_0 \simeq 0.3$, $\Delta m^2 \sim 10^{-5} - 10^{-4}\ eV^2$.

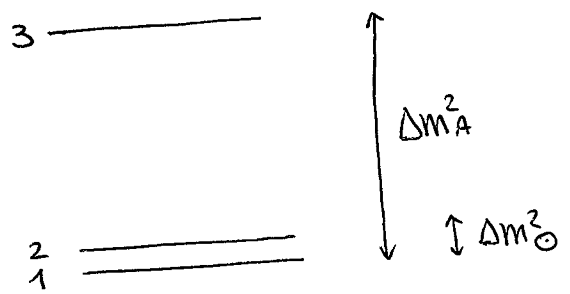
Present knowledge in ν physics

From atm, solar, reactor and accelerator ν exp,
we know

$$\Delta m^2_\odot = 8.0 \times 10^{-5}\,\mathrm{eV}^2$$
$$\Delta m^2_A = 2.5 \times 10^{-3}\,\mathrm{eV}^2$$

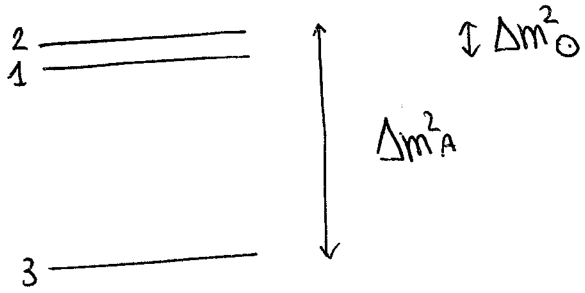
$\Rightarrow$ 3-ν mixing

ν masses can be arranged in two possible ways:



NORMAL ORDERING

$$m_1 = m_{MIN}$$
$$m_2 = \sqrt{m^2_{MIN} + \Delta m^2_\odot}$$
$$m_3 = \sqrt{m^2_{MIN} + \Delta m^2_A}$$



INVERTED ORDERING

$$m_1 = \sqrt{m^2_{MIN} + \Delta m^2_A - \Delta m^2_\odot}$$
$$m_2 = \sqrt{m^2_{MIN} + \Delta m^2_A}$$
$$m_3 = m_{MIN}$$

In order to determine the absolute ν masses,

- absolute mass scale, m_{MIN}
- type of ordering (hierarchy).

It is conventional to consider 3 different types of ν mass spectrum:

- NH : $m_1 \ll m_2 \ll m_3$

  $m_2 = \sqrt{\Delta m^2_\odot}$

  $m_3 = \sqrt{\Delta m^2_A}$

- IH : $m_3 \ll m_1 \simeq m_2$

  $m_1 \cong \sqrt{\Delta m^2_A - \Delta m^2_\odot}$

  $m_2 \cong \sqrt{\Delta m^2_A}$

- QD : $m_1 \simeq m_2 \simeq m_3 \gg \Delta m^2_\odot, \Delta m^2_A$

Summary of angles: $\sin^2\theta_\odot = 0.30$, $\sin^2\theta_A = 0.5$, $\sin^2\theta_{13} < 0.04$.

How can we express ν mass in the Lagrangian and then explain their origin?