Majorana Neutrino

S. Bilenky
JINR(Dubna)

28 February 2007

Ettore Majorana. Short biography

Majorana paper "Symmetrical theory of the electron and the positron"

Present status of neutrino Majorana (neutrinoless double β- decay)

Ettore Majorana was born in Catania on 5.08.1906. There were five children in the family. Ettore father was an engineer, specialist in telephone communications

Unusual abilities in numerical calculations Ettore showed when he was very young. At the age of four he could multiply two three-digit numbers and get correct result in seconds

In 1921 the family moved to Rome

Ettore finished Liceo in 1923 with high marks. In autumn of 1923 he entered Engineer faculty of the Rome University

Among his fellow-students and friends was E. Segre. and E. Amaldi

Segre and later Amaldi switched to physics (1927) and started to work with E. Fermi who was appointed in 1926 as a Professor of theoretical physics at Rome University

E. Majorana was famous at Engineer faculty for his extraordinary ability of solving difficult mathematical problems.

E. Segre often talked about him in the Fermi group and tried to convince Majorana to follow his example

Majorana switched to physics in the beginning of 1928 after a talk with Fermi

Fermi was developing at that time the statistical model which is known as Thomas-Fermi model. He explained Majorana the model and showed him the table with numerical values of the screening potential which he calculated

Next morning Majorana returned back to the Institute with his own table for the potential. He transformed second order nonlinear Thomas-Fermi equation into Riccati equation and solved it numerically. Majorana and Fermi results coincided

A few days later Majorana became student of the Physics faculty. He impressed everybody by his lively mind and broad interests. He was very critical person. For his criticism he was called in the Fermi group "Great Inquisitor"

Majorana received diploma in 1929. His thesis were devoted to the investigation of the structure of nuclei and to the theory of the alpha-decay. His supervisor was Fermi

After doctorate Majorana visited the Institute for a few hours every day. He spend most of his time in library studying papers of Dirac, Heisenberg, Pauli, Weil and Wigner

He was very interested in the group theory and wanted to write a book on the group theory

At that time Fermi and his group worked on problems of atomic and molecular physics

Majorana wrote six papers on the subject

The splitting of levels of electrons in different heavy atoms due to electron spin. The calculation of levels of electrons in helium. Detailed calculation of atomic spectrum of calcium. The theory of molecular ion He_2^+. The theory of H_2 molecule.

In the last paper of this period Majorana developed the theory of non adiabatic spin-flip transitions of polarized particles in rapidly varying magnetic field. Majorana considered the case $J = 1/2$. His result was generalized by Bloch and Rabi for arbitrary J. This Majorana paper is well known and is widely used

These papers demonstrated profound Majorana ability of using symmetry properties of the states

This allowed him to simplify the problem and to choose the suitable approximation, which is normal now but was not usual at that time. These papers also demonstrated perfect knowledge of experimental data.

In 1932 Majorana received teaching diploma ("libero docente") Committee (Fermi, Lo Surdo, Persico) concluded " the candidate has a complete mastery in theoretical physics"

In the end of 1931-beginning of 1932 all members of the Fermi group started to concentrate their efforts on nuclear physics. Amaldi returned from Leipzig and made a series of seminars on Retherford, Chadwick and Ellis papers. Fermi, Majorana, Rasetti, Segre and a few students attended the seminars. They were often interrupted by questions which gave Fermi opportunity for detailed blackboard explanations.

Majorana was usually silent but sometimes he made very keen observations, always to the point

F. Jolliot and I.Curies were the first who observed recoil protons with energy about 5 MeV from "penetrating radiation" produced by bombardment of Be by α particles. They interpreted their results as a anomalous interaction of γ and proton

After reading F. Jolliot and I.Curies papers Majorana noticed: "They did not understood what they observed. Proton recoil is probably produced by a heavy neutral particle"

A few days later "Nature" was received in Rome with Chadwick paper on the discovery of neutron (1932)

Appeared many models of nuclei in which n, p and also e^-, α were considered as constituents of nuclei

Majorana was one of the first who came to an idea that only p and n are constituents of nuclei

Majorana started to develop the theory of nuclear forces. He developed the theory of space exchange forces between p and n (Majorana potential)

Fermi was very interested in the idea and pushed Majorana to publish his results. However, he refused and even did not allow Fermi to mention them in his talk on atomic nuclei at the conference in Paris

Fermi managed, however, to persuade Majorana to go to Leipzig (Heisenberg) and to Copenhagen (Bohr)

Majorana was abroad during seven months, starting from January 1933

Majorana had difficulties in establishing new relations with people. Abroad he was rather lonely. However, he became friendly with Heisenberg to whom he had great respect. Heisenberg convinced Majorana to publish his paper on nuclear theory which appeared in 1933

After returning from Germany Majorana started to come to the Institute at via Panisperna rather rare and after some months did not come at all

He was at home and became interested in political economy, philosophy, construction of ships, medicine... He even wrote a paper on statistical laws in physics and social sciences which was discovered and published after his disappearence

Meanwhile new talented physicists grown up in Italy: Wick, Racah, Giovanni Gentili Jr., Pincherle , Watagin. It was time to create a new chair in theoretical physics. The competition for the chair was announced at the beginning of 1937 by the University of Palermo.

It was a problem to convince Majorana to take part in the competition

Finally Fermi, Amaldi and Segre managed to convince Majorana to take part in the competition

Majorana had no publications during several years. He sent to "Nuovo Cimento" his most important paper "Symmetrical theory of the electron and the positron"

Probably without the competition the paper would never appear

Then happened the following. By the request of Senator Giovanni Gentili Majorana for his extraordinary abilities was appointed without competition as professor at Napoli University

The commitee for Palermo chair made the short list: Wick, Racah, Giovanni Gentili Jr.

In January 1938 Majorana moved to Napoli In Napoli he had lonely life and lived in different hotels. He went to the University only when he had lectures (on Quantum mechanics). After lectures he visited Carrelli, professor of Experimental Physics with whom he became friendly and discuss different problems in physics.

Never mentioned what he was doing. Discussed his neutrino theory. Carrelli had impression that Majorana considered this theory as his most important contribution to physics

On March 23 1938 Majorana decided to go to Palermo

On March 25 Carrelli received a telegram from Majorana from Palermo. He asked Carrelli do not worry about a letter which he would receive

In the letter which came soon Majorana wrote that he found his life useless and decided to commit suicide

Carrelli called Fermi and Fermi called Luciano, Ettore brother

Luciano immediately went to Napoli. He understood that on the evening of March 25 Ettore took boat to Napoli. He was seen sleeping in his cabin when the boat was entering the bay of Napoli. He did not arrive to Napoli. His body was never found.

Fermi once noticed that if a person with such intelligence decided to disappear he will succeed to do that

During several months there was investigation conducted by family and police. Vatican tried to find out whether he entered some monastery. No traces were found

Mussolini took personal interest in the investigation. Ettore mother wrote to him with a letter from Fermi

”I have no hesitation in saying and it is no way of exaggeration, that of all the Italian and foreign scholars whom I have had the opportunity of knowing, Majorana is the one whose depth most impressed me. Capable of developing baldly hypothesis and, at the same time criticizing his own work and that of others, highly skilled in calculations and a mathematician of a great depth, who never lost sight of the true nature of the physics problems behind the vail of figures and mathematical techniques, Ettore Majorana was highly endowed with that rare combination of gifts which go to make a typical theoretician of the first rank”

Fermi (From Cocconi memories)

"There are various kind of scientists in the world. The second and third-rate ones do their best but do not get very far. There are also first-rate people who make very important discoveries which are of capital importance for the development of the science. Then there are genius like Galillo and Newton. Ettore Majorana was one of these. Majorana had greater gifts that anyone else in the world; unfortunately he lacked one quality which other men generally have: plain common sense"

From Pontecorvo memories

E. Majorana was very critical to himself and other people. He was permanently unhappy with himself. He was a pessimist but had very accute sense of humor. He was conditioned by complicated and absolutely nontrivial living rules

E. Majorana was quite rich and I (Pontecorvo) can not avoid thinking that his life might not have finished so tragically should he have been obliged to work for a living. For that reason and also because he did not like to publish the results of all investigations he had made, Majorana contribution to physics is much less than it could be

SYMMETRICAL THEORY OF THE ELECTRON AND POSITRON

The Dirac sea leads to the symmetrical theory of electrons and positrons.

Majorana was not satisfied by the methods how the symmetry was reached

In fact the symmetry is formally lost when vacuum is considered as the state with all levels of electrons with negative energies occupied

"As for electron and positrons are concerned, we should expect from the theory only a formal improvement. However, in our oppinion it is important (for possible extension of the theory) that the very notion of negative states disappears."

Dirac equation for complex spinor field $\psi(x)$

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\psi(x) = 0$$

For the conjugated field $\psi^{c}(x) = C\bar{\psi}^{T}(x)$ $(C\gamma^{T}_{\alpha}C^{-1} = -\gamma_{\alpha}; \quad C^{T} = -C)$ the same equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\psi^{c}(x) = 0$$

Majorana presented the field $\psi(x)$ in the form

$$\psi(x) = \frac{1}{\sqrt{2}}\chi_{1} + i\frac{1}{\sqrt{2}}\chi_{2}$$

$$\chi_{1}(x) = \frac{\psi(x)+\psi^{c}(x)}{\sqrt{2}}; \;\; \chi_{2}(x) = \frac{\psi(x)-\psi^{c}(x)}{\sqrt{2}i}$$

The fields $\chi_{1,2}(x)$ satisfy Dirac equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\chi_{1,2}(x) = 0$$

and additional conditions

$$\chi^{c}_{1,2}(x) = \chi_{1,2}(x)$$

Majorana used a representation in which γ^{α} are imaginary matrices and $\psi^{c}(x) = \psi^{*}(x)$. In this representation $\chi_1(x)$ and $\chi_2(x)$ are real and imaginary parts of $\psi(x)$

He constructed first quantum theory for the field $\chi_1(x)$ (or $\chi_2(x)$)

Current for the fields $\chi_1(x)$ or $\chi_2(x)$

$$j^i_\alpha(x) = \bar{\chi}_i(x)\gamma^\alpha\chi_i(x) = -\chi^T_i(x)(\gamma^\alpha)^T\bar{\chi}_i(x)^T = -\bar{\chi}_i(x)\gamma^\alpha\chi_i(x) = 0$$

$\chi_i(x)$ is the field of neutral particles without charge and magnetic moment.

Vector of the energy and momentum of the field $\chi_1(x)$ or $\chi_2(x)$

$$P^\alpha = \int \sum_r p^\alpha a^\dagger_r(p) a_r(p) d^3p$$

Operators $a_r(p)$ and $a^\dagger_r(p)$ satisfy usual anticommutation relations.

Thus, $a_r(p)$ ($a^\dagger_r(p)$) is the operator of absorption (creation) of a neutral particle with momentum p and helicity r.

$|p\rangle = a^\dagger_r(p)|0\rangle$ is state of the particle with momentum p and helicity r. No states with negative enegies, no antiparticles

In the case of the complex field
$\psi(x) = \frac{1}{\sqrt{2}}\chi_1 + i\frac{1}{\sqrt{2}}\chi_2$ the current
$j^i_\alpha(x) = \bar{\psi}_i(x)\gamma^\alpha\psi_i(x)$ different from zero.

After quantization Majorana came to symmetrical theory of particles and antiparticles with operators of total momentum and of charge

$$P^\alpha = \int \sum_r p^\alpha [c^\dagger_r(p) c_r(p) + d^\dagger_r(p) d_r(p)] d^3p$$

$$Q = e \int \sum_r [c^\dagger_r(p) c_r(p) - d^\dagger_r(p) d_r(p)] d^3p$$

$$|p\rangle = c^\dagger_r(p)|0\rangle; \quad |p\rangle = d^\dagger_r(p)|0\rangle$$

are states of particle with charge e and mass m and antiparticle with charge $-e$ and same mass m

Majorana conclusion

"A generalization of Jordan-Wigner quantization method allows not only to give symmetrical form to the electron-positron theory but also to constract an essentially new theory for particles without electric charge (neutrons and hypothetical neutrinos)"

and later in the paper

"Although it is perhaps not possible now to ask experiment to choose between the new theory and that in which the Dirac equations are simply extended to neutral particles, one should keep in mind that the new theory is introducing in the unexplored field a smaller number of hypothetical entities"

Soon after Majorana paper appeared Racah (1937) proposed a possible test of the Majorana hypothesis

The chain

$$(A,Z) \to (A,Z+1) + e^- + \nu \quad \nu + (A',Z') \to (A',Z'+1) + e^-$$

is possible if neutrino is Majorana particle and is forbidden if neutrino is Dirac particle.

At that time (and also now) it is no possibile to realize this idea

In 1938 Furry suggested that Racah chain with virtual Majorana neutrino can induce neutrinoless double β-decay of nuclei

$$(A,Z) \to (A,Z+2) + e + e$$

This is the most realistic way to search for Majorana neutrinos

MAJORANA NEUTRINO; PRESENT STATUS

Are massive neutrinos and antineutrinos identical or different particles?

This problem which had been put forward by Majorana about 70 years ago is the most fundamental problem of todays neutrino physics

Neutrino interaction is perfectly described by the Standard Model Lagrangian with CC and NC currents

$$j^{\mathrm{CC}}_{\alpha} = 2\sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_{\alpha} l_{L}$$

$$j^{\mathrm{NC}}_{\alpha} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_{\alpha}\nu_{lL}$$

Neutrino mixing

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x)$$

U is 3×3 PMNS unitary mixing matrix, $\nu_i(x)$ is the field of neutrino (Dirac or Majorana) with mass m_i

Neutrino mixing was confirmed by the observation of neutrino oscillations in SK, SNO, KamLAND, Homestake, Gallex-GNO, SAGE, K2K and MINOS experiments

Nature of neutrinos with definite masses ν_i at present unknown

Nature of neutrinos is determined by the type of neutrino mass term.

For neutrinos, particles with equal to zero electric charge, two mass terms are possible

I. Majorana mass term

$$\mathcal{L}^{\mathsf{M}} = -\frac{1}{2}\,\bar{n}_L\,M^{\mathsf{M}}(n_L)^c + \text{h.c.} =$$
$$-\frac{1}{2}\,\bar{n}_L\,M^{\mathsf{M}}C(\bar{n}_L)^T + \text{h.c.}$$

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \end{pmatrix}$$

$\nu_{s_i L}$ are sterile fields

No global gauge invariance in the case of the Majorana mass term

No conserved total lepton number which could allow to distinguish neutrinos and antineutrinos

After the diagonalization of the mass term

$$\mathcal{L}^{\mathrm{M}} == -\frac{1}{2}\sum_i m_i\,\bar{\nu}_i\nu_i$$

$\nu_i^c(x) = \nu_i(x)$ (ν_i is Majorana field)

Neutrino mixing

$$\nu_{lL} = \sum_i U_{li}\;\nu_{iL},\; \nu_{sL} = \sum_i U_{si}\;\nu_{iL}$$

What is the origin of the violation of L ?

The most natural mechanism is see-saw

From experimental data: neutrino masses are many orders of magnitude smaller than masses of quarks and leptons

The see-saw mechanism connect smallness of neutrino masses with violation of the total lepton number by right- handed Majorana mass term at a large scale

Assume that sterile fields are right-handed fields

$$\nu_{s_1 L} = (\nu_{eR})^c;\ \nu_{s_2 L} = (\nu_{\mu R})^c;\ \nu_{s_3 L} = (\nu_{\tau R})^c$$

and mass matrix has the form

$$M^{\mathrm{M+D}} = \begin{pmatrix} 0 & m_{\mathrm{D}}^T \\ m_{\mathrm{D}} & M_R \end{pmatrix},$$

$$M_R \gg m_{\mathrm{D}}$$

$$U^T\, M^{\mathrm{M+D}}\, U = \begin{pmatrix} m_\nu & 0 \\ 0 & M_R \end{pmatrix},$$

$$m_\nu = -m_{\mathrm{D}}\, M_R^{-1}\, m_{\mathrm{D}}^T$$

Neutrino masses are much smaller than masses of quarks or leptons.

Major consequences

1. Neutrinos are Majorana particles with small masses

2. Heavy Majorana particles, see-saw partners of light Majorana neutrinos, exist. CP-violating decays of these particles in the early Universe is a possible source of the barion asymmetry of the Universe .

II. Dirac mass term

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l'l} \bar{\nu}_{l'L}\, M^{\mathrm{D}}_{l'l} \nu_{lL} + \mathrm{h.c.}. = -\sum_i m_i \bar{\nu}_i \nu_i$$

Neutrino mixing

$$\nu_{lL} = \sum_i U_{li}\ \nu_{iL}$$

In the case of D mass term invariance under global gauge transformation

$$\nu'_{lL}(x) = e^{i\alpha}\ \nu_{lL}(x); \nu'_{lR}(x) = e^{i\alpha}\ \nu_{lR}(x)$$

$$l'(x) = e^{i\alpha}\ l(x); \quad q'(x) = q(x)$$

$L = L_e + L_\mu + L_\tau$ is conserved

$\nu_i(x)$ is complex 4-component field of neutrinos ($L_{\nu_i} = 1$) and antineutrinos ($L_{\bar{\nu}_i} = -1$) with mass m_i

Smallness of neutrino masses can be explained, for example, by models with large extra dimensions

Neutrino masses are suppressed by a large volume factor

$m_i \sim kv\frac{M_x}{M_P}$; $M_x \sim (1 - 10)$ TeV; $M_P \simeq 10^{16}$ TeV

How to reveal the nature of neutrinos?

Neutrino oscillations are not sensitive to the neutrino nature. We need to search for processes in which total lepton number is violated

Important theorem. For the SM weak interaction, theories with massless Dirac and Majorana neutrinos are equivalent

Effects which allow to reveal the Majorana nature of neutrinos are proportional to $\frac{m_i^2}{Q^2} \ll 1$ and strongly suppressed

The neutrinoless double β-decay

$$(A, Z) \rightarrow (A, Z+2) + e + e$$

is the most sensitive to small Majorana masses process. Large targets , small backgrounds, high energy resolutions,...

Second order in G_F process with virtual neutrino

Neutrino propagator

$$\sum_{i,k} < 0|T(\nu_{iL}(x_1)\nu_{kL}^T(x_2))|0 > U_{ei}U_{ek} \simeq m_{ee} \, \frac{i}{(2\pi)^4}\int e^{-ip(x_1-x_2)}\frac{1}{p^2}d^4p\,\frac{1-\gamma_5}{2}C$$

$$m_{ee} = \sum_i U_{ei}^2 m_i$$

The half-life of $0\nu\beta\beta$-decay

$$\frac{1}{T_{1/2}^{0\nu}(A,Z)} = |m_{ee}|^2 \, |M(A,Z)|^2 \, G^{0\nu}(E_0, Z)$$

$M(A,Z)$ is nuclear matrix element,
$G^{0\nu}(E_0,Z)$ is known phase space factor

The most stringent lower bounds

Heidelberg-Moscow experiment

$$T^{0\,\nu}_{1/2}(^{76}\mathrm{Ge}) \geq 1.9 \cdot 10^{25}\mathrm{years} \quad (90\%\mathrm{CL})$$

Cryogenic CUORICINO experiment

$$T^{0\,\nu}_{1/2}(^{130}\mathrm{Te}) \geq 1.8 \cdot 10^{24}\,\mathrm{years} \quad (90\%\mathrm{CL})$$

Taking into account different calculations of the nuclear matrix elements

$$|m_{ee}| \leq (0.3 - 1.2)\ \mathrm{eV}$$

The aim of future experiments (CUORE, MAJORANA, EXO, SUPER NEMO, and others)

$$|m_{ee}| \simeq \text{a few } 10^{-2}\ \mathrm{eV}$$

$|m_{ee}|$ strongly depends on neutrino mass spectrum and the lightest neutrino mass

Three standard neutrino mass spectra

I. Hierarchy of neutrino masses

$$m_1 \ll m_2 \ll m_3$$

$$|m_{ee}| \simeq \left| \sin^2\theta_{12}\sqrt{\Delta m_{12}^2} + e^{2i\,\alpha_{23}} \sin^2\theta_{13}\sqrt{\Delta m_{23}^2} \right|$$

$\alpha_{23} = \alpha_3 - \alpha_2$ is the difference of Majorana CP phases.

Both terms are small. Compensation is possible

Upper bound

$$|m_{ee}| \leq 6.6 \cdot 10^{-3}\ \text{eV}$$

is smaller than the expected sensitivity of future experiments

II. Inverted hierarchy of neutrino masses

$$m_3 \ll m_1 < m_2$$

$$|m_{ee}| \simeq \sqrt{|\Delta m^2_{13}|}\ (1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha_{12})^{\frac{1}{2}}$$

The only unknown parameter is $\sin^2\alpha_{12}$.

$$0.9 \cdot 10^{-2} \leq |m_{ee}| \leq 5.8 \cdot 10^{-2}\ \text{eV}$$

in the range of the anticipated sensitivities of the future experiments

III. Quasi-degenerate neutrino mass spectrum

If the lightest neutrino mass $m_0 \gg \sqrt{|\Delta m^2_{23}|}$

$$m_1 \simeq m_2 \simeq m_3$$

$$|m_{ee}| \simeq m_0\,(1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha_{12})^{\frac{1}{2}}$$

Large $|m_{ee}|$ are expected

In order to determine $|m_{ee}|$ from experimental data nuclear matrix elements must be known.

Complicated nuclear problem

Different calculations of NME for the same nuclear transition differ by factor 2-3 and more

If $0\nu\beta\beta$-decay of *different nuclei* is observed ratio of NME can be measured

This can be used as a model independent test of different calculations

Are massive neutrinos and antineutrinos identical or different ?

This problem, which has been put forward by E. Majorana about 70 years ago, is the most fundamental problem of the modern neutrino physics

Without its solution the origin of the small neutrino masses and neutrino mixing can not be revealed