Neutrinos

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neutrino ≡ fermion without strong or electromagntic interactions
(active/sterile neutrino ≡ weak-SU(2) doublet/singlet)

1. what we know: weak interactions, masses and mixing angles

2. how we know : oscillations in vacuum and matter

3. other possible observables
   - $[m^2]_{ee}$ from β decay
   - $[m]_{ee}$ from neutrinoless double beta decay $(0\nu 2\beta)$
   - ("non-standard" interactions, magnetic moments,...)

4. why are neutrino massses small : models
   - suppressed by a large mass scale and small couplings: the seesaw
   - suppressed by small couplings and loops: R_p violation in SUSY

Plots from Strumia + Vissani: hep-ph/0606054

An overview of the history of neutrinos

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- inflation (produce large scale CMB fluctuations) (?could be driven by the sneutrino?)
- baryogenesis (excess of matter over anti-matter in room/Universe) ?leptogenesis in the seesaw?
- relic density of (cold) Dark Matter (?could be (heavy) neutrinos too??? Shaposhnikov et al)
- Big Bang Nucleosynthesis (produce $H, D,^3 He,^4 He,^7 Li$ abundances at $T \sim$ MeV))
  $\Leftrightarrow$ 3 species of relativistic ν in the thermal soup
- decoupling of photons — $e + p \rightarrow H$ (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- for 10^{10} yrs —stars are born, radiate (γ, ν), and die
- supernovae explode (?grace aux ν?) spreading heavy elements
- 1930: Pauli hypothesises the "neutrino", to conserve E in $n \rightarrow p + e(+\nu)$
- 1953 Reines and Cowan: neutrino CC interactions in detector near a reactor
- invention of the Standard Model
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helicity, chirality and all that...

ψ a Dirac spinor, 4 degrees of freedom labelled by $\{\pm E, \pm s\}$.

Chiral decomposition of $\psi = \psi_L + \psi_R$,

$$\psi_L = P_L \psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \qquad , \qquad \psi_R = P_R \psi \quad \text{avec} \quad P_R = \frac{(1+\gamma_5)}{2}$$

not an observable; property of the field ($P_{L,R}$ simple to calculate with :))
independent of reference frame—but becomes helicity in the relativistic limit.
Standard Model is chiral = different gauge interactions for LH, RH fermions.

define **helicity** as $\pm \hat{s} \cdot \hat{k} = \pm 1/2$, for particle of 4-momentum $(k_0, \vec{k})$. Observable. Ugly operator.

Gauge kinetic terms for chiral fermions : $\overline{\psi}\,\gamma^\mu D_\mu \psi = \overline{\psi_L}\,\gamma^\mu D_\mu \psi_L + \overline{\psi_R}\,\gamma^\mu D_\mu \psi_R$,

but not the Dirac mass: $m\overline{\psi}\,\psi = m\overline{\psi_L}\,\psi_R + m\overline{\psi_R}\,\psi_L$

Careful about notation: $\overline{(\psi_R)} = (\overline{\psi})_L \neq (\overline{\psi})_R$

Neutrinos in the Standard Model: weak interactions

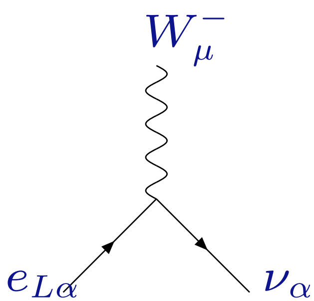
3 generations of lepton doublets in the SM: ν_R not required by observation (gauge singlets, m_ν not detected)

$$\ell_{\alpha L} \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \;,\; \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \;,\; \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

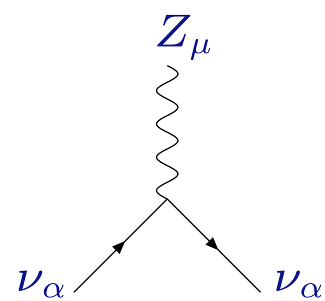
In the Lagrangian: $i\,\overline{\ell_L}^T \gamma^\mu \mathbf{D}_\mu \ell_L = i\,\overline{\ell_L}^T \gamma^\mu (\partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + ig' Y_\ell B_\mu)\,\ell_L$, $Y_\ell = \frac{1}{2}$ gives

$$-i\,\frac{g}{2} \begin{pmatrix} \overline{\nu_L} \\ \overline{e_L} \end{pmatrix}^T \gamma^\mu \left(\begin{pmatrix} W^0_\mu & \sqrt{2}\,W^+_\mu \\ \sqrt{2}\,W^-_\mu & -W^0_\mu \end{pmatrix} - \tan\theta_W B_\mu \right) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

where $\tan\theta_W = g'/g$, $A_\mu \equiv \cos\theta_W\, B_\mu + \sin\theta_W W^3_\mu$ and $Z_\mu \equiv -\sin\theta_W\, B_\mu + \cos\theta_W W^3_\mu$
Get:



$$-\,i\,\frac{g}{\sqrt{2}}\,\gamma_\mu \left(\frac{1-\gamma^5}{2}\right)$$



$$-\,i\,\frac{g}{2\cos\theta_W}\,\gamma_\mu \left(\frac{1-\gamma^5}{2}\right)$$

CC verified by lepton universality tests ($\tau \to \nu\bar{\nu}\mu$, $\tau \to \nu\bar{\nu}e$ ...),
NC: invisible decay of the Z: decays to 2.994 ± 0.012 invisible chiral fermions (precision fit)

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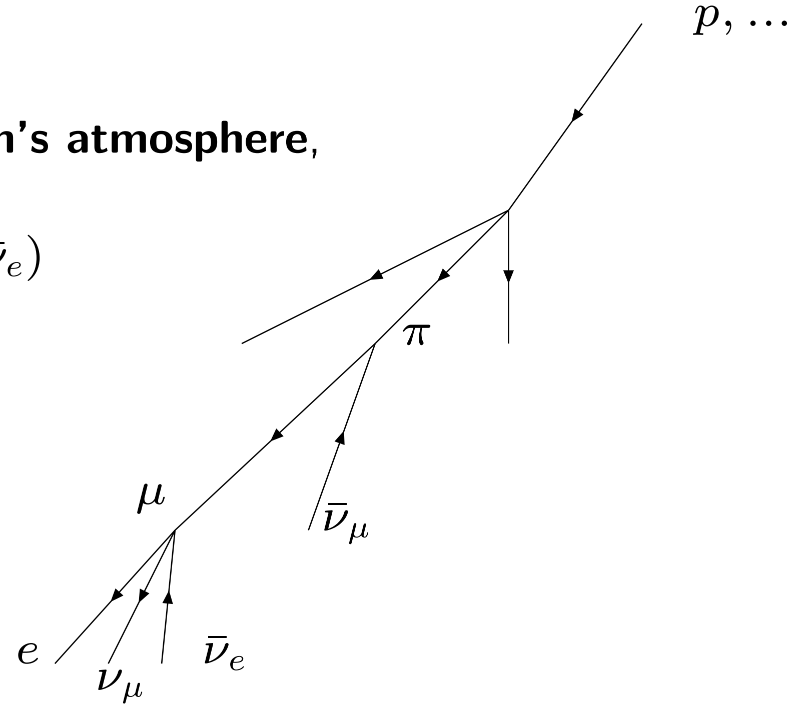
neutrino deficits—something other than weak interactions required?

Historical "problems":

1. **the sun** produces energy by a network of nuclear reactions, which should produce ν_e (lines and continuum) which escape. The energy diffuses to the surface. Observed ν_e flux $\sim .3 \to .5$ expected from solar energy output. Flux in $\sum$ flavours $\sim$ expected.
   $\Rightarrow$ new ν physics, that changes ν flavour on way out of sun:
   - magnetic moments?
   - wierd new interactions?
   - masses (and mixing angles)
   - ...
2. deficit of ν_μ arriving from **the earth's atmosphere**, produced in cosmic ray interactions:
   expect $N(\nu_\mu + \bar{\nu}_\mu) \simeq 2N(\nu_e + \bar{\nu}_e)$
   see deficit of $\nu_\mu, \bar{\nu}_\mu$ from above.



To write a mass for ν_L... Dirac or Majorana

Work in effective theory of SM below m_W. SU(2) (spontaneously) broken, so a mass term for ν_L is allowed. It must be Lorentz invariant. Allowed mass term, four-component fermion ψ:

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$$

1. **Dirac masss term**:
   SM has only ν_L, 2 dof chiral fermion $\Rightarrow$ introduce another 2 dof chiral fermion ν_R
   fermion number conserving mass term like all other SM fermions:

$$m\overline{\nu_L}\,\nu_R \;+ m\overline{\nu_R}\,\nu_L$$

2. **Majorana mass term**: the charge conjugate of ν_L is right-handed !
   $\Rightarrow$ can write a fermion number non-conserving mass term using just 2 dof of ν_L.
   No new fields, but lepton number violating mass.

Majorana mass term: the charge conjugate of ν_L is right-handed

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \quad \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi^c = -i\gamma_0\gamma_2\bar{\psi}^T = -i\gamma_0\gamma_2\gamma_0\psi^* = i\gamma_2^*\psi^* = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \psi_L^* \end{pmatrix} \\ \begin{pmatrix} \psi_R^* \end{pmatrix} \end{pmatrix}$$

$$(\nu_L)^c = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \end{pmatrix} \\ \begin{pmatrix} -i\sigma_2\nu_L^* \end{pmatrix} \end{pmatrix}$$

Allowed mass term, four-component fermion ψ: $m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R + m\overline{\psi_R}\,\psi_L$

$\Rightarrow$ with *only* the 2 dof of a chiral fermion, can write mass term:

$$m[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] = m[(\nu_L)^\dagger\gamma_0(\nu_L)^c + ((\nu_L)^c)^\dagger\gamma_0\nu_L] = -im[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L]$$

Mass matrices: Dirac or Majorana

1. **Dirac mass matrix** (Add 3 ν_R to the SM): arbitrary 3 $\times$ 3 matrix (like other SM Yukawa couplings). In charged lepton mass eigenstate basis for $\nu_L \equiv$ " flavour basis" (indices $\alpha, \beta \ldots$), diagonalise with independent transformations on SU(2) doublet/singlet indices:

$$\overline{\nu_L}_\alpha [m]_{ab} \nu_{Rb} + \overline{\nu_R}_b \, [m]^*_{b\alpha} \nu_{L\alpha} = \overline{\nu_L}_\alpha [V_L^* V_L^T m V_R^* V_R^T]_{ab} \nu_{Rb} + h.c = \overline{\nu_L}_j \, m_j \nu_{Rj} + h.c.$$

   $mm^\dagger$ hermitian, obtain V_L from $V_L^T mm^\dagger V_L^* = D_m^2$. (real eigenvals for hermitian matrices).

2. **Majorana mass matrix**:can write a majorana mass term (one generation) as

$$\frac{1}{2} m [\overline{\nu_L} (\nu_L)^c + \overline{(\nu_L)^c} \nu_L] = \frac{-im}{2} [\nu_L^\dagger \sigma_2 \nu_L^* + \nu_L^T \sigma_2 \nu_L] = \frac{m}{2} \nu_L \nu_L + h.c.$$

   With multiple generations, $[m]_{\alpha\beta}$ will be a *symmetric* matrix:

$$\frac{1}{2} \nu_{L\alpha} [m]_{\alpha\beta} \nu_{L\beta} + h.c. = \frac{1}{2} \nu_{L\alpha} [U^* U^T m U U^\dagger]_{\alpha\beta} \nu_{L\beta} + h.c. = \frac{1}{2} \nu_{Li} m_i \nu_{Li} + h.c.$$

   Yes! fermion fields anti-commute. But for ρ, σ spinor indices, $\nu^\rho_{Li} \varepsilon_{\rho\sigma} \nu^\sigma_{Lj} = -\nu^\sigma_{Lj} \varepsilon_{\rho\sigma} \nu^\rho_{Li} = \nu^\sigma_{Lj} \varepsilon_{\sigma\rho} \nu^\rho_{Li}$
   $mm^\dagger$ hermitian, obtain U from $U^T mm^\dagger U^* = D_m^2$.
   U called PMNS matrix (for Pontecorvo, Maki, Nakagawa and Sakata) : U_{PMNS}.

counting mixing matrix phases: 1 for Dirac, 3 for Majorana

- A 3×3 complex matrix has 18 real parameters
- the unitarity condition $VV^\dagger = 1$, $UU^\dagger = 1$ reduces this to 9, which can be taken as 3 angles and 6 phases.
- five of those phases are relative phases between the fields $e, \mu, \tau, \nu_1, \nu_2$ and ν_3
- ...so if we are free to choose the phases of all the LH fermions, we are left with one phase in the mixing matrix. This is the case for a dirac mass matrix ($e.g.$ quarks), where any potential phase on the masses could be absorbed by the RH fermion fields. Also the case in oscillations, where appears $mm^\dagger$.
- if ν_L have Majorana masses, between themselves and their antiparticle, it is the LH neutrino field which must absorb the phase off the Majorana mass. So in physical processes where the Majorana mass appears linearly (not as mm^*; eg $0\nu2\beta$), one can choose the phase such that the mass is real—in which case one can remove one less phase from MNS, or one can keep MNS with one phase, and allow complex masses.
- it is always possible to remove the phase from one majorana mass, by using the global overall phase of all the leptons (the sixth phase of $e, \mu, \tau, \nu_1, \nu_2$ and ν_3, which we could not use to remove phases from the lepton number conserving PMNS matrix). So in three generations, there are possibly two complex majorana neutrino masses, so two “Majorana” phases in addition to the “Dirac” phase δ of PMNS.

So,finally...observables

Majorana mixing matrix is U. Dirac neutrino mixing matrix is V:

$$U = V \cdot diag\{e^{-i\phi/2}, e^{-i\phi'/2}, 1\}$$

$$V_{\alpha i} = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix}.$$

$$\theta_{23} \simeq .7 \pm .2 \simeq \pi/4 \quad \theta_{12} \simeq .6 \pm .1 \simeq \pi/6 \quad \theta_{13} \leq .2$$

δ, ϕ, ϕ' unknown —CPV in lepton sector not observed (yet).

Three masses: in hierarchical pattern $m_1 < m_2 < m_3$, or inverse hierarchical $m_2 > m_1 > m_3$:

$$\Delta m^2_{atm} = m_3^2 - m_2^2 \simeq 2.6 \times 10^{-3}\mathrm{eV}^2 \qquad \Delta m^2_{\odot} = m_2^2 - m_1^2 \simeq 7.9 \times 10^{-5}\mathrm{eV}^2$$

with overall scale bounded above:

$$\sum_i |m_i| \leq .37 - 2 \text{ eV} \ \text{ LSS and CMB} \qquad [m^2]_{ee} \lesssim 2 \text{ eV} \quad \beta\ decay,\ Mainz$$

$$|[m]_{ee}| \begin{cases} \lesssim .35 \text{ eV} & 0\nu 2\beta, HM \\ \simeq .3 \text{ eV} & 0\nu 2\beta, KK \end{cases}$$

Tangent—diagonalising a Majorana mass matrix

To find eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$ (recall from high-school)

$$A\vec{v}_i = a_i\vec{v}_i$$

For Majorana matrix ?

Tangent—diagonalising a Majorana mass matrix... to not waste two weeks :

To find eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$

$$A\vec{v}_i = a_i\vec{v}_i$$

For Majorana matrix :

$$A\vec{v}_i = a_i\vec{v}_i^*$$

Tangent—diagonalising a Majorana mass matrix

To find eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$

$$A\vec{v}_i = a_i\vec{v}_i$$

For Majorana matrix :

$$A\vec{u}_i = a_i\vec{u}_i^*$$

hermitian : $V^\dagger AV = D_A = diag\{a_1, ...a_n\}$ (V unitary)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \left[\begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix} \right] = \left[\begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix} \right] \begin{bmatrix} a_1 & & \\ & \dots & \\ & & a_n \end{bmatrix}$$

majorana : $U^T AU = D_A \Rightarrow AU = U^* D_A$ (U unitary $UU^\dagger = 1$)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \left[\begin{pmatrix} \\ \vec{u}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_3 \\ \\ \end{pmatrix} \right] = \left[\begin{pmatrix} \\ \vec{u}_1^* \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_2^* \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_3^* \\ \\ \end{pmatrix} \right] \begin{bmatrix} a_1 & & \\ & \dots & \\ & & a_n \end{bmatrix}$$

And another curiosity about diagonalisation...

A hermitian matrix with degenerate eigenvalues is always diagonal. Not true for majorana mass matrix (due to phases on masses): its not the same to diagonalise $M^\dagger M = V^\dagger D_M^2 V$, or $M = U^T D_M U$ (for degenerate eigenvalues) Ex:

$$M = \left[\begin{array}{cc} 0 & M_1 e^{i\phi} \\ M_1 e^{i\phi} & 0 \end{array} \right] , \qquad M^\dagger M = \left[\begin{array}{cc} M_1^2 & 0 \\ 0 & M_1^2 \end{array} \right] \qquad M_1 \in \Re$$

oscillations in vaccuum (sloppy field theory)

Take a toy model with two generations: $\nu_\alpha = U_{\alpha i}\nu_i$

$$\left(\begin{array}{c} \nu_e \\ \nu_\mu \end{array}\right) = \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array}\right) \cdot \left(\begin{array}{c} \nu_1 \\ \nu_2 \end{array}\right).$$

Suppose relativistic neutrinos, produced in muon decay at $t = 0$. Amplitude to produce mass eigenstate i

$$U_{\mu i}$$

Neutrinos travel distance $L =$ time τ to a detector. Propagator in position space for(scalar) mass eigenstate:

$$G[(0,0);(L,\tau)] \propto \int \frac{d^3p}{(2\pi)^3} e^{i(E\tau - pL)}$$

Describe ν_i by a wave packet peaked at $\sim (E, \vec{k})$. $\simeq$ fixed 4-momentum and detector/source positions... so amplitude

$$\mathcal{A}_{\mu\alpha} \propto \sum_j U_{\mu j} \times e^{-i(E_j\tau - k_jL)} \times U^*_{\alpha j}$$

where, at detector, produce an e or a μ by CC scattering ($\alpha = e$ or μ)

$$\mathcal{A}_{\mu\alpha} \propto \sum_j U_{\mu j} \times e^{-i(E_j \tau - k_j L)} \times U^*_{\alpha j}$$

$m_j \ll E, p \Rightarrow L \simeq \tau$, so

$$-i(E_j\tau - p_j L) \simeq -i(E_j - p_j)L = -i\frac{E_j^2 - p_j^2}{E_j + p_j}L \simeq -i\frac{m_j^2}{2E}L$$

$$\mathcal{P}_{\mu\alpha} = |\mathcal{A}_{\mu\alpha}|^2 = |\sum_j U_{\mu j} e^{-im_j^2 L/(2E)} U^*_{\alpha j}|^2$$

In 2 generation case:

$$\begin{aligned}
\mathcal{P}_{\mu\to e}(\tau) &= \left|\sin\theta\,\cos\theta\left(e^{-im_1^2\tau/2E} - e^{-im_2^2\tau/2E}\right)\right|^2 \\
&= \sin^2(2\theta)\sin^2\left(L\,\frac{\Delta m^2}{4E}\right) \\
\mathcal{P}_{\mu\to\mu}(t) &= 1 - \sin^2(2\theta)\sin^2\left(L\,\frac{\Delta m^2}{4E}\right) = 1 - \sin^2(2\theta)\sin^2\left(1.27\frac{L}{km}\,\frac{\Delta m^2}{\mathrm{eV}^2}\frac{\mathrm{GeV}}{4E}\right)
\end{aligned}$$

E is ν energy, L is distance from source- detector.

Comment : quarks are different—how?

Produce at source a flavour eigenstate wave packet (superpositions of mass eigenstates)
Mass eigenstates remain “superposed” over $L \sim (E/GeV)(eV^2/\Delta m^2)$ km $\Leftrightarrow$ averaging

$\mathcal{P}_{\alpha\to\alpha}(t) = 1 - \sin^2(2\theta)\sin^2\left(L\,\frac{\Delta m^2}{4E}\right)$

over E_i of eigenstates has negligeable effect.

For $L \gg E/\Delta m^2$, decoherence of wavepacket

$$\begin{aligned}\mathcal{P}_{\alpha\to\alpha}(t) &= 1 - \frac{1}{2}\sin^2(2\theta) = 1 - 2\sin^2(\theta)\cos^2(\theta) = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2(\theta)\cos \\ &= \sin^4\theta + \cos^4\theta = \left(|\langle\alpha|1\rangle|^2\right)^2 + \left(|\langle\alpha|2\rangle|^2\right)^2\end{aligned}$$

$\leftrightarrow$ propagating mass eigenstates

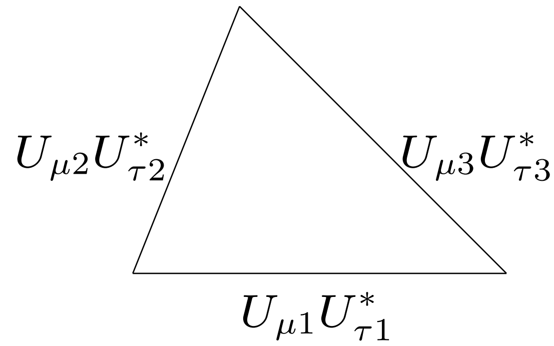
1. L is a classical distance for neutrinos ($\ll 10^{-6}$ cm for quarks)
2. ν can travel distance L before interacting (quarks have strong/electromagntic interactions)

Comment : why two (not three) flavour analysis is relevant?

Amplitude to oscillate from flavour α to β over distance L:

$$\mathcal{A}_{\alpha\beta}(L) = U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2}e^{-i(m_2^2-m_1^2)\tau/(2E)} + U_{\alpha 3}U^*_{\beta 3}e^{-i(m_3^2-m_1^2)\tau/(2E)}$$

at $L=0$, unitarity: $\Rightarrow \mathcal{A}_{\alpha\beta} = 1$ for $\alpha = \beta$
$\mathcal{A}_{\alpha\beta} = 0$ for $\alpha \neq \beta$
$\Leftrightarrow$ unitarity triangle(in complex plane)



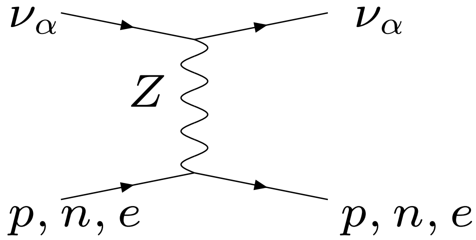
At $L = \tau \neq 0$, two of the vectors rotate in the complex plane, with frequencies $(m_j^2 - m_1^2)/2E$
(oscillations $\leftrightarrow$ time-dependent non-unitarity)

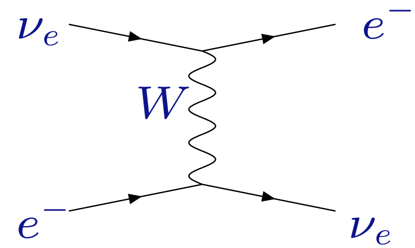
- "Atmospheric" neutrinos (oscillations via Δm^2_{31}): $U_{\mu 3}U^*_{\tau 3}$ oscillates on timescale $\tau = L \sim (m_3^2 - m_1^2)/E$, but $U_{\mu 2}U^*_{\tau 2} \sim$ stationary.
- "Solar" neutrinos (survival of ν_e over $L \leftrightarrow (m_2^2 - m_1^2)/2E$): 2 ν approx works because θ_{13} is small ($U_{e3} = sin\theta_{13}$):

$$\begin{aligned}\mathcal{A}_{ee} &= |U_{e1}|^2 + |U_{e2}|^2 e^{-i(m_2^2-m_1^2)\tau/(2E)} + |U_{e3}|^2 e^{-i(m_3^2-m_1^2)L/(2E)} \\ &\simeq |U_{e1}|^2 + |U_{e2}|^2 e^{-i(m_2^2-m_1^2)\tau/(2E)}\end{aligned}$$

Oscillations in matter

Coherent forward scattering of ν in matter give extra contribution to the Hamiltonian:





NC are same for all generations of ν (add unit matrix to H—irrelevant for oscillations)
CC for ν_e only (no μ or τ in the matter)
In two generations: write $H_{\rm mat}$ in flavour basis $(\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2})$:

$$H_{\rm mat} \quad = \quad \ldots + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \Delta m^2/(2E) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} V_e & 0 \\ 0 & 0 \end{bmatrix}$$

$$V_e = \sqrt{2} G_F n_e \simeq 8\,\mathrm{eV} \times Y_e \frac{\rho}{10^{14} g/cm^3} \;\;,\;\; Y_e = \frac{n_e}{n_n + n_p} \;\;,\;\; \rho = \begin{cases} 10 g/cm^3 & \text{earth core} \\ 100 g/cm^3 & \text{solar core} \\ 10^{14} g/cm^3 & \text{supernova} \end{cases}$$

NB: “large” V_e suppresses oscillations; its flavour diagonal

Oscillations in matter — ctd

In two generations: write $H_{\rm mat}$ in flavour basis $(\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2})$:

$$H_{\rm mat} \quad = \quad \ldots + \left[\begin{array}{cc} -\frac{\Delta m^2}{4E}\cos 2\theta + V_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{array} \right]$$

Define $U^T_{mat} H_{mat} U^*_{mat} =$ diagonal... so the mixing angle in matter is:

$$\tan(2\theta_{\rm mat}) = \frac{\Delta m^2 \sin(2\theta_{vac})}{2EV_e - \Delta m^2 \cos(2\theta_{vac})} \stackrel{2EV_e \to \Delta m^2 c2\theta}{\longrightarrow} \text{large}$$

- for $2EV_e \ll \Delta m^2 \cos(2\theta_{vac})$, matter effects are negligeable
  (in the sun $E <$ few MeV)
- $\theta_{mat} \longrightarrow \pi/4$ ("resonance") at $2EV_e = \Delta m^2 \cos(2\theta_{vac})$
  (V of opposite sign for anti-neutrinos. *eg*, coming out of SN, resonance for ν, or $\bar{\nu}$)
- for $2EV \gg \Delta m^2 \cos(2\theta_{vac})$, mixing angle is suppressed ($\nu_e \sim$ mass eigenstate)

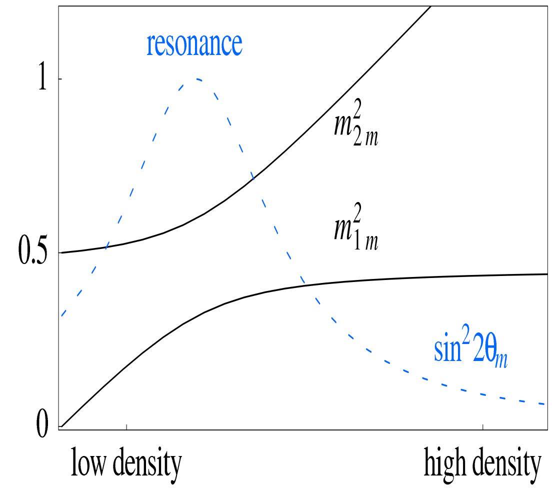
matter of varying density

Recall $V_e \simeq 8Y_e\frac{\rho}{10^{14}g/cm^3}\ \mathrm{eV}$. For varying $\rho(r)$, have t-dep Hamiltonian:

$$\begin{bmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{bmatrix}$$

Matter mixing angle θ_{mat} time dependent...two limits:

1. **adiabatic case**: neglect $\dot{\theta}_{mat}$,
   instantaneous mass eigenstates ν_i do not mix.
   NB: $\mathcal{P}_{ee} \longrightarrow 0$ possible
2. **non-adiabatic** $\Leftrightarrow$ level hopping

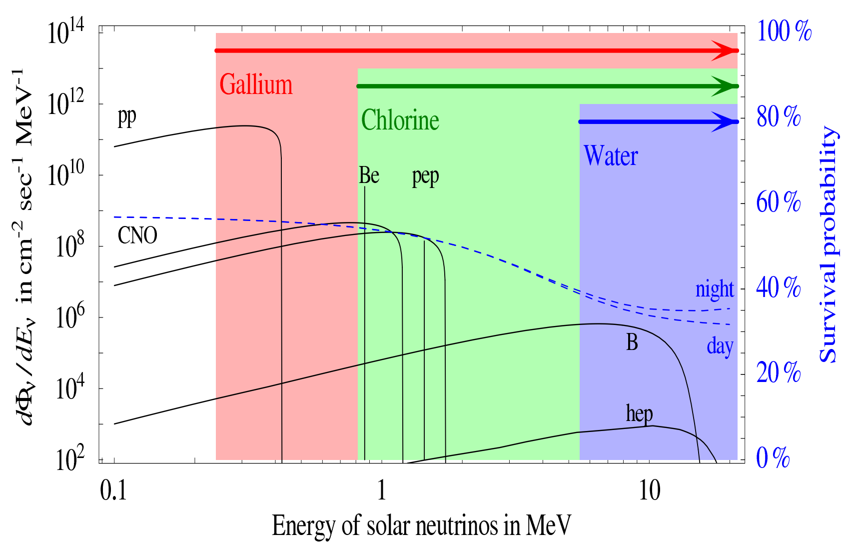


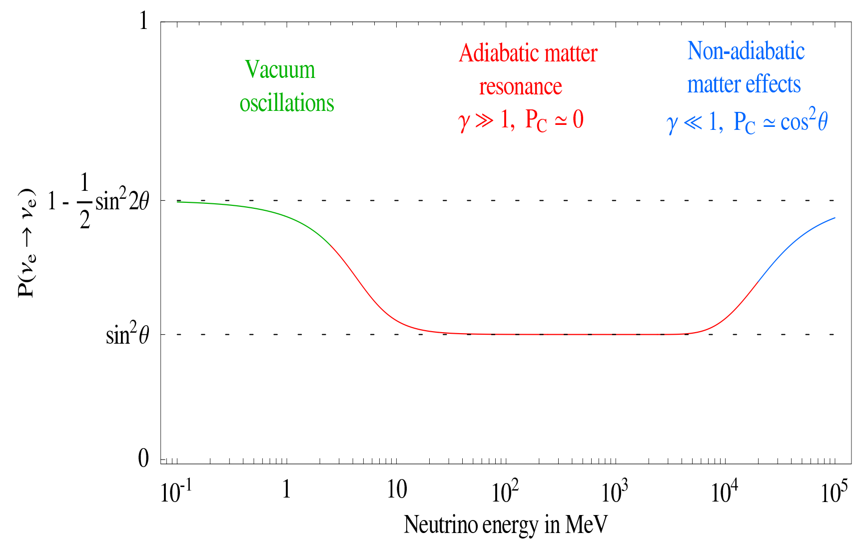
The sun and the bathtub

- produce ν_e at the core of the sun: .4 MeV $\lesssim E \lesssim$ 10 MeV.
- matter oscillation length $<$ vacuum oscillation length $\sim 10\frac{E}{MeV}\frac{10^{-4}eV^2}{\Delta m^2} \ll R_{sun}$. So oscillations decohere $\Leftrightarrow$ propagate mass eigenstates.
- Matter effects negligeable for $E \lesssim$ few MeV:

$$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta_{vac} > \frac{1}{2}$$

- adiabatic matter effects for $E \gtrsim$ few MeV, allows $P_{ee} < .5$.





Summary

There are three "left-handed" massless neutrinos, with charged current and neutral current weak interactions.

That's it in the Standard Model, so in the SM, three lepton flavours L_e, L_μ, L_τ are conserved.

Beyond the Standard Model physics! Neutrinos have (very small $\lesssim$ eV) masses $\Leftrightarrow$ mixing matrix U_{PMNS}. Two mixing angles are large ($\simeq \pi/4, \pi/6$) , one unmeasured ($\lesssim .2$).

Masses can be

- Dirac — add light sterile fermions (right-handed neutrinos) to the SM. Lepton number conserved. One phase in U.
- Majorana — lepton number non-conserving masses. Three phases in U_{PMNS}.

We know about the mixing angles and mass differences because we see $oscillations$: flavour change in propagation. In matter, there is contribution to ee element of "mass matrix" from interactions with matter.