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An introduction to Neutrino physics and oscillations

Neutrino masses and mixing in the Standard Model

Interaction Lagrangians and currents

The Standard Model (SM) Lagrangians for charged and neutral current neutrino interactions are assumed to be

$$\mathcal{L}_I^{\mathsf{CC}}(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{\mathsf{CC}}(x) W^\alpha(x) + \mathsf{H.c.} \quad \text{and} \quad \mathcal{L}_I^{\mathsf{NC}}(x) = -\frac{g}{2\cos\theta_{\mathsf{W}}} j_\alpha^{\mathsf{NC}}(x) Z^\alpha(x).$$

Here g is the $SU(2)$ (electro-weak) gauge coupling constant

$$g^2 = 4\sqrt{2} M_W^2 G_F, \quad g\sin\theta_{\mathsf{W}} = |e|$$

and θ_{W} is the weak mixing (Weinberg) angle ($\sin^2\theta_{\mathsf{W}}(M_Z) = 0.23120$).
The leptonic charged current and neutrino neutral current are given by the expressions:

$$j_\alpha^{\mathsf{CC}}(x) = 2 \sum_{\ell=e,\mu,\tau,\ldots} \overline{\nu}_{\ell,L}(x)\gamma_\alpha \ell_L(x) \quad \text{and} \quad j_\alpha^{\mathsf{NC}}(x) = \sum_{\ell=e,\mu,\tau,\ldots} \overline{\nu}_{\ell,L}(x)\gamma_\alpha \nu_{\ell,L}(x).$$

The currents may include (yet unknown) heavy neutrinos and corresponding charged leptons. The left- and right-handed fermion fields are defined as usually:

$$\nu_{\ell,L/R}(x) = \left(\frac{1\pm\gamma_5}{2}\right)\nu_\ell(x) \quad \text{and} \quad \ell_{L/R}(x) = \left(\frac{1\pm\gamma_5}{2}\right)\ell(x).$$

Note that the kinetic term of the Lagrangian includes both L and R handed neutrinos and moreover, it can include other sterile neutrinos:

$$\begin{aligned}\mathcal{L}_0 =& \frac{i}{2}\left[\overline{\nu}(x)\gamma^\alpha\partial_\alpha\nu(x) - \partial_\alpha\overline{\nu}(x)\gamma^\alpha\nu(x)\right] \equiv \frac{i}{2}\overline{\nu}(x)\overleftrightarrow{\boldsymbol{\partial}}\nu(x)\\ =& \frac{i}{2}\left[\overline{\nu}_L(x)\overleftrightarrow{\boldsymbol{\partial}}\nu_L(x) + \overline{\nu}_R(x)\overleftrightarrow{\boldsymbol{\partial}}\nu_R(x)\right],\end{aligned}$$

$$\boldsymbol{\nu}(x) = \boldsymbol{\nu}_L(x) + \boldsymbol{\nu}_R(x) = \begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\\ \nu_\tau(x)\\ \cdot\\ \cdot\\ \cdot\end{pmatrix}, \quad \boldsymbol{\nu}_{L/R}(x) = \begin{pmatrix}\nu_{e,L/R}(x)\\ \nu_{\mu,L/R}(x)\\ \nu_{\tau,L/R}(x)\\ \cdot\\ \cdot\\ \cdot\end{pmatrix} = \frac{1\pm\gamma_5}{2}\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\\ \nu_\tau(x)\\ \cdot\\ \cdot\\ \cdot\end{pmatrix}.$$

Neutrino chirality: $\gamma_5\nu_L = -\nu_L$ and $\gamma_5\nu_R = +\nu_R$.

The Lagrangian of the theory with massless neutrinos is invariant with respect to the global gauge transformations $\nu_\ell(x) \to e^{i\Lambda_\ell}\nu_\ell(x)$, $\ell(x) \to e^{i\Lambda_\ell}\ell(x)$ with $\Lambda_\ell = \text{const}$. This leads to conservation of the individual lepton flavor numbers L_ℓ (electron, muon, tauon,...). It is not the case for massive neutrinos.

There are two types of possible neutrino mass terms: Dirac and Majorana.

Dirac neutrinos

The Dirac mass term has the form

$$\mathcal{L}_{\mathsf{D}}(x) = -\overline{\boldsymbol{\nu}}_R(x)\mathbf{M}_{\mathsf{D}}\boldsymbol{\nu}_L(x) + \text{H.c.},$$

where $\mathbf{M}_{\mathsf{D}}$ is a $N \times N$ complex *nondiagonal* matrix. In general, $N \geq 3$ that is the column $\boldsymbol{\nu}_L$ may include the heavy *active* neutrino fields as well as *sterile* neutrino fields which do not enter into the standard charged and neutral currents.

An arbitrary complex matrix can be diagonalized by means of an appropriate *biunitary* transformation. One has

$$\mathbf{M}_{\mathsf{D}} = \tilde{\mathbf{V}}\mathbf{m}\mathbf{V}^\dagger, \quad \mathbf{m} = ||m_{kl}|| = ||m_k\delta_{kl}||,$$

where $\mathbf{V}$ and $\tilde{\mathbf{V}}$ are unitary matrices and $m_k \geq 0$. Therefore

$$\mathcal{L}_{\mathsf{D}}(x) = -\overline{\boldsymbol{\nu}'}_R(x)\mathbf{m}\boldsymbol{\nu}'_L(x) + \text{H.c.} = -\overline{\boldsymbol{\nu}'}(x)\mathbf{m}\boldsymbol{\nu}'(x) = -\sum_{k=1}^{N} m_k\overline{\nu}_k(x)\nu_k(x),$$

$$\boldsymbol{\nu}'_L(x) = \mathbf{V}^\dagger\boldsymbol{\nu}_L(x), \quad \boldsymbol{\nu}'_R(x) = \tilde{\mathbf{V}}^\dagger\boldsymbol{\nu}_R(x), \quad \boldsymbol{\nu}'(x) = (\nu_1, \nu_2, \ldots, \nu_N)^T.$$

It is easy to prove that the kinetic term in the neutrino Lagrangian is transformed to

$$\mathcal{L}_0 = \frac{i}{2}\overline{\boldsymbol{\nu}'}(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}'(x) = \frac{i}{2}\sum_k\overline{\nu}_k(x)\overleftrightarrow{\boldsymbol{\partial}}\nu_k(x).$$

Hence, one can conclude that $\nu_k(x)$ is the field of a Dirac neutrino with the mass m_k and the flavor LH neutrino fields $\nu_{\ell,L}(x)$ present in the standard weak lepton currents are linear combinations of the LH components of the fields of neutrinos with definite masses:

$$\boldsymbol{\nu}_L = \mathbf{V}\boldsymbol{\nu}'_L \quad \text{or} \quad \nu_{\ell,L} = \sum_k V_{\ell k}\nu_{k,L}.$$

The matrix $\mathbf{V}$ is sometimes referred to as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino (vacuum) mixing matrix.[a]

It can be proved that the Lagrangian of the theory with the Dirac mass term is invariant with respect to the global gauge transformation

$$\nu_k(x) \to e^{i\Lambda}\nu_k(x), \quad \ell(x) \to e^{i\Lambda}\ell(x), \quad \Lambda = \text{const}.$$

This means that the lepton charge

$$L = \sum_{\ell=e,\mu,\tau,\ldots} L_\ell$$

common to all charged leptons and all neutrinos ν_k is conserved. However

the individual lepton flavor numbers L_ℓ are no longer conserved.

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[a] Of course it is not the same as the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. It seems however that the PMNS and CKM matrices are, in a sense, *complementary* (see below).

Parametrization of mixing matrix for Dirac neutrinos

It is well known that a complex $n \times n$ unitary matrix depends on n^2 *real* parameters. The classical result by Murnagham[a] states that the matrices from the unitary group $U(n)$ are products of a diagonal phase matrix

$$\boldsymbol{\Gamma} = \text{diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, \ldots, e^{i\alpha_n}\right),$$

containing n phases α_k, and $n(n-1)/2$ matrices whose main building blocks have the form

$$\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ -\sin\theta\, e^{+i\phi} & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{+i\phi} \end{pmatrix} \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{Euler rotation}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{pmatrix}.$$

Therefore any $n \times n$ unitary matrix can be parametrized in terms of

$n(n-1)/2$ "angles" (taking values within $[0, \pi/2]$)

and

$n(n+1)/2$ "phases" (taking values within $[0, 2\pi)$).

The usual parametrization of both the CKM and PMNS matrices is of this type.

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[a] F. D. Murnagham, "The unitary and rotation groups," Washington, DC: Sparta Books (1962).

One can reduce the number of the phases further by taking into account that the Lagrangian with the Dirac mass term is invariant with respect to the transformation

$$\ell \mapsto e^{ia_\ell}\ell, \quad \nu_k \mapsto e^{ib_\ell}\nu_k, \quad V_{\ell k} \mapsto e^{i(b_k - a_\ell)}V_{\ell k},$$

and to the global gauge transformation

$$\ell \mapsto e^{i\Lambda}\ell, \quad \nu_k \mapsto e^{i\Lambda}\nu_k.$$

Therefore $2N-1$ phases are unphysical and the number of physical (*Dirac*) phases is

$$n_\mathsf{D} = \frac{N(N+1)}{2} - (2N-1) = \frac{N^2-3N+2}{2} = \frac{(N-1)(N-2)}{2} \qquad (N \geq 2);$$

$$n_\mathsf{D}(2) = 0, \quad n_\mathsf{D}(3) = 1, \quad n_\mathsf{D}(4) = 3, \ldots$$

The nonzero phases lead to the CP and T violation in the neutrino sector.

Three-neutrino case

In the most interesting (today!) case of three lepton generations one defines the orthogonal rotation matrices in the ij-planes which depend upon the mixing angles θ_{ij}:

$$\mathbf{O}_{12} = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar matrix}}, \quad \mathbf{O}_{13} = \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}_{\text{Bona vacantia (as yet)}}, \quad \mathbf{O}_{23} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric matrix}},$$

(where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$) and the diagonal matrix with the Dirac phase factor:

$$\boldsymbol{\Gamma}_{\text{D}} = \text{diag}\left(1, 1, e^{i\delta}\right).$$

The parameter δ is commonly referred to as the Dirac CP-violation phase.

Finally, by taking into account the Murnagham theorem, the PMNS mixing matrix for the Dirac neutrinos can be parametrized as[a]

$$\begin{aligned} \mathbf{V}_{(\text{D})} &= \mathbf{O}_{23}\boldsymbol{\Gamma}_{\text{D}}\mathbf{O}_{13}\boldsymbol{\Gamma}_{\text{D}}^{\dagger}\mathbf{O}_{12} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \end{aligned}$$

[a] This is the *Chau–Keung presentation* advocated by the PDG for both CKM and PMNS matrices.

Since the Dirac mass term violates conservation of the individual lepton numbers L_e, L_μ and L_τ, it allows many lepton family number violating processes, like

$$\mu^\pm \to e^\pm + \gamma, \quad \mu^\pm \to e^\pm + e^+ + e^-,$$
$$K^+ \to \pi^+ + \mu^\pm + e^\mp, \quad K^- \to \pi^- + \mu^\pm + e^\mp,$$
$$\mu^- + (A,Z) \to e^- + (A,Z), \quad \tau^- + (A,Z) \to \mu^- + (A,Z), \ldots$$

However the neutrinoless double beta decay[a] $(A,Z) \to (A,Z+2) + 2e^-$ and the processes like $K^+ \to \pi^- + \mu^+ + e^+$, $K^- \to \pi^+ + \mu^- + e^-$, etc. are forbidden as a consequence of the total lepton charge conservation.

Table 1: Current limits on the simplest lepton family number violating μ and τ decays.
[From S. Eidelman *et al.* (Particle Data Group), "Review of particle physics," Phys. Lett. B **592** (2004) 1–1109.].

Decay Modes	Fraction	C.L.		Decay Modes	Fraction	C.L.
$\mu^- \to \nu_e \overline{\nu}_\mu$	$< 1.2\%$	90%		$\tau^- \to e^- \gamma$	$< 2.7 \times 10^{-6}$	90%
$\mu^- \to e^- \gamma$	$< 1.2 \times 10^{-11}$	90%		$\tau^- \to \mu^- \gamma$	$< 1.1 \times 10^{-6}$	90%
$\mu^- \to e^- e^+ e^-$	$< 1.0 \times 10^{-12}$	90%		$\tau^- \to e^- \pi^0$	$< 3.7 \times 10^{-6}$	90%
$\mu^- \to 2\gamma$	$< 7.2 \times 10^{-11}$	90%		$\tau^- \to \mu^- \pi^0$	$< 4.0 \times 10^{-6}$	90%

[a]Hereafter abbreviated as $(\beta\beta)_{0\nu}$.

Neutrinoless muon decay

The L_μ and L_e violating muon decay $\mu^- \to e^-\gamma$ is allowed if $V^*_{\mu k}V_{ek} \neq 0$ for $k = 1, 2$ or 3. The corresponding Feynman diagrams include W loops and thus the decay width is strongly suppressed by the neutrino to W boson mass ratios:

$$R = \frac{\Gamma\left(\mu^- \to e^-\gamma\right)}{\Gamma\left(\mu^- \to e^-\nu_\mu\overline{\nu}_e\right)} = \frac{3\alpha}{32\pi}\left|\sum_k V^*_{\mu k}V_{ek}\frac{m_k}{m_W}\right|^2.$$

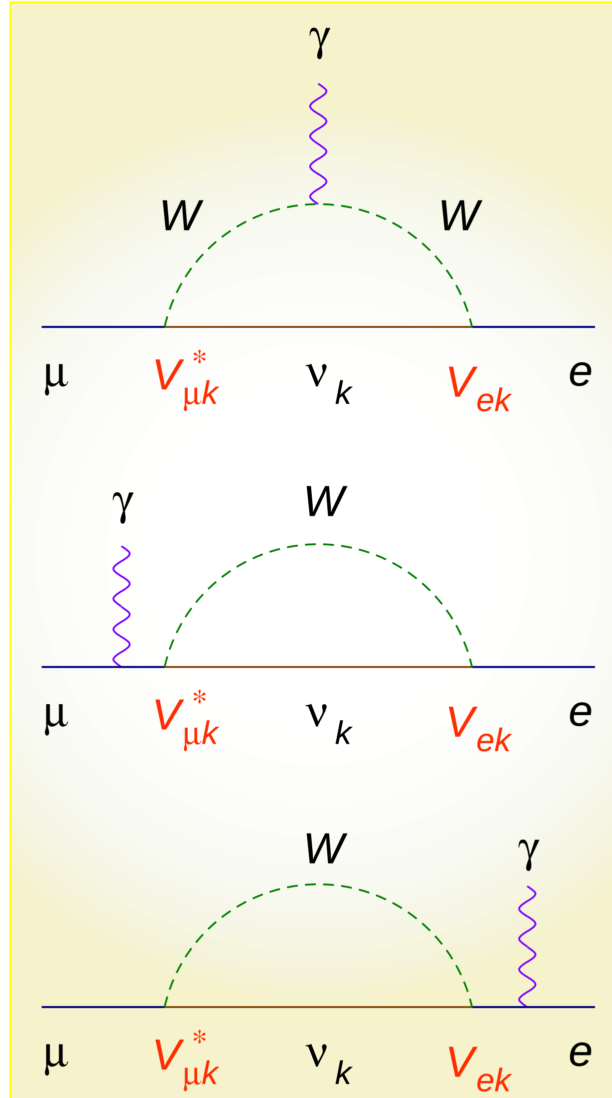
Since $m_k/m_W = 1.24 \times 10^{-11}\,(m_k/1\text{ eV})$, the ratio can be estimated as

$$R \approx 3.37 \times 10^{-26}\left|\sum_k V^*_{\mu k}V_{ek}\left(\frac{m_k}{1\text{ eV}}\right)\right|^2,$$

while the current experimental upper limit is (at least!) 15 orders of magnitude larger:

$$R_{(\text{exp})} < 1.2 \times 10^{-11} \text{ at } 90\% \text{ C.L.}$$

(see Table 1) $\Longrightarrow$ **NO GO**...



Majorana neutrinos

The charge conjugated bispinor field ψ^c is defined by the transformation

$$\psi \longmapsto \psi^c = C\overline{\psi}^T, \quad \overline{\psi} \longmapsto \overline{\psi^c} = -\psi^T C,$$

where C is the charge-conjugation matrix which satisfies the conditions

$$C\gamma_\alpha^T C^\dagger = -\gamma_\alpha, \quad C\gamma_5^T C^\dagger = \gamma_5, \quad C^\dagger = C^{-1} = C, \quad C^T = -C,$$

and thus coincides (up to a phase factor) with the inversion of the axes x^0 and x^2:

$$C = \gamma_0\gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$$

Reminder:

The Pauli matrices:

$$\sigma_0 \equiv 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The Dirac matrices:

$$\gamma^0 = \gamma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix}, \quad \gamma^k = -\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad k = 1,2,3, \quad \gamma^5 = \gamma_5 = -\begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix}.$$

Clearly a charged fermion field $\psi(x)$ is different from the charge conjugated field $\psi^c(x)$. But for a *neutral* fermion field $\nu(x)$ the equality $\boxed{\nu^c(x) = \nu(x)}$ is not forbidden. This is the *Majorana condition*[a] (Majorana neutrino and antineutrino coincide). In the chiral representation

$$\nu = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \nu^c = C\overline{\nu}^T = \begin{pmatrix} -\sigma_2\chi^* \\ +\sigma_2\phi^* \end{pmatrix}.$$

According to the Majorana condition

$$\phi = -\sigma_2\chi^* \quad \text{and} \quad \chi = \sigma_2\phi^* \quad \Longrightarrow \quad \phi + \chi = \sigma_2\left(\phi - \chi\right)^*.$$

(The Majorana neutrino is two-component, i.e. needs only one chiral projection). Then

$$\nu_L = \left(\frac{1+\gamma_5}{2}\right)\nu = \begin{pmatrix} \phi - \chi \\ \chi - \phi \end{pmatrix} \quad \text{and} \quad \nu_R = \left(\frac{1-\gamma_5}{2}\right)\nu = \begin{pmatrix} \phi + \chi \\ \phi + \chi \end{pmatrix} = \nu_L^c.$$

Therefore

$$\boxed{\nu = \nu_L + \nu_R = \nu_L + \nu_L^c.}$$

Now we can construct the Majorana mass term in the general N-neutrino case. It is

$$\mathcal{L}_{\mathsf{M}}(x) = -\frac{1}{2}\overline{\boldsymbol{\nu}}_L^c(x)\mathbf{M}_{\mathsf{M}}\boldsymbol{\nu}_L(x) + \text{H.c.},$$

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[a] More generally, $\nu^c(x) = e^{i\phi}\nu(x)$ ($\phi = \text{const}$).

where $\mathbf{M}_{\mathsf{M}}$ is a $N \times N$ complex *nondiagonal* matrix and, in general, $N \geq 3$.

It can be proved that the $\mathbf{M}_{\mathsf{M}}$ should be symmetric,

$$\mathbf{M}_{\mathsf{M}}^{T} = \mathbf{M}_{\mathsf{M}}.$$

If one assume for a simplification that its spectrum is nondegenerated, the mass matrix can be diagonalized by means of the following transformation

$$\mathbf{M}_{\mathsf{M}} = \mathbf{V}^{*}\mathbf{m}\mathbf{V}^{\dagger}, \quad \mathbf{m} = ||m_{kl}|| = ||m_k\delta_{kl}||,$$

where $\mathbf{V}$ is a unitary matrix and $m_k \geq 0$. Therefore

$$\mathcal{L}_{\mathsf{M}}(x) = -\frac{1}{2}\left[(\overline{\boldsymbol{\nu}'}_L)^c\mathbf{m}\boldsymbol{\nu}'_L + \overline{\boldsymbol{\nu}'}_L\mathbf{m}(\boldsymbol{\nu}'_L)^c\right] = -\frac{1}{2}\overline{\boldsymbol{\nu}'}\mathbf{m}\boldsymbol{\nu}' = -\frac{1}{2}\sum_{k=1}^{N} m_k\overline{\nu}_k\nu_k,$$

$$\boldsymbol{\nu}'_L = \mathbf{V}^{\dagger}\boldsymbol{\nu}_L, \quad (\boldsymbol{\nu}'_L)^c = C\left(\overline{\boldsymbol{\nu}'_L}\right)^T, \quad \boldsymbol{\nu}' = \boldsymbol{\nu}'_L + (\boldsymbol{\nu}'_L)^c.$$

The last equality means that the fields $\nu_k(x)$ are Majorana neutrino fields.

Considering that the kinetic term in the neutrino Lagrangian is transformed to

$$\mathcal{L}_0 = \frac{i}{4}\overline{\boldsymbol{\nu}'}(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}'(x) = \frac{i}{4}\sum_k \overline{\nu}_k(x)\overleftrightarrow{\boldsymbol{\partial}}\nu_k(x),$$

one can conclude that $\nu_k(x)$ is the field with the definite mass m_k.

The flavor LH neutrino fields $\nu_{\ell,L}(x)$ present in the standard weak lepton currents are linear combinations of the LH components of the fields of neutrinos with definite masses:

$$\boldsymbol{\nu}_L = \mathbf{V}\boldsymbol{\nu}'_L \quad \text{or} \quad \nu_{\ell,L} = \sum_k V_{\ell k}\nu_{k,L}.$$

Of course neutrino mixing matrix $\mathbf{V}$ is not the same as in the case of Dirac neutrinos.

There is no global gauge transformations under which the Majorana mass term (in its most general form) could be invariant. This implies that there are no conserved lepton charges that could allow us to distinguish Majorana νs and $\overline{\nu}$s. In other words,

the Majorana neutrinos are truly neutral fermions.

Parametrization of mixing matrix for Majorana neutrinos

Since the Majorana neutrinos are not rephasable, there may be a lot of extra phase factors in the mixing matrix. The Lagrangian with the Majorana mass term is invariant with respect to the transformation

$$\ell \mapsto e^{ia_\ell}\ell, \quad V_{\ell k} \mapsto e^{-ia_\ell}V_{\ell k}$$

Therefore N phases are unphysical and the number of the physical phases now is

$$\frac{N(N+1)}{2} - N = \frac{N(N-1)}{2} = \underbrace{\frac{(N-1)(N-2)}{2}}_{\text{Dirac phases}} + \underbrace{(N-1)}_{\text{Majorana phases}} = n_{\text{D}} + n_{\text{M}};$$

$$n_{\text{M}}(2) = 1, \quad n_{\text{M}}(3) = 2, \quad n_{\text{M}}(4) = 3, \ldots$$

In the case of three lepton generations one defines the diagonal matrix with the extra phase factors: $\boldsymbol{\Gamma}_{\text{M}} = \text{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$, where $\alpha_{1,2}$ are commonly referred to as the Majorana CP-violation phases. Then the PMNS matrix can be parametrized as

$$\mathbf{V}_{(\text{M})} = \mathbf{O}_{23}\boldsymbol{\Gamma}_{\text{D}}\mathbf{O}_{13}\boldsymbol{\Gamma}_{\text{D}}^{\dagger}\mathbf{O}_{12}\boldsymbol{\Gamma}_{\text{M}} = \mathbf{V}_{(\text{D})}\boldsymbol{\Gamma}_{\text{M}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

Neither L_ℓ nor $L = \sum_\ell L_\ell$ is conserved allowing a lot of new processes, for example, $\tau^- \to e^+(\mu^+)\pi^-\pi^-$, $\tau^- \to e^+(\mu^+)\pi^-K^-$, $\pi^- \to \mu^+\overline{\nu}_e$, $K^+ \to \pi^-\mu^+e^+$, $K^+ \to \pi^0e^+\overline{\nu}_e$, $D^+ \to K^-\mu^+\mu^+$, $B^+ \to K^-e^+\mu^+$, $\Xi^- \to p\mu^-\mu^-$, $\Lambda_c^+ \to \Sigma^-\mu^+\mu^+$, etc.

No one was discovered yet but (may be!?) the $(\beta\beta)_{0\nu}$ decay. Thus we have to discuss this issue with some details.

→ Course B