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An introduction to Neutrino physics and oscillations

Theory of neutrino oscillations

- Weak eigenstates do not have to coincide with mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \Rightarrow U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$$where\ c_{ij} = \cos\theta_{ij},\ and\ s_{ij} = \sin\theta_{ij}$$

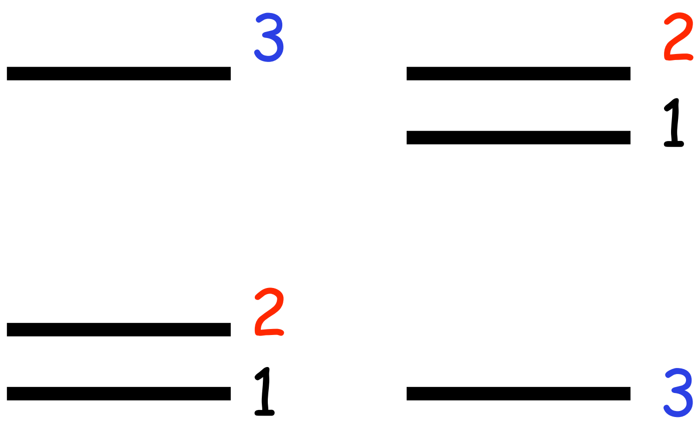
Neutrino mixing matrix (also known as Pontecorvo-Maki-Nakagawa-Sakata, PMNS matrix) is similar to CKM matrix of quark sector.

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

States: $\left|\nu_\alpha\right\rangle = \sum_i U_{\alpha i}\left|\nu_i\right\rangle$ where $\alpha = e, \mu, \tau$ and $i = 1,2,3$

Masses: labels and splittings

Consensus labels: doublet=(ν_1, ν_2), with ν_2 heaviest in both hierarchies



$$\delta m^2 = m_2^2 - m_1^2 > 0$$

Sign of smallest splitting: conventional.
The relative ν_e content of ν_1 and ν_2 is instead physical (given by MSW effect)

$$\text{Note}: \quad |m_3^2 - m_1^2| = \begin{cases} \text{largest splitting (N.H.)} \\ \text{next–to–largest splitting (I.H.)} \end{cases}$$

$$\Rightarrow \Delta m_{31}^2 \text{ (or } \Delta m_{32}^2\text{) change physical meaning from NH to IH}$$

We prefer to define the 2nd independent splitting as:

$$\Delta m^2 = \left| \frac{\Delta m^2_{31} + \Delta m^2_{32}}{2} \right| = \left| m^2_3 - \frac{m^2_1 + m^2_2}{2} \right|$$

so that the largest and next-to-largest splittings, in both NH & IH, are given by: $\Delta m^2 \pm \dfrac{\delta m^2}{2}$

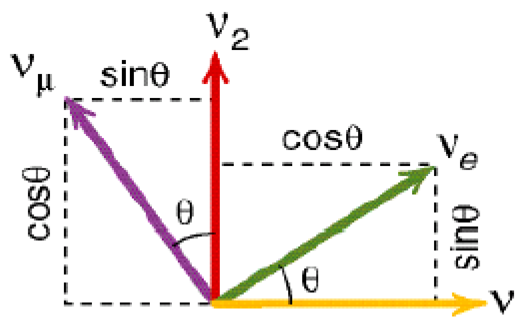
and only one physical sign distinguishes NH (+) from IH (-), as it should be:

$$(m^2_1, m^2_2, m^2_3) = \frac{m^2_2 + m^2_1}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm \Delta m^2 \right)$$

Theory of neutrino oscillations (cont)

- Assume only mixing between two neutrinos:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

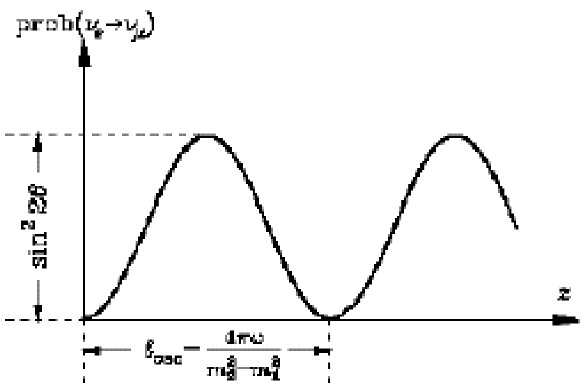


- The mass states evolve in time according to their momentum:

$$|\nu_\alpha(t)\rangle = e^{i\vec{p}\cdot\vec{r}}\left[e^{-iE_1 t}|\nu_1\rangle + e^{-iE_2 t}|\nu_2\rangle\right] \qquad \text{with: } E_i = \sqrt{p^2+m_i^2} \approx |p| + \frac{m_i^2}{2|p|}$$

$$\Rightarrow P_{\nu_\alpha\to\nu_\beta}(x) = \left|\langle\nu_\beta|\nu_\alpha(x)\rangle\right|^2 = \sin^2 2\theta \sin^2\left[\frac{\Delta m_{12}^2}{4E}x\right] = \sin^2 2\theta \sin^2\left[\pi\frac{x}{L_{12}}\right]$$

with the oscillation length: $$L_{ij} = \frac{4\pi E}{\Delta m_{ij}^2} \equiv \frac{4\pi E}{|m_i^2 - m_j^2|} = 2.48\,(m)\frac{E(MeV)}{\Delta m_{ij}^2(eV^2)}$$



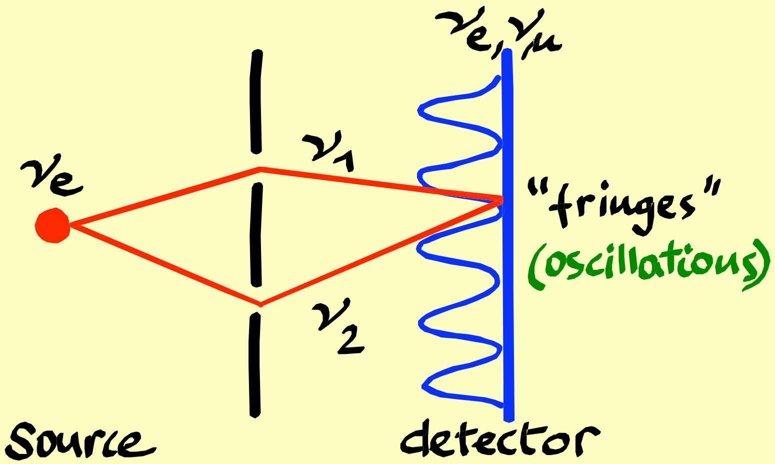
2ν oscillations in vacuum

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}=\begin{pmatrix}c_\theta & s_\theta\\ -s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}\quad ; \quad \Delta m^2 = m_2^2 - m_1^2$$

$$P(\nu_e\to\nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

see tutorials

Analogy with 2-slit expt.:



Length scales:

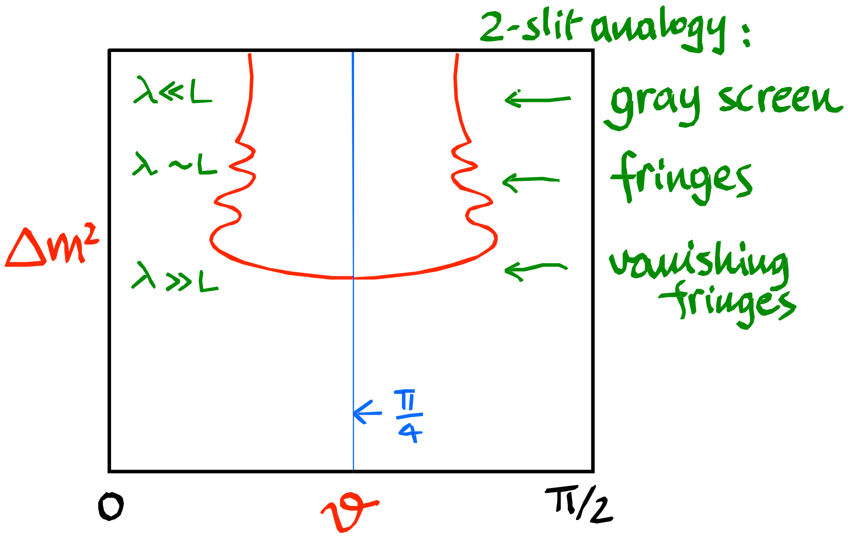
L = baseline

$\lambda = \frac{4\pi E}{\Delta m^2}$ = osc. length

Fringes may not be visible for $\lambda \ll L$ ("fast oscillations) or large expt. smearing ($\Delta E/E$ etc.)

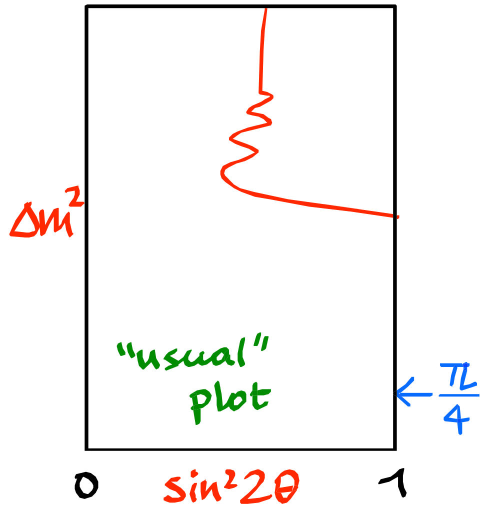
$$\to \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle \sim \frac{1}{2}$$

Typical iso-$\langle P_{e\mu} \rangle$ contours



Octant symmetry: $\theta \to \frac{\pi}{2} - \theta$ in $P_{e\mu}$

If 2nd octant folded onto the 1st one:

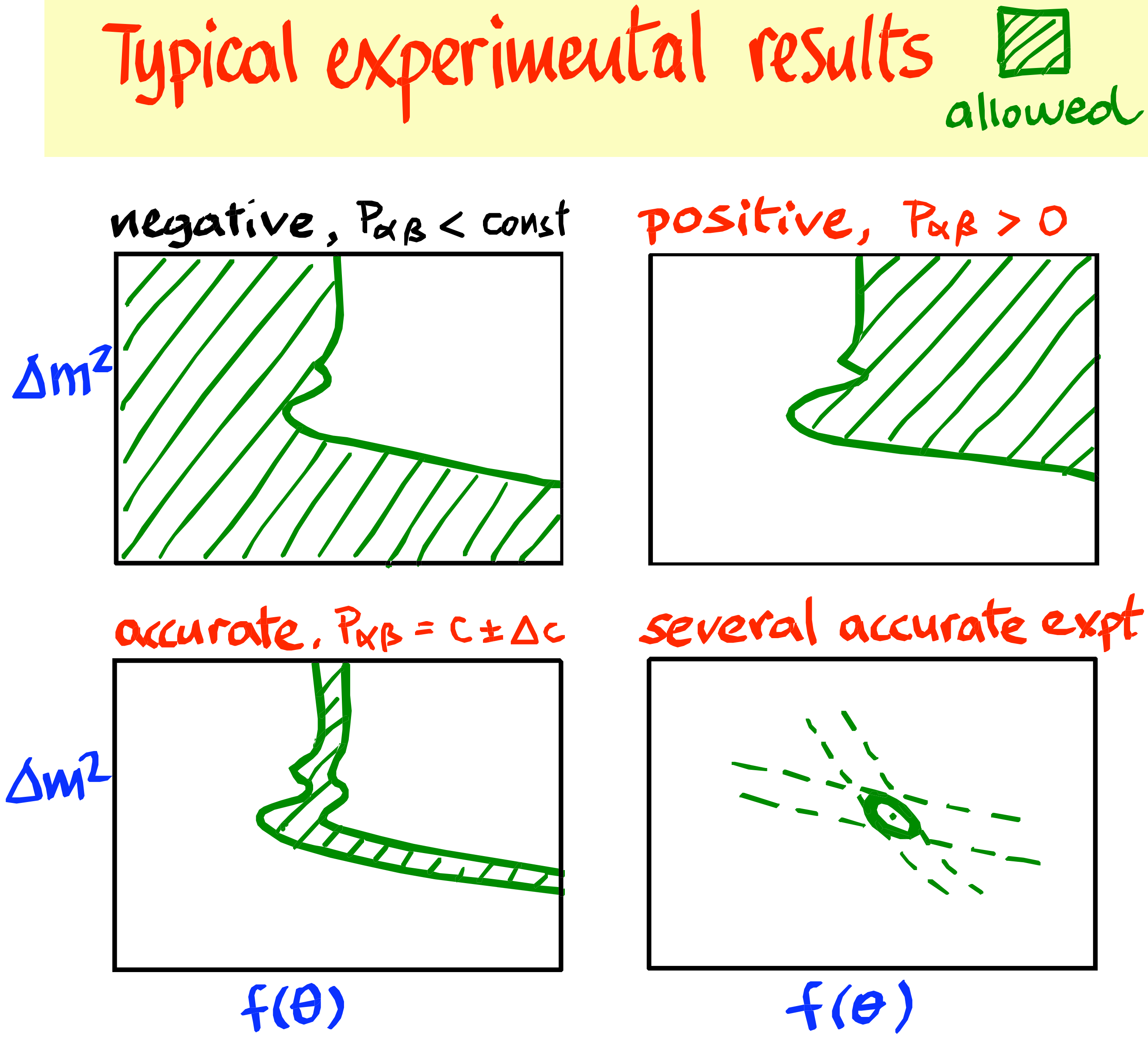


Basically obsolete

$(\Delta m^2, \sin^2 2\theta)$ plot still used for pure 2ν $\nu_\mu \to \nu_\tau$ oscillations (they are vacuum-like even in matter)

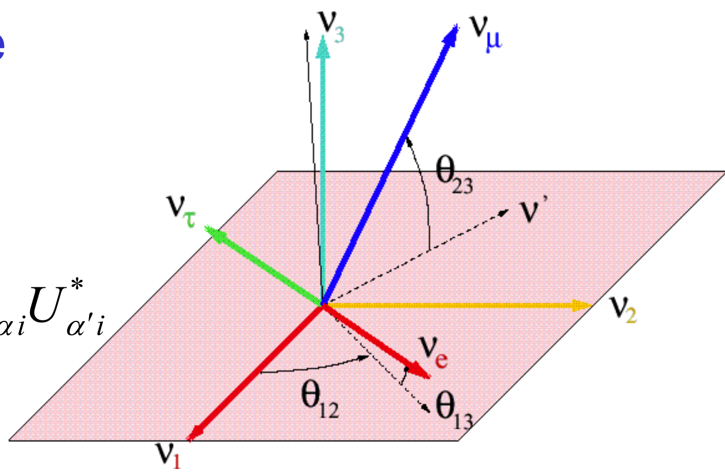
In general, better to use $\log \tan^2\theta$ or $\sin^2\theta$ (preserve octant-symmetry)

Typical experimental results



Theory of neutrino oscillations (cont)

- With three generations the formalism is more complicated, but it is still a rotation in space:



$$\left|\nu_\alpha(t)\right\rangle = e^{i\vec{p}\cdot\vec{r}}\sum_i e^{-iE_it}U_{\alpha i}\left|\nu_i\right\rangle$$

$$\left\langle\nu_{\alpha'}\middle|\nu_\alpha(t)\right\rangle = e^{i\vec{p}\cdot\vec{r}}\sum_{i,j}\left\langle\nu_j\middle|U^+_{j\alpha'}e^{-iE_it}U_{\alpha i}\middle|\nu_i\right\rangle = e^{i\vec{p}\cdot\vec{r}}\sum_i e^{-iE_it}U_{\alpha i}U^*_{\alpha' i}$$

- Oscillation formula:

$$P_{\nu_\alpha\to\nu_{\alpha'}}(t) = \left|\left\langle\nu_{\alpha'}\middle|\nu_\alpha(t)\right\rangle\right|^2 = \sum_{i,j}\left|U_{\alpha i}U^*_{\alpha' i}U^*_{\alpha j}U_{\alpha' j}\right|\cos\left[(E_i - E_j)t - \varphi_{\alpha\alpha' ij}\right]$$

with $\varphi_{\alpha\alpha' ij} = \arg\left(U_{\alpha i}U^*_{\alpha' i}U^*_{\alpha j}U_{\alpha' j}\right)$ If $U_{\alpha i}$ real, then: $\varphi_{\alpha\alpha' ij} = 0$

therefore: $$P_{\nu_\alpha\to\nu_{\alpha'}}(x) = \sum_{i,j}\left|U_{\alpha i}U^*_{\alpha' i}U^*_{\alpha j}U_{\alpha' j}\right|\cos\left[\frac{2\pi x}{L_{ij}} - \varphi_{\alpha\alpha' ij}\right]$$

with the oscillation length: $$L_{ij} = \frac{4\pi E}{\Delta m^2_{ij}} \equiv \frac{4\pi E}{\left|m_i^2 - m_j^2\right|} = 2.48\,(m)\frac{E(MeV)}{\Delta m^2_{ij}(eV^2)}$$

Theory of neutrino oscillations (cont)

- Oscillations of three neutrino families, if: $\left|\Delta m_{12}^2\right| << \left|\Delta m_{23}^2\right|, \left|\Delta m_{13}^2\right| \approx \left|\Delta m_{23}^2\right|$

$$P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)}(x) = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left[\frac{\Delta m_{23}^2}{4E}x\right]$$

$$P_{\nu_e\nu_\tau(\bar{\nu}_e\bar{\nu}_\tau)}(x) = c_{23}^2 \sin^2 2\theta_{13} \sin^2\left[\frac{\Delta m_{23}^2}{4E}x\right]$$

$$P_{\nu_\mu\nu_\tau(\bar{\nu}_\mu\bar{\nu}_\tau)}(x) = c_{13}^4 \sin^2 2\theta_{23} \sin^2\left[\frac{\Delta m_{23}^2}{4E}x\right]$$

with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

- Oscillations, if $\left|\Delta m_{12}^2\right|$ not negligible:

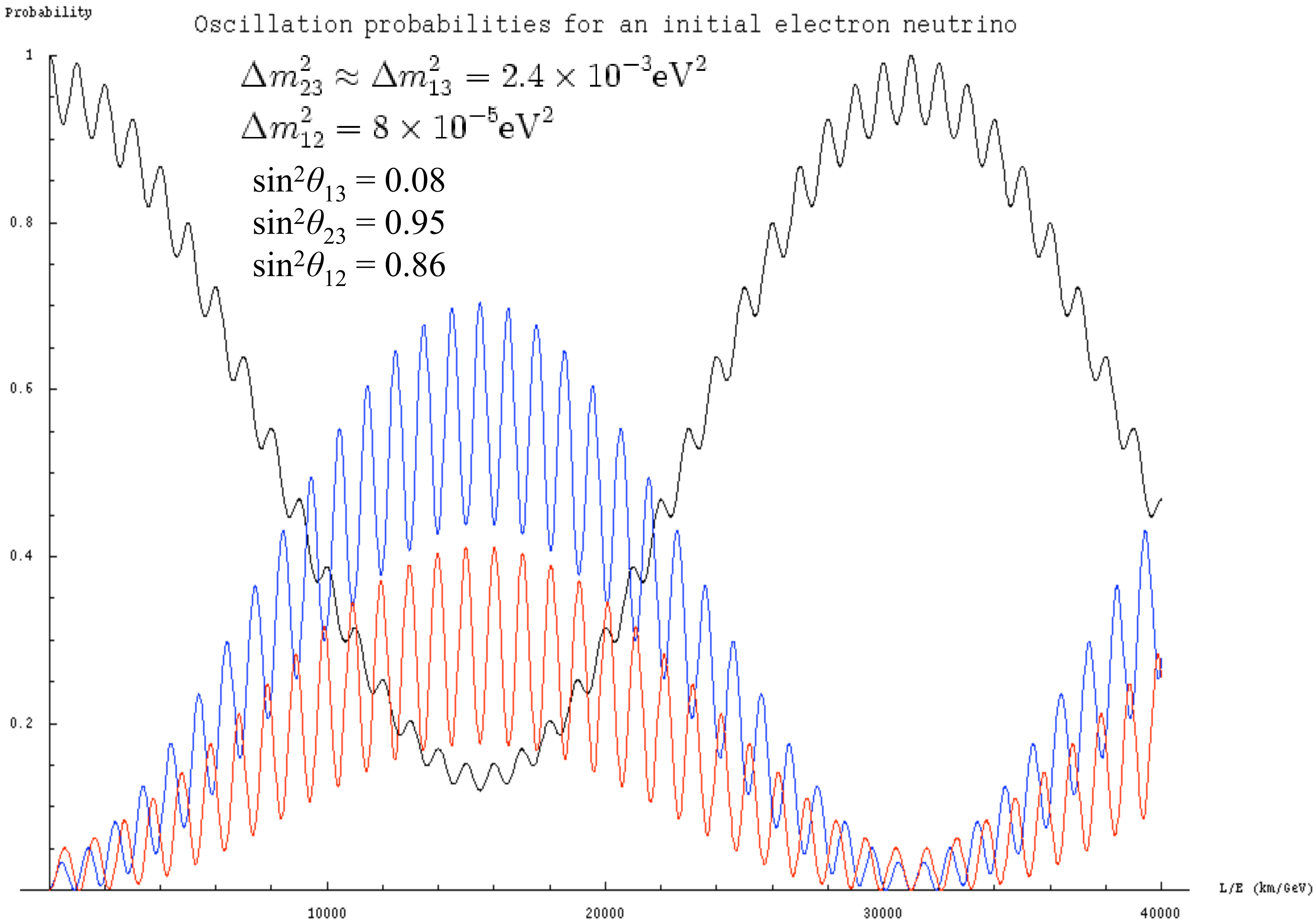
$$P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)}(x) = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left[\frac{\Delta m_{13}^2}{4E}x\right] + c_{23}^2 \sin^2 2\theta_{12} \sin^2\left[\frac{\Delta m_{12}^2}{4E}x\right] +$$

$$+\tilde{J}\cos\left[\pm\delta - \frac{\Delta m_{13}^2}{4E}x\right]\left(\frac{\Delta m_{12}^2}{4E}x\right)\sin\left[\frac{\Delta m_{13}^2}{4E}x\right]$$

where $\pm$ is for $\nu, \bar{\nu}$

$\tilde{J} \equiv c_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}$

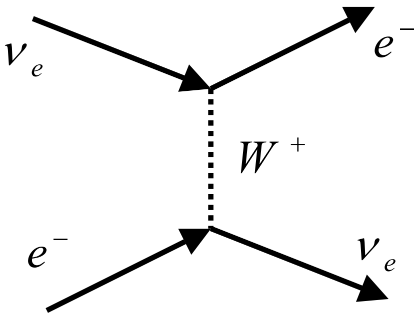
(Jarlskog coefficient for CP violation)

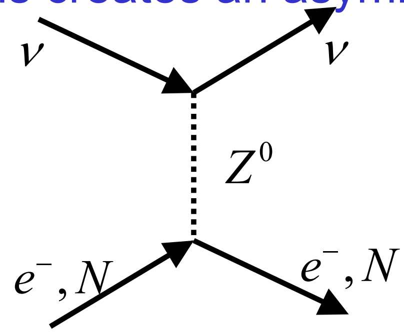


Probability
Oscillation probabilities for an initial electron neutrino
$\Delta m^2_{23} \approx \Delta m^2_{13} = 2.4 \times 10^{-3}\mathrm{eV}^2$
$\Delta m^2_{12} = 8 \times 10^{-5}\mathrm{eV}^2$
$\sin^2\theta_{13} = 0.08$
$\sin^2\theta_{23} = 0.95$
$\sin^2\theta_{12} = 0.86$
1
0.8
0.6
0.4
0.2
10000
20000
30000
40000
L/E (km/GeV)

Oscillations in matter (MSW effect)

- When electron neutrinos travel through matter (ie. sun, earth) they can encounter charged and neutral current interactions, while other neutrinos can only have neutral current interactions. This creates an asymmetry in the cross-sections.





- Coherent neutrino scattering at θ=0º modifies propagation of neutrinos in matter with respect to vacuum.
- Hamiltonian in vacuum (Schrodinger equation):

$$i\frac{d}{dt}\begin{pmatrix}\nu_1(t)\\ \nu_2(t)\end{pmatrix}=\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\begin{pmatrix}\nu_1(t)\\ \nu_2(t)\end{pmatrix} \qquad \begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}=U\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

$$\Rightarrow i\frac{d}{dt}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix}=H_V\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} \quad H_V=UHU^+=E+\frac{m_1^2+m_2^2}{4E}+\frac{\Delta m_{12}^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

$$\tan 2\theta=\frac{2H_{21}}{H_{22}-H_{11}}$$

Oscillations in matter (MSW effect) (cont)

- Hamiltonian in matter:

$$H_M = H_V + V_Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + V_W \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad V_W = \sqrt{2} G_F n_e \approx 7.6 \times 10^{-14} \frac{Z}{A} \rho(g/cm^3)\, eV$$

$n_e =$ **electron number density**

$$H_M = \left(E + \frac{m_1^2 + m_2^2}{4E} + V_Z \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{\Delta m_{12}^2}{4E} \cos 2\theta + \sqrt{2} G_F n_e & \frac{\Delta m_{12}^2}{4E} \sin 2\theta \\ \frac{\Delta m_{12}^2}{4E} \sin 2\theta & \frac{\Delta m_{12}^2}{4E} \cos 2\theta \end{pmatrix}$$

$$\tan 2\theta_M = \frac{2H_{21}}{H_{22} - H_{11}} = \frac{\Delta m_{12}^2 \sin 2\theta}{\Delta m_{12}^2 \cos 2\theta - A} \qquad A \equiv 2\sqrt{2} G_F n_e E$$

- Resonance condition: $A = \Delta m_{12}^2 \cos 2\theta \Rightarrow \theta_M = \frac{\pi}{4}$ (maximal mixing even if θ_V small)
- Mass eigenvalues:

$$E_i = E + V_Z + \frac{\tilde{m}_i^2}{2E} \qquad \tilde{m}_{1,2}^2 = \frac{1}{2}\left(\left(m_1^2 + m_2^2 + A \right) \mp \sqrt{\left(\Delta m_{12}^2 \cos 2\theta - A \right)^2 + \Delta m_{12}^4 \sin^2 2\theta} \right)$$

(Mikheyev, Smirnov, Wolfenstein resonance condition: MSW effect)

Evaluation of V_{CC}^{ee}

$$H_{cc} = \underbrace{\frac{G_F}{\sqrt{2}} \bar{e}\gamma^\mu(1-\gamma_5)\nu_e}_{J^{cc}} \otimes \underbrace{\bar{\nu}_e\gamma_\mu(1-\gamma_5)e}_{J^{cc}} \overset{\text{Fierz}}{=\!=} \underbrace{\frac{G_F}{\sqrt{2}} \bar{e}\gamma^\mu(1-\gamma_5)e}_{J_e} \underbrace{\bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e}_{J_\nu}$$

From the ν viewpoint, the e^- is ~nonrelativistic and ~unpolarized

→ Dirac representation, $e \simeq \begin{bmatrix} \xi \\ 0 \end{bmatrix}$

$$\bar{e}\gamma^\mu(1-\gamma_5)e \simeq (\underbrace{\xi^\dagger\xi}_{\substack{\text{density} \\ N_e}}, \underbrace{\xi^\dagger\vec{\sigma}\xi}_{\substack{\text{polarization} \\ \sim 0}}) \simeq N_e\,\delta_{\mu 0}$$

$$H_{cc} = \frac{G_F}{\sqrt{2}} N_e \bar{\nu}_e\gamma_0(1-\gamma_5)\nu_e = \underbrace{\sqrt{2}G_F N_e}_{\text{coupling}} \underbrace{\bar{\nu}_{eL}\gamma_0\nu_{eL}}_{\text{"static" term}}$$

$$\boxed{V_{CC}^{ee} = \sqrt{2}\, G_F N_e}$$

Exercise : prove that

$$\frac{A}{eV^2} = 1.526 \times 10^{-7} \left(\frac{N_e}{mol/cm^3}\right)\left(\frac{E}{MeV}\right)$$

where $A = 2EV = 2\sqrt{2}\, G_F N_e E$

hint : remember that

$G_F = 1.16637 \times 10^{-5}\ GeV^{-2}$

and use (see tutorials)

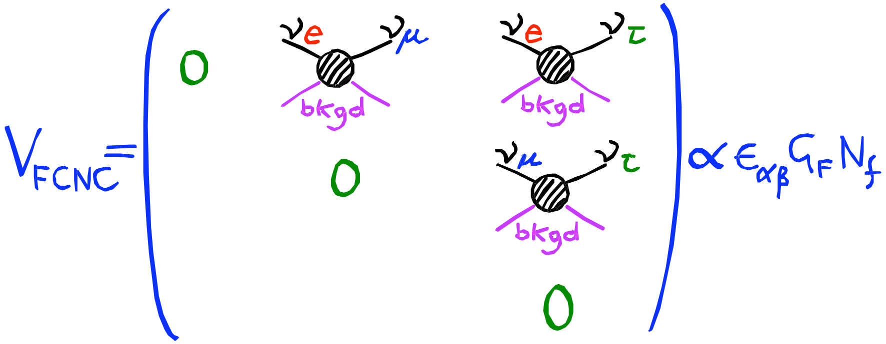
$$1\ \frac{mol}{cm^3} = 4.627 \times 10^{-9}\ MeV^3$$

Back to 3 massless ν in matter

Standard EW inter.
+ ordinary matter $\rightarrow H = \begin{pmatrix} p+V_{cc} & & \\ & p & \\ & & p \end{pmatrix}$

$\rightarrow$ no off-diagonal elements in flavor basis
$\rightarrow$ flavor is conserved

However, flavor changing neutral currents may arise in theories beyond the standard model:



(e.g. SUSY with R-parity breaking, violations or equivalence principle ...)

In such cases, flavor transitions could take place even for massles ν

More on standard EW interaction energies

ν type	bkgd matter	Interaction energy V
ν_e	e	$\frac{1}{\sqrt{2}} G_F (4s_W^2+1)(N_e - N_{\bar{e}})$
$\nu_{\mu,\tau}$	e	$\frac{1}{\sqrt{2}} G_F (4s_W^2-1)(N_e - N_{\bar{e}})$
$\nu_{e,\mu,\tau}$	n	$\frac{1}{\sqrt{2}} G_F (N_{\bar{n}} - N_n)$
$\nu_{e,\mu,\tau}$	p	$\frac{1}{\sqrt{2}} G_F (1-4s_W^2)$
ν_s	e, p, n	0

for $\nu \to \bar{\nu}$:
$V \to -V$

In ordinary matter : $N_e = N_p$, $N_{\bar{e}} = N_{\bar{p}} = N_{\bar{n}} = 0$

$$V_e - V_{\mu,\tau} = \sqrt{2}\, G_F N_e$$ } as before

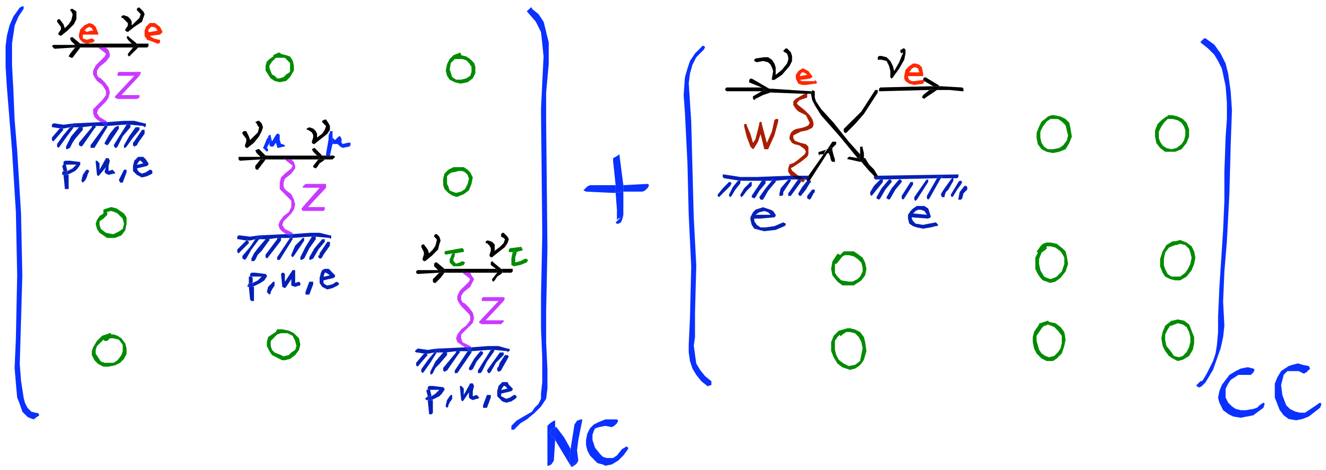
$$V_\mu - V_\tau = 0$$ } vacuum-like

$$V_s - V_{\mu,\tau} = \sqrt{2}\, G_F \frac{N_n}{2}$$

$$V_s - V_e = \sqrt{2}\, G_F \left(N_e - \frac{1}{2} N_n\right)$$

} relevant for sterile ν phenomenology

$V =$

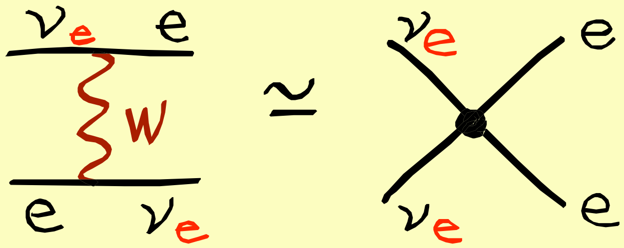


$V = V_{NC} + V_{CC}$ with $V_{NC} \propto \mathbb{1}$

(up to small higher-order corrections)

→ Relevant term is the interaction energy difference V_{CC}

$V_{CC}^{ee} =$



Oscillations in matter (MSW effect) (cont)

- Resonant oscillation length:

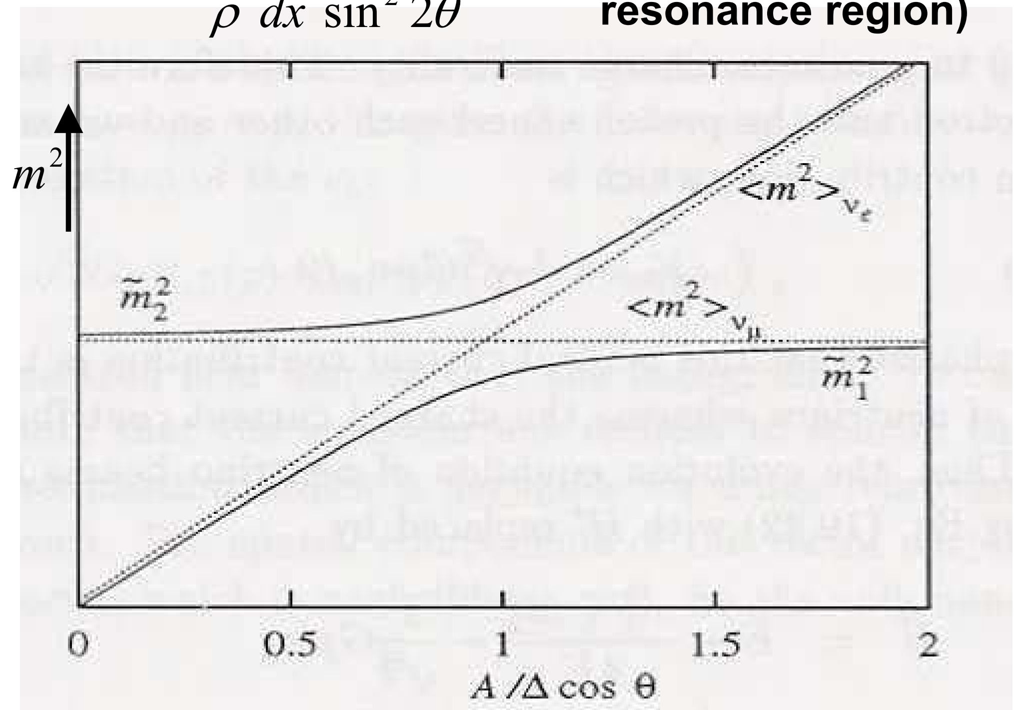
$$L_M = L_V \frac{\Delta m_{12}^2}{\sqrt{\left(\Delta m_{12}^2 \cos 2\theta - A\right)^2 + \Delta m_{12}^4 \sin^2 2\theta}} \Rightarrow L_M = \frac{L_V}{\sin 2\theta} = \frac{4\pi E}{\Delta m_{12}^2 \sin 2\theta}$$ **(at resonance)**

- Ideally, matter density constant (ρ=const), but if not, also valid if slowly varying matter density (adiabaticity condition): $\frac{1}{\rho}\frac{d\rho}{dx}\frac{L_V}{\sin^2 2\theta} << 1$ **(many oscillations in resonance region)**

- Adiabatic condition:

  In vacuum (A=0) the light eigenstate ν_1 is almost all ν_e and for very dense media (A>>Δcos2θ) the lighter eigenstate is almost all ν_μ. The effective mass squared value of ν_e starts lower than ν_μ but as A increases then the effective mass squared value of ν_μ is greater than ν_e (level crossing).

  $(\Delta = \Delta m_{12}^2)$



Adiabatic evolution

At each point x :

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\tilde\theta(x) & \sin\tilde\theta(x)\\ -\sin\tilde\theta(x) & \cos\tilde\theta(x)\end{pmatrix}\begin{pmatrix}\tilde\nu_1(x)\\ \tilde\nu_2(x)\end{pmatrix}$$

with $P(\tilde\nu_1 \to \tilde\nu_2)$ "no crossing"

Typically, $\tilde\lambda \ll L \to$ phase information lost

$\to$ can propagate "probabilities" (rather than amplitudes)

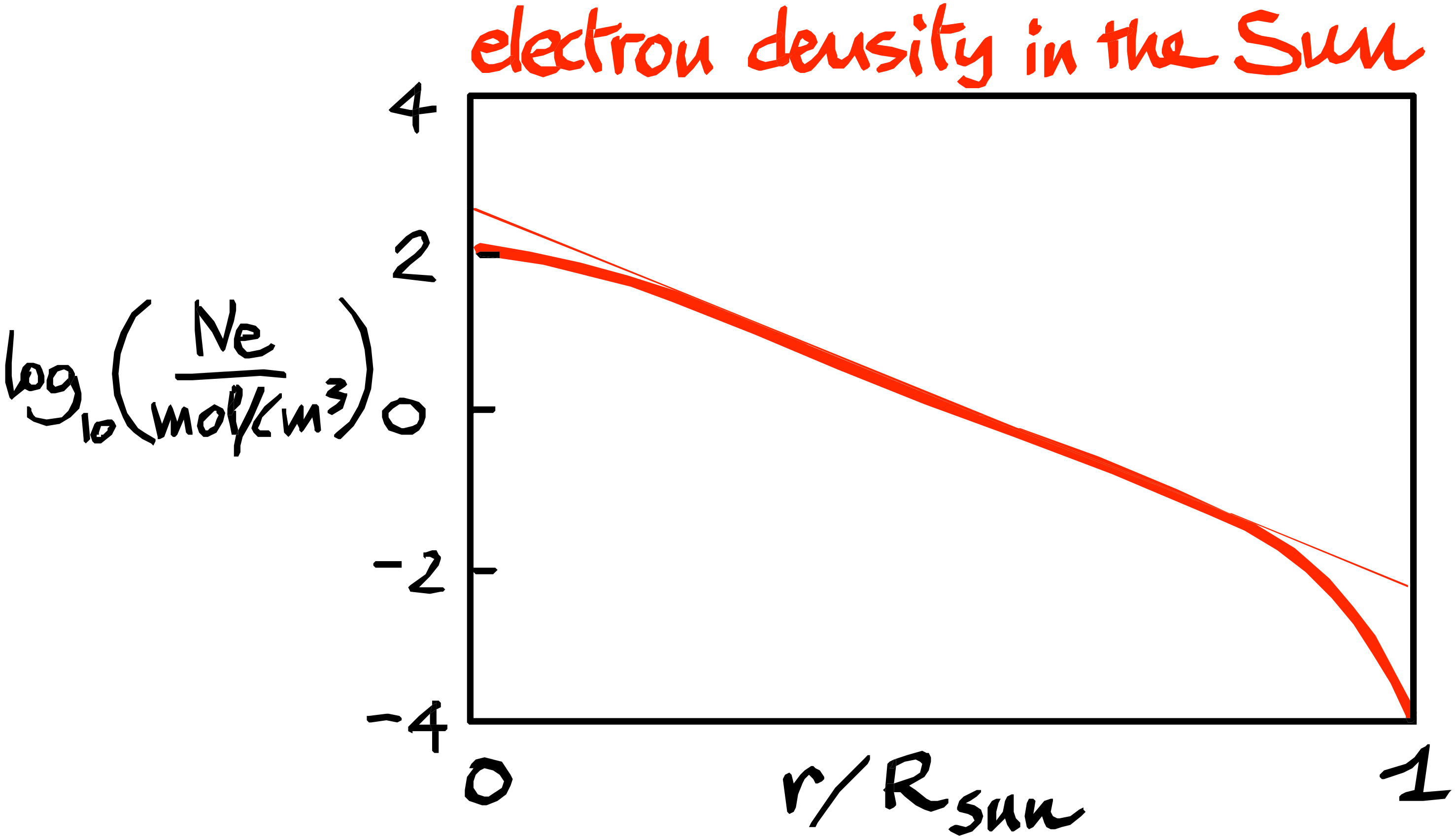
$$P(\nu_e\to\nu_e) = \underset{\uparrow\ \text{final}\ \nu_e}{(1,0)}\underset{\uparrow\ \text{rotate back at}\ x=x_f}{\begin{pmatrix}\cos^2\tilde\theta_f & \sin^2\tilde\theta_f\\ \sin^2\tilde\theta_f & \cos^2\tilde\theta_f\end{pmatrix}}\underset{\uparrow\ \text{no crossing}}{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}\underset{\uparrow\ \text{rotate at}\ x=x_i\ \text{to}\ \tilde\nu_{1,2}\ \text{basis}}{\begin{pmatrix}\cos^2\tilde\theta_i & \sin^2\tilde\theta_i\\ \sin^2\tilde\theta_i & \cos^2\tilde\theta_i\end{pmatrix}}\underset{\uparrow\ \text{initial}\ \nu_e}{\begin{pmatrix}1\\ 0\end{pmatrix}}$$

From right to left :

$$\boxed{P_{ee} = \frac{1}{2}\left(1+\cos 2\tilde\theta_i \cos 2\tilde\theta_f\right)}$$

For solar neutrinos: $\tilde\theta_f = \theta$ (vacuum), up to Earth matter effects

$$P_{ee}^{\odot} = \frac{1}{2}\left(1+\cos 2\underset{\uparrow\ \text{production point}}{\tilde\theta(x)}\cos 2\theta\right)$$



Exp. approximation:

$N_e \simeq N_e(0)\, e^{-x/r_0}$

$N_e(0) \simeq 245\ mol/cm^3$

$r_0 \simeq R_\odot/10.54$

But in true SSM:

$N_e(0) \simeq 100\ mol/cm^3$

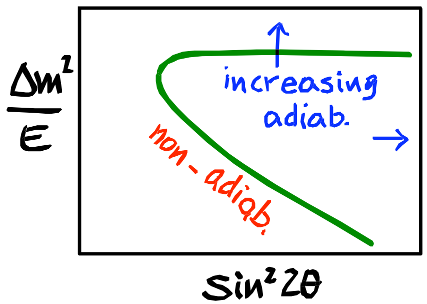
Nonadiabatic corrections

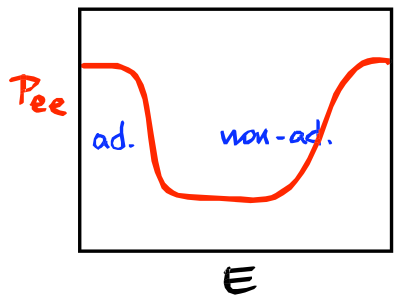
In $(\tilde{\nu}_1, \tilde{\nu}_2)$ basis: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1-P_c & P_c \\ P_c & 1-P_c \end{pmatrix}$ P_c = crossing prob. $\tilde{\nu}_1 \to \tilde{\nu}_2$ "tunnelling"

$$\boxed{P_{ee} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\tilde{\theta}_i \cos 2\tilde{\theta}_f}$$

enormous literature on P_c evaluation

Historically relevant in solar ν solutions:





MSW "triangle", zone of small P_{ee}

Strong difference from vacuum case

Oscillations in matter (MSW effect) (cont)

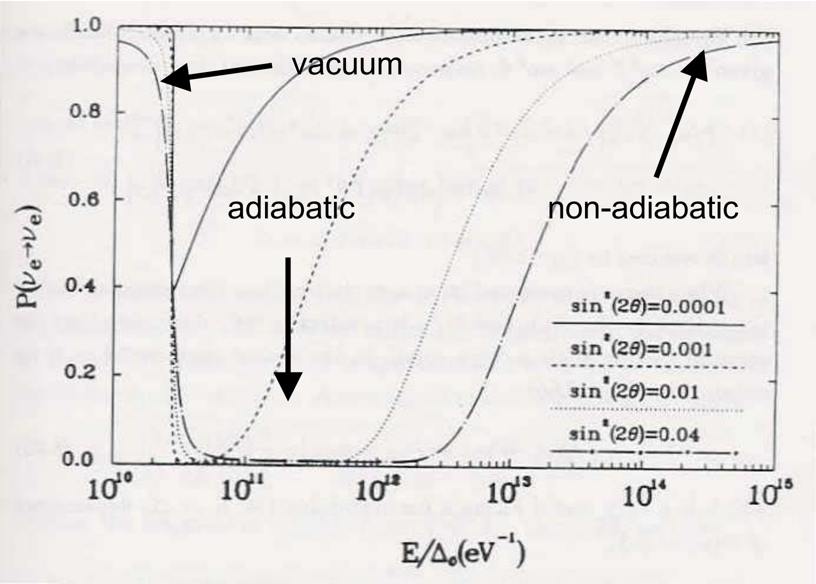
- If adiabatic condition not met: $\frac{1}{\rho}\frac{d\rho}{dx}\frac{L_V}{\sin^2 2\theta} \approx 1$ **(hardly any oscillations in resonance region)**

$$P_{\nu_e\nu_e} = \frac{1}{2}\left[1+\left(1-2P_{LZ}\right)\cos 2\tilde{\theta}_{M,0}\cos 2\theta\right]$$

$$\tilde{\theta}_{M,0} = \theta_M$$ **at production point t=t_0**

$$P_{LZ} = \exp\left(-\frac{\pi}{4}Q\right)$$ **(Landau-Zener probability)**

$$Q = \frac{\Delta m_{12}^2 \sin^2 2\theta}{E\cos 2\theta\left|\frac{1}{\rho}\frac{d\rho}{dx}\right|}$$ **(adiabaticity parameter)**



- Matter oscillation results for three neutrinos:

$$P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)}(x) = s_{23}^2\sin^2 2\theta_{13}\left(\frac{\Delta m_{13}^2}{2EB_\mp}\right)\sin^2\left[\frac{B_\mp}{2}x\right] + c_{23}^2\sin^2 2\theta_{12}\left(\frac{\Delta m_{12}^2}{A}\right)\sin^2\left[\frac{A}{4E}x\right] +$$

$$+\tilde{J}\cos\left[\pm\delta - \frac{\Delta m_{13}^2}{4E}x\right]\left(\frac{\Delta m_{12}^2}{A}\frac{\Delta m_{13}^2}{2EB_\mp}\right)\sin\left[\frac{A}{4E}x\right]\sin\left[\frac{B_\mp}{2}x\right]$$

where $\pm$ is for $\nu,\bar{\nu}$

$$with \quad B_\mp \equiv \frac{1}{2E}\sqrt{\left(\Delta m_{13}^2\cos 2\theta_{13} \mp A\right)^2 + \Delta m_{13}^4\sin^2 2\theta_{13}}$$

Calculation of the crossing probability

- A widely used formula for the calculation of the crossing probability is the *double-exponential*:

$$P(\nu_a^m \rightarrow \nu_b^m) = P_c(k, \theta) \cong \frac{\exp(2\pi r_0 k \cos^2\theta) - 1}{\exp(2\pi r_0 k) - 1}$$

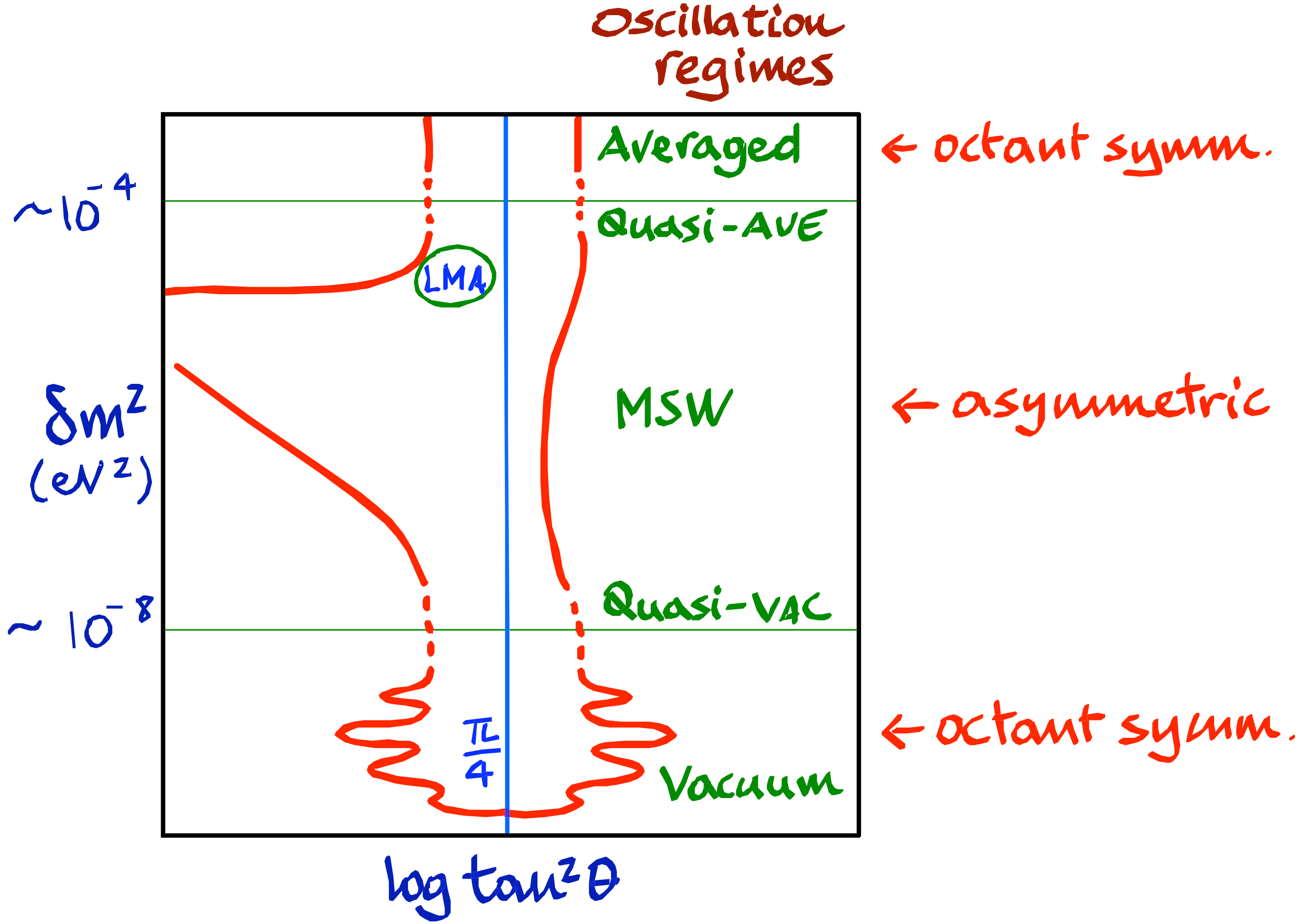
where $k = \Delta m_{ab}^2/2E$ is the vacuum oscillation wavenumber and r_0 is a *scale factor*, i.e., the inverse of the logarithmic derivate of the potential $V(x)$ in the *crossing point* x_p:

$$r_0 = \left| \frac{1}{V(x)} \frac{dV(x)}{dx} \right|^{-1}_{x=x_c}$$

- The crossing point x_c should be chosen as the point where the adiabaticity condition is maximally violated. It can be proved that this point correspond with a good approximation to the point where:

$$\boxed{V(x_p) = k}$$

Solar Pee suppression: param. space



Generalization to 3 flavor case

- Take the evolution equation in matter: $$i\frac{d\nu_\alpha}{dx} = \mathbf{H}_\alpha{}^\beta \, \nu_\beta$$

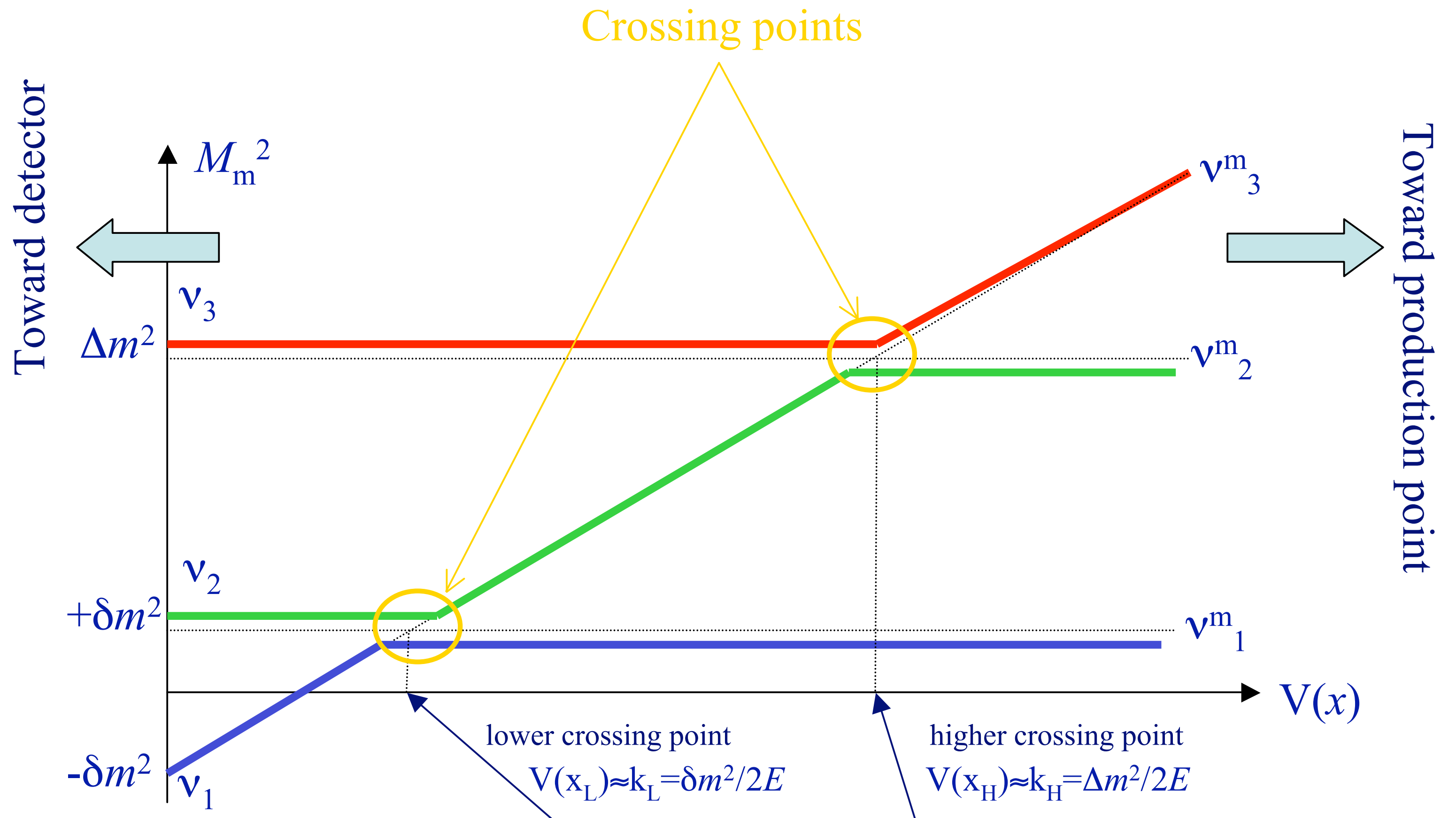
- Diagonalize the Hamiltonian: $$\tilde{\mathbf{U}}^+ \cdot \mathbf{H}(x) \cdot \tilde{\mathbf{U}} = \mathrm{diag}\left(M_1(x)^2 , M_2(x)^2 , M_3(x)^2\right)$$

- Write the neutrino "eigenstates" in matter: $$\nu^m{}_i = \left(\tilde{\mathbf{U}}^+\right)_i{}^\alpha \, \nu_\alpha$$

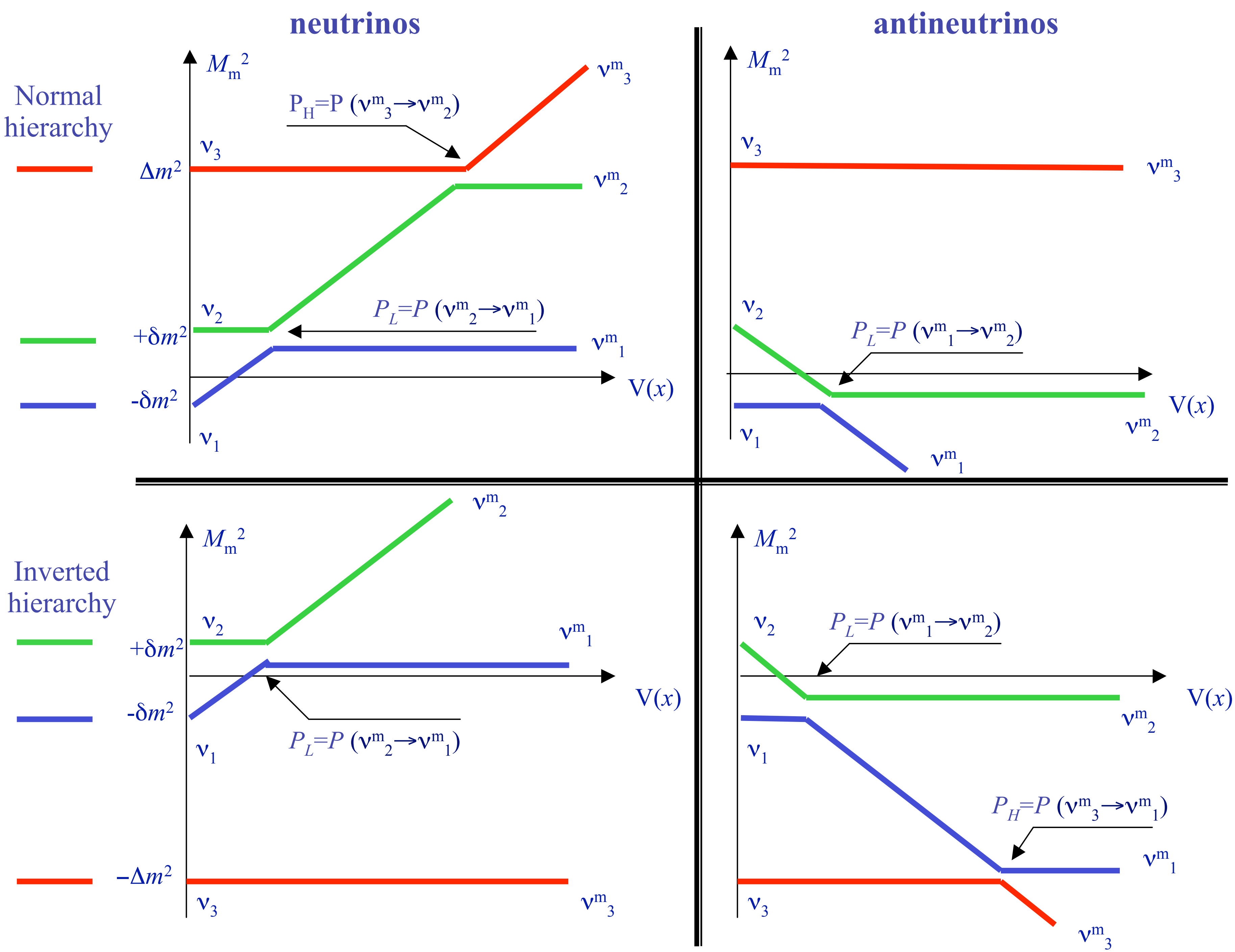
- Rewrite the evolution equation in this basis: $$i\frac{d\nu^m{}_i}{dx} = \frac{1}{2E}\left\{\Sigma M_i{}^2 \, \delta_i{}^j - \left(\tilde{\mathbf{U}}^+ \frac{d\tilde{\mathbf{U}}}{dx}\right)_{ij}\right\}\nu^m{}_j$$

- The term $\tilde{\mathbf{U}}^+ d\tilde{\mathbf{U}}/dx$ is generally much smaller than the first, except near to the points when $M_1(x_L)^2 \sim M_2(x_L)^2$ (*lower resonance*) and $M_2(x_H)^2 \sim M_3(x_H)^2$ (*higher resonance*). Far from these two points, the eigenstates in matter are conserved (*adiabatic propagation*).

- In the lower (higher) resonance points there is a non-zero probability for a $\nu^m{}_1 \rightarrow \nu^m{}_2$ ($\nu^m{}_2 \rightarrow \nu^m{}_3$) transition.



$$P_{ee} \cong [1,\ 0,\ 0] \cdot \underbrace{\begin{pmatrix} U_{e1}^2 & U_{e2}^2 & U_{e3}^2 \\ U_{\mu1}^2 & U_{\mu2}^2 & U_{\mu3}^2 \\ U_{\tau1}^2 & U_{\tau2}^2 & U_{\tau3}^2 \end{pmatrix}}_{\substack{\text{rotation to the} \\ \text{original flavor basis}}} \cdot \underbrace{\begin{pmatrix} 1-P_L & P_L & 0 \\ P_L & 1-P_L & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{lower transition} \\ P_L = P_c(\nu^m{}_2 \to \nu^m{}_1)}} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-P_H & P_H \\ 0 & P_H & 1-P_H \end{pmatrix}}_{\substack{\text{higher transition} \\ P_H = P_c(\nu^m{}_3 \to \nu^m{}_2)}} \cdot \underbrace{\begin{pmatrix} \tilde{U}_{e1}^2 & \tilde{U}_{\mu1}^2 & \tilde{U}_{\tau1}^2 \\ \tilde{U}_{e2}^2 & \tilde{U}_{\mu2}^2 & \tilde{U}_{\tau2}^2 \\ \tilde{U}_{e3}^2 & \tilde{U}_{\mu3}^2 & \tilde{U}_{\tau3}^2 \end{pmatrix}}_{\substack{\text{rotation to eigenstates} \\ \text{in matter} \\ \text{(at the production point)}}} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



neutrinos
antineutrinos
Normal hierarchy
Inverted hierarchy
M_m^2
Δm^2
$+\delta m^2$
$-\delta m^2$
$-\Delta m^2$
ν_1
ν_2
ν_3
ν^m_1
ν^m_2
ν^m_3
$P_H=P(\nu^m_3 \to \nu^m_2)$
$P_L=P(\nu^m_2 \to \nu^m_1)$
$P_L=P(\nu^m_1 \to \nu^m_2)$
$P_H=P(\nu^m_3 \to \nu^m_1)$
V(x)

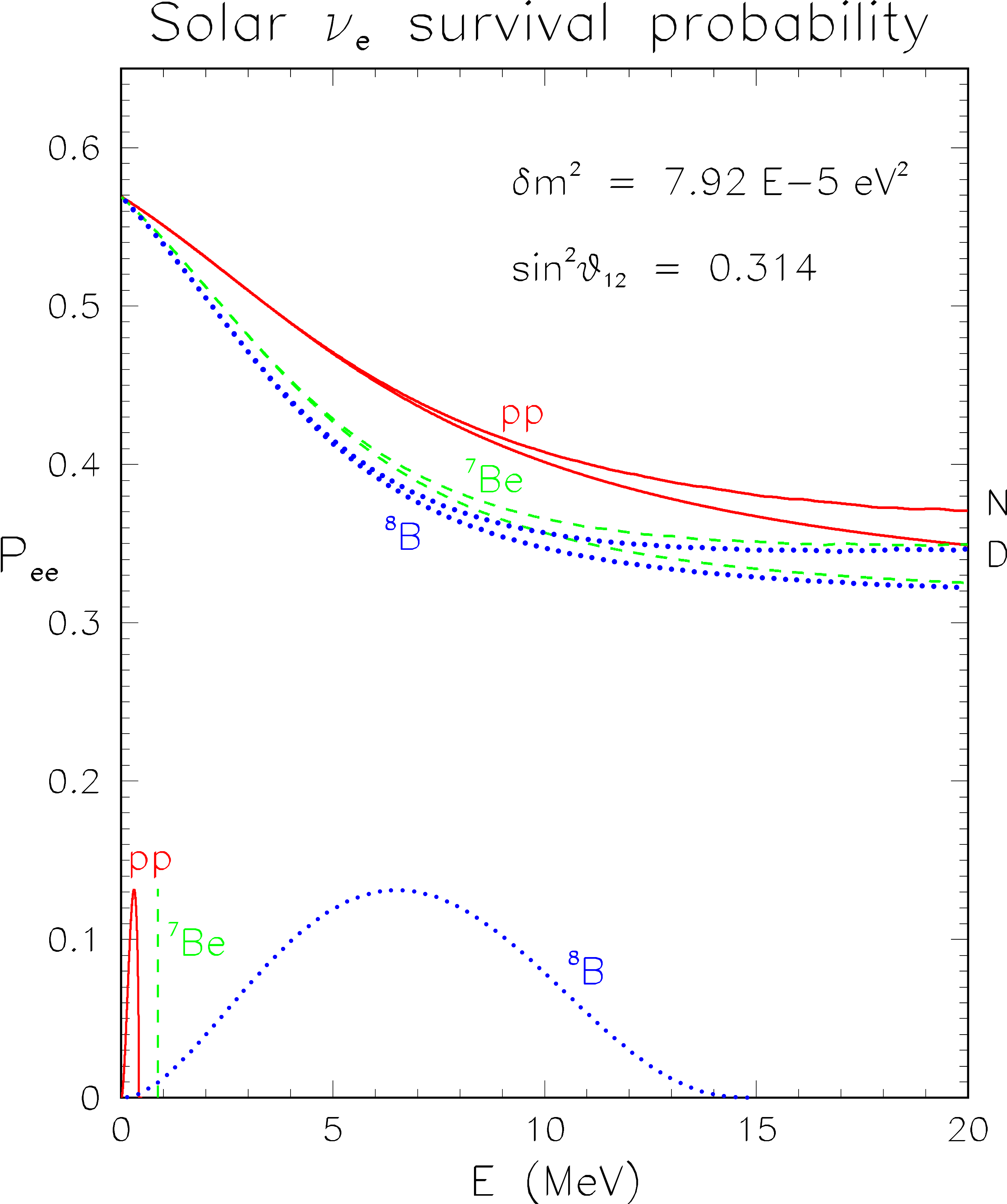
Since solar density is low, neutrinos never crossing the high resonace. In this case the electron survival probability reduces to:

$$P_{ee}^{3\nu} \simeq \cos^4\theta_{13} P_{2\nu} + \sin^4\theta_{13}$$

where:

$$P_{ee}^{2\nu} = \frac{1}{2} + \left(\frac{1}{2} - P_L\right) \cos 2\theta_{12} \cos 2\tilde{\theta}_{12}$$

As we will see in the next lecture, in the current phenomenology the neutrino oscillations in the Sun can be assumed completely adiabatic, i.e., P_L=0.



Solar ν_e survival probability
$\delta m^2 = 7.92$ E−5 eV^2
$\sin^2\vartheta_{12} = 0.314$
pp
^{7}Be
^{8}B
N
D
P_{ee}
E (MeV)
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
pp
^{7}Be
^{8}B