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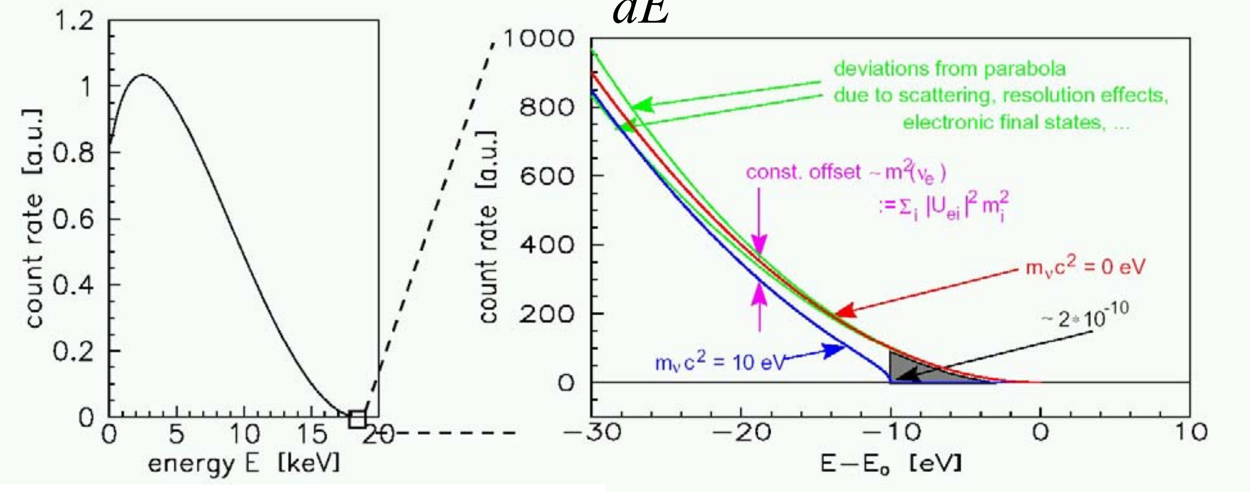
An introduction to Neutrino physics and oscillations

Direct mass measurements

- Beta decay spectrum for direct mass of ν_e: $(A,Z) \rightarrow (A,Z+1) + e^- + \overline{\nu}_e$
- Spectral shape: $$\frac{dn}{dE} = F(Z,E)\,p_e E (E_0 - E)\left((E_0 - E)^2 - m_\nu^2\right)^{1/2}$$



Kurie plot:

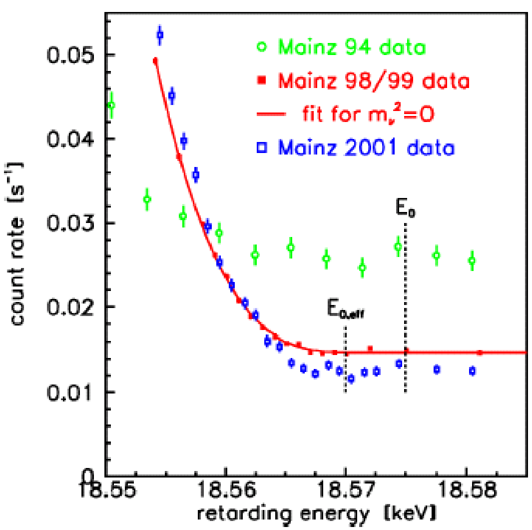
$$K(E) = \frac{1}{F p_e E}\frac{dn}{dE}$$

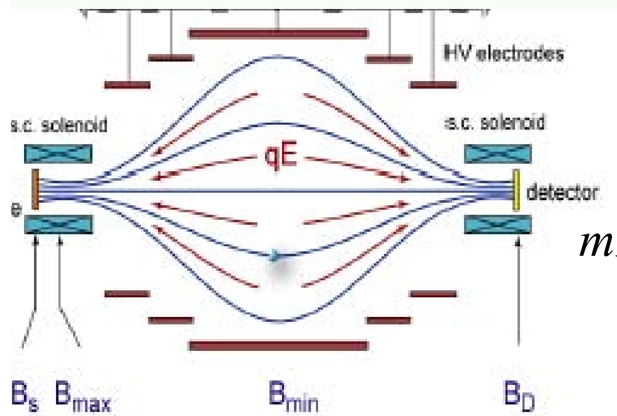
Tritium:

$$^3H \rightarrow {}^3He + e^- + \overline{\nu}_e$$

$$E_0 = M(^3H) - M(^3He) - m_e$$
$$= 18.574\ keV$$

$$\tau_{1/2} = 12.3\ years$$





$$m_\nu^2 = -1.2 \pm 2.2(\text{stat}) \pm 2.1(\text{syst})\ \text{eV}^2$$

Current limit:
m_{ν_e} < 2.2 eV (95% CL)
(Mainz, Troitsk)

Nuclear beta decay

The method of measurement of the (anti)neutrino mass through the investigation of the high-energy part of the β-spectrum was proposed by Perrin (1933) and Fermi (1934).
The first experiments on the measurement of the neutrino mass with this method have been done by Curran, Angus and Cockcroft (1948) and Hanna and Pontecorvo (1949).
The energy spectrum of electrons in the decay $(A,Z) \to (A,Z+1) + e^- + \overline{\nu}_e$ is[a]

$$\frac{d\Gamma}{dT} = \sum_k |V_{ek}|^2 \frac{d\Gamma_k}{dT}, \tag{1}$$

$$\frac{d\Gamma_k}{dT} = \frac{(G_F \cos\theta_C)^2}{2\pi^3}\, p p_k \,(T+m_e)\,(Q-T)\,|\mathcal{M}|^2\, F(T)\theta\,(Q-T-m_k). \tag{2}$$

Here G_F is the Fermi constant, θ_C is the Cabibbo angle, m_e, p and T are the mass, magnitude of the momentum and kinetic energy of the electron, respectively,

$$p_k = \sqrt{E_k^2 - m_k^2} = \sqrt{(Q-T)^2 - m_k^2}$$

is the magnitude of the neutrino momentum, Q is the energy released in the decay (the endpoint of the β spectrum in case of zero neutrino mass), $\mathcal{M}$ is the nuclear matrix element, and $F(T)$ is the Fermi function, which describes the Coulomb interaction of the final particles. The step function in Eq. (2) ensures that a neutrino

[a] The recoil of the final nucleus and radiative corrections (luckily small) are neglected.

state ν_k is only produced if its total energy is larger than its mass: $E_k = Q - T \geq m_k$. As it is seen from Eq. (1), the largest distortion of the β-spectrum due to neutrino masses can be observed in the region

$$Q - T \sim m_k. \tag{3}$$

However, for $\max(m_k) \simeq 1$ eV only a very small part (about 10^{-13}) of the decays give contribution to the region (3). This is the reason why in the analysis of the results of the measurement of the β-spectrum a relatively large part of the spectrum is used.[b] Taking this into account and applying unitarity of the mixing matrix, we can write

$$\sum_k |V_{ek}|^2 \, p_k \approx \sum_k |V_{ek}|^2 \, (Q - T) \left[1 - \frac{m_k^2}{2(Q-T)^2}\right]$$
$$= (Q - T) \left[1 - \frac{1}{2(Q-T)^2} \sum_k |V_{ek}|^2 \, m_k^2\right] \approx \sqrt{(Q-T)^2 - m_\beta^2},$$

where the effective neutrino mass m_β is defined by

$$m_\beta^2 = \sum_k |V_{ek}|^2 \, m_k^2$$

and it was assumed that $\max(m_k^2) \ll 4(Q-T)^2$.

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[b]For example, in the Mainz tritium experiment (see below) the last 70 eV of the spectrum is used.

Finally, the β-spectrum that is used for fitting the data can be presented as

$$\frac{d\Gamma}{dT} \propto p\,(T + m_e)\,|\mathcal{M}|^2\,F(T)K^2(T),$$

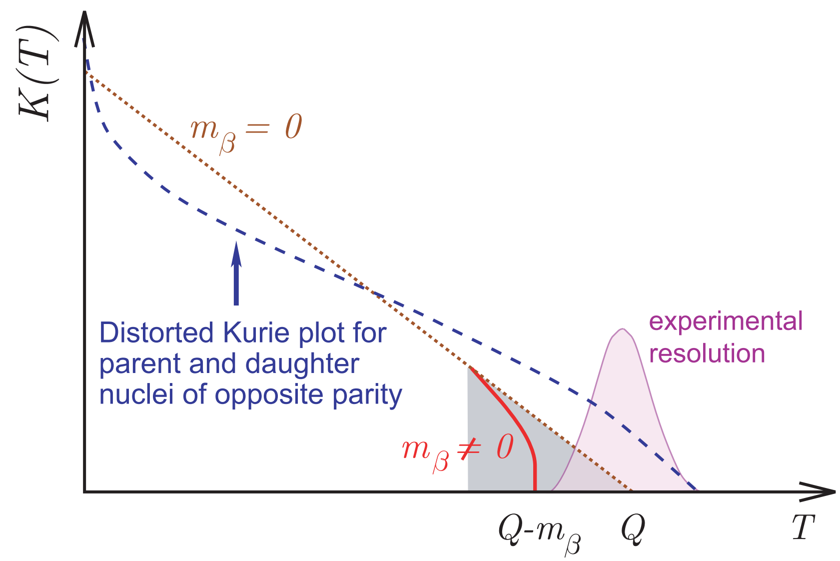
where

$$K(T) \propto \sqrt{\frac{d\Gamma/dT}{p\,(T + m_e)\,|\mathcal{M}|^2\,F(T)}}$$

$$\approx (Q - T)\left[1 - \frac{m_\beta^2}{(Q-T)^2}\right]^{1/4}$$

is the so-called *Kurie plot*.

Unfortunately, the real situation is much more complicated.



Kurie plot for allowed processes is a sensitive test of the effective neutrino mass m_β while the first order forbidden processes should have a distorted Kurie plot.

In an actual experiment, the measurable quantity is a sum of β spectra, leading each with probability $P_n = P_n(E_0 - V_n - E)$ to a final state n of excitation energy V_n:

$$\frac{d\Gamma(T,Q)}{dT} \longmapsto \sum_n P_n\,(E_0 - V_n - E)\,\frac{d\Gamma\,(T, E_0 - V_n)}{dT}.$$

Here $E_0 = Q - \mathcal{E}$ and $\mathcal{E}$ is the recoil energy of the daughter nucleus.

Example: Tritium beta decay. [a]

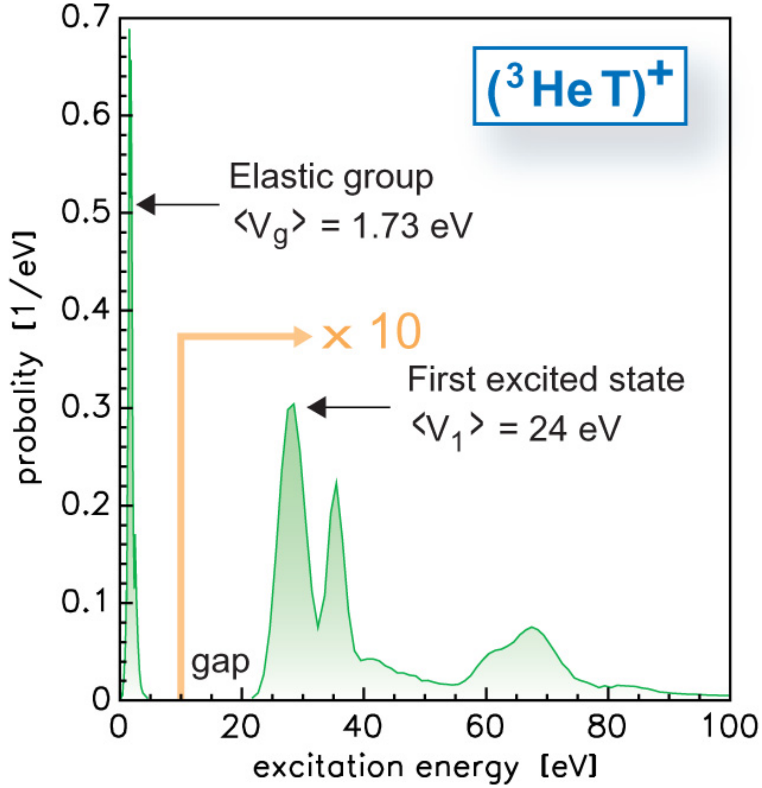
An important issue is the decay of molecular tritium $\mathsf{T}_2 \to ({}^3\mathsf{HeT})^+ + e^- + \overline{\nu}_e$. Considering the most precise direct determination of the mass difference

$$m(\mathsf{T}) - m\left({}^3\mathsf{He}\right) = (18590.1 \pm 1.7)\ \mathsf{eV}/c^2$$

and taking into account the recoil and apparative effects (these are taken for the Mainz experiment) one derives an endpoint energy of the molecular ion $({}^3\mathsf{HeT})^+$ ground state:

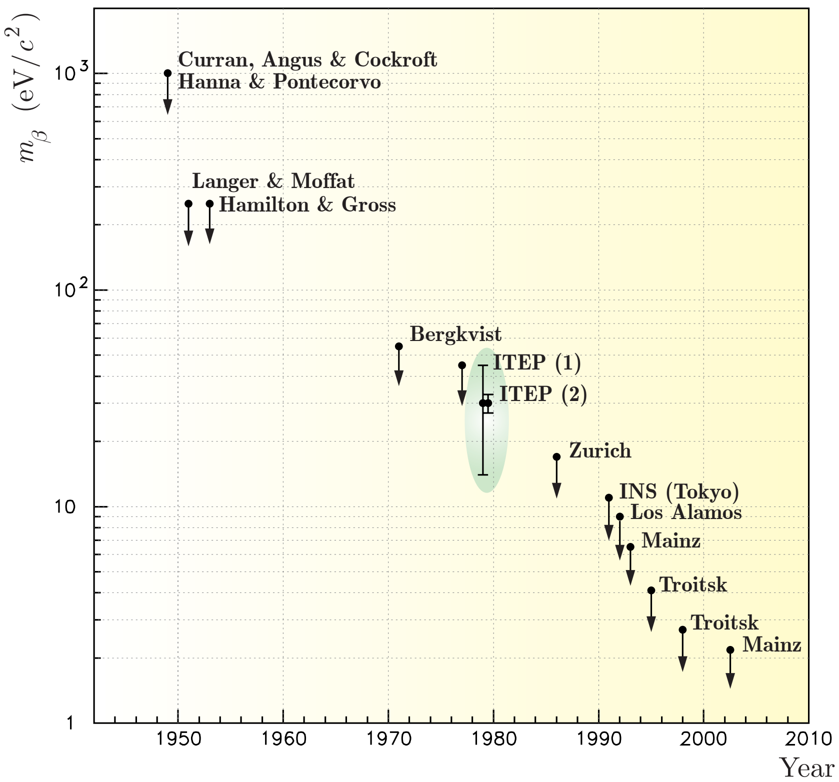
$$E_0 = (18574.3 \pm 1.7)\ \mathsf{eV}.$$

The excitation spectrum is shown in the figure. The first group concerns rotational and vibrational excitation of the molecule in its electronic ground state; it comprises a fraction of $P_g = 57.4\%$ of the total rate.



Excitation spectrum of the daughter molecular ion $({}^3\mathsf{HeT})^+$ in β decay of molecular tritium.

[a] For more details, see C. Kraus *et al.*, "Final results from phase II of the Mainz neutrino mass search in tritium β decay," Eur. Phys. J. C **40** (2005) 447–468 (hep-ex/0412056).



Progress of the neutrino mass measurements in tritium β decay, including the final Mainz phase II upper limit (see below).

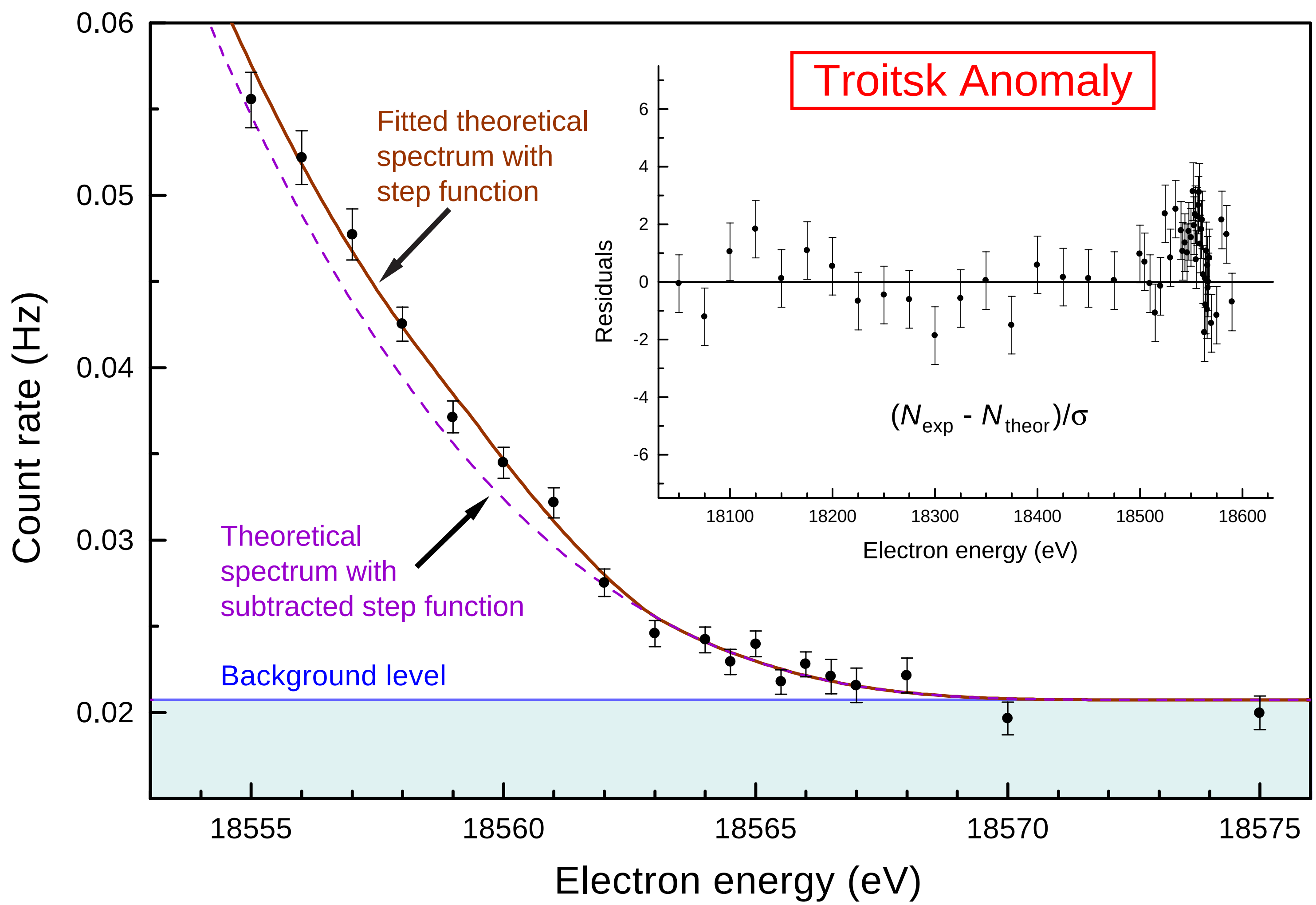
[The compilation is taken from V. M. Lobashev, "Direct search for mass of neutrino," in Proceedings of the 18th International Conference on Physics in Collision ("PIC 98"), Frascati, June 17–19, 1998, pp. 179–194.]

The history of the search for the neutrino mass in the tritium β decay counts almost 55 years (see figure). In 1980, the steady improvement of the upper limit was suddenly speeded up by a report of the ITEP group (Moscow) on the observation of the nonzero neutrino mass effect in the β-spectrum in the valine molecule ($C_5H_{11}NO_2$). The reported result was[a]

$$14 \le m_\beta \le 46 \text{ eV}/c^2 \quad (99\% \text{ C.L.})$$

This research stimulated more than 20 experimental proposals with an intention to check this clime. Alas!... in several years the experimental groups from Zurich, Tokyo, Los Alamos, and then Livermore refuted the ITEP result.

[a] V. A. Lyubimov *et al.*, "An estimate of the ν_e mass from the β-spectrum of tritium in the valine molecule," Phys. Lett. B **94** (1980) 266–268 (327 citations in SPIRES!)



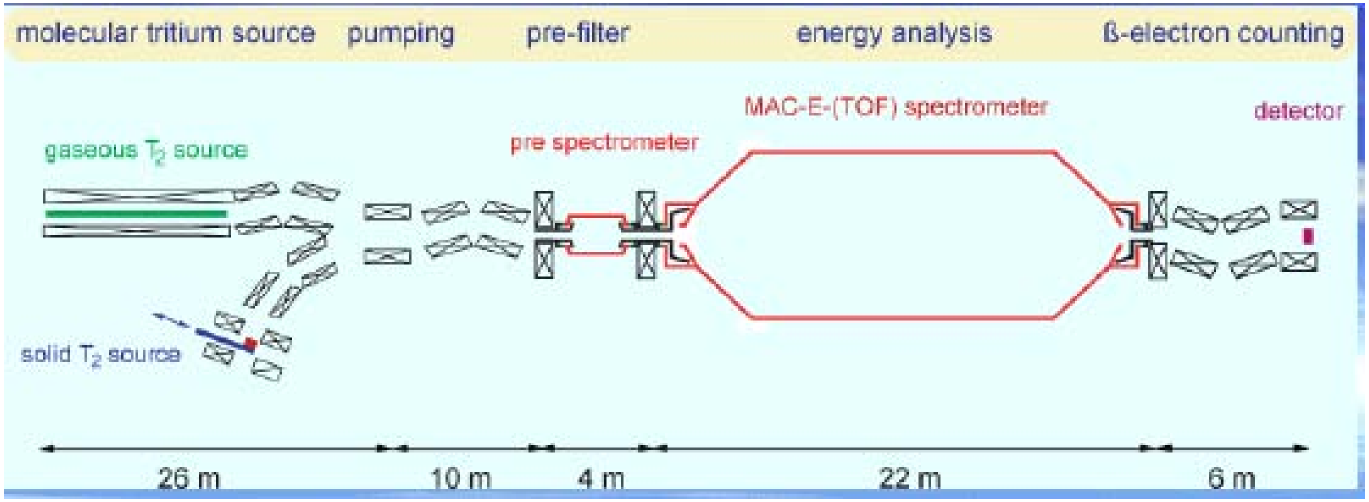
Part of experimental β-spectrum in the Troitsk ν-mass experiment near the end-point.

[V. M. Lobashev *et al.*, "Direct search for mass of neutrino and anomaly in the tritium beta-spectrum," Phys. Lett. B **460** (1999) 227–235.]

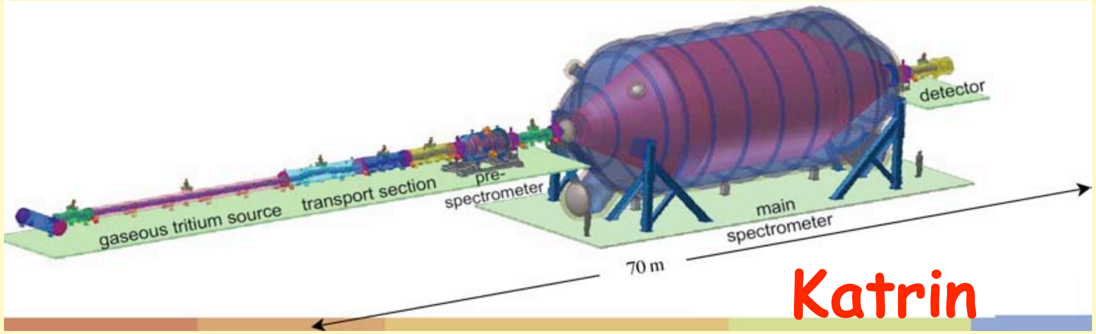
Direct mass measurements (cont.)

- Katrin (Kalsruhe Tritium Neutrino Experiment): due to start 2008.

Expected limit: $m_{\nu_e} < 0.2$ eV (90% c.l.)
Discovery potential: $m_{\nu_e} = 0.35$ eV at 5σ



… to huge (ultimate?) magnetic spectrometers



Direct mass measurements (cont)

- Direct mass of ν_μ from pion decay at rest: $\pi^+ \rightarrow \mu^+ + \nu_\mu$

  Experiment at PSI, Switzerland (Phys Rev D53, (1996) 6065.):

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2m_\pi \left(p_\mu^2 + m_\mu^2\right)^{1/2}$$

$$m_\pi = 139.56995 \pm 0.00035\ MeV$$

$$m_\mu = 105.658357 \pm 0.000005\ MeV$$

$$p_\mu = 29.79200 \pm 0.00011\ MeV$$

$$\Rightarrow m_{\nu_\mu}^2 = \left(-0.016 \pm 0.023\right) MeV^2$$

Measure muon momentum

π^- bound to $_{12}Mg^{24}$ nuclei:
ΔA(4f->3d)=23.9 keV

μ^+ magnetic moment = $\dfrac{ge\hbar}{2m_\mu}$

Current limit: m_{ν_μ} < 190 keV (90% c.l.)

Direct mass measurements (cont)

- Direct mass of ν_τ from multi-prong tau decay:

Most sensitive: ALEPH (LEP)

$$e^+e^- \to \tau^+\tau^-$$

$$\tau^- \to e^-\overline{\nu}_e\nu_\tau, \mu^-\overline{\nu}_\mu\nu_\tau, \pi^-\nu_\tau, \pi^-\pi^0\nu_\tau \text{ (one prong)}$$

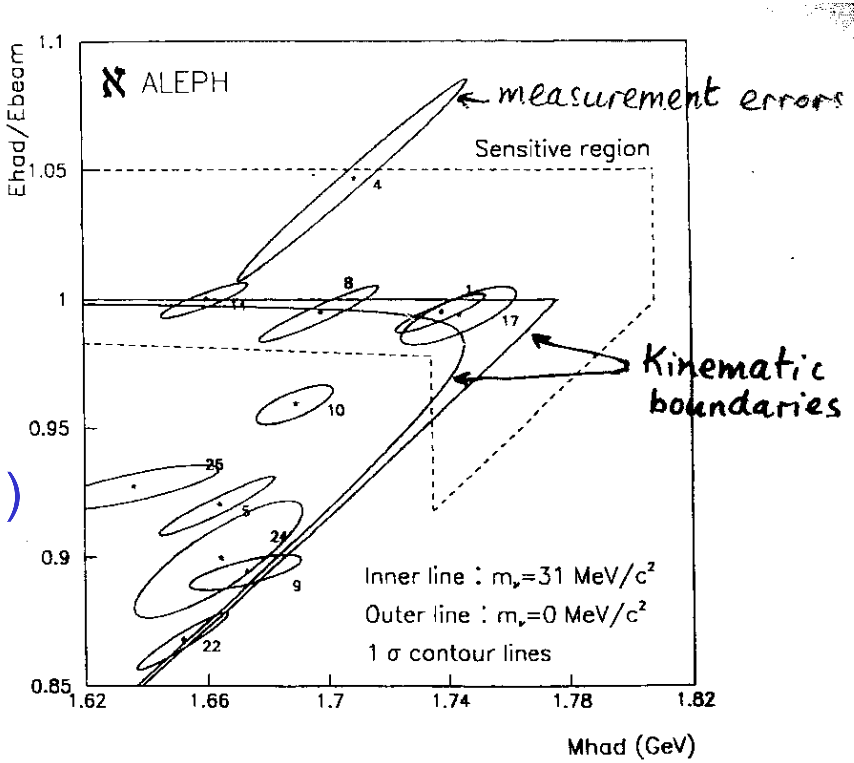
$$\tau^+ \to \pi^+\pi^+\pi^+\pi^-\pi^-\overline{\nu}_\tau, \pi^+\pi^+\pi^+\pi^-\pi^-\pi^0\overline{\nu}_\tau \text{ (multi-prong)}$$

Invariant mass multi-pion system:

$$M_{had}^2 = E_{had}^2 - \left|p_{had}^2\right|$$

$$E_{had} < E_\tau = E_{beam}$$

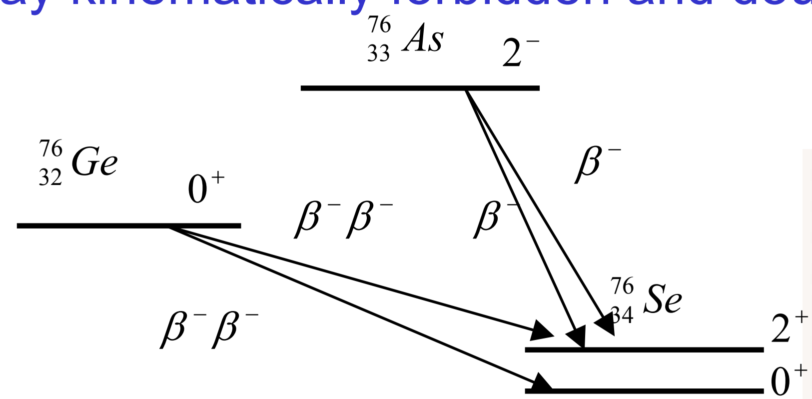
$$M_{had} < m_\tau$$



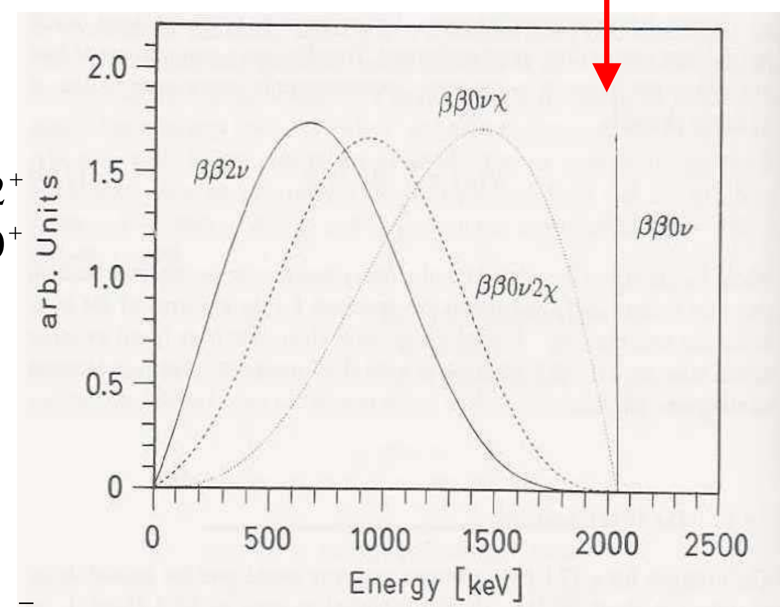
Current limit: m_{ν_τ} < 18.2 MeV (90% c.l.)

Double beta decay

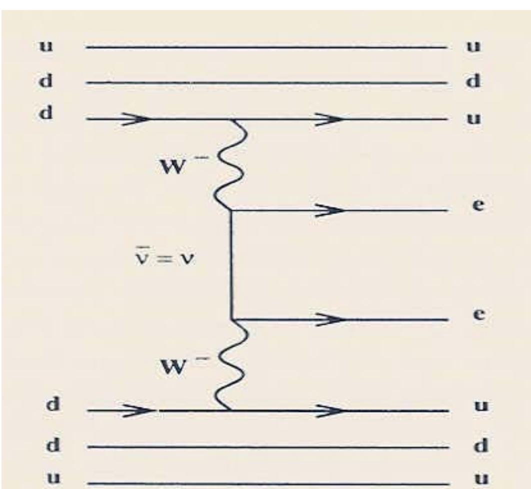
- Double beta decay: $(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\overline{\nu}_e$
- 35 possible beta emitters (even-even nuclei). Very rare (half-life more than 10^{20} years) since second order process.
- Beta decay kinematically forbidden and double beta kinematically allowed



Spike at end of spectrum



- If neutrino Majorana particle, neutrinoless double beta decay ($0\nu\beta\beta$) allowed:



$$\nu_e = \overline{\nu}_e$$

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-}$$

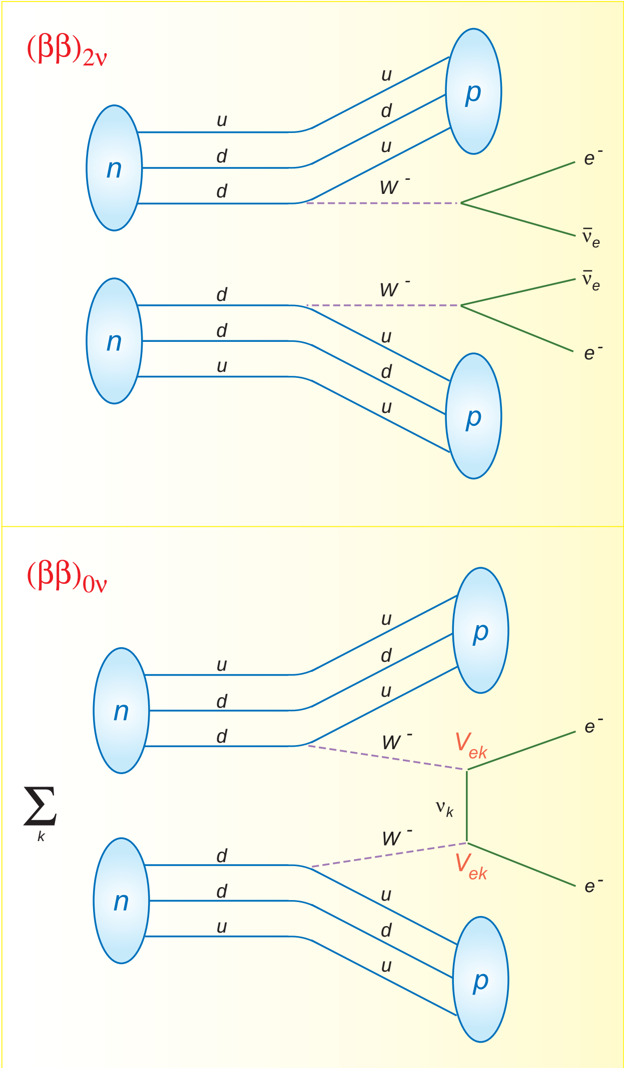
Neutrinoless double beta decay

The theory with Majorana neutrinos allows the decay $(A, Z) \rightarrow (A, Z+2) + 2e^-$ with $\Delta L = 2$. The decay rate for this process is expressed as follows:

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_Z^{0\nu} \left|m_{\beta\beta}\right|^2 \left|\mathcal{M}_{\mathsf{F}}^{0\nu} - (g_A/g_V)^2 \mathcal{M}_{\mathsf{GT}}^{0\nu}\right|^2,$$

where $G_Z^{0\nu}$ is the two-body phase-space factor including coupling constant, $\mathcal{M}_{\mathsf{F/GT}}^{0\nu}$ are the Fermi/Gamow-Teller nuclear matrix elements. The constants g_V and g_A are the vector and axial-vector relative weak coupling constants, respectively. The complex parameter $m_{\beta\beta}$ is the *effective Majorana electron neutrino mass* given by

$$m_{\beta\beta} = \sum_k V_{ek}^2 m_k$$
$$= \left|V_{e1}\right|^2 m_1 + \left|V_{e2}\right|^2 m_2 e^{i\phi_2} + \left|V_{e3}\right|^2 m_3 e^{i\phi_3},$$

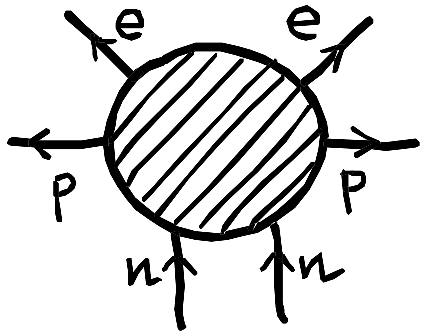


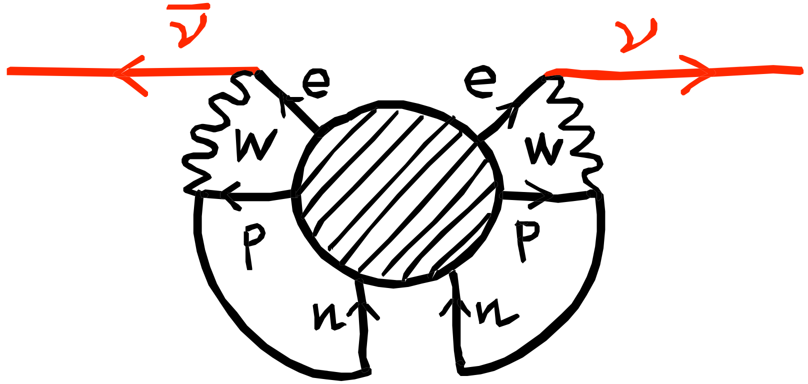
Here $\phi_2 = \alpha_2 - \alpha_1$ (pure Majorana phase) and $\phi_3 = -(\alpha_2 + 2\delta)$ (mixture of Dirac and Majorana CP-violation phases).

Deep link between $0\nu 2\beta$ decay and Majorana ν

Independently of the mechanism for $0\nu 2\beta$ decay...

... get a Majorana neutrino mass term if $0\nu 2\beta$ occurs



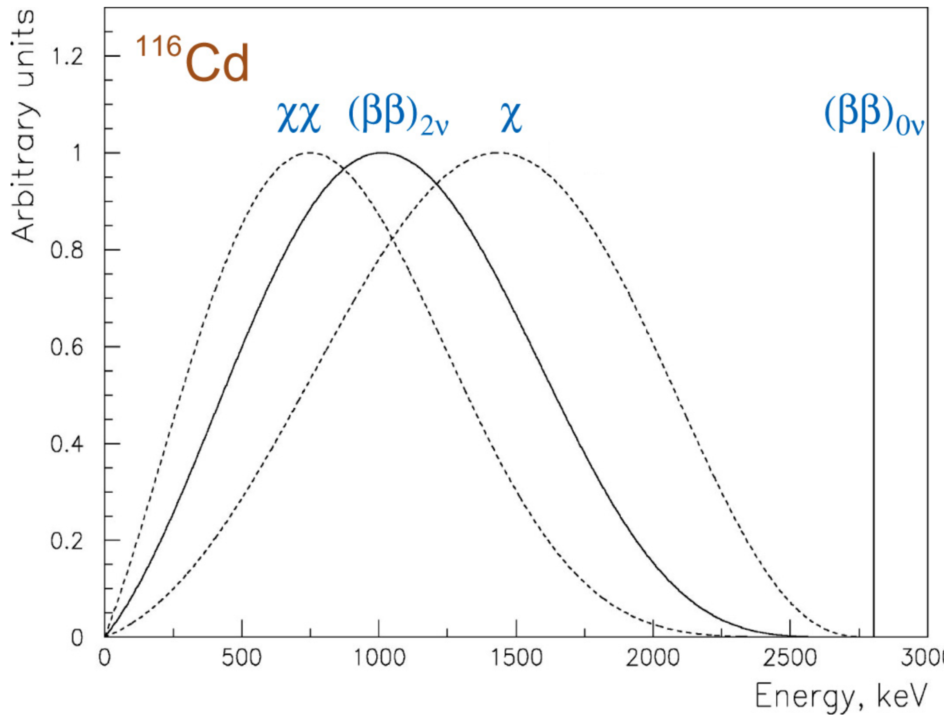


The electron sum energy spectrum of the $(\beta\beta)_{2\nu}$ mode as well as of the exotic modes with one or two majorons in final state,

$$(A, Z) \to (A, Z+2) + 2e^- + \chi,$$

$$(A, Z) \to (A, Z+2) + 2e^- + 2\chi,$$

is *continuous* because the available energy release ($Q_{\beta\beta}$) is shared between the electrons and other final state particles. In contrast, the two electrons from the $(\beta\beta)_{0\nu}$ decay carry the full available energy, and hence the electron sum energy spectrum has a sharp peak at the $Q_{\beta\beta}$ value. This feature allows one to distinguish the $(\beta\beta)_{0\nu}$ decay signal from the background.



The electron sum energy spectra calculated for the different β decay modes of cadmium-116.

[From Y. Zdesenko, "Colloquium: The future of double beta decay research," Rev. Mod. Phys. **74** (2003) 663–684.]

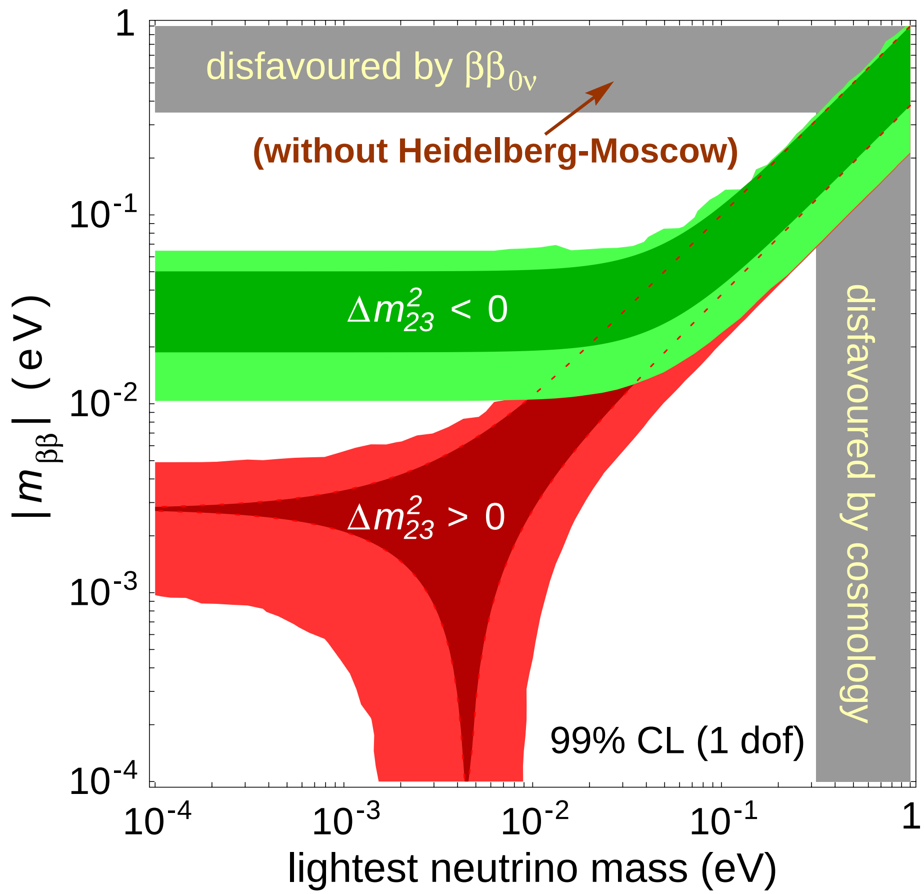
Majoron is a Nambu-Goldstone boson, – a hypothetical neutral pseudoscalar zero-mass particle which couples to Majorana neutrinos and may be emitted in the neutrinoless β decay. It is a consequence of the spontaneous breaking of the global $B - L$ symmetry.

Table 2: Summary of the most recent $\beta\beta_{2\nu}$ experiments and calculations.
[From E. Fiorini, "Experimental prospects of neutrinoless double beta decay," Phys. Scripta **T121** (2005) 86–93].

Element	Isotope	$T_{1/2}^{2\nu}$ (years) Measured	Calculated
Calcium	$^{48}_{20}$Ca	$4.2^{+2.1}_{-1.0} \times 10^{19}$	$6 \times 10^{18} - 5 \times 10^{20}$
Germanium	$^{76}_{32}$Ge	$1.42^{+0.09}_{-0.07} \times 10^{21}$	$7 \times 10^{19} - 6 \times 10^{22}$
Selenium	$^{82}_{34}$Se	$(0.9 \pm 0.1) \times 10^{20}$	$3 \times 10^{18} - 6 \times 10^{21}$
Zirconium	$^{96}_{40}$Zr	$2.1^{+0.8}_{-0.4} \times 10^{19}$	$3 \times 10^{17} - 6 \times 10^{20}$
Molybdenum	$^{100}_{42}$Mo	$(8.0 \pm 0.7) \times 10^{18}$	$1 \times 10^{17} - 2 \times 10^{22}$
Molybdenum	$^{100}_{42}$Mo (0^{+*})	$(6.8 \pm 1.2) \times 10^{20}$	$5 \times 10^{19} - 2 \times 10^{21}$
Cadmium	$^{116}_{48}$Cd	$3.3^{+0.4}_{-0.3} \times 10^{19}$	$3 \times 10^{18} - 2 \times 10^{21}$
Tellurium	$^{128}_{52}$Te	$(2.5 \pm 0.4) \times 10^{24}$	$9 \times 10^{22} - 3 \times 10^{25}$
Tellurium	$^{130}_{52}$Te	$(9.0 \pm 1.5) \times 10^{20}$	$2 \times 10^{19} - 7 \times 10^{20}$
Neodymium	$^{150}_{60}$Nd	$(7.0 \pm 1.7) \times 10^{18}$	$6 \times 10^{16} - 4 \times 10^{20}$
Uranium	$^{238}_{92}$U	$(2.0 \pm 0.6) \times 10^{21}$	$1.2 \times 10^{19} - ? \times 10^{21}$

The standard $(\beta\beta)_{2\nu}$ is observed for 10 isotopes with $T_{1/2}^{2\nu} \sim 10^{19-25}$ years (see table).

The figure summarizes the present knowledge of the absolute Majorana mass scale. Shown are the 99% CL regions allowed by the neutrino oscillation data in the plane of $m_{\beta\beta}$ and m_L – the mass of lightest neutrino. The two bands marked with $\Delta m^2_{23} > 0$ and $\Delta m^2_{23} < 0$ correspond to the *normal mass hierarchy* (i.e. $m_1 \ll m_2 \ll m_3$) and *inverted mass hierarchy* (i.e. $m_3 \ll m_1 \approx m_2$), respectively.
For a given m_L the range of $m_{\beta\beta}$ is determined by variations of the Majorana phase and uncertainties in the neutrino oscillation parameters.

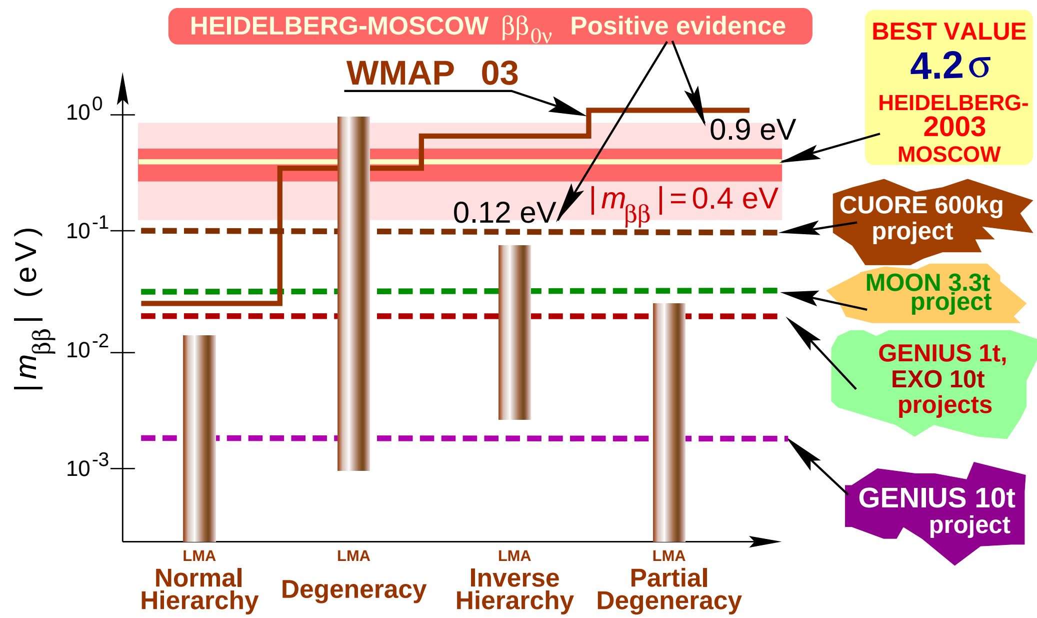


The darker regions show how the $m_{\beta\beta}$ range would shrink if the present best-fit values of the oscillation parameters were confirmed with negligible error.

[From A. Strumia and F. Vissani, "Implications of neutrino data circa 2005," Nucl. Phys. B **726** (2005) 294–316 (hep-ph/0503246).]

The only evidence for the $(\beta\beta)_{0\nu}$ decay has been obtained by the Heidelberg-Moscow (HM) (**sub**)collaboration in the Gran Sasso lab. The HM best value of the effective neutrino mass is $|m_{\beta\beta}| = 0.4$ eV. Allowing conservatively for an uncertainty of the nuclear matrix element of $\pm 50\%$ the 3σ confidence range may widen to $(0.1 - 0.9)$ eV.

The bars in the figure denote allowed ranges of $|m_{\beta\beta}|$ in different neutrino mass scenarios, still allowed by neutrino oscillation experiments. All models except the *degenerate* one are excluded by the new $(\beta\beta)_{0\nu}$ decay result. Also shown is the exclusion line from WMAP, plotted for $\sum_k m_k < 1.0$ eV (which is perhaps too strict). WMAP does not rule out any of the neutrino mass schemes.



Further shown are the expected sensitivities for the future potential $(\beta\beta)_{0\nu}$ decay experiments CUORE, MOON, EXO and GENIUS.

[From H. V. Klapdor-Kleingrothaus, "First evidence for neutrinoless double beta decay and world status of double beta experiments," hep-ph/0512263.]

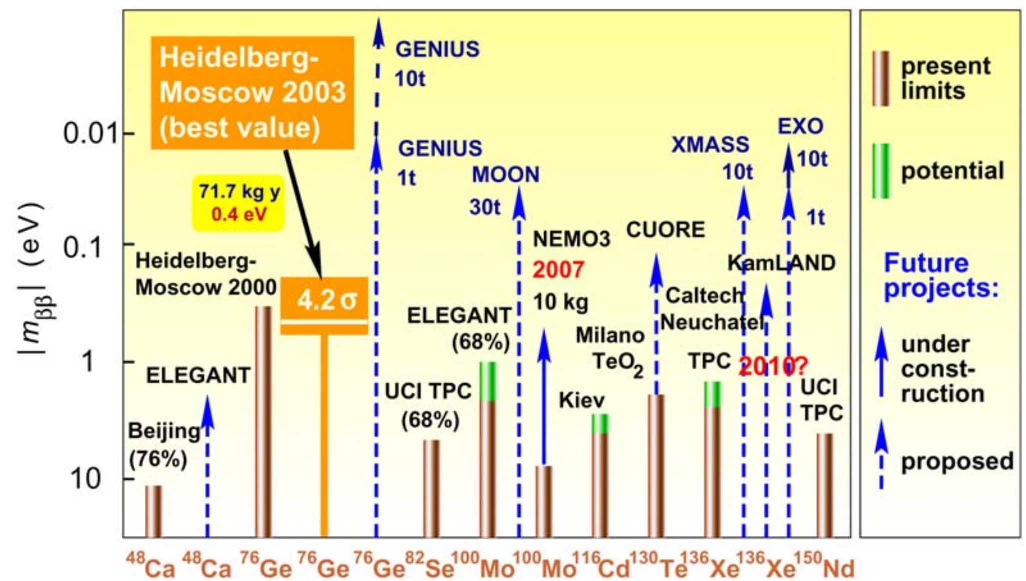
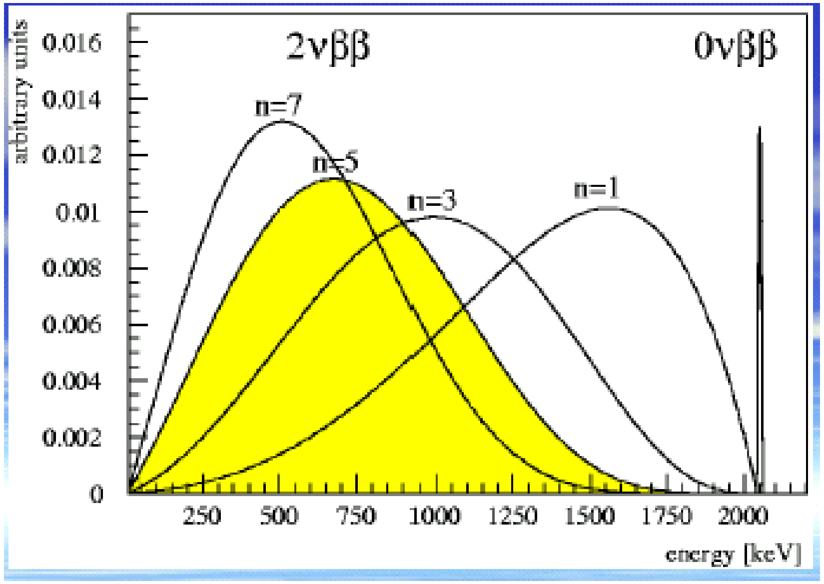


Figure shows the HM-2000 and HM-2003 results in comparison with the potential of the most promising other $(\beta\beta)_{0\nu}$ experiments as well as the expected potential of several future projects. Given are limits for $|m_{\beta\beta}|$, except for the HM-2003 experiment where the measured *value* is given (confidence range and best value).

[The histogram is built by combining the data from papers H. V. Klapdor-Kleingrothaus *et al.*, "Latest results from the Heidelberg-Moscow double beta decay experiment," Eur. Phys. J. A **12** (2001) 147–154 (hep-ph/0103062) and H. V. Klapdor-Kleingrothaus, "First evidence for neutrinoless double beta decay and world status of double beta experiments," in Proceedings of the 11th International Workshop on Neutrino Telescopes, Venice, Feb. 22–25, 2005, edited by M. Baldo-Ceolin, pp. 215–237 (hep-ph/0512263).]

New approaches and considerably enlarged experiments are required to fix the $(\beta\beta)_{0\nu}$ half life with higher accuracy. This will, however, only marginally improve the precision of the deduced effective neutrino mass $|m_{\beta\beta}|$ (or its upper limit), because of the uncertainties in the nuclear matrix elements, which probably hardly can be reduced to less than 50%.

Double beta decay (cont)

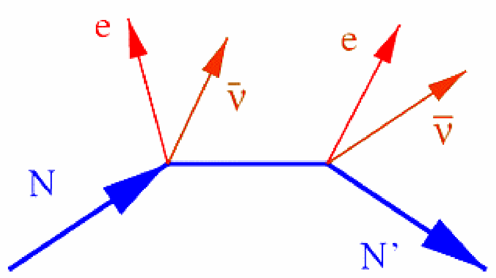


$$\langle m_\nu \rangle = m_{ee} = \left| \sum_k U_{ek}^2 m_k \right| = \left| \sum_k |U_{ek}|^2 e^{i\alpha_{ek}} m_k \right|$$

$$m_k = mass\ eigenstates$$

Isotope	$T_{1/2}^{0\nu}$ (y)	$\langle m_\nu \rangle$ (eV)
^{48}Ca	$> 9.5 \times 10^{21}$ (76%)	< 8.3
^{76}Ge	$> 1.9 \times 10^{25}$	< 0.35
	$> 1.6 \times 10^{25}$	$< 0.33 - 1.35$
^{82}Se	$> 2.7 \times 10^{22}$ (68%)	< 5
^{100}Mo	$> 5.5 \times 10^{22}$	< 2.1
^{116}Cd	$> 7 \times 10^{22}$	< 2.6
^{128,130}Te	$\frac{T_{1/2}(130)}{T_{1/2}(128)} = (3.52 \pm 0.11) \times 10^{-4}$ (geochemical)	$< 1.1 - 1.5$
^{128}Te	$> 7.7 \times 10^{24}$	$< 1.1 - 1.5$
^{130}Te	$> 1.4 \times 10^{23}$	$< 1.1 - 2.6$
^{136}Xe	$> 4.4 \times 10^{23}$	$< 1.8 - 5.2$
^{150}Nd	$> 1.2 \times 10^{21}$	< 3

2ν ββ decay: a standard process in nuclear physics

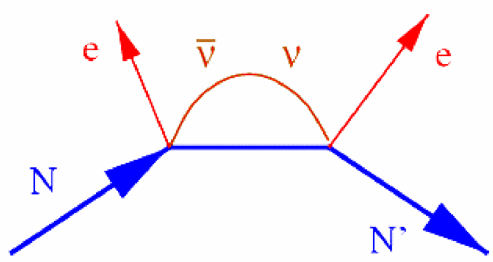


0ν ββ decay: a hypothetical process

→ $m_\nu \neq 0$

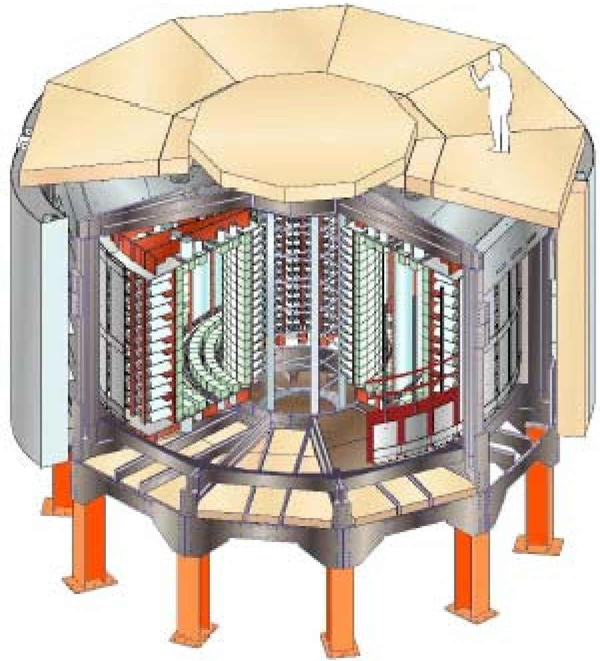
→ $\bar{\nu} = \nu$

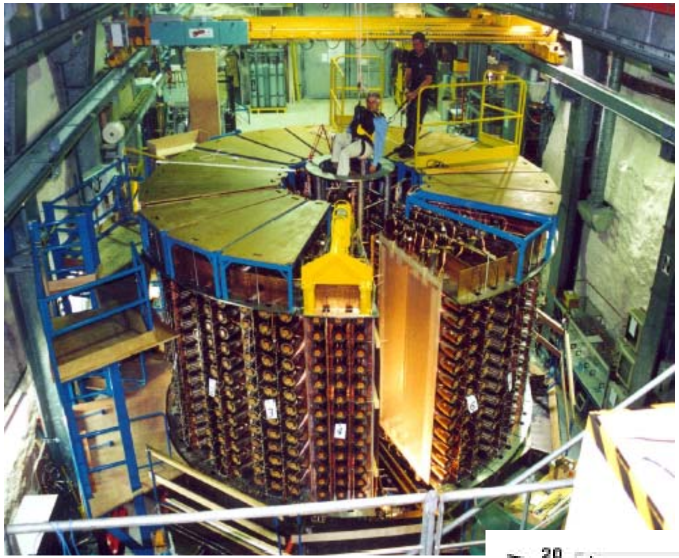
since helicity has to "flip"



Double beta decay (cont)

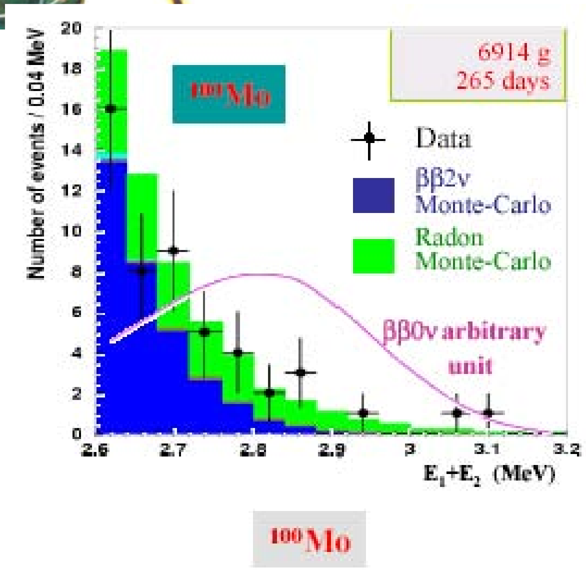
- NEMO3 double beta decay experiment: Frejus tunnel (4800 mwe)





- 10 kg of different ββ emitting isotopes, tracking chamber in B field and calorimeter

$$^{100}\mathrm{Mo}: \quad T_{1/2}(\beta\beta 0\nu) < 3\times 10^{23}\ \mathrm{yr.}$$



Double beta decay (cont)

- Heidelberg-Moscow experiment: Gran Sasso Laboratory

Five Ge diodes (10.9 kg) – 71.7 kg yr data

Isotopically enriched (86%) in ^{76}Ge

Pb and Cu shield and nitrogen purging

Q-value of reaction: 2040.71 keV

Controversial claim for discovery:

H.V. Klapdor-Kleingrothaus et al.

Mod Phys Lett A 16 (2001), 2409

Phys Lett B 586 (2004), 198.

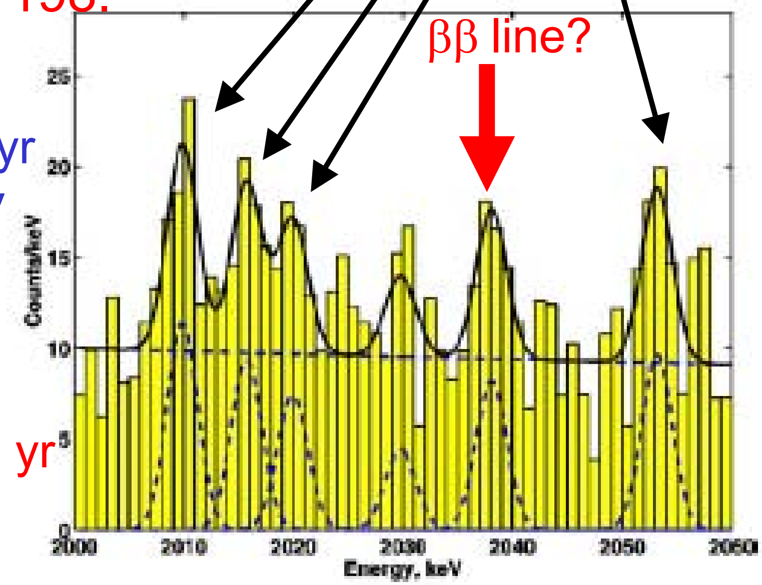
4.2σ signal:

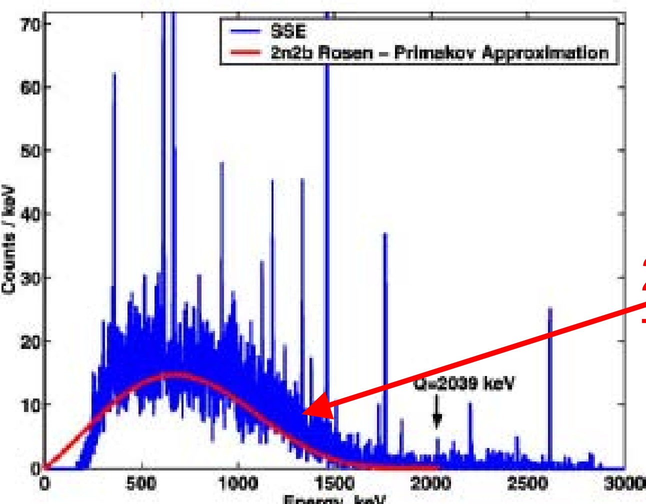
$T_{1/2}$ = (0.69-4.18)x10^{25} yr

$\Rightarrow$ <m> = 0.24-0.58 eV

^{214}Bi lines

ββ line?





2νββ spectrum:

$T_{1/2}$ = (1.74+-0.18)x10^{21} yr