Global Overview of Mixing and Masses

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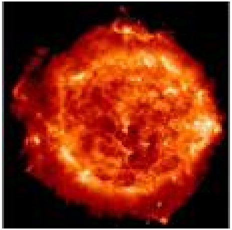
Neutrino 08, May 27, Christchurch, New Zealand

A Fairly Exotic Effect ...

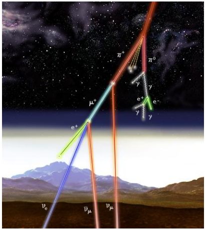
"In as far as the neutrino masses are negligeable compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effects such as neutrino oscillations."

(Froggatt and Nielsen, 1978

Several Evidences that Nature Likes *Exotic*!



Solar ν



Atmospheric ν

Reactor ν

Accelerator ν

The Standard Framework: masses, mixings and phases

ν_e, ν_μ, ν_τ (*flavor eigenstates*) $\neq$ ν_1, ν_2, ν_3 (mass eigenstates)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad U = V \, \mathrm{diag}(1, e^{i\alpha_2/2}, e^{i(\alpha_3+2\delta)/2})$$

Majorana CP violating Phases

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$ $\quad s_{ij} \equiv \sin\theta_{ij}$ $\quad \theta_{ij} \in [0, \pi/2]$ $\quad \delta \in [0, 2\pi]$ $\quad \alpha_i \in [0, 2\pi]$

Oscillation in Vacuum and Matter

In the flavor basis the ultrarelativistic neutrino propagation is described by

$$\mathbf{H} = \frac{1}{2\mathbf{E}} \mathbf{U}\,\mathbf{M}^2\,\mathbf{U}^\dagger + \mathbf{V}_{\rm mat} \qquad \mathbf{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta \mathbf{m}^2_{21} & 0 \\ 0 & 0 & \Delta \mathbf{m}^2_{31} \end{pmatrix}$$

$$\mathbf{V}_{\rm mat} = diag(\sqrt{2}\, G_F\, n_e(x), 0, 0) \qquad \Delta \mathbf{m}^2_{ij} = \mathbf{m}^2_i - \mathbf{m}^2_j$$

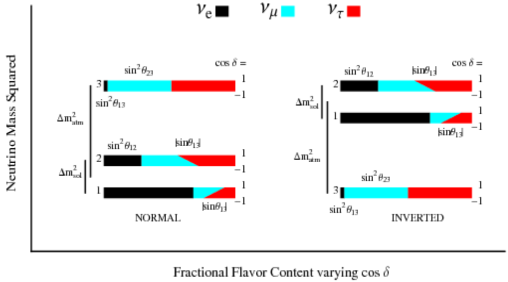
$$\bar{\nu} : \mathbf{U} \to \mathbf{U}^*, \mathbf{V}_{\rm mat} \to -\mathbf{V}_{\rm mat}$$

Mass Scales and Hierarchies

Current experimental results imply:

$$\Delta \mathrm{m}^2_{12} = \Delta \mathrm{m}^2_{\odot} << \Delta \mathrm{m}^2_{\mathrm{atm}} = |\Delta \mathrm{m}^2_{32}| \approx |\Delta \mathrm{m}^2_{31}|$$

Two possible hierarchies:



[H. Nunokawa, S. Parke, J.W.F. Valle, Prog. Part. Nucl. Phys. 60, 338 (2008)] $\sin^2 2\theta_{13} = 0.1$

General Features From Experimental Data

- $\Delta m^2_\odot / \Delta m^2_{\rm atm} \approx 0.03$
- $\sin^2\theta_{13} < 0.04$ **(CHOOZ)**
- **2-generation analysis is a very good description!**
  but unfortunatelly no sensitivity to δ or hierarchy...
- **Atmospheric and Accelerator Experiments ($\nu_\mu \to \nu_\tau$):** $\Delta m^2_{\rm atm}$ **and** $\sin^2\theta_{23}$
- **Solar and Reactor Experiments ($\nu_e \to \nu_x$):** $\Delta m^2_\odot$ **and** $\sin^2\theta_{12}$

however 3-generation analysis performed since 2001
Let's see what we have learned from them ...

Sub-Leading Effect in Atmospheric

Effects of: $\Delta m^2_\odot, \sin^2\theta_{12}$, θ_{13} **hierarchy and** δ
For atmospheric the e-excess at sub or multi-GeV energies is given by ($r = \Phi_{\nu_\mu}/\Phi_{\nu_e}$**)**

$$\frac{N_e}{N_e^0} \simeq 1 + \delta_1 + \delta_2 + \delta_3$$

$$\delta_1 \simeq \sin^2 2\tilde{\theta}_{13}\ \sin^2\left(\Delta m^2_{31}\frac{\sin 2\theta_{13}}{\sin 2\tilde{\theta}_{13}}\frac{L}{4E}\right)(r\, s^2_{23} - 1)$$

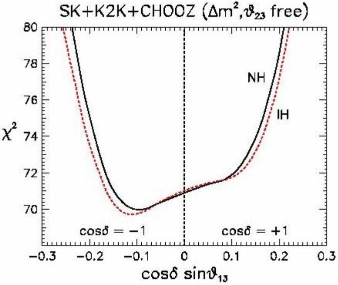
$$\delta_2 \simeq \sin^2 2\tilde{\theta}_{12}\ \sin^2\left(\Delta m^2_\odot\frac{\sin 2\theta_{12}}{\sin 2\tilde{\theta}_{12}}\frac{L}{4E}\right)(r\, c^2_{23} - 1)$$

$$\delta_3 \simeq \sin^2 2\tilde{\theta}_{12}\ \sin^2\left(\Delta m^2_\odot\frac{\sin 2\theta_{12}}{\sin 2\tilde{\theta}_{12}}\frac{L}{4E}\right) r\, s_{13}\cos\delta\, c^2_{13}\frac{\sin 2\theta_{23}}{\tan 2\tilde{\theta}_{12}}$$

Sub-Leading Effect in LBL

Effet of θ_{13}

$$P^{3\nu}_{\nu_\mu\nu_\mu} \simeq s^2_{13}\,\frac{\cos 2\theta_{23}}{c^2_{23}} + \left(1 - s^2_{13}\,\frac{\cos 2\theta_{23}}{c^2_{23}}\right)\, P^{2\nu}_{\nu_\mu\nu_\mu}(\Delta m^2_{32}, \theta_{23})$$



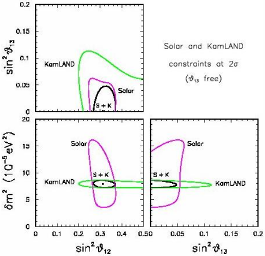
Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006)

Sub-Leading Effect in Solar+KamLAND

Effet of θ_{13}

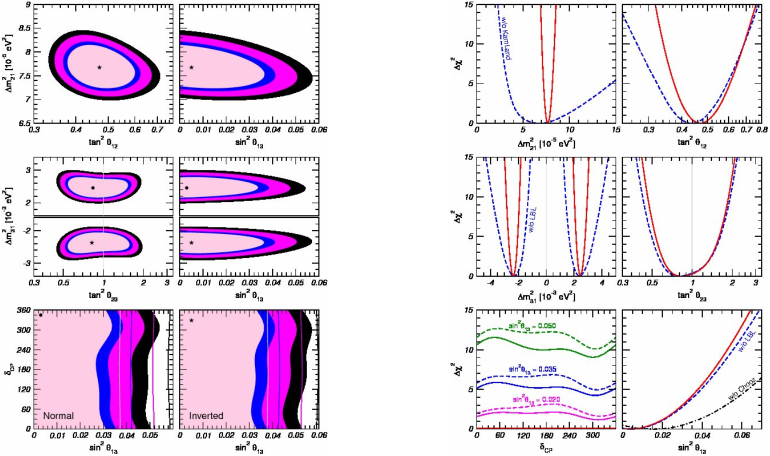
$$\mathbf{P}_{3\nu} = \mathbf{c}_{13}^{4}\ \mathbf{P}_{2\nu} + \mathbf{s}_{13}^{4} \quad \text{(KamLAND)}$$

$$\mathbf{P}_{3\nu} \simeq \mathbf{c}_{13}^{4}\ \mathbf{P}'_{2\nu} + \mathbf{s}_{13}^{4} \qquad \mathbf{P}'_{2\nu} = \mathbf{P}_{2\nu}\big|_{\mathbf{V}\to\mathbf{c}_{13}^{2}\mathbf{V}} \quad \text{(Solar)}$$



Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006)

Sub-leading Effects in Global Analysis (3 ν)



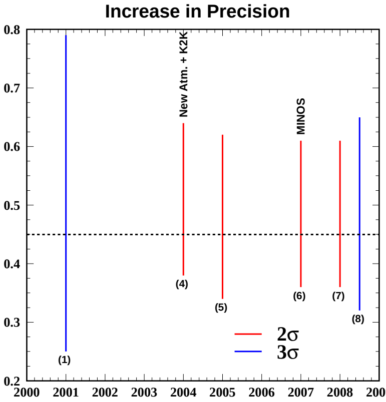
Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)

Determination of $\sin^2\theta_{23}$

(1) Gonzalez-Garcia *et al.*, Phys. Rev. D **63**, 033005 (2001)
(2) Fogli *et al.*, Phys. Rev. D **66**, 053010 (2002)
(3) Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003)
(4) Maltoni *et al.*, New. J. Phys. **6**, 122 (2004)
(5) Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006)
(6) Fogli *et al.*, Phys. Rev. D **75**, 053001 (2007)
(7) Fogli *et al.*, arXiv:0805.2517 (2008)
(8) Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)

$$\sin^2\theta_{23} = 0.45^{+0.16}_{-0.09} \quad (35\%)$$

$$\sin^2\theta_{23} = 0.45^{+0.20}_{-0.13} \quad (44\%)$$



Determination of $|\Delta m^2_{31}| \times 10^{-3}/\mathrm{eV}^2$

(1) Gonzalez-Garcia *et al.*, Phys. Rev. D **63**, 033005 (2001)
(2) Fogli *et al.*, Phys. Rev. D **66**, 053010 (2002)
(3) Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003)
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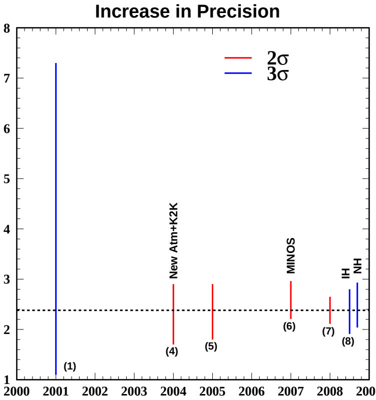
(7) Fogli *et al.*, arXiv:0805.2517 (2008)
(8) Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)

$$\Delta m^2_{31} = \begin{cases} +2.46^{+0.47}_{-0.42} \times 10^{-3}\ \mathrm{eV}^2 \\ -2.37^{+0.43}_{-0.46} \times 10^{-3}\ \mathrm{eV}^2 \end{cases}$$

$(\sim 19\%)$



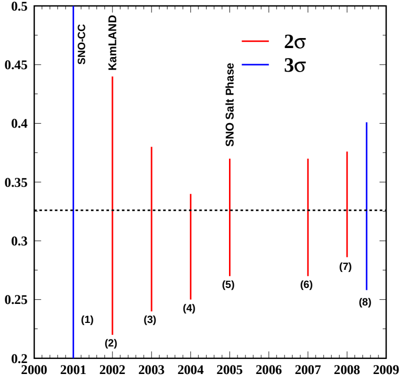
Determination of $\sin^2\theta_{12}$

(1) Gonzalez-Garcia *et al.*, Phys. Rev. D **63**, 033005 (2001)
(2) Fogli *et al.*, Phys. Rev. D **66**, 053010 (2002)
(3) Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003)
(4) Maltoni *et al.*, New. J. Phys. **6**, 122 (2004)
(5) Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006)
(6) Fogli *et al.*, Phys. Rev. D **75**, 053001 (2007)
(7) Fogli *et al.*, arXiv:0805.2517 (2008)
(8) Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)

$$\sin^2\theta_{12} = 0.33^{+0.05}_{-0.04} \quad (15\%)$$

$$\sin^2\theta_{12} = 0.32^{+0.08}_{-0.06} \quad (25\%)$$

Increase in Precision



Determination of $|\Delta m^2_{21}| \times 10^{-5}/\mathrm{eV}^2$

(1) Gonzalez-Garcia *et al.*, Phys. Rev. D **63**, 033005 (2001)
(2) Fogli *et al.*, Phys. Rev. D **66**, 053010 (2002)
(3) Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003)
(4) Maltoni *et al.*, New. J. Phys. **6**, 122 (2004)
(5) Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006)
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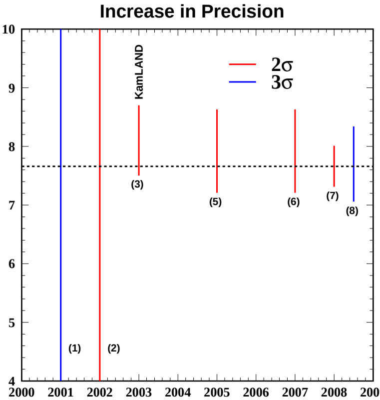
(7) Fogli *et al.*, arXiv:0805.2517 (2008)
(8) Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)

$$\Delta m^2_{21} = (7.66 \pm 0.35) \times 10^{-5}\ \mathrm{eV}^2$$

$(\sim 5\%)$

$$\Delta m^2_{21} = 7.67^{+0.67}_{-0.61} \times 10^{-5}\ \mathrm{eV}^2$$

$(\sim 9\%)$



Determination of $\sin^2\theta_{13}$

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(2) Fogli *et al.*, Phys. Rev. D **66**, 053010 (2002)
(3) Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003)
(4) Maltoni *et al.*, New. J. Phys. **6**, 122 (2004)
(5) Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006)
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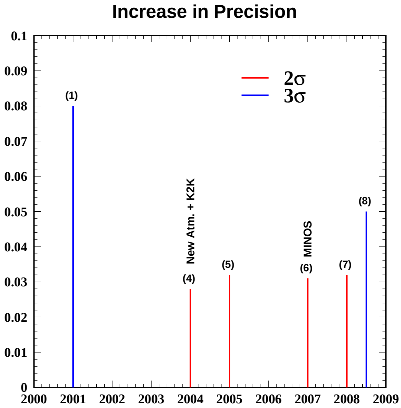
(8) Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)

$$\sin^2\theta_{13} < 0.03$$

$$\sin^2\theta_{13} < 0.05$$



Evaluation of the Mixing Matrix Entries

Take the values in 3 σ range (w/o correlations)

$$|V| = \begin{pmatrix} 0.76 - 0.86 & 0.50 - 0.62 & 0.00 - 0.22 \\ 0.42 - 0.51 & 0.34 - 0.71 & 0.57 - 0.79 \\ 0.29 - 0.42 & 0.49 - 0.71 & 0.58 - 0.82 \end{pmatrix}$$

Do a more sofisticated evaluation [Gonzalez-Garcia and Maltoni, Phys. Rep. 460, 1 (2008)]

$$|V|_{3\sigma} = \begin{pmatrix} 0.77 - 0.86 & 0.50 - 0.63 & 0.00 - 0.22 \\ 0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80 \\ 0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82 \end{pmatrix}$$

Tri-bimaximal Mixing Very Good Approximation

$$|\mathbf{V}_{\rm tbm}| = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

Masses and Hierarchies

Current experimental results imply:

$$\Delta m^2_{21} = \Delta m^2_{\odot} << \Delta m^2_{\rm atm} = |\Delta m^2_{32}| \approx |\Delta m^2_{31}|$$

Two possible hierarchies: $m_0, \Delta m^2{}_{\odot}, \Delta m^2{}_{\rm atm}$

NORMAL

$$m_1 = m_0$$
$$m_2 = \sqrt{m_0^2 + \Delta m^2_{\odot}}$$
$$m_3 = \sqrt{m_0^2 + \Delta m^2_{\odot} + \Delta m^2_{\rm atm}}$$

INVERTED

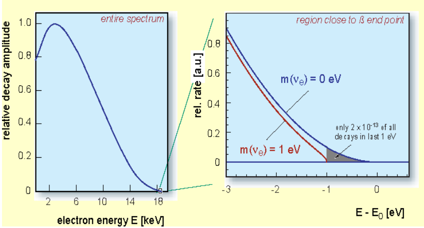
$$m_1 = \sqrt{m_0^2 - \Delta m^2_{\odot} + \Delta m^2_{\rm atm}}$$
$$m_2 = \sqrt{m_0^2 + \Delta m^2_{\rm atm}}$$
$$m_3 = m_0$$

Effective ν_e mass (m_β)

Single β-decay probes

$$\mathbf{m}_\beta = \sqrt{\sum_{\mathbf{i}} \mathbf{m}_{\mathbf{i}}^2 |\mathbf{U}_{\mathbf{ei}}|^2} = \sqrt{\mathbf{c}_{13}^2 \mathbf{c}_{12}^2 \mathbf{m}_1^2 + \mathbf{c}_{13}^2 \mathbf{s}_{12}^2 \mathbf{m}_2^2 + \mathbf{s}_{13}^2 \mathbf{m}_3^2}$$

Endpoint of the decay: $^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$



Absolute Mass Scale: Single β Decay

Most stringent limits today are quite weak

$m_\beta < 1.8\ (2.2)\ \mathrm{eV}$ **from Mainz+Troitsk (Mainz)**

unfortunatelly these bounds have little impact at the present moment

In the future Katrin may lower this down to 0.25 eV

Effective Majorana Mass $(m_{\beta\beta})$

Neutrinoless $\beta\beta$-decay – $(Z, A) \to (Z+2, A) + 2e^-$ – probes

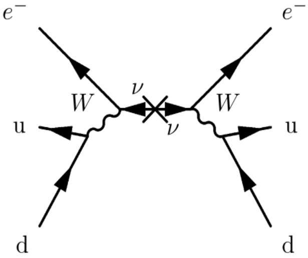
(if only Majorana mass term contributes)

$$\mathbf{m}_{\beta\beta} = \left|\sum_i U_{ei}^2 m_i\right| = \left|m_1 c_{13}^2 c_{12}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_2} + m_3 s_{13}^2 e^{i\alpha_3}\right|$$

if occurs through exchange of light ν

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} |M_{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

$M_{0\nu}$ = nuclear matrix elements
$G^{0\nu}$ = phase space integral



Absolute Mass Scale: Neutrinoless $\beta\beta$ Decay

- **part of Heidelberg-Moscow group has reported a signal in ^{76}Ge → $T^{0\nu}_{1/2} = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yrs claiming 6σ CL (controversial)**
- **Cuoricino group reported $T^{0\nu}_{1/2} > 2.5 \times 10^{24}$ yrs for ^{130}Te half life at 95% CL**

using nuclear matrix elements and uncertainties estimated by **Rodin, Faessler, Simkovic and Vogel, Nucl. Phys. A 766, 107 (2006)**

Currently Available Limits (2σ)

$$0.16 < \mathbf{m}_{\beta\beta}/\mathrm{eV} < 0.52 \;\; \text{(HM)}$$
$$0 \leq \mathbf{m}_{\beta\beta}/\mathrm{eV} \leq 0.23 \;\; \text{(Cuoricino A)}$$
$$0 \leq \mathbf{m}_{\beta\beta}/\mathrm{eV} \leq 0.85 \;\; \text{(Cuoricino B)}$$

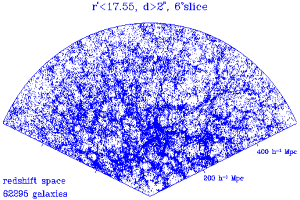
[Fogli *et al.*, arXiv:0805.2517]

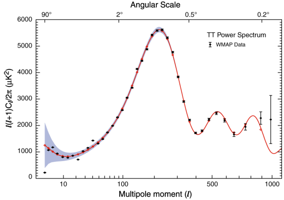
Neutrino Masses and Cosmology

Neutrinos contribute to energy density of our Universe + influence large scale structure formation

$$\Omega_\nu \mathbf{h}^2 = \Sigma/(\mathbf{94}\ \mathrm{eV}) \quad \Longrightarrow \Sigma = \mathbf{m_1} + \mathbf{m_2} + \mathbf{m_3}$$

bounds depend on data set included (CMB, LSS, BAO, Lymann-α etc.), priors and statistical treatment





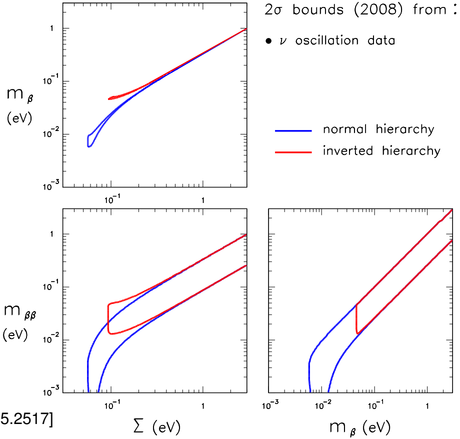
Absolute Mass Scale: Cosmological Limits

Very Strong Limits (but depend on data sets, priors and statistical treatment) (2σ)

(1) $\Sigma < 1.3$ eV **(WMAP5)**
(2) $\Sigma < 1.19$ eV **(WMAP5+ACBAR+VSA+CBI+BOOMERANG)**
(3) $\Sigma < 0.75$ eV **(CMB+HST+SN-Ia)**
(4) $\Sigma < 0.60$ eV **(CMB+HST+SN-Ia+BAO)**
(5) $\Sigma < 0.19$ eV **(CMB+HST+SN-Ia+BAO+Lyα)**

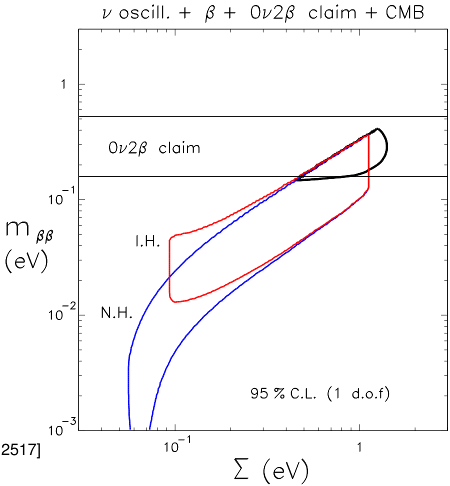
[Fogli *et al.*, arXiv:0805.2517]

Regions Allowed by Neutrino Oscillation Data



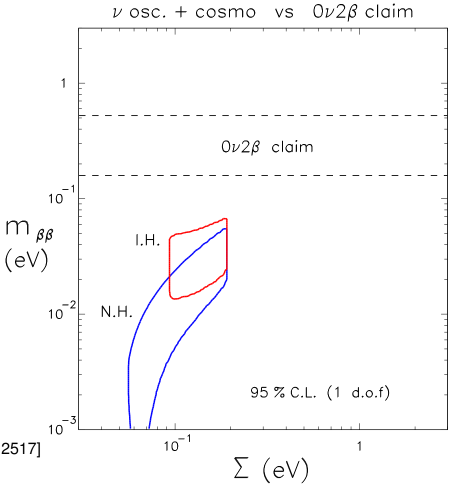
[Fogli *et al.*, arXiv:0805.2517]

Only CMB - Case (2)



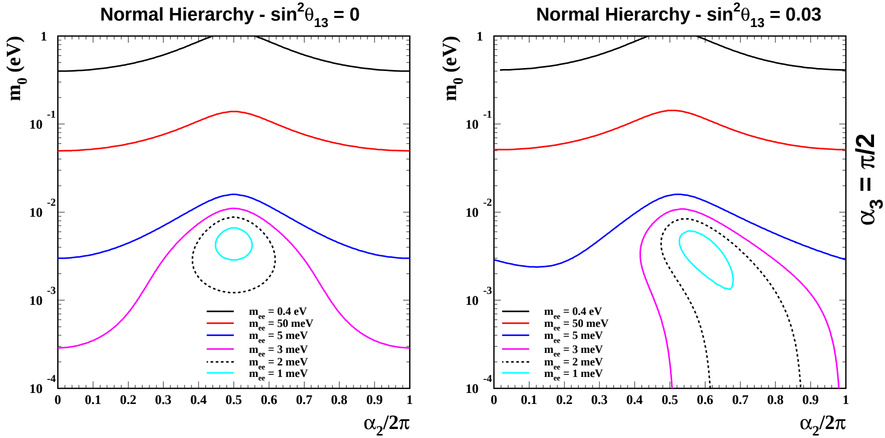
[Fogli *et al.*, arXiv:0805.2517]

All Cosmological Data - Case (5)



[Fogli *et al.*, arXiv:0805.2517]

α_2 and α_3



[Nunokawa, Teves, RZF, Phys. Rev. D. **66**, 093010 (2002)] - updated

General Conclusions

- **We have started the precision era of OE**
- Δm^2_{21} **know to better than 10% at** 3σ
- $|\Delta m^2_{31}|$ **know to better than 20% at** 3σ
- $\sin^2\theta_{12}$ **know to better than 25% at** 3σ
- $\sin^2\theta_{23}$ **know to better than 45% at** 3σ
- $\sin^2\theta_{13} < 0.05$ **at** 3σ

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- **mass hierarchy and CP-violating phases still unknown (see talk by H. Minakata)**
- **Results from NOE already provide important constraints. There combination with OE constraints can be very powerfull in the future**
- **Majorana phases seem out of reach**
- **It may be the case we will need more agressive** β**-decay experiments in the future to acess** m_0

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General Conclusions

- Other solutions have been suggested to explain the neutrino data, but *The Paradigm* seems to be at present the best solution, i.e. the leading effect
- As precision increases, it is important to check for sub-leading effects as we may encounter new surprises (see talk by M. Maltoni)