Introduction to Neutrino Interaction Physics
NUFACT08 Summer School

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University of Glasgow

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Benasque, Spain
Paul Soler

4. Quasi-elastic, resonant, coherent and diffractive scattering

4.1 Motivation

4.2 Charged current quasi-elastic scattering

4.3 Neutral current elastic scattering

4.4 Resonant pion production

4.5 Coherent pion production

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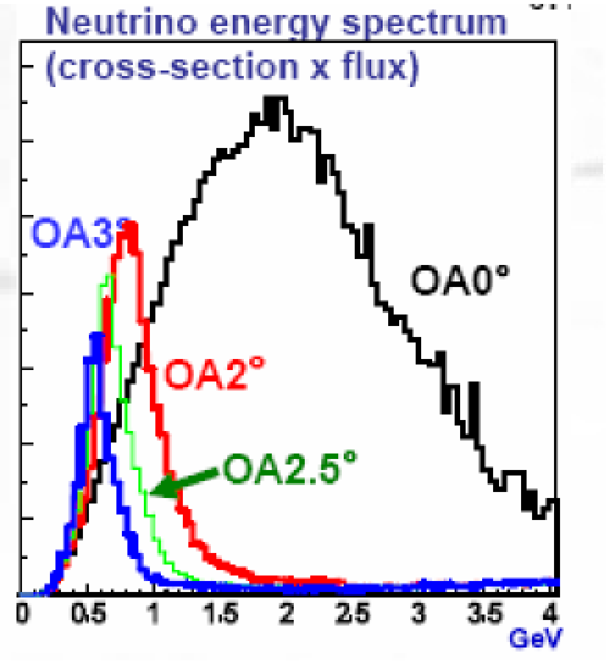
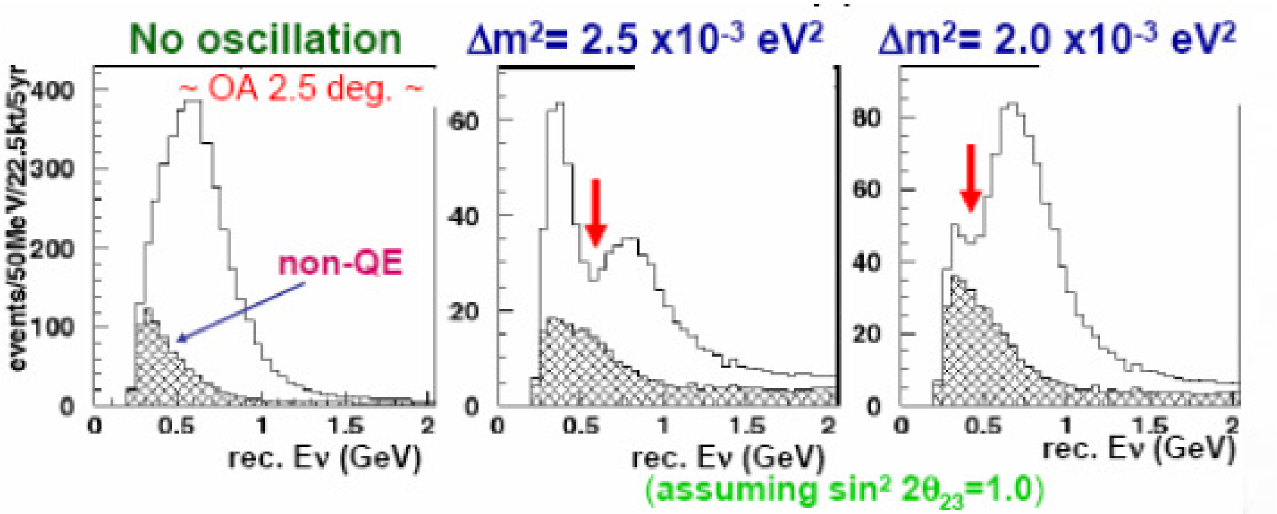
4.1 Motivation

- Many neutrino oscillation experiments need to achieve E/L ~ 10^{-3} GeV/km, so for distances ~1000 km, we need interactions around 1 GeV.
- For example, T2K, MINOS, atmospheric experiments require knowledge of cross-section between 0.4 and 2 GeV/c to perform accurate Δm^2_{23} and θ_{23} analysis

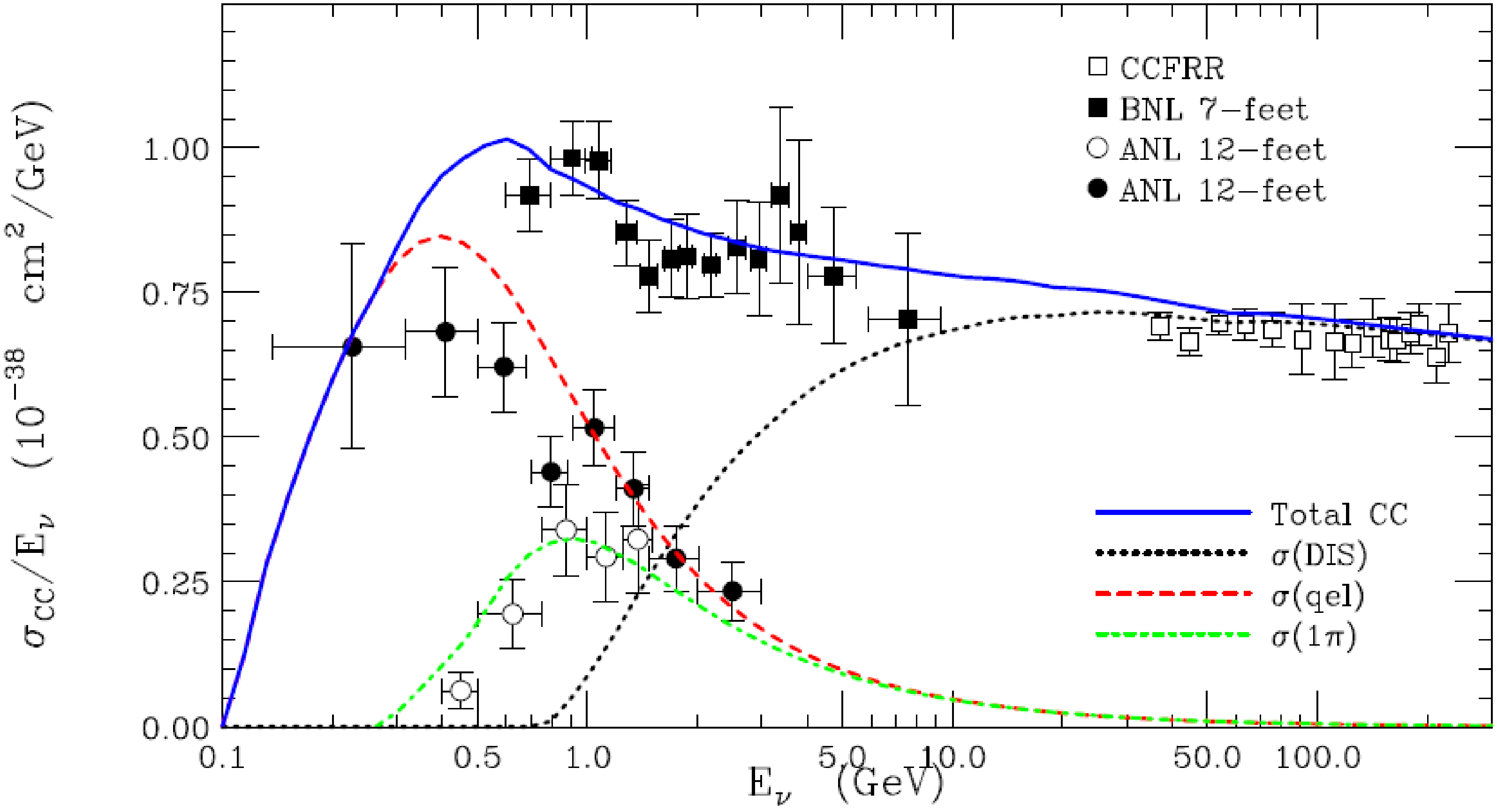


T2K



4.1 Motivation

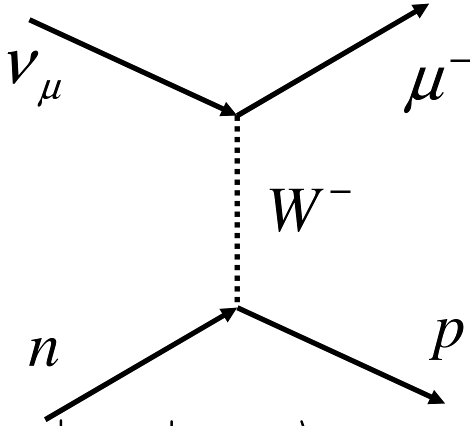
- Around 1 GeV there is a complicated region where deep inelastic scattering (DIS), quasi-elastic (QEL) scattering and resonance production (for example, 1π production) co-exist



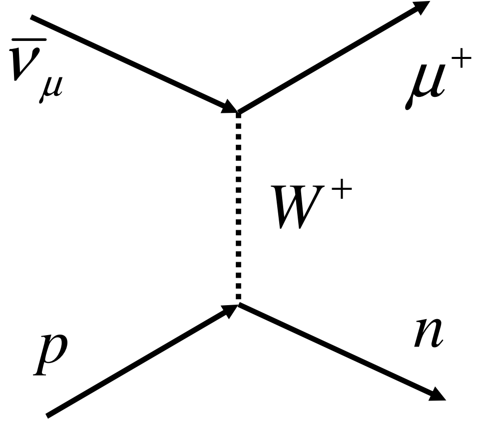
4.2 Charged current quasi-elastic scattering

- Quasi-elastic neutrino-nucleon scattering reactions (small q^2): affects nucleon as a whole

$$\nu_\mu + n \rightarrow \mu^- + p$$



$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$



$$M = \langle \mu^-, p | H_{eff} | \nu_\mu, n \rangle =$$

$$\frac{G_F \cos\theta_c}{\sqrt{2}} \left[\bar{\mu}\gamma^\mu (1-\gamma_5)\nu_\mu \right] \left[\bar{p}\gamma_\mu \left(F_V(q^2) + F_A(q^2)\gamma_5 \right) n \right]$$

$F_V(q^2) = \text{vector form factor}$

$F_A(q^2) = \text{axial} - \text{vector form factor}$

$\cos\theta_C = 0.975 \text{ (Cabbibo angle)}$

4.2 Charged current quasi-elastic scattering

- In reality, it is more complicated and we need Llewelyn-Smith formalism to calculate QE differential cross-sections:

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2} = \frac{G_F^2 M^2}{8\pi E_\nu^2}\left[A \mp \frac{(s-u)}{M^2} B + \frac{(s-u)^2}{M^4} C \right]$$

$$(s-u) = 4ME_\nu - Q^2 - m_\mu^2$$

- A, B, C are complicated functions of two vector form factors $F_1^V(Q^2)$, $F_2^V(Q^2)$, the axial form factor $F_A(Q^2)$ and the pseudoscalar form factor $F_P(Q^2)$. **See Zeller, hep-ex/0312061, for details**

$$A = \frac{(m_\mu^2 + Q^2)}{M^2}\left[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 - \frac{m_\mu^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - \left(\frac{Q^2}{M^2}+4\right)F_P^2 \right) \right]$$

$$B = \frac{Q^2}{M^2} F_A (F_1 + F_2)$$

$$C = \frac{1}{4}\left(F_A^2 + F_1^2 + \tau F_2^2 \right)$$

$$\tau = \frac{Q^2}{4M^2}$$

$$F_1^V(Q^2) = \frac{1+\tau(1+\mu_p-\mu_n)}{(1+\tau)\left(1+\frac{Q^2}{m_V^2}\right)^2}$$

$$F_2^V(Q^2) = \frac{1+\tau(1+\mu_p-\mu_n)}{(1+\tau)\left(1+\frac{Q^2}{m_V^2}\right)^2}$$

$$F_A(Q^2) = \frac{g_A}{\left(1+\frac{Q^2}{m_A^2}\right)^2}$$

Form factors: assume dipole approximation

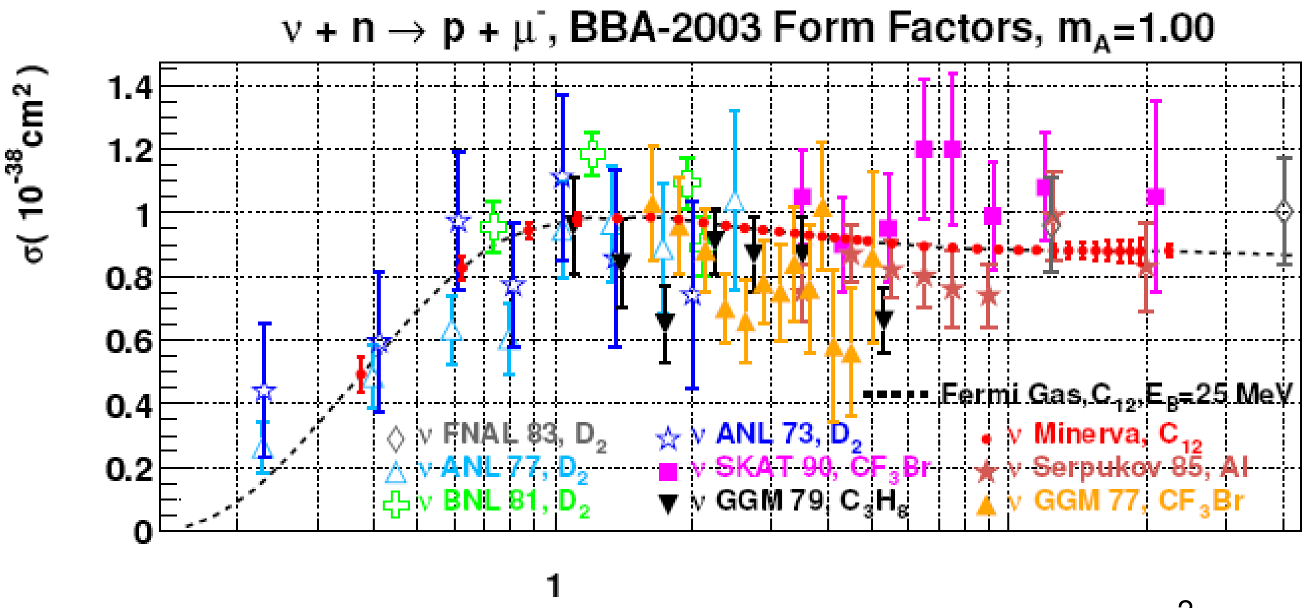
$$F_P(Q^2) = \frac{2M^2}{m_\pi^2 + Q^2} F_A(Q^2)$$

$$F_A(0) = g_A = -1.2573 \pm 0.028$$

$$\mu_p = 1.793\mu_N \quad and \quad \mu_n = -1.913\mu_N$$

4.2 Charged current quasi-elastic scattering

- Form factors introduced since proton, neutron not elementary.
- Depends on vector and axial weak charges of the proton and neutron.
- Conservation of Vector Current (CVC) relates form factors to electron scattering
- Main physics to be extracted from QE scattering data are empirical form factor parameters (fits to m_A, m_V, deviations from dipole approximation)



$$F_V(q^2) = \frac{F_V(0)}{\left(1 - q^2/m_V^2\right)^2}$$

$$F_A(q^2) = \frac{F_A(0)}{\left(1 - q^2/m_A^2\right)^2}$$

$$m_V = 0.84\ GeV$$

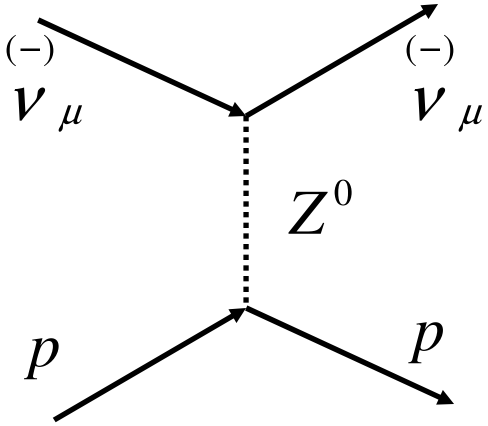
$$m_A = 1.032\ GeV$$

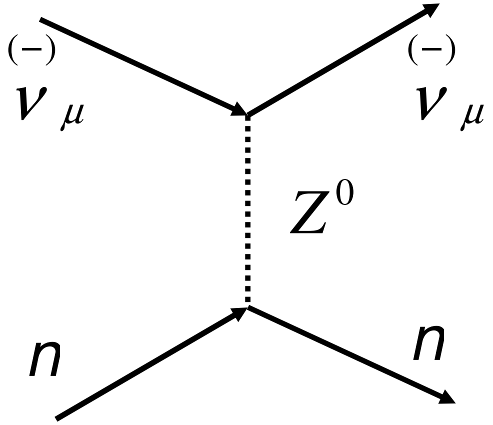
$$\sigma(\nu_e n) = \sigma(\overline{\nu}_e p) \approx 0.975 \times 10^{-38} \left(\frac{E}{1\,GeV}\right)^2$$

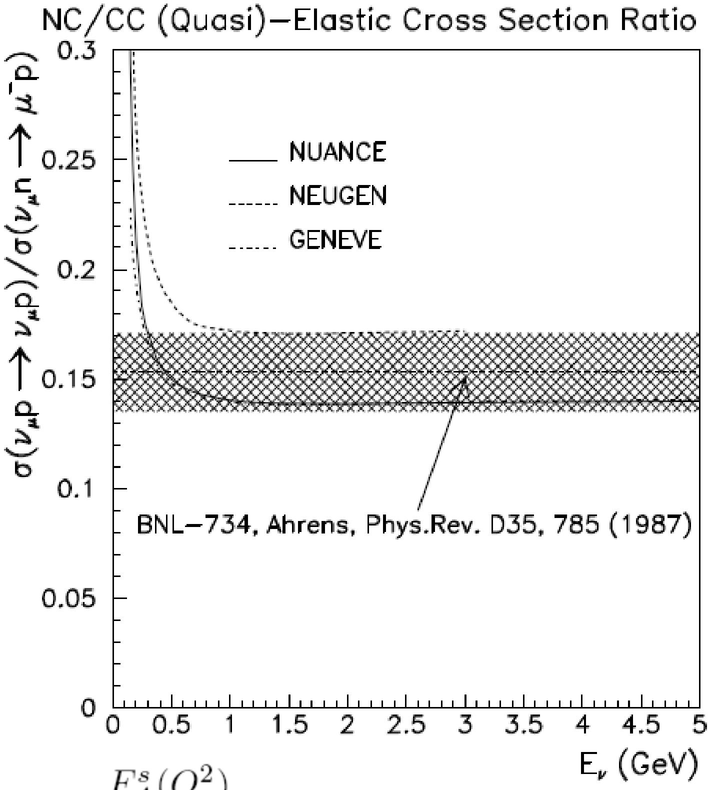
4.3 Neutral current elastic scattering

- Neutral current elastic neutrino-nucleon scattering reactions are related to the CC quasielastic (small q^2): about 15% of CC QEL

$$\overset{(-)}{\nu}_{\mu} + p \rightarrow \overset{(-)}{\nu}_{\mu} + p \qquad \overset{(-)}{\nu}_{\mu} + n \rightarrow \overset{(-)}{\nu}_{\mu} + n$$







$$\begin{aligned}
F_1(Q^2) &= \left(\frac{1}{2} - \sin^2\theta_W\right)\left[\frac{\tau_3(1+\tau\,(1+\mu_p-\mu_n))}{(1+\tau)\,(1+Q^2/m_V^2)^2}\right] \\
&\quad - \sin^2\theta_W\left[\frac{1+\tau\,(1+\mu_p+\mu_n)}{(1+\tau)\,(1+Q^2/m_V^2)^2}\right] - \frac{F_1^s(Q^2)}{2} \\
F_2(Q^2) &= \left(\frac{1}{2} - \sin^2\theta_W\right)\frac{\tau_3\,(\mu_p-\mu_n)}{(1+\tau)\left(1+\frac{Q^2}{m_V^2}\right)^2} \\
&\quad - \sin^2\theta_W \frac{\mu_p+\mu_n}{(1+\tau)\left(1+\frac{Q^2}{m_V^2}\right)^2} - \frac{F_2^s(Q^2)}{2}
\end{aligned}$$

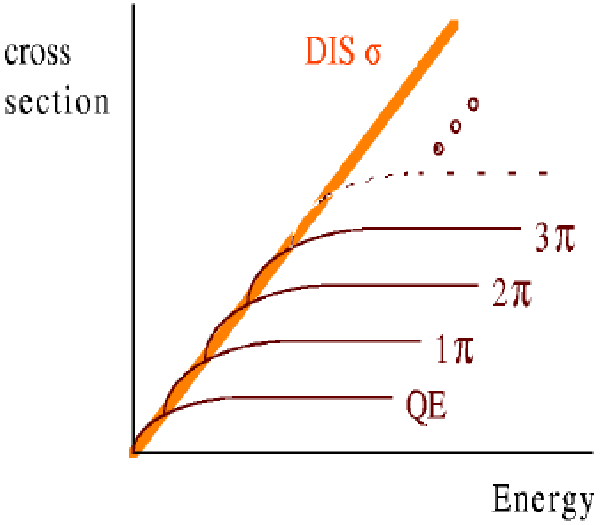
Also need to calculate form factors

$$F_A(Q^2) = \frac{g_A\,\tau_3}{2\left(1+\frac{Q^2}{m_A^2}\right)^2} + \frac{F_A^s(Q^2)}{2}$$

4.4 Resonant pion production

- Between the elastic and inelastic region is an area associated with pion production through the excitation of baryon resonances

$$\nu_l + N \rightarrow l + N^* \quad and \quad N^* \rightarrow \pi + N'$$



- Invariant mass squared:

$$W^2 = M_T^2 + 2M_T\nu(1-x)$$

- If x=1 then quasi-elastic scattering but if x<1 then you can excite different pion states:

$$W^2 = (M_T + m_\pi)^2, (M_T + 2m_\pi)^2, \ldots$$

- Rein and Sehgal's model describes low energy pion production by a coherent superposition of all possible resonances
- Cross-section: $$\frac{d\sigma}{dQ^2 dW} = \frac{1}{32ME^2}\frac{1}{2}\sum_{spins}\left|T(\nu N \rightarrow lN^*)\right|^2 \Gamma(W-M)$$

with: $$\Gamma(W-M) = \frac{1}{2\pi}\frac{\Gamma_0}{(W-M)^2 + \Gamma^2/4}$$

4.4 Resonant pion production

- For example, possible resonances are Δ^{++} or Δ^{+}

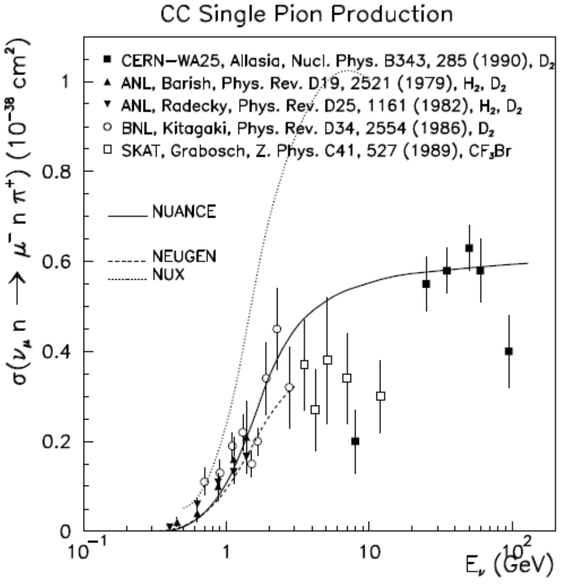
$$\nu_\mu + N \rightarrow \mu^- + \Delta^{++} \rightarrow \mu^- + p + \pi^+$$

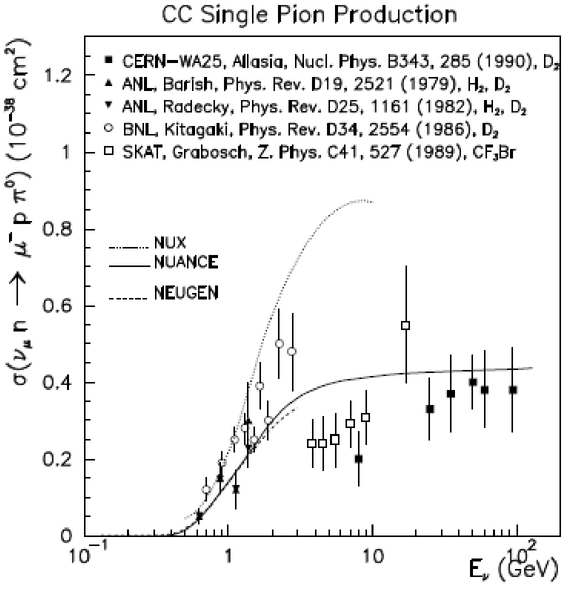
$$\nu_\mu + N \rightarrow \mu^- + \Delta^{+} \rightarrow \mu^- + n + \pi^+$$

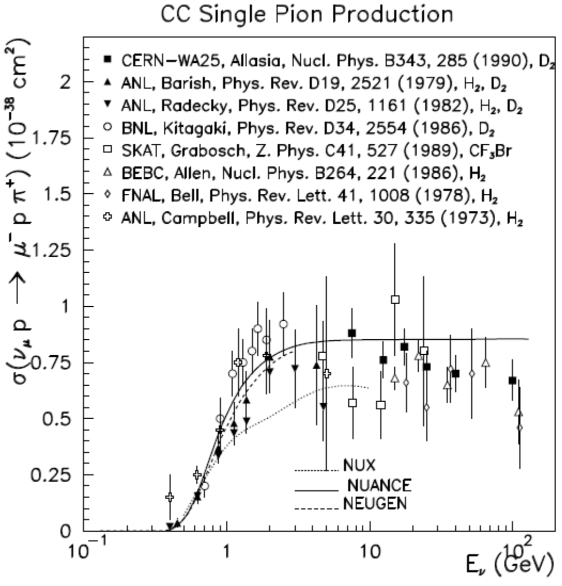
- All possible channels: 3 in CC and 4 in NC

$$\begin{array}{lcl}
\nu_\mu\, p & \rightarrow & \mu^-\, p\, \pi^+ \\
\nu_\mu\, n & \rightarrow & \mu^-\, p\, \pi^0 \\
\nu_\mu\, n & \rightarrow & \mu^-\, n\, \pi^+
\end{array}
\qquad
\begin{array}{lcl}
\nu_\mu\, p & \rightarrow & \nu_\mu\, p\, \pi^0 \\
\nu_\mu\, p & \rightarrow & \nu_\mu\, n\, \pi^+ \\
\nu_\mu\, n & \rightarrow & \nu_\mu\, n\, \pi^0 \\
\nu_\mu\, n & \rightarrow & \nu_\mu\, p\, \pi^-
\end{array}$$

Very little data, has large statistical errors, mainly from old bubble chamber experiments







4.4 Resonant pion production

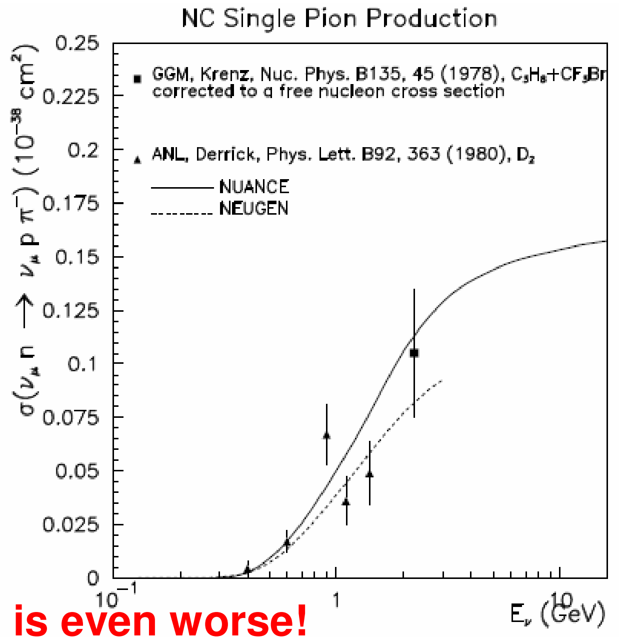
- For example, possible resonances are Δ^{++} or Δ^{+}

$$\nu_\mu + N \to \mu^- + \Delta^{++} \to \mu^- + p + \pi^+$$

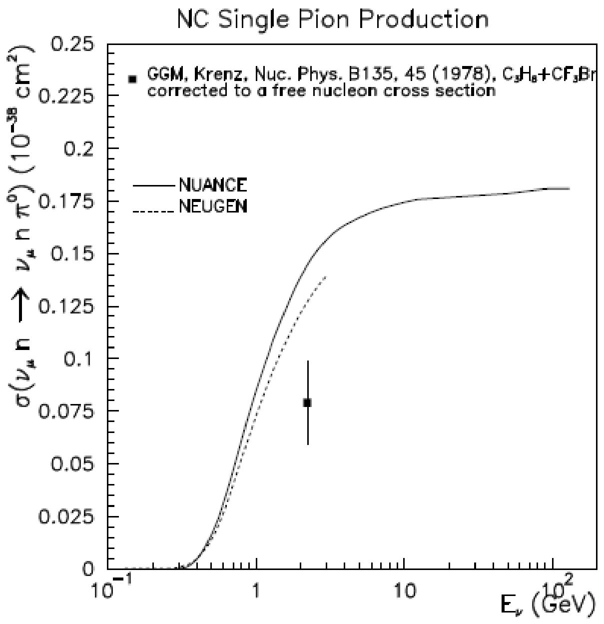
$$\nu_\mu + N \to \mu^- + \Delta^{+} \to \mu^- + n + \pi^+$$

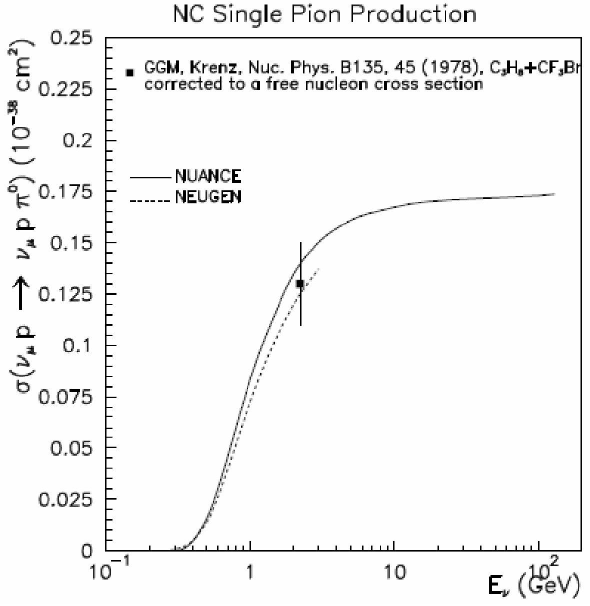
- All possible channels: 3 in CC and 4 in NC

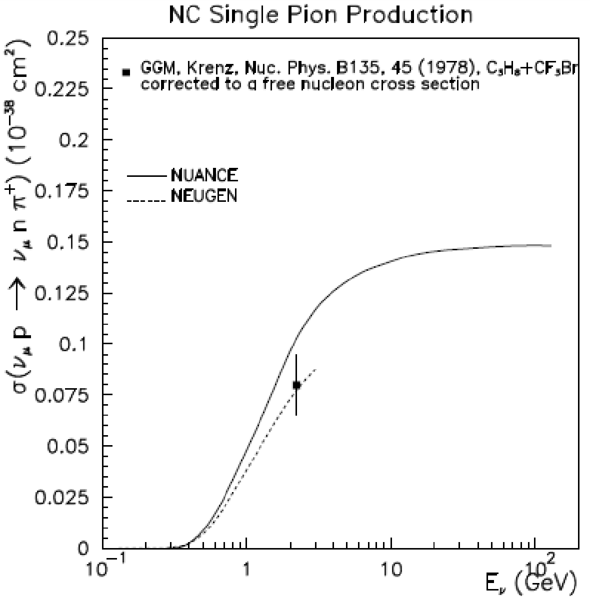
$$\begin{array}{lcl}
\nu_\mu\, p & \to & \mu^-\, p\, \pi^+ \\
\nu_\mu\, n & \to & \mu^-\, p\, \pi^0 \\
\nu_\mu\, n & \to & \mu^-\, n\, \pi^+
\end{array}
\qquad
\begin{array}{lcl}
\nu_\mu\, p & \to & \nu_\mu\, p\, \pi^0 \\
\nu_\mu\, p & \to & \nu_\mu\, n\, \pi^+ \\
\nu_\mu\, n & \to & \nu_\mu\, n\, \pi^0 \\
\nu_\mu\, n & \to & \nu_\mu\, p\, \pi^-
\end{array}$$



NC data is even worse!



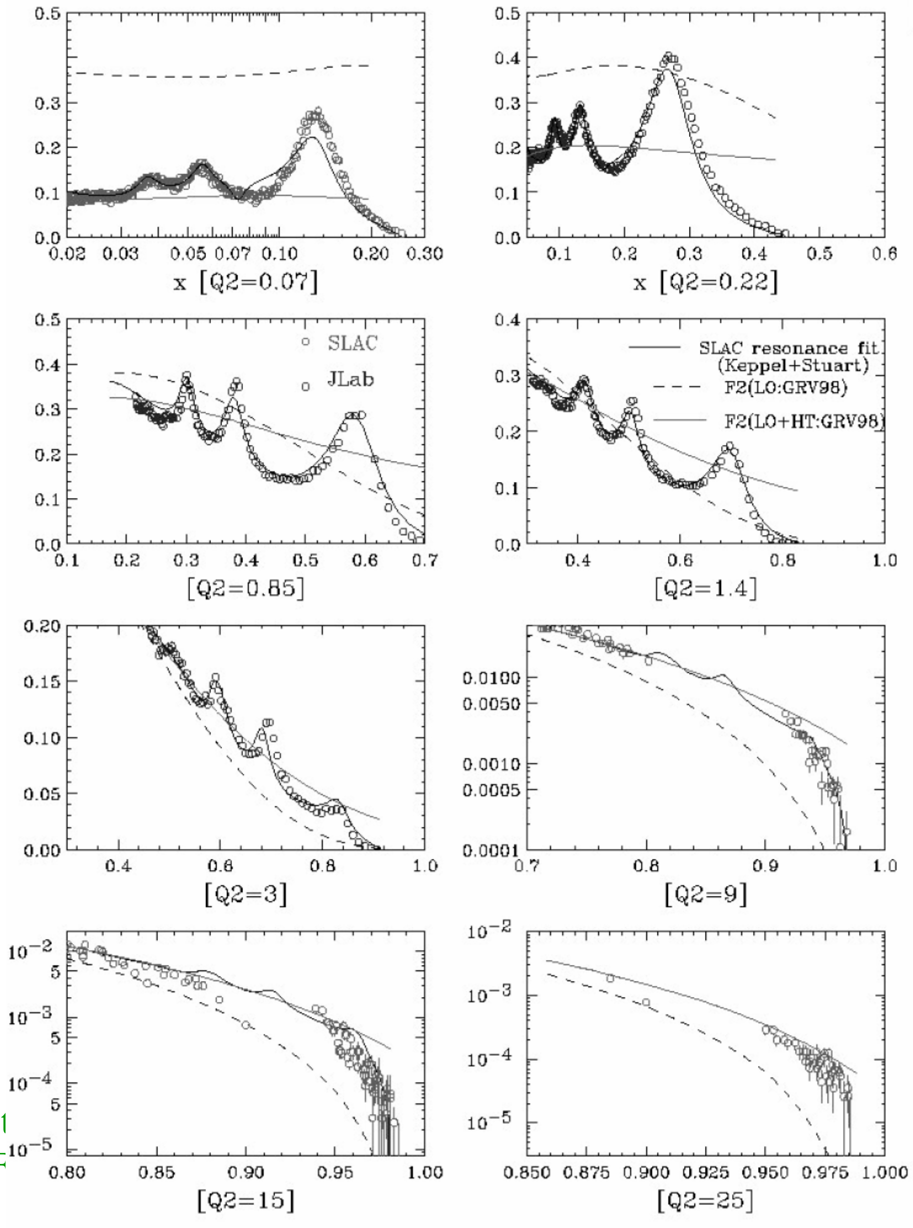




4.4 Resonant pion production

- Duality: use electron scattering data to improve precision of model
- Can observe individual resonances with good agreement data and model

Bodek and Yang



4.5 Coherent pion production

- Neutrinos can also produce pions coherently (low Q^2 and high ν)
- The neutrino coherently scatters off the whole nucleus with negligible energy transfer to the whole nucleus of mass A
- This results in a forward scattered single pion (background in oscillation searches because forward peaked)
- Neutral and charged current processes are possible:

$$\nu_\mu + A \rightarrow \nu_\mu + A + \pi^0$$

$$\nu_\mu + A \rightarrow \mu^- + A + \pi^+$$

- Rein and Sehgal's model also describes coherent pion production:
- Cross-section:

$$\frac{d\sigma}{dQ^2dydt} = \frac{G^2M}{2\pi^2} f_\pi^2 A^2 E_\nu (1-y) \frac{1}{16\pi} \left(\sigma_{tot}^{\pi N}\right)^2 (1+r^2) \left(\frac{m_A^2}{m_A^2+Q^2}\right)^2 e^{-b|t|} F_{abs}$$

$f_{\pi N}(0) = \text{pion - nucleon scattering amplitude}$

$$r \equiv \frac{\text{Re}[f_{\pi N}(0)]}{\text{Im}[f_{\pi N}(0)]}$$

$f_\pi = 0.93 m_\pi = \text{pion decay constant}$

$F_{abs} = e^{-\langle x \rangle / \lambda} = \text{pion absorption}$

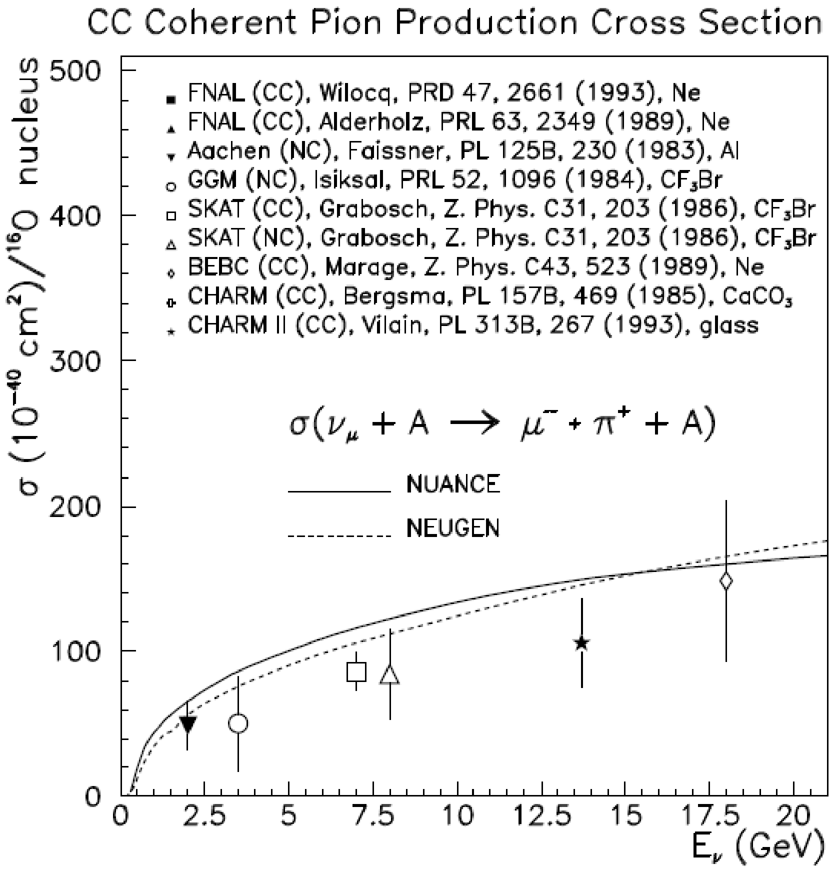
$$t = -(q-p_\pi)^2 \approx \left(\sum_i (E_i - p_i^{||})\right)^2 - \left(\sum_i (p_i^{\perp})\right)^2$$

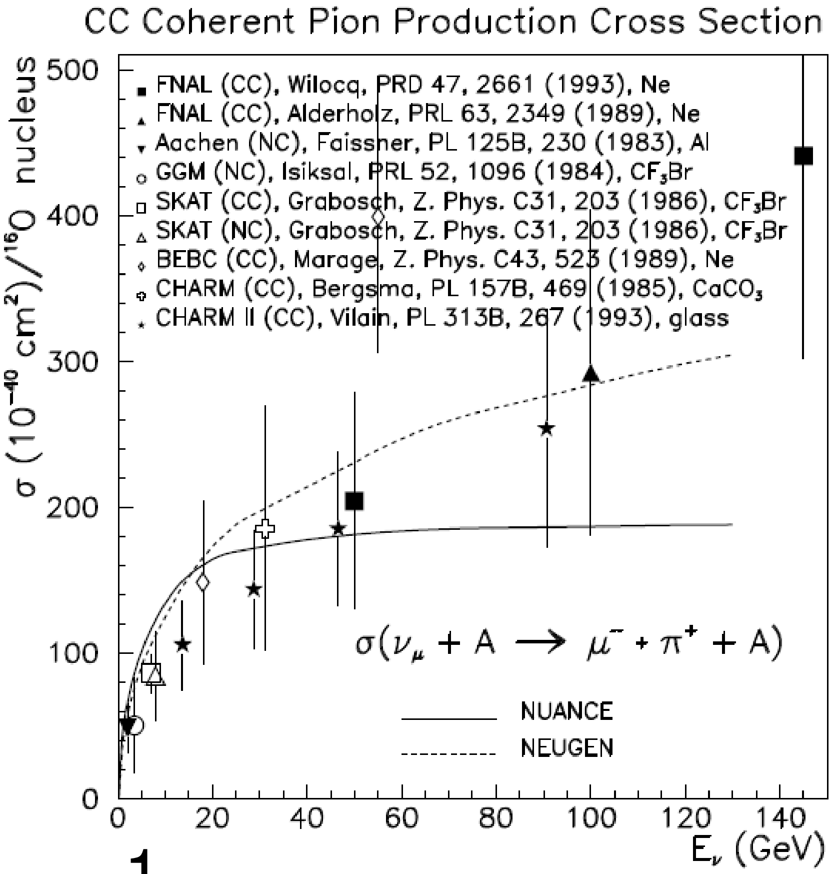
$b = (1/3)R^{2/3} = \text{impact parameter}$

Exponential in |t| distribution

4.5 Coherent pion production

- Charged current single pion coherent cross-section:





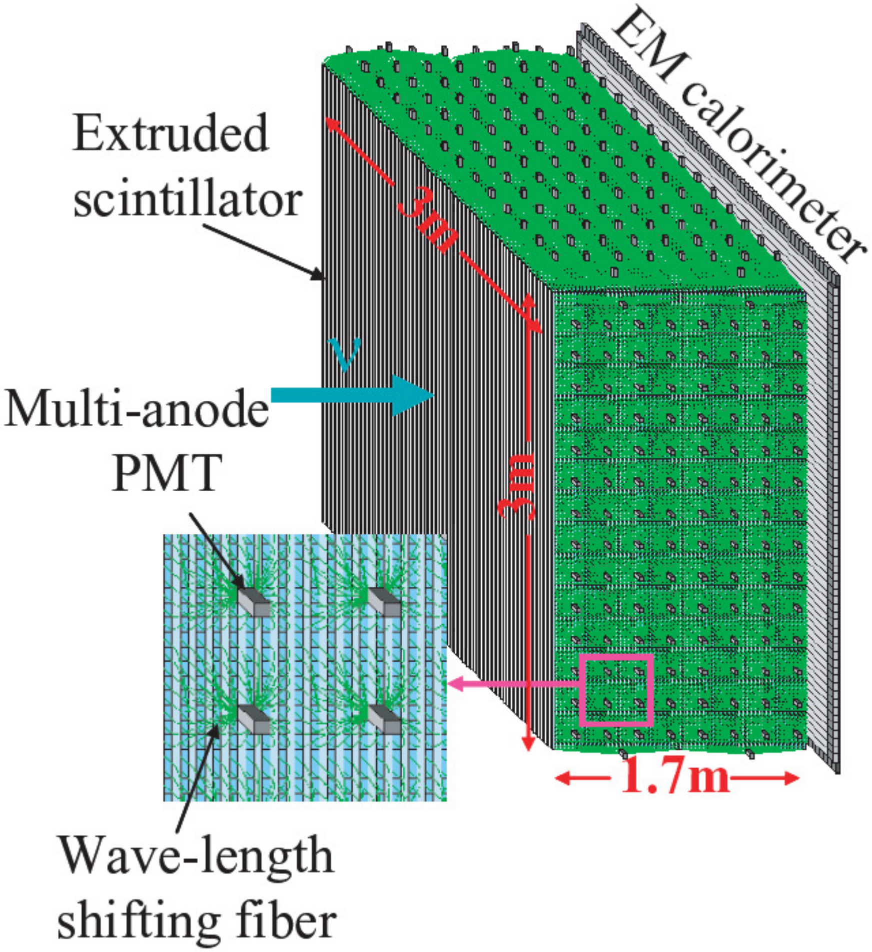
- NC cross-section is half of CC: $\sigma_{NC}^{coh} = \frac{1}{2}\sigma_{CC}^{coh}$

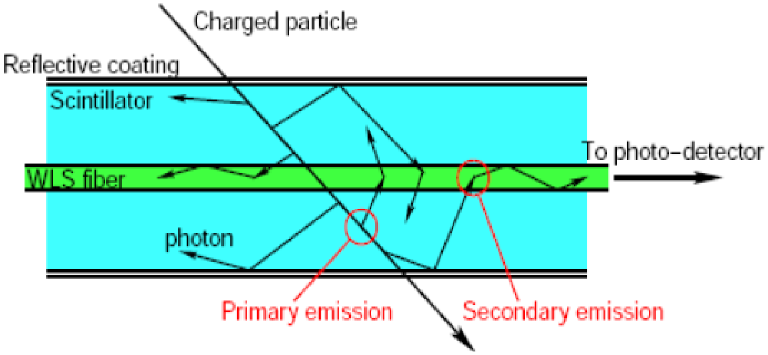
4.6 Experiments

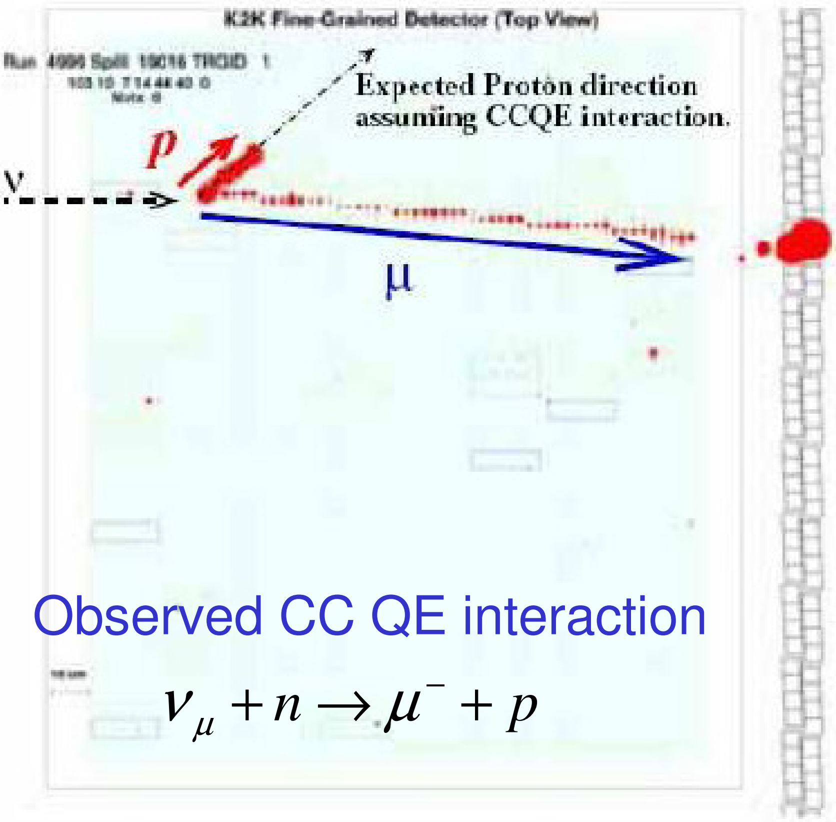
- Recent experiments carrying out measurements in the ~1GeV region:
  - K2K near detectors (ie. SciBar): completed
  - MINOS near detector: running
  - MiniBoone: running
  - SciBoone: moved SciBar to Fermilab, operating at the Booster beamline
  - Minerva (under construction)
  - T2K (under construction)

4.6 Experiments

☐ K2K SciBar and SciBoone







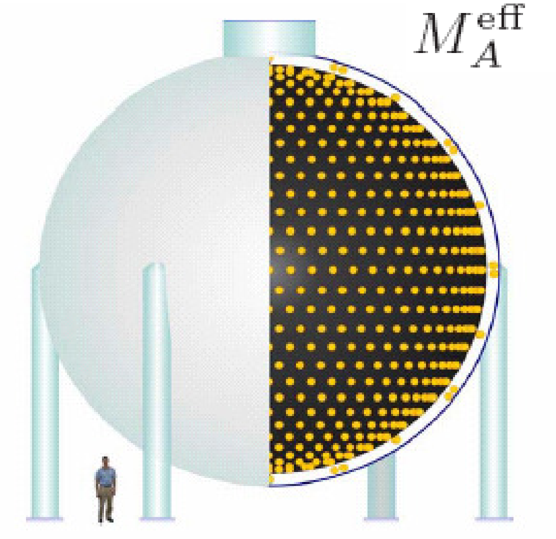
Observed CC QE interaction

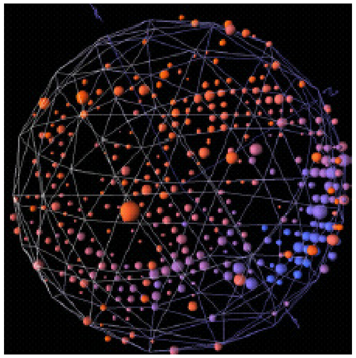
$$\nu_\mu + n \rightarrow \mu^- + p$$

4.6 Experiments

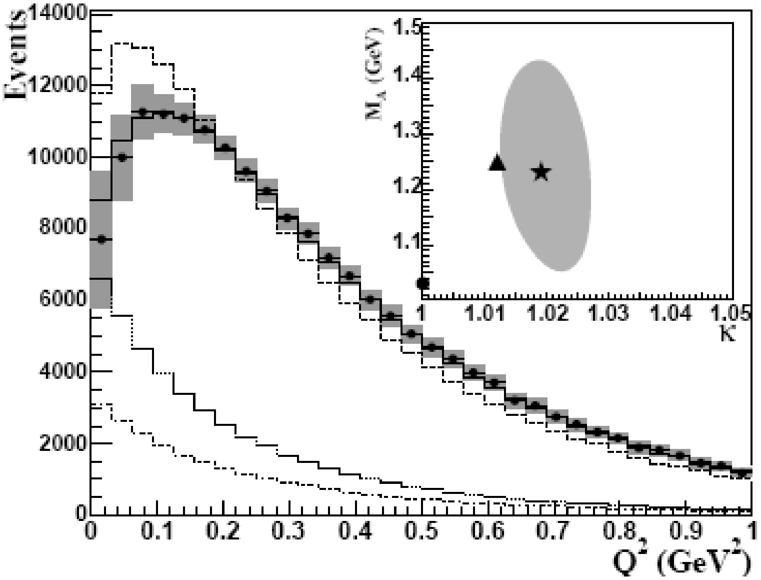
- MiniBoone: measurement of CCQE scattering
  - Fitted form factor, effective axial mass:

$$M_A^{\text{eff}} = 1.23 \pm 0.20 \text{ GeV}$$

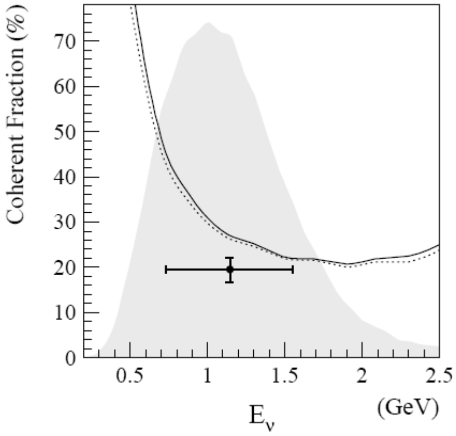




π^0 event

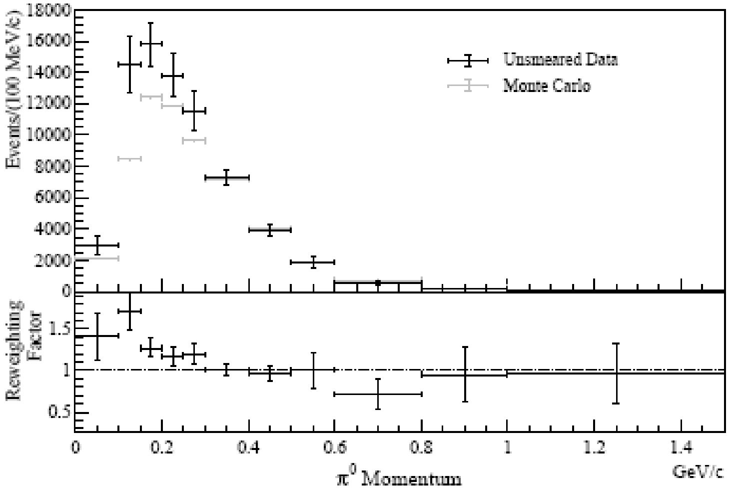


  - NC π^0 measurement: 28,000 events



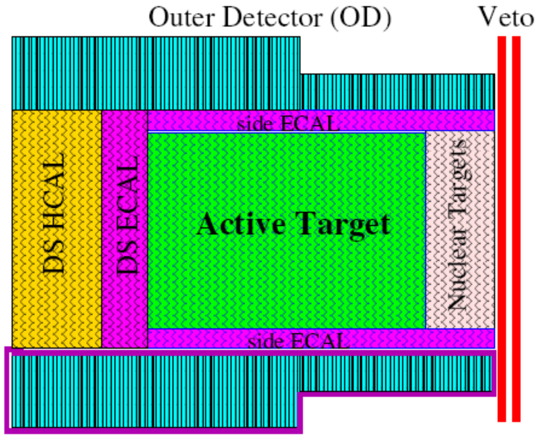
Ratio coherent/non-coherent:

$$(19.5 \pm 1.1(\text{stat}) \pm 2.5(\text{sys}))\%$$

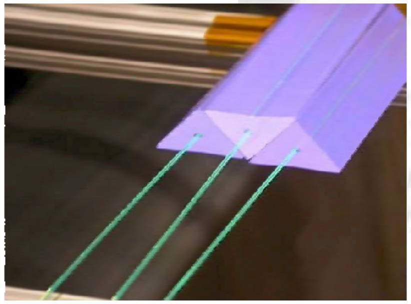


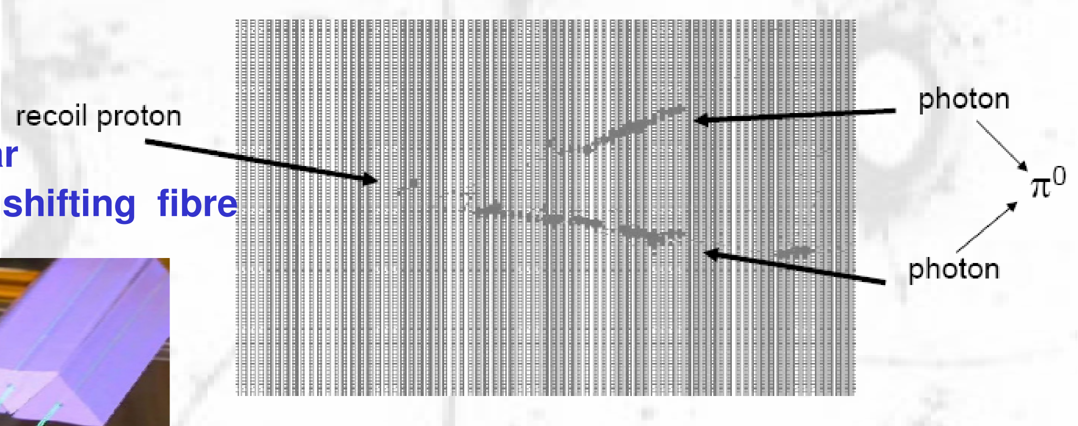
4.6 Experiments

- Minerva: a detector for precision interaction physics at Fermilab

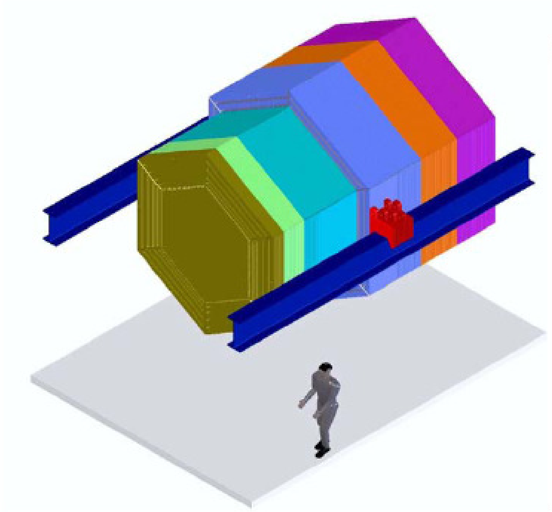


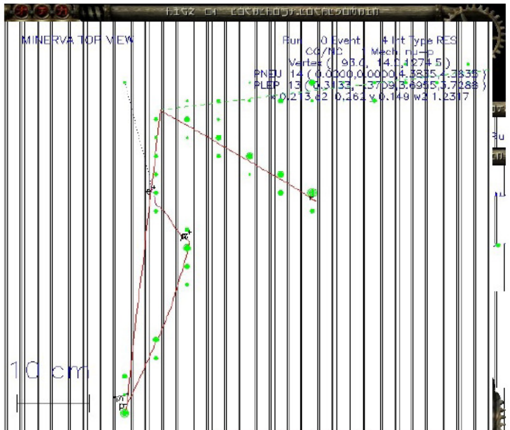
Scintillator bar
+ wavelength shifting fibre





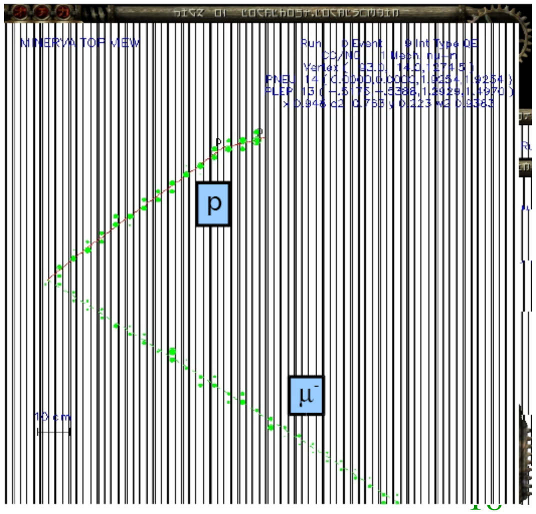
$$\nu_\mu p \to \nu_\mu p \pi^0,\ \pi^0 \to \gamma\gamma$$





$$\nu_\mu + p \to \mu^- + \Delta^{++} \to \mu^- + p + \pi^+$$

Resonance event



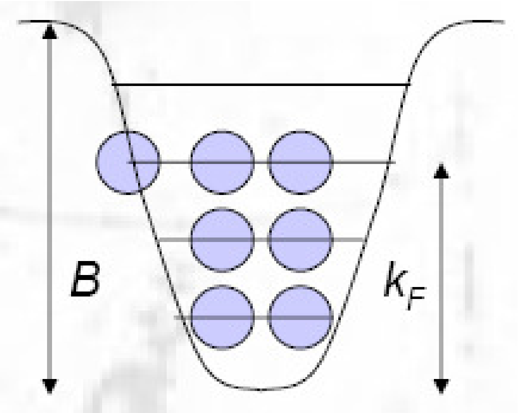
CCQE event $\nu_\mu + n \to \mu^- + p$

5. Nuclear Effects

5.1 Fermi smearing and Pauli blocking

5.2 Nuclear re-interactions

5.1 Fermi smearing and Pauli blocking

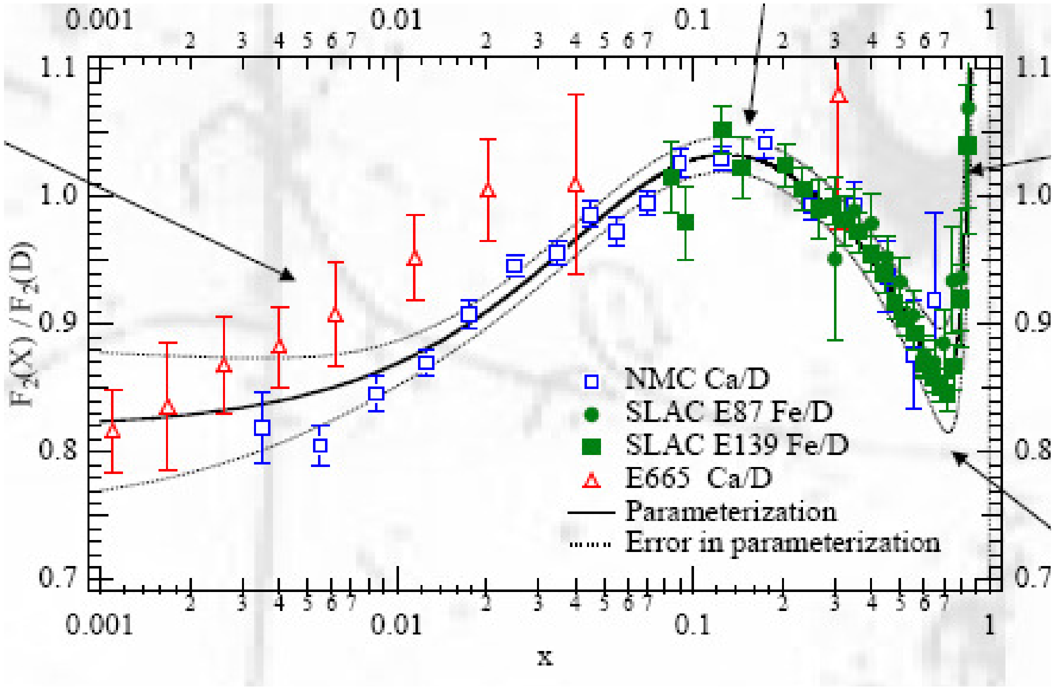


- Nuclear effects in neutrino scattering:
  - In a nucleus, the target nucleon has a momentum which modifies scattering
  - Modelled as “Fermi gas” that fills up all available states until some initial state Fermi momentum, k_F
  - The Pauli exclusion principle ensures that states cannot occupy states that are already filled (Pauli blocking)
  - Particles that escape nuclear medium may be re-scattered and deflected by the Fermi momentum, especially at low energies.
  - We need better understanding of the Fermi motion
  - For example, MiniBoone have already published a paper suggesting a modification to the Fermi gas model based on matching QE scattering in all values of Q^2 with their data.

5.1 Fermi smearing and Pauli blocking

- Effects on Structure Functions:
  - In charged lepton scattering, have observed shadowing and modifications to PDFs due to nucleons.
  - At small x, coherent interaction of a hadronic component of the virtual photon with target nucleus - shadowing
  - It is not clear if this is also present in neutrino structure functions since at low x, dominated by axial current

These effects need to be studied in detail with high statistics neutrino scattering



Anti-shadowing

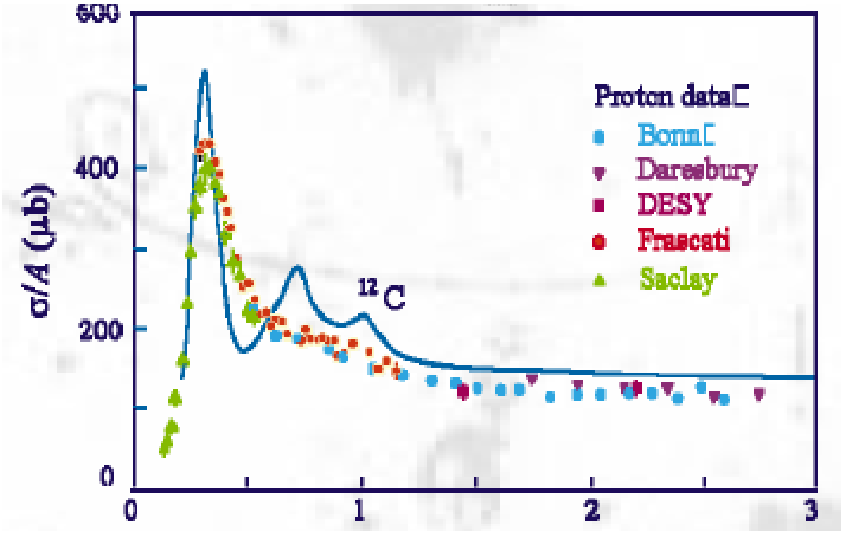
Shadowing

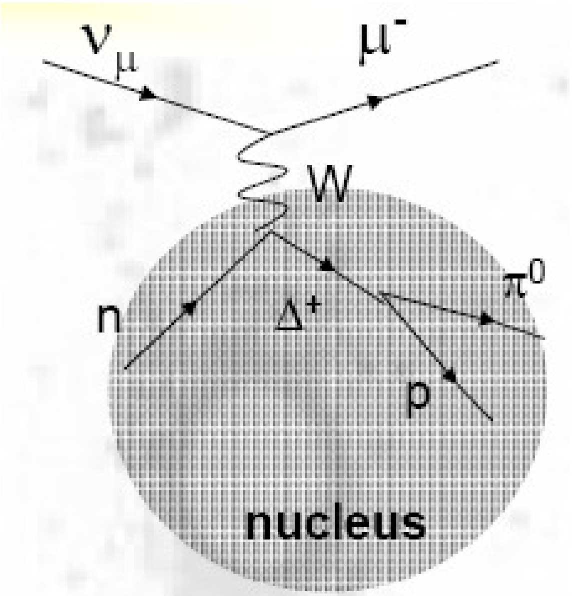
Fermi motion

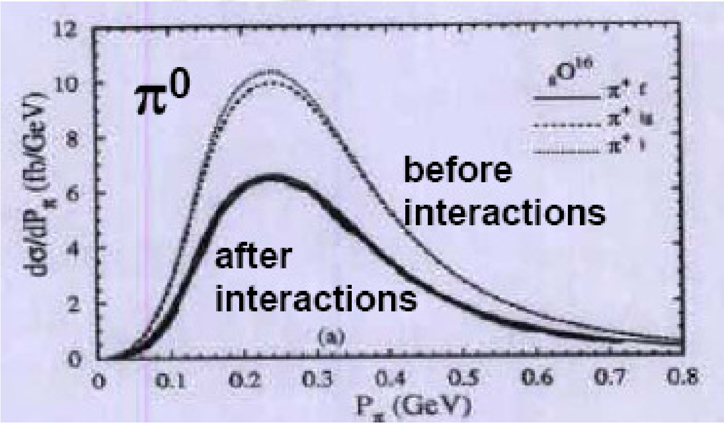
EMC effect

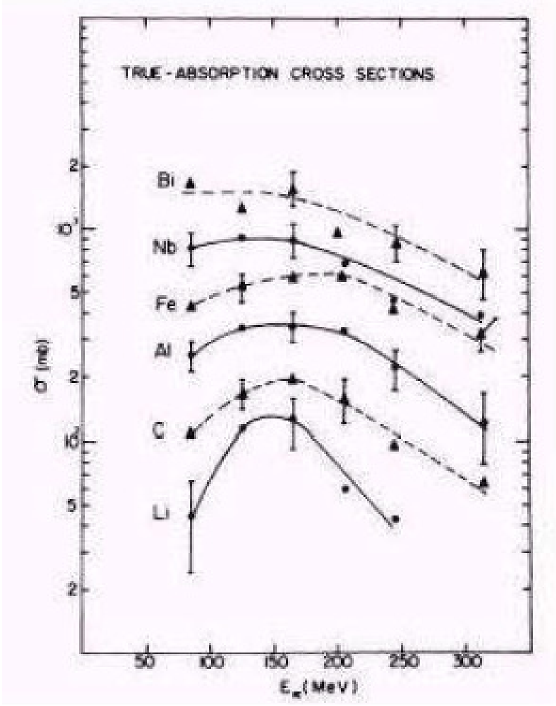
5.2 Re-interactions

- Nuclear effects in resonance region:
  - Production of resonance may be affected by nuclear medium (see plot of photoabsorption data)
  - Resonant structure gets washed out
  - Pions may either rescatter or be absorbed. This needs to be measured









Conclusions

- Neutrino interactions have provided valuable insight into the theory of weak interactions
  - Maximal parity violation, V-A theory and finally the Glashow-Weinberg-Salam electroweak theory were developed in part from information on neutrino interactions
  - Neutrino interaction data is used to probe the electroweak theory, such as in the measurements of $\sin^2\theta_W$.
- Neutrino interactions have also provided information on the structure of nucleons
  - Structure function measurements and scaling violations have been observed (F_3 is only accessible through neutrino interactions)
- Neutrino oscillations allow us to probe the grand unification energy scale, but it is crucial that we understand further the ~1 GeV energy region to be able to exploit oscillation experiments to the maximum
- A new generation of experiments is commencing to lead the way towards a new precision era in neutrino interaction physics