NEUTRINOS: THEORY

Contents

I. INTRODUCTION

It is already five decades since the first neutrino was observed by Cowan and Reines [1] in 1956 in a reactor experiment, and more than seventy five years since its existence was postulated by Wolfgang Pauli [2], in 1930, in order to reconcile the observed continuous spectrum of nuclear beta decay with energy conservation. It has been a long and winding road that has lead us from these pioneering times to the present overwhelming proof that neutrinos are massive and leptonic flavors are not symmetries of Nature. A road in which both theoretical boldness and experimental ingenuity have walked hand by hand to provide us with the first evidence of physics beyond the Standard Model. From the desperate solution of Pauli to the cathedral-size detectors built to capture and study in detail the elusive particle.

Neutrinos are copiously produced in natural sources: in the burning of the stars, in the interaction of cosmic rays...even as relics of the Big Bang. Starting from the 1960's, neutrinos produced in the sun and in the atmosphere were observed. In 1987, neutrinos from a supernova in the Large Magellanic Cloud were also detected. Indeed an important leading role in this story was played by the neutrinos produced in the sun and in the atmosphere. The experiments that measured the flux of atmospheric neutrinos found results that suggested the disappearance of muon-neutrinos when propagating over distances of order hundreds (or more) kilometers. Experiments that measured the flux of solar neutrinos found results that suggested the disappearance of electron-neutrinos while propagating within the Sun or between the Sun and the Earth.

These results called back to 1968 when Gribov and Pontecorvo [3, 4] realized that flavor oscillations arise if neutrinos are massive and mixed. The disappearance of both atmospheric ν_μ's and solar ν_e's was most easily explained in terms of neutrino oscillations. The emerging picture was that at least two neutrinos were massive and mixed, unlike what it is predicted in the Standard Model.

In the last decade this picture became fully established with the upcome of a set of precise experiments. In particular, during the last five years the results obtained with solar and atmospheric neutrinos have been confirmed in experiments using terrestrial beams in which neutrinos produced in nuclear reactors and accelerators facilities have been detected at distances of the order of hundred kilometers.

Neutrinos were introduced in the Standard Model as truly massless fermions, for which no gauge invariant renormalizable mass term can be constructed. Consequently, in the Standard Model there is neither mixing nor CP violation in the leptonic sector. Therefore, the experimental evidence for neutrino masses and mixing provided an unambiguous signal of new physics.

At present the phenomenology of massive neutrinos is in a very interesting moment. On the one hand many extensions of the Standard Model anticipated ways in which neutrinos may have small, but definitely non-vanishing masses. The better determination of the flavor structure of the leptons at low energies is of vital importance as, at present, it is our only source of positive information to pin-down the high energy dynamics implied by the neutrino masses. Needless to say that its potential will be further expanded and complemented if a positive signal on the absolute value of the mass scale is observed in kinematic searches or or in neutrinoless double beta decay as well as if the observations from a positive evidence in the precision cosmological data.

The purpose of these lectures is to quantitatively summarize the present status of the phenomenology of massive neutrinos . I will present the low energy formalism for adding neutrino masses to the SM and the induced leptonic mixing, and then we describe the phenomenology associated with neutrino oscillations in vacuum and in matter. I will also describe the status of the existing probes to the absolute neutrino mass scale. Time allowing I will present some expected collider signatures in some of the models which provided a plausible explanation for the observed neutrino masses.

The field of neutrino phenomenology and its forward-looking perspectives is rapidly evolving and these lectures are only a partial introduction. For more details I suggest to consult the review articles, Refs. [5–16], and text books, Refs. [17–23].

II. STANDARD MODEL OF MASSLESS NEUTRINOS

The greatest success of modern particle physics has been the establishment of the connection between forces mediated by spin-1 particles and local (gauge) symmetries. Within the Standard Model, the strong, weak and electromagnetic interactions are connected to, respectively, $SU(3)$, $SU(2)$ and $U(1)$ gauge groups. The characteristics of the different interactions are explained by the symmetry to which they are related. For example, the way

in which the fermions exert and experience each of the forces is determined by their representation under the corresponding symmetry group (or simply their charges in the case of Abelian gauge symmetries).

Once the gauge invariance is elevated to the level of fundamental physics principle, it must be verified by all terms in the Lagrangian, including the mass terms. This, as we will see, has important implications for the neutrino.

The Standard Model (SM) is based on the gauge group

$$G_{\mathrm{SM}} = SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}, \tag{1}$$

with three matter fermion generations. Each generation consists of five different representations of the gauge group:

$$\left(1, 2, -\frac{1}{2}\right), \quad \left(3, 2, \frac{1}{6}\right), \quad (1, 1, -1), \quad \left(3, 1, \frac{2}{3}\right), \quad \left(3, 1, -\frac{1}{3}\right) \tag{2}$$

where the numbers in parenthesis represent the corresponding charges under the group (1). In this notation the electric charge is given by

$$Q_{\mathrm{EM}} = T_{L3} + Y . \tag{3}$$

The matter content is shown in Table I, and together with the corresponding gauge fields it constitutes the full list of fields required to describe the observed elementary particle interactions. In fact, these charge assignments have been tested to better than the percent level for the light fermions [24]. The model also contains a single Higgs boson doublet, $\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$ with charges $(1, 2, 1/2)$, whose vacuum expectation value breaks the gauge symmetry,

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \Longrightarrow \quad G_{\mathrm{SM}} \to SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{EM}}. \tag{4}$$

This is the only piece of the SM model which still misses experimental confirmation. Indeed, the search for the Higgs boson, remains one of the premier tasks of present and future high energy collider experiments.

As can be seen in Table I neutrinos are fermions that have neither strong nor electromagnetic interactions (see Eq. (3)), *i.e.* they are singlets of $SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{EM}}$. We will refer as *active* neutrinos to neutrinos that, such as those in Table I, reside in the lepton doublets,

TABLE I: Matter contents of the SM.

$L_L(1,2,-\frac{1}{2})$ $Q_L(3,2,\frac{1}{6})$	$E_R(1,1,-1)$ $U_R(3,1,\frac{2}{3})$ $D_R(3,1,-\frac{1}{3})$
$\begin{pmatrix} c\nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L$	$e_R \qquad u_R \qquad d_R$
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L$	$\mu_R \qquad c_R \qquad s_R$
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$	$\tau_R \qquad t_R \qquad b_R$

that is, that have weak interactions. Conversely *sterile* neutrinos are defined as having no SM gauge interactions (their charges are $(1,1,0)$), that is, they are singlets of the full SM gauge group.

The SM has three active neutrinos accompanying the charged lepton mass eigenstates, e, μ and τ, thus there are weak charged current (CC) interactions between the neutrinos and their corresponding charged leptons given by

$$-\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \sum_\ell \bar{\nu}_{L\ell} \gamma^\mu \ell_L^- W_\mu^+ + \text{h.c.}. \tag{5}$$

In addition, the SM neutrinos have also neutral current (NC) interactions,

$$-\mathcal{L}_{\text{NC}} = \frac{g}{2\cos\theta_W} \sum_\ell \bar{\nu}_{L\ell} \gamma^\mu \nu_{L\ell} Z_\mu^0. \tag{6}$$

The SM as defined in Table I, contains no sterile neutrinos.

Thus, within the SM, Eqs. (5) and (6) describe all the neutrino interactions. From Eq. (6) one can determine the decay width of the Z^0 boson into neutrinos which is proportional to the number of light (that is, $m_\nu \leq m_Z/2$) left-handed neutrinos. At present the measurement of the invisible Z width yields $N_\nu = 2.984 \pm 0.008$ [24] which implies that whatever the extension of the SM we want to consider, it must contain three, and only three, light active neutrinos.

An important feature of the SM, which is relevant to the question of the neutrino mass, is the fact that the SM with the gauge symmetry of Eq. (1) and the particle content of Table I presents an accidental global symmetry:

$$G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}. \tag{7}$$

$U(1)_B$ is the baryon number symmetry, and $U(1)_{L_e, L_\mu, L_\tau}$ are the three lepton flavor symmetries, with total lepton number given by $L = L_e + L_\mu + L_\tau$. It is an accidental symmetry because we do not impose it. It is a consequence of the gauge symmetry and the representations of the physical states.

In the SM, fermions masses arise from the Yukawa interactions which couple a right-handed fermion with its left-handed doublet and the Higgs field,

$$-\mathcal{L}_{\text{Yukawa}} = Y_{ij}^d \bar{Q}_{Li} \phi D_{Rj} + Y_{ij}^u \bar{Q}_{Li} \tilde{\phi} U_{Rj} + Y_{ij}^\ell \bar{L}_{Li} \phi E_{Rj} + \text{h.c.}, \tag{8}$$

(where $\tilde{\phi} = i\tau_2 \phi^\star$) which after spontaneous symmetry breaking lead to charged fermion masses

$$m_{ij}^f = Y_{ij}^f \frac{v}{\sqrt{2}}. \tag{9}$$

However, since no right-handed neutrinos exist in the model, the Yukawa interactions of Eq. (8) leave the neutrinos massless.

In principle neutrino masses could arise from loop corrections if these corrections induced effective operators of the form

$$\frac{Z_{ij}^\nu}{v} \left(\bar{L}_{Li} \tilde{\phi} \right) \left(\tilde{\phi}^T L_{Lj}^C \right) + \text{h.c.}, \tag{10}$$

In the SM, however, this cannot happen because this operator violates the total lepton symmetry by two units. As mentioned above total lepton number is a global symmetry of the model and therefore L-violating terms cannot be induced by loop corrections. Furthermore, the $U(1)_{B-L}$ subgroup of $G_{\text{SM}}^{\text{global}}$ is non-anomalous. and therefore $B - L$-violating terms cannot be induced even by nonperturbative corrections.

It follows that the SM predicts that neutrinos are precisely massless. In order to add a mass to the neutrino the SM has to be extended.

III. INTRODUCING MASSIVE NEUTRINOS

As discussed above, with the fermionic content and gauge symmetry of the SM one cannot construct a renormalizable mass term for the neutrinos. So in order to introduce a neutrino mass one must either extend the particle contents of the model or abandon gauge invariance and/or renormalizability.

In what follows we illustrate the different types of neutrino mass terms by assuming that we keep the gauge symmetry and we explore the possibilities that we have to introduce a neutrino mass term if one adds to the SM an arbitrary number m of sterile neutrinos $\nu_{si}(1,1,0)$ or extends the scalar sector.

With the particle contents of the SM and the addition of an arbitrary m number of sterile neutrinos one can construct two types mass terms that arise from gauge invariant renormalizable operators:

$$-\mathcal{L}_{M_\nu} = M_{Dij}\bar{\nu}_{si}\nu_{Lj} + \frac{1}{2}M_{Nij}\bar{\nu}_{si}\nu^c_{sj} + \text{h.c.}. \tag{11}$$

Here ν^c indicates a charge conjugated field, $\nu^c = C\bar{\nu}^T$ and C is the charge conjugation matrix. M_D is a complex $m \times 3$ matrix and M_N is a symmetric matrix of dimension $m \times m$.

The first term is a Dirac mass term. It is generated after spontaneous electroweak symmetry breaking from Yukawa interactions

$$Y^\nu_{ij}\bar{\nu}_{si}\tilde{\phi}^\dagger L_{Lj} \Rightarrow M_{Dij} = Y^\nu_{ij}\frac{v}{\sqrt{2}} \tag{12}$$

similarly to the charged fermion masses. It conserves total lepton number but it breaks the lepton flavor number symmetries.

The second term in Eq. (11) is a Majorana mass term. It is different from the Dirac mass terms in many important aspects. It is a singlet of the SM gauge group. Therefore, it can appear as a bare mass term. Furthermore, since it involves two neutrino fields, it breaks lepton number by two units. More generally, such a term is allowed only if the neutrinos carry no additive conserved charge.

In general Eq. (11) can be rewritten as:

$$-\mathcal{L}_{M_\nu} = \frac{1}{2}\overline{\vec{\nu}^c}M_\nu\vec{\nu} + \text{h.c.}, \tag{13}$$

where

$$M_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}, \tag{14}$$

and $\vec{\nu} = (\vec{\nu}_L, \vec{\nu}_s^c)^T$ is a $(3+m)$-dimensional vector. The matrix M_ν is complex and symmetric. It can be diagonalized by a unitary matrix of dimension $(3 + m)$, V^ν, so that

$$(V^\nu)^T M_\nu V^\nu = \text{Diag}(m_1, m_2, \ldots, m_{3+m}). \tag{15}$$

In terms of the resulting $3 + m$ mass eigenstates

$$\vec{\nu}_{\text{mass}} = (V^{\nu})^{\dagger}\vec{\nu}, \tag{16}$$

Eq. (13) can be rewritten as:

$$-\mathcal{L}_{M_{\nu}} = \frac{1}{2}\sum_{k=1}^{3+m} m_k \left(\bar{\nu}^c_{\text{mass},k}\nu_{\text{mass},k} + \bar{\nu}_{\text{mass},k}\nu^c_{\text{mass},k}\right) = \frac{1}{2}\sum_{k=1}^{3+m} m_k \bar{\nu}_{Mk}\nu_{Mk}, \tag{17}$$

where

$$\nu_{Mk} = \nu_{\text{mass},k} + \nu^c_{\text{mass},k} = (V^{\nu\dagger}\vec{\nu})_k + (V^{\nu\dagger}\vec{\nu})^c_k \tag{18}$$

which obey the Majorana condition

$$\nu_M = \nu^c_M \tag{19}$$

and are refereed to as Majorana neutrinos. Notice that this condition implies that there is only one field which describes both neutrino and antineutrino states. Thus a Majorana neutrino can be described by a two-component spinor unlike the charged fermions, which are Dirac particles, and are represented by four-component spinors.

From Eq. (18) we find that the weak-doublet components of the neutrino fields are:

$$\nu_{Li} = L\sum_{j=1}^{3+m} V^{\nu}_{ij}\nu_{Mj} \quad i = 1,2,3, \tag{20}$$

where L is the left-handed projector.

In the rest of this section we will discuss three interesting cases.

A. $M_N = 0$: Dirac Neutrinos

Forcing $M_N = 0$ is equivalent to imposing lepton number symmetry on the model. In this case, only the first term in Eq. (11), the Dirac mass term, is allowed. For $m = 3$ we can identify the three sterile neutrinos with the right-handed component of a four-spinor neutrino field. In this case the Dirac mass term can be diagonalized with two 3×3 unitary matrices, V^{ν} and V^{ν}_R as:

$$V^{\nu\dagger}_R M_D V^{\nu} = \text{Diag}(m_1, m_2, m_3). \tag{21}$$

The neutrino mass term can be written as:

$$-\mathcal{L}_{M_{\nu}} = \sum_{k=1}^{3} m_k \bar{\nu}_{Dk}\nu_{Dk} \tag{22}$$

where

$$\nu_{Dk} = (V^{\nu\dagger}\vec{\nu}_L)_k + (V_R^{\nu\dagger}\vec{\nu}_s)_k \,, \tag{23}$$

so the weak-doublet components of the neutrino fields are

$$\nu_{Li} = L \sum_{j=1}^{3} V_{ij}^{\nu} \nu_{Dj} \,, \qquad i = 1, 2, 3 \,. \tag{24}$$

Let's point out that in this case the SM is not even a good low-energy effective theory since both the matter content and the assumed symmetries are different. Furthermore there is no explanation to the fact that neutrino masses happen to be much lighter than the corresponding charged fermion masses as in this case all acquire their mass via the same mechanism.

B. $M_N \gg M_D$: The Type I see-saw mechanism

In this case the scale of the mass eigenvalues of M_N is much higher than the scale of electroweak symmetry breaking $\langle\phi\rangle$. The diagonalization of M_ν leads to three light, ν_l, and m heavy, N, neutrinos:

$$-\mathcal{L}_{M_\nu} = \frac{1}{2}\bar{\nu}_l M^l \nu_l + \frac{1}{2}\bar{N}M^h N \tag{25}$$

with

$$M^l \simeq -V_l^T M_D^T M_N^{-1} M_D V_l, \qquad M^h \simeq V_h^T M_N V_h \tag{26}$$

and

$$V^\nu \simeq \begin{bmatrix} \left(1 - \frac{1}{2}M_D^\dagger M_N^{*}{}^{-1}M_N^{-1}M_D\right)V_l & M_D^\dagger M_N^{*}{}^{-1}V_h \\ -M_N^{-1}M_D V_l & \left(1 - \frac{1}{2}M_N^{-1}M_D M_D^\dagger M_N^{*}{}^{-1}\right)V_h \end{bmatrix} \tag{27}$$

where V_l and V_h are 3×3 and $m \times m$ unitary matrices respectively. So the heavier are the heavy states, the lighter are the light ones. This is the *Type I see-saw mechanism* [25–29]. Also as seen from Eq. (27) the heavy states are mostly right-handed while the light ones are mostly left-handed. Both the light and the heavy neutrinos are Majorana particles. Two well-known examples of extensions of the SM that lead to a Type I see-saw mechanism for neutrino masses are SO(10) GUTs [26–28] and left-right symmetry [29].

In this case the SM is a good effective low energy theory. Indeed the Type I see-saw mechanism is a particular realization of the general case of a full theory which leads to the SM with three light Majorana neutrinos as its low energy effective realization as we discuss in Sec III E.

C. Light sterile neutrinos

This appears if the scale of some eigenvalues of M_N is not higher than the electroweak scale. As in the case with $M_N = 0$, the SM is not even a good low energy effective theory: there are more than three light neutrinos, and they are admixtures of doublet and singlet fields. Again both light and heavy neutrinos are Majorana particles.

As we will see the analysis of neutrino oscillations is the same whether the light neutrinos are of the Majorana- or Dirac-type. From the phenomenological point of view, only in the discussion of neutrinoless double beta decay the question of Majorana versus Dirac neutrinos is crucial. However, as we have tried to illustrate above, from the theoretical model building point of view, the two cases are very different.

D. Majorana ν_L masses: Type II see-saw

In order to be able to construct a gauge invariant neutrino mass term involving only left handed neutrinos one has to extend the Higgs sector of the Standard Model to include besides the doublet ϕ, an $SU(2)_L$ scalar triplet $\Delta \sim (1, 3, 1)$. We write the triplet in the matrix representation as

$$\Delta = \begin{pmatrix} \Delta^0 & -\Delta^+/\sqrt{2} \\ -\Delta^+/\sqrt{2} & -\Delta^{++} \end{pmatrix}. \tag{28}$$

The neutrino mass term arises from the Lagrangian:

$$\mathcal{L}_Y = -f_{\nu ij} \, \bar{L}^C_{L,i} \Delta \, L_{Lj} + \text{h.c.}, \tag{29}$$

When the neutral component of the triple acquires a vev $\langle \Delta_0 \rangle = v_\Delta/\sqrt{2}$, the 3 left handed neutrinos acquire a Majorana mass

$$M_\nu = f_\nu \, v_\Delta . \tag{30}$$

It is clear that v_Δ breaks L by two units. If the scalar potential preserves L the breaking is "spontaneous". In this case the model contains a massless Goldstone boson, *the triple Majoron*. Because it is part of a $SU(2)_L$ triplet, the triplet Majoron couples to the Z boson and it would contribute to its invisible decay. At present this is ruled out by the precise measurement of the Z decay width.

It could also be that the scalar potential breaks L explicitly. In this case there is no massless Goldstone boson. This explicit breaking can be induced by a triple-double mixing term in the scalar potential which would contain among others the following two terms:

$$M_\Delta^2 \, \text{Tr}(\Delta^\dagger \Delta) \; + \; \left(\mu \, \tilde{\phi}^T \, \Delta \, \tilde{\phi} \; + \; h.c. \right)$$

In this case the minimization can lead to a vev for the triple

$$v_\Delta = \frac{\mu \, v^2}{\sqrt{2} \, M_\Delta^2}. \tag{31}$$

So if $M_\Delta^2 \gg \mu \, v$, then $v_\Delta \ll v$ which gives an explanation to the smallness of the neutrino mass. This mechanism is labeled in the literature as Type II see-saw [30, 31].

E.   Neutrino Masses from Non-renormalizable Operators

In general, if the SM is an effective low energy theory valid up to the scale Λ_{NP}, the gauge group, the fermionic spectrum, and the pattern of spontaneous symmetry breaking of the SM are still valid ingredients to describe Nature at energies $E \ll \Lambda_{\text{NP}}$. But because it is an effective theory, one must also consider non-renormalizable higher dimensional terms in the Lagrangian whose effect will be suppressed by powers $1/\Lambda_{\text{NP}}^{\text{dim}-4}$. In this approach the largest effects at low energy are expected to come from dim$= 5$ operators.

There is no reason for generic NP to respect the accidental symmetries of the SM (7). Indeed, there is a single set of dimension-five terms that is made of SM fields and is consistent with the gauge symmetry, and this set violates (7). It is given by

$$\mathcal{O}_5 = \frac{Z_{ij}^\nu}{\Lambda_{\text{NP}}} \left(\bar{L}_{Li} \tilde{\phi} \right) \left(\tilde{\phi}^T L_{Lj}^C \right) + \text{h.c.}, \tag{32}$$

which violate total lepton number by two units and leads, upon spontaneous symmetry breaking, to:

$$-\mathcal{L}_{M_\nu} = \frac{Z_{ij}^\nu}{2} \frac{v^2}{\Lambda_{\text{NP}}} \bar{\nu}_{Li} \nu_{Lj}^c + \text{h.c.} \, . \tag{33}$$

Comparing with Eq. (13) we see that this is a Majorana mass term built with the left-handed neutrino fields and with:

$$(M_\nu)_{ij} = Z_{ij}^\nu \frac{v^2}{\Lambda_{\text{NP}}}. \tag{34}$$

Since Eq. (34) would arise in a generic extension of the SM, we learn that neutrino masses are very likely to appear if there is NP. As mentioned above, a theory with SM plus m heavy

sterile neutrinos leads to three light mass eigenstates and an effective low energy interaction of the form (32). In particular, the scale Λ_{NP} is identified with the mass scale of the heavy sterile neutrinos, that is the typical scale of the eigenvalues of M_N.

Furthermore, comparing Eq. (34) and Eq. (9), we find that the scale of neutrino masses is suppressed by v/Λ_{NP} when compared to the scale of charged fermion masses providing an explanation not only for the existence of neutrino masses but also for their smallness. Finally, Eq. (34) breaks not only total lepton number but also the lepton flavor symmetry $U(1)_e \times U(1)_\mu \times U(1)_\tau$. Therefore, as we shall see in Sec. IV, we should expect lepton mixing and CP violation unless additional symmetries are imposed on the coefficients Z_{ij}.

IV.   LEPTON MIXING

The possibility of arbitrary mixing between two massive neutrino states was first introduced in Ref. [32]. In the general case, we denote the neutrino mass eigenstates by $(\nu_1, \nu_2, \nu_3, \ldots, \nu_n)$ and the charged lepton mass eigenstates by (e, μ, τ). The corresponding interaction eigenstates are denoted by (e^I, μ^I, τ^I) and $\vec{\nu} = (\nu_{Le}, \nu_{L\mu}, \nu_{L\tau}, \nu_{s1}, \ldots, \nu_{sm})$. In the mass basis, leptonic charged current interactions are given by

$$-\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}}(\bar{e}_L,\, \bar{\mu}_L,\, \bar{\tau}_L)\gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \\ \nu_n \end{pmatrix} W_\mu^+ - \mathrm{h.c.}. \tag{35}$$

Here U is a $3 \times n$ matrix [31, 33, 34] which verifies

$$UU^\dagger = I_{3\times 3} \tag{36}$$

but in general $U^\dagger U \neq I_{n\times n}$.

The charged lepton and neutrino mass terms and the neutrino mass in the interaction basis are:

$$-\mathcal{L}_M = [(\bar{e}_L^I,\, \bar{\mu}_L^I,\, \bar{\tau}_L^I)M_\ell \begin{pmatrix} e_R^I \\ \mu_R^I \\ \tau_R^I \end{pmatrix} + \mathrm{h.c.}] - \mathcal{L}_{M_\nu} \tag{37}$$

with $\mathcal{L}_{M_\nu}$ given in Eq. (13). One can find two 3×3 unitary diagonalizing matrices for the charge leptons, V^ℓ and V_R^ℓ, such that

$$V^{\ell\dagger} M_\ell V_R^\ell = \text{Diag}(m_e, m_\mu, m_\tau).$$ (38)

The charged lepton mass term can be written as:

$$-\mathcal{L}_{M_\ell} = \sum_{k=1}^{3} m_{\ell_k} \bar{\ell}_k \ell_k$$ (39)

where

$$\ell_k = (V^{\ell\dagger} \ell_L^I)_k + (V_R^{\ell\dagger} \ell_R^I)_k$$ (40)

so the weak-doublet components of the charge lepton fields are

$$\ell_{Li}^I = L \sum_{j=1}^{3} V_{ij}^\ell \ell_j, \qquad i = 1, 2, 3$$ (41)

From Eqs. (20), (24) and (41) we find that U is:

$$U_{ij} = P_{\ell,ii} V_{ik}^{\ell\dagger} V_{kj}^\nu (P_{\nu,jj}).$$ (42)

P_ℓ is a diagonal 3×3 phase matrix, that is conventionally used to reduce by three the number of phases in U. P_ν is a diagonal $n \times n$ phase matrix with additional arbitrary phases which can chosen to reduce the number of phases in U by $n-1$ only for Dirac states. For Majorana neutrinos, this matrix is simply a unit matrix. The reason for that is that if one rotates a Majorana neutrino by a phase, this phase will appear in its mass term which will no longer be real. Thus, the number of phases that can be absorbed by redefining the mass eigenstates depends on whether the neutrinos are Dirac or Majorana particles. Altogether for Majorana [Dirac] neutrinos the U matrix contains a total of $6(n-2)$ $[5n-11]$ real parameters, of which $3(n-2)$ are angles and $3(n-2)$ $[2n-5]$ can be interpreted as physical phases.

In particular, if there are only three Majorana neutrinos, U is a 3×3 matrix analogous to the CKM matrix for the quarks [35] but due to the Majorana nature of the neutrinos it depends on six independent parameters: three mixing angles and three phases. In this case the mixing matrix can be conveniently parametrized as:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (43)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The angles θ_{ij} can be taken without loss of generality to lie in the first quadrant, $\theta_{ij} \in [0, \pi/2]$ and the phases δ_{CP}, $\eta_i \in [0, 2\pi]$. This is to be compared to the case of three Dirac neutrinos, where the Majorana phases, η_1 and η_2, can be absorbed in the neutrino states and therefore the number of physical phases is one (similarly to the CKM matrix). In this case the mixing matrix U takes the form [24]:

$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{i\delta_{\mathrm{CP}}} & c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{i\delta_{\mathrm{CP}}} & c_{13}\, s_{23} \\ s_{12}\, s_{23} - c_{12}\, s_{13}\, c_{23}\, e^{i\delta_{\mathrm{CP}}} & -c_{12}\, s_{23} - s_{12}\, s_{13}\, c_{23}\, e^{i\delta_{\mathrm{CP}}} & c_{13}\, c_{23} \end{pmatrix}. \tag{44}$$

Note, however, that the two extra Majorana phases are very hard to measure since they are only physical if neutrino mass is non-zero and therefore the amplitude of any process involving them is suppressed a factor m_ν/E to some power where E is the energy involved in the process which is typically much larger than the neutrino mass. The most sensitive experimental probe of Majorana phases is the rate of neutrinoless $\beta\beta$ decay.

If no new interactions for the charged leptons are present we can identify their interaction eigenstates with the corresponding mass eigenstates after phase redefinitions. In this case the charged current lepton mixing matrix U is simply given by a $3 \times n$ sub-matrix of the unitary matrix V^ν.

It worth noticing that while for the case of 3 light Dirac neutrinos the procedure leads to a fully unitary U matrix for the light states, generically for three light Majorana neutrinos this is not the case when the full spectrum contains heavy neutrino states which have been integrated out as can be seen, from Eq. (27). Thus, strictly speaking, the parametrization in Eq. (43) does not hold to describe the flavor mixing of the three light Majorana neutrinos in the type I see-saw mechanism. However, as seen in Eq. (27), the unitarity violation is of the order $\mathcal{O}(M_D/M_N)$ and it is expected to be very small (at it is also severely constrained experimentally). Consequently in what follows we will ignore this effect.

V. NEUTRINO OSCILLATIONS IN VACUUM

If neutrinos have masses, the weak eigenstates, ν_α, produced in a weak interaction are, in general, linear combinations of the mass eigenstates ν_i

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i}|\nu_i\rangle \tag{45}$$

where n is the number of light neutrino species and U is the the mixing matrix. (Implicit in our definition of the state $|\nu\rangle$ is its energy-momentum and space-time dependence). After traveling a distance L (or, equivalently for relativistic neutrinos, time t), a neutrino originally produced with a flavor α evolves as:

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{n} U^*_{\alpha i} |\nu_i(t)\rangle , \tag{46}$$

and it can be detected in the charged-current (CC) interaction $\nu_\alpha(t) N' \rightarrow \ell_\beta N$ with a probability

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = |\sum_{i=1}^{n} \sum_{j=1}^{n} U^*_{\alpha i} U_{\beta j} \langle \nu_j | \nu_i(t) \rangle|^2 , \tag{47}$$

where E_i and m_i are, respectively, the energy and the mass of the neutrino mass eigenstate ν_i.

Using the standard approximation that $|\nu\rangle$ is a plane wave $|\nu_i(t)\rangle = e^{-i E_i t} |\nu_i(0)\rangle$, that neutrinos are relativistic with $p_i \simeq p_j \equiv p \simeq E$

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p + \frac{m_i^2}{2E} \tag{48}$$

and the orthogonality relation $\langle \nu_j | \nu_i \rangle = \delta_{ij}$, we get the following transition probability

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i<j}^{n} \mathrm{Re}[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin^2 X_{ij} + 2 \sum_{i<j}^{n} \mathrm{Im}[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin 2 X_{ij} , \tag{49}$$

where

$$X_{ij} = \frac{(m_i^2 - m_j^2) L}{4E} = 1.27 \frac{\Delta m_{ij}^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{m/MeV}} . \tag{50}$$

Here $L = t$ is the distance between the production point of ν_α and the detection point of ν_β. The first line in Eq. (49) is CP conserving while the second one is CP violating and has opposite sign for neutrinos and antineutrinos.

The transition probability, Eq. (49), has an oscillatory behavior, with oscillation lengths

$$L^{\mathrm{osc}}_{0,ij} = \frac{4\pi E}{\Delta m_{ij}^2} \tag{51}$$

and amplitudes that are proportional to elements in the mixing matrix. Thus, in order to undergo flavor oscillations, neutrinos must have different masses ($\Delta m_{ij}^2 \neq 0$) and they must mix ($U_{\alpha i} U_{\beta i} \neq 0$). Also, as can be seen from Eq. (49), the Majorana phases cancel out in the oscillation probability as expected because flavor oscillation is a total lepton number conserving process.

A neutrino oscillation experiment is characterized by the typical neutrino energy E and by the source-detector distance L. But in general, neutrino beams are not monoenergetic and, moreover, detectors have finite energy resolution. Thus, rather than measuring $P_{\alpha\beta}$, the experiments are sensitive to the average probability

$$
\begin{aligned}
\langle P_{\alpha\beta} \rangle &= \frac{\int dE \frac{d\Phi}{dE} \sigma_{CC}(E) P_{\alpha\beta}(E) \epsilon(E)}{\int dE \frac{d\Phi}{dE} \sigma_{CC}(E) \epsilon(E)} \\
&= \delta_{\alpha\beta} - 4 \sum_{i<j}^{n} \text{Re}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \langle \sin^2 X_{ij} \rangle \\
&\quad + 2 \sum_{i<j}^{n} \text{Im}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \langle \sin 2 X_{ij} \rangle ,
\end{aligned}
\tag{52}
$$

where Φ is the neutrino energy spectrum, σ_{CC} is the cross section for the process in which the neutrino is detected (in general, a CC interaction), and $\epsilon(E)$ is the detection efficiency. The range of the energy integral depends on the energy resolution of the experiment.

In order to be sensitive to a given value of Δm_{ij}^2, the experiment has to be set up with $E/L \approx \Delta m_{ij}^2$ ($L \sim L_{0,ij}^{\text{osc}}$). The typical values of L/E for different types of neutrino sources and experiments and the corresponding ranges of Δm^2 to which they can be most sensitive are summarized in Table II.

Generically if $(E/L) \gg \Delta m_{ij}^2$ ($L \ll L_{0,ij}^{\text{osc}}$), the oscillation phase does not have time to give an appreciable effect because $\sin^2 X_{ij} \ll 1$. Conversely if $L \gg L_{0,ij}^{\text{osc}}$, the oscillating phase goes through many cycles before the detection and is averaged to $\langle \sin^2 X_{ij} \rangle = 1/2$. Maximum sensitivity to the oscillation phase – and correspondingly to Δm^2 – is obtained when the set up is such that:

- $E/L \approx \Delta m_{ij}^2$,

- the energy resolution of the experiment is good enough, $\Delta E \ll L \Delta m_{ij}^2$,

- the experiment is sensitive to different values of L with $\Delta L \ll E/\Delta m^2$.

For a two-neutrino case, the mixing matrix depends on a single parameter,

$$
U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} ,
\tag{53}
$$

and there is a single mass-squared difference Δm^2. Then $P_{\alpha\beta}$ of Eq. (49) takes the well known form

$$
P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 X .
\tag{54}
$$

TABLE II: Characteristic values of L and E for various neutrino sources and experiments and the corresponding ranges of Δm^2 to which they can be most sensitive.

Experiment		L (m)	E (MeV)	Δm^2 (eV^2)
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	10^2–10^5	$10^{-1} - 10^{-4}$
Reactor	SBL	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^2	10^3–10^4	> 0.1
	LBL	$10^5 - 10^6$	10^4	$10^{-2} - 10^{-3}$

The physical parameter space is covered with $\Delta m^2 \geq 0$ and $0 \leq \theta \leq \frac{\pi}{2}$ (or, alternatively, $0 \leq \theta \leq \frac{\pi}{4}$ and either sign for Δm^2).

Changing the sign of the mass difference, $\Delta m^2 \to -\Delta m^2$, and changing the octant of the mixing angle, $\theta \to \frac{\pi}{2} - \theta$, amounts to redefining the mass eigenstates, $\nu_1 \leftrightarrow \nu_2$: $P_{\alpha\beta}$ must be invariant under such transformation. Eq. (54) reveals, however, that $P_{\alpha\beta}$ is actually invariant under each of these transformations separately. This situation implies that there is a two-fold discrete ambiguity in the interpretation of $P_{\alpha\beta}$ in terms of two-neutrino mixing: the two different sets of physical parameters, $(\Delta m^2, \theta)$ and $(\Delta m^2, \frac{\pi}{2} - \theta)$, give the same transition probability in vacuum. One cannot tell from a measurement of, say, $P_{e\mu}$ in vacuum whether the larger component of ν_e resides in the heavier or in the lighter neutrino mass eigenstate. This symmetry is lost when neutrinos travel through regions of dense matter and/or for when there are more than two neutrinos mixed in the neutrino evolution.

VI.   PROPAGATION OF MASSIVE NEUTRINOS IN MATTER

When neutrinos propagate in dense matter, the interactions with the medium affect their properties. These effects can be either coherent or incoherent. For purely incoherent inelastic ν-p scattering, the characteristic cross section is very small:

$$\sigma \sim \frac{G_F^2 s}{\pi} \sim 10^{-43} \text{ cm}^2 \left(\frac{E}{\text{MeV}}\right)^2. \tag{55}$$

On the contrary, in coherent interactions, the medium remains unchanged and it is possible to have interference of scattered and unscattered neutrino waves which enhances the effect.

Coherence further allows one to decouple the evolution equation of the neutrinos from the equations of the medium. In this approximation, the effect of the medium is described by an effective potential which depends on the density and composition of the matter [36].

Taking this into account, the evolution equation for n ultrarelativistic neutrinos propagating in matter written in the mass basis can be casted in the following form (there are several derivations in the literature of the evolution equation of a neutrino system in matter, see for instance Ref. [37–39]):

$$i\frac{d\vec{\nu}}{dx} = H\,\vec{\nu}, \qquad H = H_m + U^{\nu\dagger}\,V\,U^{\nu}\,, \tag{56}$$

where $\vec{\nu} \equiv (\nu_1,\, \nu_2,\, \ldots,\, \nu_n)^T$, H_m is the Hamiltonian for the kinetic energy,

$$H_m = \frac{1}{2E}\mathrm{Diag}(m_1^2,\, m_2^2,\, \ldots,\, m_n^2), \tag{57}$$

and V is the effective potential that describes the coherent forward interactions of the neutrinos with matter in the interaction basis. U^{ν} is the $n \times n$ submatrix of the unitary V^{ν} matrix corresponding to the n ultrarelativistic neutrino states.

Let's consider the evolution of ν_e in a medium with electrons, protons and neutrons with corresponding n_e, n_p and n_n number densities. The effective low-energy Hamiltonian describing the relevant neutrino interactions is given by

$$H_W = \frac{G_F}{\sqrt{2}}\left[J^{(+)\alpha}(x)J_\alpha^{(-)}(x) + \frac{1}{4}J^{(N)\alpha}(x)J_\alpha^{(N)}(x)\right], \tag{58}$$

where the J_α's are the standard fermionic currents

$$J_\alpha^{(+)}(x) = \bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)e(x)\,, \tag{59}$$

$$J_\alpha^{(-)}(x) = \bar{e}(x)\gamma_\alpha(1 - \gamma_5)\nu_e(x)\,, \tag{60}$$

$$\begin{aligned} J_\alpha^{(N)}(x) = &\ \bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)\nu_e(x) \\ &- \bar{e}(x)[\gamma_\alpha(1 - \gamma_5) - 4\sin^2\theta_W\gamma_\alpha]e(x) \\ &+ \bar{p}(x)[\gamma_\alpha(1 - g_A^{(p)}\gamma_5) - 4\sin^2\theta_W\gamma_\alpha]p(x) \\ &- \bar{n}(x)\gamma_\alpha(1 - g_A^{(n)}\gamma_5)n(x)\,, \end{aligned} \tag{61}$$

and $g_A^{(n,p)}$ are the axial couplings for neutrons and protons, respectively.

Consider first the effect of the charged current interactions. The effective CC Hamiltonian

due to electrons in the medium is

$$
\begin{aligned}
H_C^{(e)} &= \frac{G_F}{\sqrt{2}} \int d^3 p_e f(E_e, T) \\
&\times \Big\langle \langle e(s, p_e) | \bar{e}(x)\gamma^\alpha(1 - \gamma_5)\nu_e(x)\bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)e(x) | e(s, p_e) \rangle \Big\rangle \\
&= \frac{G_F}{\sqrt{2}}\bar{\nu}_e(x)\gamma_\alpha(1 - \gamma_5)\nu_e(x) \\
&\int d^3 p_e f(E_e, T) \Big\langle \langle e(s, p_e) | \bar{e}(x)\gamma_\alpha(1 - \gamma_5)e(x) | e(s, p_e) \rangle \Big\rangle,
\end{aligned}
\tag{62}
$$

where s is the electron spin and p_e its momentum. The energy distribution function of the electrons in the medium, $f(E_e, T)$, is assumed to be homogeneous and isotropic and is normalized as

$$
\int d^3 p_e f(E_e, T) = 1 \,.
\tag{63}
$$

By $\Big\langle \dots \Big\rangle$ we denote the averaging over electron spinors and summing over all electrons in the medium. Notice that coherence implies that s, p_e are the same for initial and final electrons. To calculate the averaging we notice that the axial current reduces to the spin in the non-relativistic limit and therefore averages to zero for a background of non-relativistic electrons. The spatial components of the vector current cancel because of isotropy and therefore the only non trivial average is

$$
\int d^3 p_e f(E_e, T) \Big\langle \langle e(s, p_e) | \bar{e}(x)\gamma_0 e(x) | e(s, p_e) \rangle \Big\rangle = n_e(x)
\tag{64}
$$

which gives a contribution to the effective Hamiltonian

$$
H_C^{(e)} = \sqrt{2}G_F n_e \bar{\nu}_{eL}(x)\gamma_0 \nu_{eL}(x) \,.
\tag{65}
$$

This can be interpreted as a contribution to the ν_{eL} potential energy

$$
V_C = \sqrt{2}G_F n_e \,.
\tag{66}
$$

A more detailed derivation of the matter potentials can be found, for example, in Ref. [21].

For ν_μ and ν_τ, the potential due to its CC interactions is zero for most media since neither μ's nor $\tau's$ are present.

In the same fashion one can derive the effective potential for any active neutrino due to the neutral current interactions to be

$$
V_{NC} = \frac{\sqrt{2}}{2}G_F \left[-n_e(1 - 4\sin^2\theta_w) + n_p(1 - 4\sin^2\theta_w) - n_n \right] \,.
\tag{67}
$$

For neutral matter $n_e = n_p$ so the contribution from electrons and protons cancel each other and we are left only with the neutron contribution

$$V_{NC} = -1/\sqrt{2} G_F n_n \tag{68}$$

Altogether we can write the evolution equation for the three SM active neutrinos with purely SM interactions in a neutral medium with electrons, protons and neutrons as Eq. (56) with $U^\nu \equiv U$, and the effective potential:

$$V = \text{Diag}\left(\pm\sqrt{2} G_F n_e(x),\, 0,\, 0\right) \equiv \text{Diag}\left(V_e,\, 0,\, 0\right). \tag{69}$$

In Eq. (69), the sign $+$ ($-$) refers to neutrinos (antineutrinos), and $n_e(x)$ is the electron number density in the medium, which in general changes along the neutrino trajectory and so does the potential. For example, at the Earth core $V_e \sim 10^{-13}$ eV while at the solar core $V_e \sim 10^{-12}$ eV. Notice that the neutral current potential Eq. (68) is flavor diagonal and therefore it can be eliminated from the evolution equation as it only contributes to an overall phase which is unobservable.

The instantaneous mass eigenstates in matter, ν_i^m, are the eigenstates of H for a fixed value of x, which are related to the interaction basis by

$$\vec{\nu} = \tilde{U}(x)\vec{\nu^m}, \tag{70}$$

while $\mu_i(x)^2/(2E)$ are the corresponding instantaneous eigenvalues with $\mu_i(x)$ being the instantaneous effective neutrino masses.

For the simplest case of the evolution of a neutrino state which is an admixture of only two neutrino species $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$

$$\mu_{1,2}^2(x) = \frac{m_1^2 + m_2^2}{2} + E[V_\alpha + V_\beta] \mp \frac{1}{2}\sqrt{[\Delta m^2 \cos 2\theta - A]^2 + [\Delta m^2 \sin 2\theta]^2}, \tag{71}$$

and $\tilde{U}(x)$ can be written as Eq. (53) with the instantaneous mixing angle in matter given by

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}. \tag{72}$$

The quantity A is defined by

$$A \equiv 2E(V_\alpha - V_\beta). \tag{73}$$

Notice that for a given sign of A (which depends on the composition of the medium and on the flavor composition of the neutrino state) the mixing angle in matter is larger or smaller

than in vacuum depending on whether this last one lies on the first or the second octant. Thus the symmetry present in vacuum oscillations is broken by matter potentials.

Generically matter effects are important when for some of the states the corresponding potential difference factor, A, is comparable to their mass difference term $\Delta m^2 \cos 2\theta$. Most relevant, as seen in Eq. (72), the mixing angle $\tan \theta_m$ changes sign if in some point along its path the neutrino passes by some matter density region verifying the *resonance condition*

$$A_R = \Delta m^2 \cos 2\theta \, . \tag{74}$$

Thus if the neutrino is created in a region where the relevant potential verifies $A_0 > A_R$, then the effective mixing angle in matter at the production point verifies that $\mathrm{sgn}(\cos 2\theta_{m,0}) = -\mathrm{sgn}(\cos 2\theta)$, this is, the flavor component of the mass eigenstates is inverted as compared to their composition in vacuum. For example for $A_0 = 2A_R$ $\theta_{m,0} = \frac{\Pi}{2} - \theta$. Asymptotically, for $A_0 \gg A_R$, $\theta_{m,0} \to \frac{\pi}{2}$.

In other words, if in vacuum the lightest mass eigenstate has a larger projection on the flavor α while the heaviest has it on the flavor β, once inside a matter potential with $A > A_R$ the opposite holds. Thus for a neutrino system which is traveling across a monotonically varying matter potential the dominant flavor component of a given mass eigenstate changes when crossing the region with $A = A_R$. This phenomenon is known as *level crossing*.

In the instantaneous mass basis the evolution equation reads:

$$i\frac{d\vec{\nu}^m}{dx} = \left[\frac{1}{2E} \mathrm{Diag}\left(\mu_1^2(x), \, \mu_2^2(x), \, \dots, \, \mu_n^2(x) \right) - i\, \tilde{U}^\dagger(x) \frac{d\tilde{U}(x)}{dx} \right] \vec{\nu}^m \, . \tag{75}$$

Because of the last term, Eq. (75) constitute a system of coupled equations which implies that the instantaneous mass eigenstates, ν_i^m, mix in the evolution and are not energy eigenstates. For constant or slowly enough varying matter potential this last term can be neglected. In this case the instantaneous mass eigenstates, ν_i^m, behave approximately as energy eigenstates and they do not mix in the evolution. This is the *adiabatic* transition approximation. On the contrary, when the last term in Eq. (75) cannot be neglected, the instantaneous mass eigenstates mix along the neutrino path so there can be *level-jumping* [40, 41] and the evolution is *non-adiabatic*.

The oscillation probability takes a particularly simple form for adiabatic evolution in matter and it can be cast very similarly to the vacuum oscillation expression, Eq. (49). For

example, neglecting CP violation:

$$P_{\alpha\beta} = \left| \sum_i \tilde{U}_{\alpha i}(0)\tilde{U}_{\beta i}(L) \exp\left(-\frac{i}{2E} \int_0^L \mu_i^2(x')dx' \right) \right|^2. \tag{76}$$

In general $P_{\alpha\beta}$ has to be evaluated numerically although there exist in the literature several analytical approximations for specific profiles of the matter potential [42].

A.   The MSW Effect for Solar Neutrinos

As an illustration of the matter effects discussed in the previous section we describe now the propagation of a $\nu_e - \nu_X$ neutrino system in the matter density of the Sun where X is some superposition of μ and τ.

The solar density distribution decreases monotonically with the distance R to the center of the Sun. For $R < 0.9R_\odot$ it can be approximated by an exponential

$$n_e(R) = n_e(0) \exp\left(-R/r_0 \right) \tag{77}$$

with $r_0 = R_\odot/10.54 = 6.6 \times 10^7$ m $= 3.3 \times 10^{14}$ eV^{-1}. After traversing this density the dominant component of the exiting neutrino state depends on the value of the mixing angle in vacuum, and on the relative size of $\Delta m^2 \cos 2\theta$ versus $A_0 = 2\,E\,G_F\,n_{e,0}$ (at the neutrino production point) as we describe next:

- If $\Delta m^2 \cos 2\theta \gg A_0$ matter effects are negligible and the propagation occurs as in vacuum with the oscillating phase averaged out due to the large value of L. In this case the survival probability at the sunny surface of the Earth is

$$P_{ee}(\Delta m^2 \cos 2\theta \gg A_0) = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}. \tag{78}$$

- If $\Delta m^2 \cos 2\theta \gtrsim A_0$ the neutrino does not pass any resonance region but its mixing is affected by the solar matter. This effect is well described by an adiabatic propagation, Eq. (76). Using

$$\tilde{U}(0) = \begin{pmatrix} \cos\theta_{m,0} & \sin\theta_{m,0} \\ -\sin\theta_{m,0} & \cos\theta_{m,0} \end{pmatrix}, \qquad \tilde{U}(L) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{79}$$

(where $\theta_{m,0}$ is the mixing angle in matter at the production point) we get

$$P_{ee} = \cos^2 \theta_{m,0} \cos^2 \theta + \sin^2 \theta_{m,0} \sin^2 \theta$$

$$+ \frac{1}{2} \sin^2 2\theta_{m,0} \sin^2 2\theta \cos \left(\frac{\int_0^L \mu_2^2(x') - \mu_1^2(x')}{2E} dx' \right) . \quad (80)$$

For all practical purposes, the oscillation term in Eq. (80) is averaged out in the regime $\Delta m^2 \cos 2\theta \gtrsim A_0$ and then the resulting probability reads

$$P_{ee}(\Delta m^2 \cos 2\theta \geq A_0) = \cos^2 \theta_{m,0} \cos^2 \theta + \sin^2 \theta_{m,0} \sin^2 \theta$$
$$= \frac{1}{2} \left[1 + \cos 2\theta_{m,0} \cos 2\theta \right] . \quad (81)$$

The physical interpretation of this expression is straightforward. An electron neutrino produced at A_0 consists of an admixture of ν_1 with fraction $P_{e1,0} = \cos^2 \theta_{m,0}$ and ν_2 with fraction $P_{e2,0} = \sin^2 \theta_{m,0}$. At the exit ν_1 consists of ν_e with fraction $P_{1e} = \cos^2 \theta$ and ν_2 consists of ν_e with fraction $P_{2e} = \sin^2 \theta$ so [43–45]

$$P_{ee} = P_{e1,0}P_{1e} + P_{e2,0}P_{2e} \quad (82)$$

which reproduces Eq. (81). Notice that as long as $A_0 < A_R$ the resonance is not crossed and consequently $\cos 2\theta_{m,0}$ has the same sign as $\cos 2\theta$ and the corresponding survival probability is also larger than $1/2$.

- If $\Delta m^2 \cos 2\theta < A_0$ the neutrino can cross the resonance on its way out if, in the convention of positive Δm^2, $\cos 2\theta > 0$ ($\theta < \pi/4$). In this case, at the production point ν_e is a combination of ν_1^m and ν_2^m with larger ν_2^m component while outside of the Sun the opposite holds. More quantitatively for $\Delta m^2 \cos 2\theta \ll A_0$ (density at the production point much higher than the resonant density),

$$\theta_{m,0} = \frac{\pi}{2} \quad \Rightarrow \quad \cos 2\theta_{m,0} = -1 . \quad (83)$$

Depending on the particular values of Δm^2 and the mixing angle, the evolution can be adiabatic or non-adiabatic. Presently we know that the oscillation parameters are such that the transition is indeed adiabatic for all ranges of solar neutrino energies. Thus the survival probability at the sunny surface of the Earth is

$$P_{ee}(\Delta m^2 \cos 2\theta < A_0) = \frac{1}{2} \left[1 + \cos 2\theta_{m,0} \cos 2\theta \right] = \sin^2 \theta \quad (84)$$

where we have used Eq. (83). Thus in this regime P_{ee} can be much smaller than $1/2$ because $\cos 2\theta_{m,0}$ and $\cos 2\theta$ have opposite signs. This is the MSW effect [36, 46] which plays a crucial role in the interpretation of the solar neutrino data.

VII.   GLOBAL 3ν ANALYSIS OF OSCILLATION DATA

In these lectures I am not going to discuss the experimental situation on neutrino oscillation searches which I assume will be covered in other lectures. However, for consistency, I am including in these notes the output of the combined global combined analysis of the present oscillation search experiments.

The minimum joint description of solar, atmospheric, long-baseline, and reactor data, requires that all three known neutrinos take part in the oscillations. In this case, the mixing parameters are encoded in the 3×3 lepton mixing matrix [32, 35] which can be conveniently parametrized in the standard form of Eq. (44), since the two Majorana phases in Eq. (43) do not affect neutrino oscillations.

The results of the global combined analysis including all dominant and subdominant oscillation effects are summarized in Fig. 1 and Fig. 2 in which we show different projections of the allowed 6-dimensional parameter space. New to previous analysis is the inclusion of the latest MINOS and of the SK-II atmospheric data and the inclusion in the analysis of the effect of $\delta_{\rm CP}$.

In Fig. 1 we plot the correlated bounds from the global analysis several pairs of parameters. The regions in each panel are obtained after marginalization of $\chi^2_{\rm global}$ with respect to the three undisplayed parameters. The different contours correspond to regions defined at 90%, 95%, 99% and 3σ CL for 2 d.o.f. ($\Delta\chi^2 = 4.61, 5.99, 9.21, 11.83$) respectively. From the figure we see that the stronger correlation appears between θ_{13} and Δm^2_{31} as a reflection of the CHOOZ bound. In the lower panels we show the allowed regions in the ($\sin^2 \theta_{13}$, $\delta_{\rm CP}$) plane. As seen in the figure, the sensitivity to the CP phase at present is marginal but we find that the present bound on $\sin^2 \theta_{13}$ can vary by about $\sim 30\%$ depending on the exact value of $\delta_{\rm CP}$. This is a pure 3ν mixing effects and arises from the interference of θ_{13} and Δm^2_{21} effects in the atmospheric neutrino observables. To illustrate this point we plot in the same panel the bounds on $\sin^2 \theta_{13}$ at 90%, 95%, 99% and 3σ CL if the atmospheric data is not included in the analysis (the vertical lines).
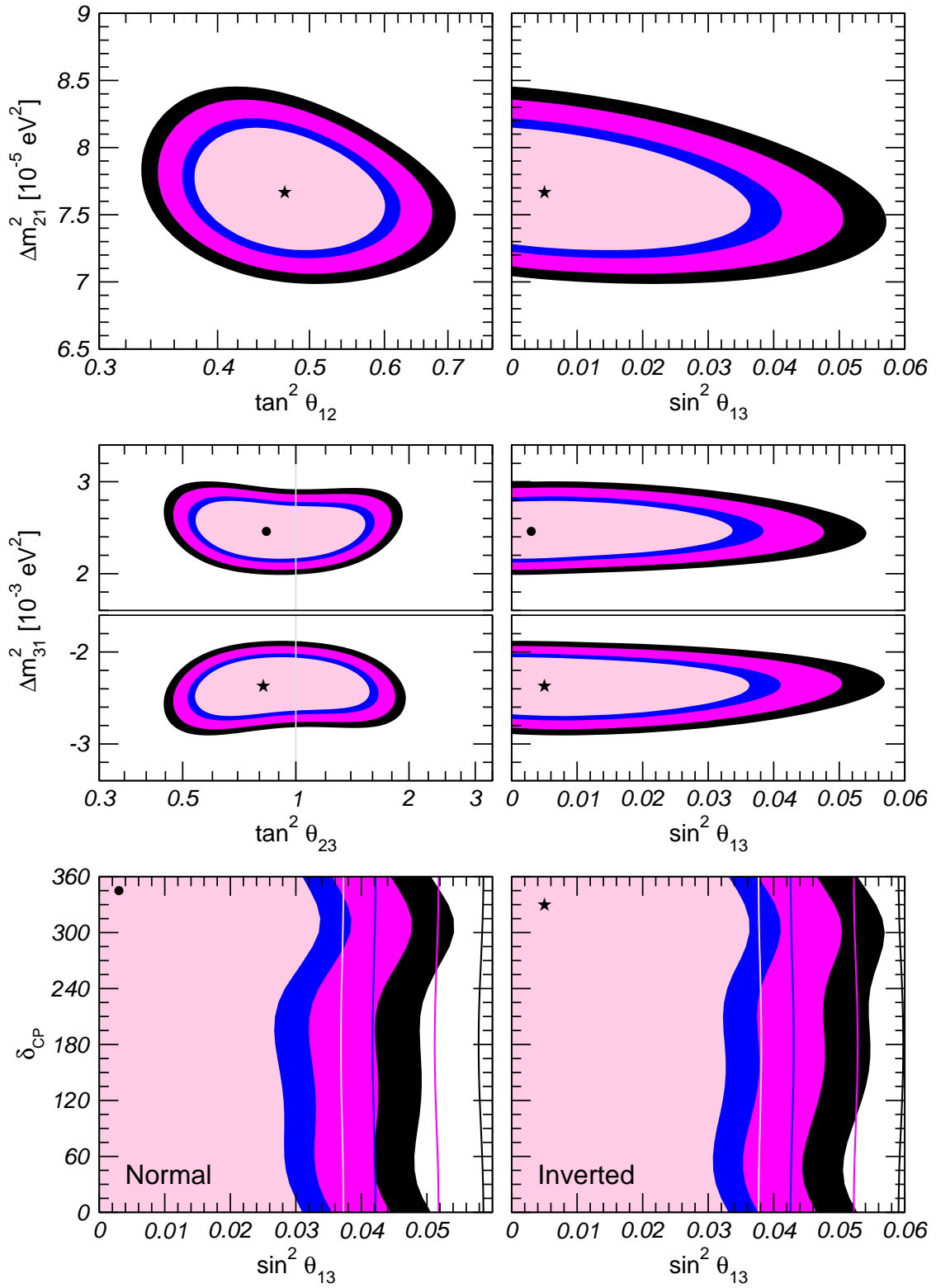
FIG. 1: Global 3ν oscillation analysis. Each panels shows 2-dimensional projection of the allowed 5-dimensional region after marginalization with respect to the undisplayed parameters. The different contours correspond to the two-dimensional allowed regions at 90%, 95%, 99% and 3σ CL. In the lowest panel the vertical lines correspond to the regions without inclusion of the atmospheric neutrino data.

In Fig. 2 we plot the individual bounds on each of the six relevant parameters derived from the global analysis (full line). To illustrate the impact of the LBL and KamLAND data we also show the corresponding bounds when KamLAND and the LBL data are not included in the analysis respectively. In each panel, except the lower left one, the displayed χ^2 has been marginalized with respect to the other five parameters. The lower left panel shows the χ^2 (marginalized over all parameters but θ_{13}) dependence of δ_{CP} for fixed values of θ_{13}.

The derived ranges for the six parameters at 1σ (3σ) are:

$$\Delta m_{21}^2 = 7.67\,^{+0.22}_{-0.21}\,\left(^{+0.67}_{-0.61}\right) \times 10^{-5} \text{ eV}^2\,,$$

$$\Delta m_{31}^2 = \begin{cases} -2.37 \pm 0.15 \,\left(^{+0.43}_{-0.46}\right) \times 10^{-3} \text{ eV}^2 & \text{(inverted hierarchy)}\,, \\ +2.46 \pm 0.15 \,\left(^{+0.47}_{-0.42}\right) \times 10^{-3} \text{ eV}^2 & \text{(normal hierarchy)}\,, \end{cases}$$

$$\theta_{12} = 34.5 \pm 1.4 \,\left(^{+4.8}_{-4.0}\right)\,,$$

$$\theta_{23} = 42.3\,^{+5.1}_{-3.3}\,\left(^{+11.3}_{-7.7}\right)\,,$$

$$\theta_{13} = 0.0\,^{+7.9}_{-0.0}\,\left(^{+12.9}_{-0.0}\right)\,,$$

$$\delta_{CP} \in [0,\ 360]\,. \tag{85}$$

Figure 2 illustrates that the dominant effect of the inclusion of the laboratory experiments KamLAND, K2K and MINOS is the better determination of the corresponding mass differences Δm_{21}^2 and Δm_{31}^2 while the mixing angle θ_{12} is dominantly determined by the solar data and the mixing angle θ_{23} is still most precisely measured in the atmospheric neutrino experiments. The non-maximality of the best θ_{23} observed in Eq. (85) is a pure 3-ν oscillation effect associated to the inclusion of the Δm_{21}^2 effects in the atmospheric neutrino analysis.

The values on the magnitude of the elements of the leptonic mixing matrix, at 90% CL are:

$$|U|_{90\%} = \begin{pmatrix} 0.80 \to 0.84 & 0.53 \to 0.60 & 0.00 \to 0.17 \\ 0.29 \to 0.52 & 0.51 \to 0.69 & 0.61 \to 0.76 \\ 0.26 \to 0.50 & 0.46 \to 0.66 & 0.64 \to 0.79 \end{pmatrix} \tag{86}$$
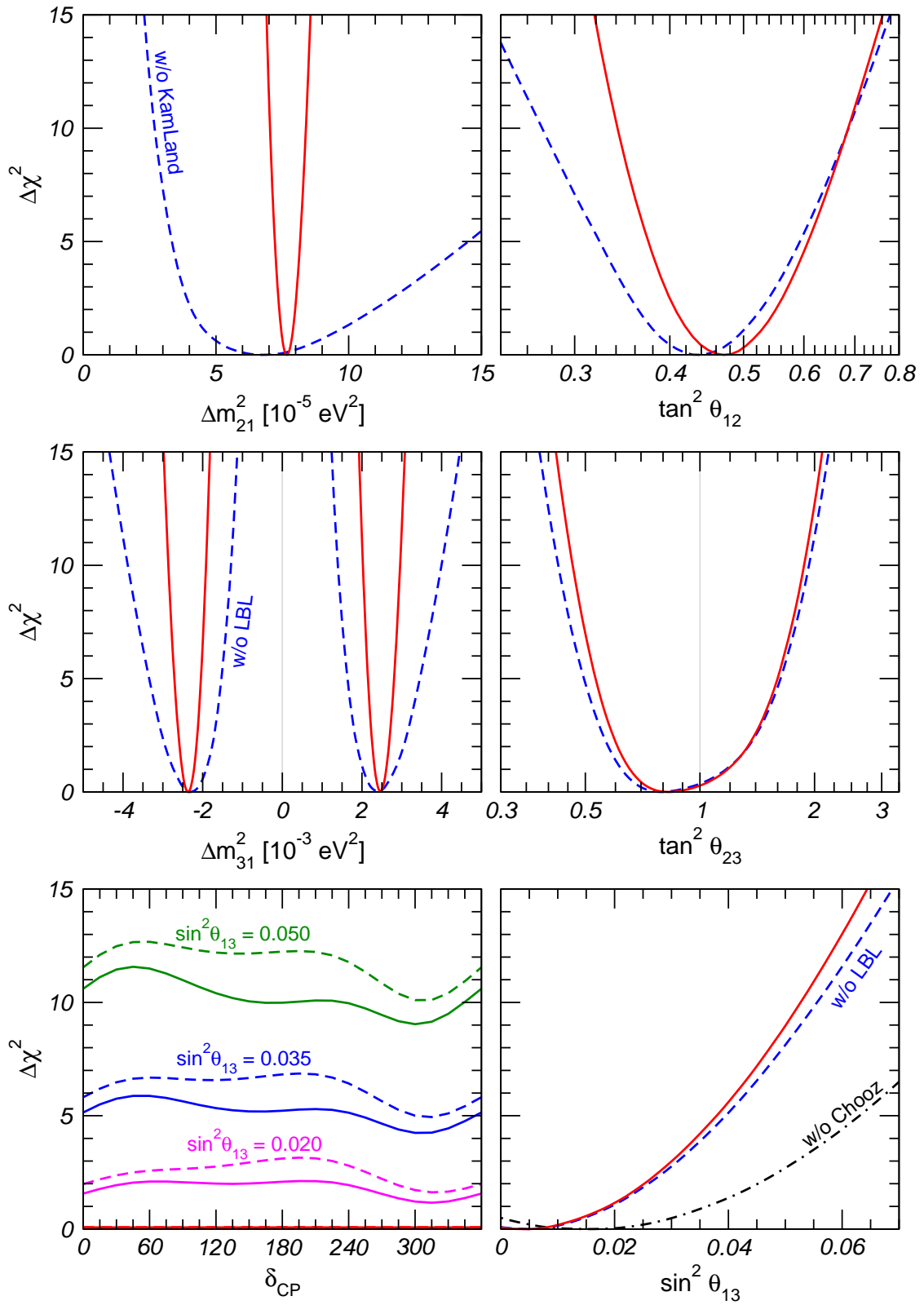
FIG. 2: Global 3ν oscillation analysis. Each panels shows the dependence of $\Delta\chi^2$ on each of the parameters from the global analysis (full line) compared to the bound prior to the KamLAND data (dashed lines in the first row) and LBL data (dashed lines in the second rows). In the lowest panel we show the dependence of $\Delta\chi^2$ on θ_{13} for different sets of data as labeled in the curves. The individual 1σ (3σ) bounds in Eqs. (85) can be read from the corresponding panel with the condition $\Delta\chi^2 \leq 1$ (9).

and at the 3σ level:

$$|U|_{3\sigma} = \begin{pmatrix} 0.77 \rightarrow 0.86 & 0.50 \rightarrow 0.63 & 0.00 \rightarrow 0.22 \\ 0.22 \rightarrow 0.56 & 0.44 \rightarrow 0.73 & 0.57 \rightarrow 0.80 \\ 0.21 \rightarrow 0.55 & 0.40 \rightarrow 0.71 & 0.59 \rightarrow 0.82 \end{pmatrix}. \qquad (87)$$

By construction these limits are obtained under the assumption that U is unitary. In other words, the ranges in the different entries of the matrix are correlated due to the fact that, in general, the result of a given experiment restricts a combination of several entries of the matrix, as well as to the constraints imposed by unitarity. As a consequence choosing a specific value for one element further restricts the range of the others.

VIII.   DIRECT DETERMINATION OF m_ν: KINEMATIC CONSTRAINTS

Oscillation experiments have provided us with important information on the differences between the neutrino masses-squared, Δm_{ij}^2, and on the leptonic mixing angles, U_{ij}. But they are insensitive to the absolute mass scale for the neutrinos, m_i.

Of course, the results of an oscillation experiment do provide a lower bound on the heavier mass in Δm_{ij}^2, $|m_i| \geq \sqrt{\Delta m_{ij}^2}$ for $\Delta m_{ij}^2 > 0$. But there is no upper bound on this mass. In particular, the corresponding neutrinos could be approximately degenerate at a mass scale that is much higher than $\sqrt{\Delta m_{ij}^2}$. Moreover, there is neither upper nor lower bound on the lighter mass m_j. In this section we briefly summarize the most sensitive probes of the absolute mass scale for the neutrinos.

It was Fermi who first proposed a kinematic search for the neutrino mass from the hard part of the beta spectra in ^{3}H beta decay $^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$. In the absence of leptonic mixing this search provides a measurement of the electron neutrino mass.

^{3}H beta decay is a superallowed transition, which means that the nuclear matrix elements do not generate any energy dependence, so that the electron spectrum is given by the phase space alone

$$\frac{dN}{dE} = C\, p\, E\, (Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\, F(E) \equiv R(E)\sqrt{(E_0 - E)^2 - m_{\nu_e}^2}. \qquad (88)$$

where $E = T + m_e$ is the total electron energy, p its momentum, $Q \equiv E_0 - m_e$ is the maximum kinetic energy of the electron and $F(E)$ is the Fermi function which incorporates

final state Coulomb interactions. In the second equality we have included in a $R(E)$ all the m_ν-independent factors.

Plotted in terms of the Curie function $K(T) \equiv \sqrt{\frac{dN}{dE} \frac{1}{pEF(E)}}$ a non-vanishing neutrino mass m_ν provokes a distortion from the straight-line T-dependence at the end point: for $m_\nu = 0 \Rightarrow T_{\max} = Q$ whereas for $m_{\nu_e} \neq 0 \Rightarrow T_{\max} = Q - m_{\nu_e}$. ^{3}H beta decay has a a very small energy release $Q = 18.6$ KeV which makes it particularly sensitive to this kinematic effect.

At present the most precise determination from the Mainz [47] and Troitsk [48] experiments give no indication in favor of $m_{\nu_e} \neq 0$ and one sets an upper limit

$$m_{\nu_e} < 2.2 \text{ eV} \tag{89}$$

at 95% confidence level (CL). For the other flavors the present limits are [24]

$$m_{\nu_\mu} < 190 \text{ keV}(90\%\text{CL}) \qquad \text{from} \qquad \pi^- \to \mu^- + \bar{\nu}_\mu , \tag{90}$$

$$m_{\nu_\tau} < 18.2 \text{ MeV}(95\%\text{CL}) \qquad \text{from} \qquad \tau^- \to n\pi + \nu_\tau . \tag{91}$$

In the presence of mixing these limits have to be modified and in general they involve more than one flavor parameter. For neutrinos with small mass differences the distortion of the beta spectrum is given by the weighted sum of the individual spectra [49]:

$$\frac{dN}{dE} = R(E) \sum_i |U_{ei}|^2 \sqrt{(E_0 - E)^2 - m_i^2} \, \Theta(E_0 - E - m_i) . \tag{92}$$

The step function, $\Theta(E_0 - E - m_i)$, reflects the fact that a given neutrino can only be produced if the available energy is larger than its mass. According to Eq. (92), there are two important effects, sensitive to the neutrino masses and mixings, on the electron energy spectrum: (i) Kinks at the electron energies $E_e^{(i)} = E \sim E_0 - m_i$ with sizes that are determined by $|U_{ei}|^2$; (ii) A shift of the end point to $E_{\text{ep}} = E_0 - m_1$, where m_1 is the lightest neutrino mass. The situation is slightly more involved when the finite energy resolution of the experiment is considered [50, 51].

In general for most realistic situations the distortion of the spectrum can be effectively described by a single parameter, m_β if for all neutrino states $E_0 - E = Q_0 - T \gg m_i$. In

this case one can expand Eq. (92) as:

$$\frac{dN}{dE} \simeq R(E) \sum_i |U_{ei}|^2 (E_0 - E)\left(1 - \frac{m_i^2}{2(E_0 - E)}\right)$$

$$= R(E) \sum_i |U_{ei}|^2 (E_0 - E)\left(1 - \frac{1}{2(E_0 - E)}\frac{\sum_i |U_{ei}|^2 m_i^2}{\sum_i |U_{ei}|^2}\right) \qquad (93)$$

$$\simeq R(E) \sum_i |U_{ei}|^2 \sqrt{(E_0 - E)^2 - m_\beta^2},$$

with

$$m_\beta^2 = \frac{\sum_i m_i^2 |U_{ei}|^2}{\sum_i |U_{ei}|^2} = \sum_i m_i^2 |U_{ei}|^2, \qquad (94)$$

where the second equality holds if unitarity is assumed. So the distortion of the end point of the spectrum is described by a single parameter which is bounded to be

$$m_\beta = \sqrt{\sum_i m_i^2 |U_{ei}|^2} < 2.2 \text{ eV}. \qquad (95)$$

A new experimental project, KATRIN [52], is under construction with an estimated sensitivity limit: $m_\beta \sim 0.3$ eV.

IX.   NEUTRINOLESS DOUBLE BETA DECAY

Direct information on neutrino masses can also be obtained from neutrinoless double beta decay ($0\nu\beta\beta$) searches:

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-. \qquad (96)$$

Schematically in the presence of neutrino masses and mixing the process in Eq. (96) can be induced by the diagram shown in Fig. 3. The amplitude of this process is proportional to the product of the two leptonic currents

$$M_{\alpha\beta} \propto [\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e]\,[\bar{e}\gamma_\beta(1 - \gamma_5)\nu_e]. \qquad (97)$$

which can only lead to a neutrino propagator from the contraction $\langle 0\,|\,\nu_e(x)\nu_e(y)^T\,|\,0\rangle$. If the neutrino is a Dirac particle ν_e field annihilates a neutrino states and creates an antineutrino state which are different. Therefore the contraction $\langle 0\,|\,\nu_e(x)\nu_e(y)^T\,|\,0\rangle = 0$ and $M_{\alpha\beta} = 0$. On the contrary, if ν_e is a Majorana particle, neutrino and antineutrino are the same state and $\langle 0\,|\,\nu_e(x)\nu_e(y)^T\,|\,0\rangle \neq 0$.
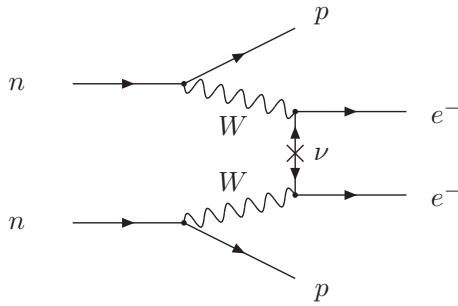
FIG. 3: Feynman diagram for neutrinoless double-beta decay.

Thus in order to induce the $0\nu\beta\beta$ decay, ν_e must be a Majorana particle. This is also obvious as the process (96) violates L by two units. The opposite also holds, if $0\nu\beta\beta$ decay is observed, neutrinos must be massive Majorana particles [53].

However, Majorana neutrino masses are not the only mechanism which can induce neutrinoless double beta decay. In general, in models beyond the standard model there may be other sources of total lepton number violation which can induce $0\nu\beta\beta$ decay. Consequently the observation or limitation of the neutrinoless double beta decay reaction rate can only be related to a bound on the neutrino mass and mixing under some assumption about the source of total lepton number violation in the model.

For the case in which the only effective lepton number violation at low energies is induced by the Majorana mass term for the neutrinos, the rate of $0\nu\beta\beta$ decay is proportional to the *effective Majorana mass of* ν_e,

$$m_{ee} = \left| \sum_i m_i U_{ei}^2 \right| \tag{98}$$

which, in addition to the masses and mixing parameters that affect the tritium beta decay spectrum, depends also on the leptonic CP violating phases.

Experimentally, what it is measured is the half-life of the decay. In $0\nu\beta\beta$ decay, the experimental signal is two electrons in the final state, whose energies add up to the Q-value of the nuclear transition while for the double beta decay with neutrinos ($2\nu\beta\beta$) (which constitute an intrinsic background) the energy spectrum of both electrons will be continuous as part of the Q is carried by the outgoing neutrinos. Also the decay rates for $0\nu\beta\beta$ and $2\nu\beta\beta$ have very different dependence on the available Q being the dependence much weaker for $0\nu\beta\beta$. For this reason, the sensitivity is better for isotopes with a high Q-value.

In the case that the only source of lepton number violation at low energies is induced by

the Majorana neutrino mass, the decay half-life is given by:

$$(T_{1/2}^{0\nu})^{-1} = G^{0\nu} \left| M^{0\nu} \right|^2 \left(\frac{m_{ee}}{m_e} \right)^2 \tag{99}$$

where $G^{0\nu}$ is the phase space integral and $|M^{0\nu}|$ is the nuclear matrix element of the transition.

The strongest bound from $0\nu\beta\beta$ decay was imposed by the Heidelberg-Moscow group [54] which used 11 kg of enriched Ge. After 53.9 kg yr of data taking they found no signal which allowed them to set a bound on the half-life of $T_{1/2}^{0\nu} > 1.9 \times 10^{25}$ yr (90% CL). This implies (for a given prediction of the nuclear matrix element):

$$m_{ee} < 0.26 \ (0.34) \ \text{eV} \quad \text{at} 68\% (90\%) \text{CL}. \tag{100}$$

Taking into account the possible uncertainties in the prediction of the nuclear matrix elements, the bound may be weaken by a factor of about 3 [55].

The final result of the Heidelberg-Moscow experiment quoted above was that no positive signal was observed, but a subgroup of the collaboration found a small peak at some value of Q [56, 57] which, if real, would imply a non vanishing range for the effective Majorana mass between 0.2–0.6 eV (the range might be widened by nuclear matrix uncertainties). However, the statistical analysis used, as well as the assumed background subtraction in order to establish this evidence at the claimed CL has been subject of severe criticisms [58–61] which renders the claimed signal as controversial.

Currently only two large scale experiments are running. CUORICINO at the Gran Sasso Underground Laboratory in Italy which uses uses bolometers running at very low temperature and searches for ^{120}Te decay with a Q-value of 2530 keV. The obtained half-life limit [62] $T_{1/2}^{0\nu}(^{120}\text{Te}) > 2.2 \times 10^{24}$ yr (90% CL) implies and upper bound on the effective Majorana neutrino mass $m_{ee} < 0.2$–1.1 eV. The second experiment, NEMO-3 [63] in the Frejus Underground Laboratory, is built in form of time projection chambers where the double beta emitter is either the filling gas of the chamber or is included in thin foils. It has obtained a half-life bound for ^{100}Mo $T_{1/2}^{0\nu}(^{100}\text{Mo}) > 5.6 \times 10^{23}$ yr (90% CL) which result in an upper effective Majorana mass bound $m_{ee} < 0.6$–2 eV.

A series of new experiments is planned with sensitivity of up to $m_{ee} \sim 0.01$ eV. For a review of the proposed experimental techniques see Ref. [64].

X. COLLIDER SIGNATURES OF ν MASS MODELS

The simplest and most straightforward lesson of the evidence for neutrino masses is also the most striking one: there is NP beyond the SM. This is the first experimental result that is inconsistent with the SM.

Given the relation (34), $m_\nu \sim v^2/\Lambda_{\rm NP}$, it is straightforward to use measured neutrino masses to estimate the scale of NP that is relevant to their generation. In particular, if there is no quasi-degeneracy in the neutrino masses, the heaviest of the active neutrino masses can be estimated,

$$m_h = m_3 \sim \sqrt{\Delta m_{\rm atm}^2} \approx 0.05 \text{ eV}. \tag{101}$$

(In the case of inverted hierarchy the implied scale is $m_h = m_2 \sim \sqrt{|\Delta m_{\rm atm}^2|} \approx 0.05$ eV). It follows that the scale in the non-renormalizable term (32) is given by

$$\Lambda_{\rm NP} \sim v^2/m_h \approx 10^{15} \text{ GeV}. \tag{102}$$

We should clarify two points regarding Eq. (102):

1. There could be some level of degeneracy between the neutrino masses that are relevant to the atmospheric neutrino oscillations. In such a case Eq. (101) is modified into a lower bound and, consequently, Eq. (102) becomes an upper bound on the scale of NP.

2. It could be that the Z_{ij} couplings of Eq. (32) are much smaller than one. In such a case, again, Eq. (102) becomes an upper bound on the scale of NP.

The crucial issue is to understand the origin of this operator in a given extension of the SM in order to identify the dimensionless coupling Z and the mass scale $\Lambda_{\rm NP}$ at which the new physics enters. For illustration I will comment, what is the *standard* expectation on two simple renormalizable extensions of the Standard Model with minimal addition to generate neutrino Majorana masses:

- *Type I see-saw mechanism* discussed in Sec. III B. One can adds fermionic singlets N_i and the neutrino masses are $m_\nu \sim Y_\nu^2 v^2/M_N$, where M_N is the right-handed neutrino mass, which sets the new physics scale $\Lambda_{\rm NP}$. From the discussion above we see that if $Y_\nu \simeq 1$ and $M_N \approx 10^{14-15}$ GeV, one obtains the natural value for the neutrino masses.

- *Type II see-saw mechanism* discussed in Sec. III D. The Higgs sector of the Standard Model is extended by adding an $SU(2)_L$ Higgs triplet Δ. The neutrino masses are

$m_\nu \approx f_\nu v_\Delta$, where v_Δ is the vacuum expectation value (vev) of the neutral component of the triplet and f_ν is the Yukawa coupling. With a doublet and triplet mixing via a dimensional parameter μ, the EWSB leads to a relation $v_\Delta \sim \mu v_0^2 / M_\Delta^2$, where M_Δ is the mass of the triplet. In this case the scale $\Lambda_{\mathrm{NP}} = M_\Delta^2 / \mu$, and a natural setting would be for $f_\nu \approx 1$ and $\mu \sim M_\Delta \approx 10^{14-15}$ GeV.

To directly test the above scenarios one needs to search for example for direct observations of the new heavy states responsible for the see-saw mechanism. As discussed above, most generically the characteristic mass scales of the new states are very large, rendering the new states experimentally inaccessible in the foreseeable future. However, one can think of scenarios in which this may not be necessary the case.

As an example let's take the Type II see-saw introduced in Sec. III D (see [65] and references therein for a recent detailed study from which I have taken the following discussion). Sec. III D. The scalar potential in the Type II see-saw is given by

$$V(H,\Delta) = -m_H^2 \, \phi^\dagger \phi + \frac{\lambda}{4}(\phi^\dagger \phi)^2 + M_\Delta^2 \, Tr\Delta^\dagger\Delta + \left(\mu \, \tilde{\phi}^T \, \Delta \, \tilde{\phi} + h.c. \right) +$$
$$+\lambda_1 \, (\phi^\dagger\phi)Tr\Delta^\dagger\Delta + \lambda_2 \left(Tr\Delta^\dagger\Delta\right)^2 + \lambda_3 \, Tr\left(\Delta^\dagger\Delta\right)^2 + \lambda_4 \, \tilde{\phi}^T\Delta\Delta^\dagger\tilde{\phi}^*. \quad (103)$$

The key ingredient that makes this model testable at LHC is to assume a very small doublet-triplet mixing

$$\mu \ll M_\Delta \quad (104)$$

and so the Higgs triplet is heavy, typically $M_\Delta^2 > v^2/2$, but not much heavier than this bound.

Imposing the conditions of global minimum one finds that (one can neglect the contributions coming from the terms proportional to λ_1, λ_2, λ_3 and λ_4).

$$-m_H^2 + \frac{\lambda}{4}v^2 - \sqrt{2} \, \mu \, v_\Delta = 0, \qquad \text{and} \qquad v_\Delta = \frac{\mu \, v_0^2}{\sqrt{2} \, M_\Delta^2}, \quad (105)$$

where v and v_Δ are the vacuum expectation values of the Higgs doublet and triplet, respectively, with $v^2 + v_\Delta^2 \approx (246 \text{ GeV})^2$. Due to the simultaneous presence of the Yukawa coupling f_ν in Eq. (29), and the term proportional to the μ parameter in Eq. (103), the lepton number is explicitly broken in this theory. The present constraints form the electroweak precision data yields

$$v_\Delta \lesssim 1 \text{ GeV} \quad (106)$$

Once the neutral component in Δ gets the vev, v_Δ as in Eq. (105), the neutrinos acquire a Majorana mass given by the following expression:

$$M_\nu = \sqrt{2}\, f_\nu\, v_\Delta = f_\nu\, \frac{\mu\, v^2}{M_\Delta^2}, \tag{107}$$

We notice that in the case of small doublet-triplet mixing, Eq. (104) one can have small neutrino masses even with M_Δ as light as

$$M_\Delta \sim 110\ \text{Gev} \tag{108}$$

without conflicting with any existing data.

After the electroweak symmetry breaking, there are seven physical massive Higgs bosons left in the spectrum:

$$H_1 = \cos\theta_0\, h^0 + \sin\theta_0\, \delta^0, \quad H_2 = -\sin\theta_0\, h^0 + \cos\theta_0 \delta^0, \quad \text{with } \theta_0 \approx \frac{2v_\Delta}{v}, \tag{109}$$

$$A = -\sin\alpha\, \xi^0 + \cos\alpha\, \eta^0, \quad \text{with} \quad \alpha \approx \frac{2v_\Delta}{v}, \tag{110}$$

$$H^\pm = -\sin\theta_\pm\, \phi^\pm + \cos\theta_\pm\, \Delta^\pm, \quad \text{with} \quad \theta_+ \approx \frac{\sqrt{2}v_\Delta}{v}, \tag{111}$$

$$\text{and} \quad H^{\pm\pm} = \Delta^{\pm\pm}, \tag{112}$$

where the neutral component of the Higgs doublet and triplet are $\phi_0 = h^0 + i\eta^0$ and $\Delta_0 = \delta_0 + i\xi^0$ respectively.

H_1 is SM-like (doublet) while the rest of the Higgs states are all Δ-like (triplet), and

$$M_{H_2} \simeq M_A \simeq M_{H^\pm} \simeq M_{H^{++}} = M_\Delta.$$

All these states are within reach of the LHC.

In the physical basis for the fermions the Yukawa interactions of the single and double charged scalars can be written as

$$-\mathcal{L} = Y_{ij}^+\, \bar{\nu}_{Li}^C\, e_{Lj}\ H^+ + \bar{Y}_{ij}^{++}\, \bar{e}_{Li}^C\, e_{Lj}\ H^{++}, \tag{113}$$

where

$$Y^+ = \cos\theta_+\, \frac{m_\nu^{diag}}{v_\Delta}\, U_{LEP}^\dagger, \quad \text{and} \quad Y^{++} = U_{LEP}^*\, \frac{m_\nu^{diag}}{\sqrt{2}\, v_\Delta}\, U_{LEP}^\dagger = f_\nu\,. \tag{114}$$

We see that the values of the couplings Y^+ and Y^{++} are determined by the spectrum and mixing angles for the active neutrinos. Therefore, one can expect that the lepton-number

violating decays of the Higgs bosons, $H^{++} \to e_i^+ e_j^+$ and $H^+ \to e_i^+ \bar{\nu}$ ($e_i = e, \mu, \tau$) will be characteristically different in each spectrum for neutrino masses.

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