Lecture #2

a) Basic nuclear physics of the ββ decay
b) Brief history of ββ decay
c) Decay rate formulae
d) Quantum mechanics of Majorana particles
e) See-saw

Basic nuclear physics issues

Whether a nucleus is stable or undergoes weak decay is determined by the dependence of the **atomic** mass M_A of the isotope (Z,A) on the nuclear charge Z.

Near its minimum this function is a parabola

$$M_A = \text{const} + b_{sym}(N-Z)^2/4A^2 + b_{coul}\, Z^2/A^{1/3} + m_e Z + \delta$$

Here δ describes nuclear pairing, coupling of nn or pp pairs to $I^\pi = 0^+$.

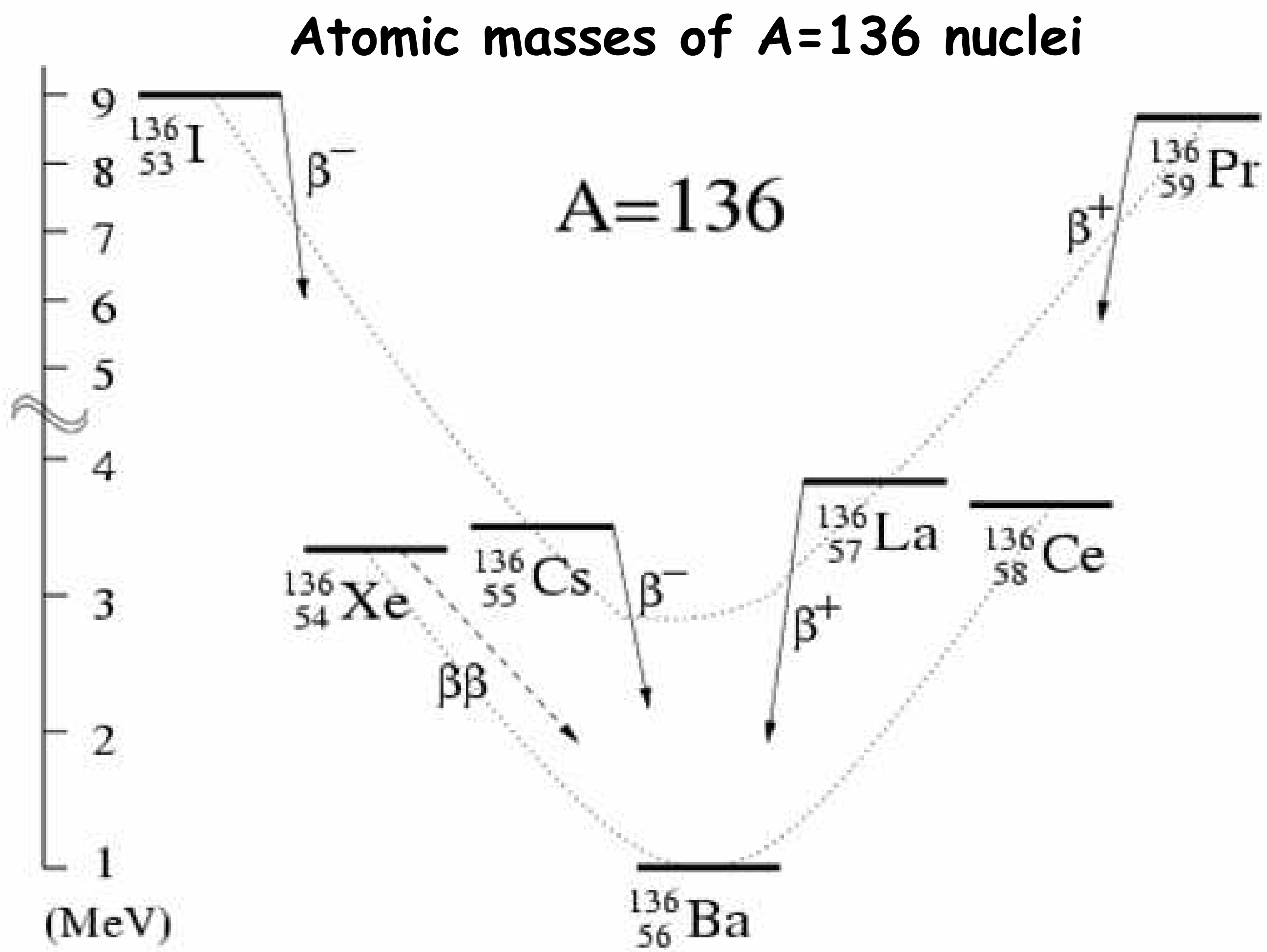
For odd A $\delta = 0$ (only one parabola)

For even A $\delta \sim +12/A^{1/2}$ MeV for odd N and odd Z

$\delta \sim -12/A^{1/2}$ MeV for even N, even Z

Thus for even A there are two shifted parabolas

Double ββ decay is observable because even-even nuclei are more bound than the odd-odd ones (due to the pairing interaction)



^{136}Xe and ^{136}Ce are stable against β decay, but unstable against ββ decay ($\beta^-\beta^-$ for ^{136}Xe and $\beta^+\beta^+$ for ^{136}Ce)

Candidate Nuclei for Double Beta Decay

	Q (MeV)	Abund.(%)
$^{48}Ca \rightarrow ^{48}Ti$	4.271	0.187
$^{76}Ge \rightarrow ^{76}Se$	2.040	7.8
$^{82}Se \rightarrow ^{82}Kr$	2.995	9.2
$^{96}Zr \rightarrow ^{96}Mo$	3.350	2.8
$^{100}Mo \rightarrow ^{100}Ru$	3.034	9.6
$^{110}Pd \rightarrow ^{110}Cd$	2.013	11.8
$^{116}Cd \rightarrow ^{116}Sn$	2.802	7.5
$^{124}Sn \rightarrow ^{124}Te$	2.228	5.64
$^{130}Te \rightarrow ^{130}Xe$	2.533	34.5
$^{136}Xe \rightarrow ^{136}Ba$	2.479	8.9
$^{150}Nd \rightarrow ^{150}Sm$	3.367	5.6

All candidate nuclei on this list have Q > 2MeV. The nuclei with an arrow are used in the present or planned large mass experiments. For most of the nuclei in this list the $2\nu\beta\beta$ decay has been observed

Transition (A,Z) -> (A,Z+2) + $2e^-$ + (possibly other neutral particles) can ago if **$M_A(Z,A) > M_A(Z+2,A)$, while $M_A(Z,A) < M_A(Z+1,A)$**

Transitions (A,Z) -> (A,Z-2) can go three possible ways:

(A,Z) -> (A,Z-2) + $2e^+$ + ... if **$M_A(Z,A) > M_A(Z-2,A) + 4m_e$**
(two positron emission)

(A,Z) -> (A,Z-2) + e^+ + EC + ... if **$M_A(Z,A) > M_A(Z-2,A) + 2m_e + B_e$**
(one positron emission + one electron capture)

(A,Z) -> (A,Z-2) + 2EC + ... if **$M_A(Z,A) > M_A(Z-2,A) + B_e(1) + B_e(2)$**
(two electron captures)

Thus the decays with positron emission are disfavored as far as the phase space is concerned. None was observed so far.

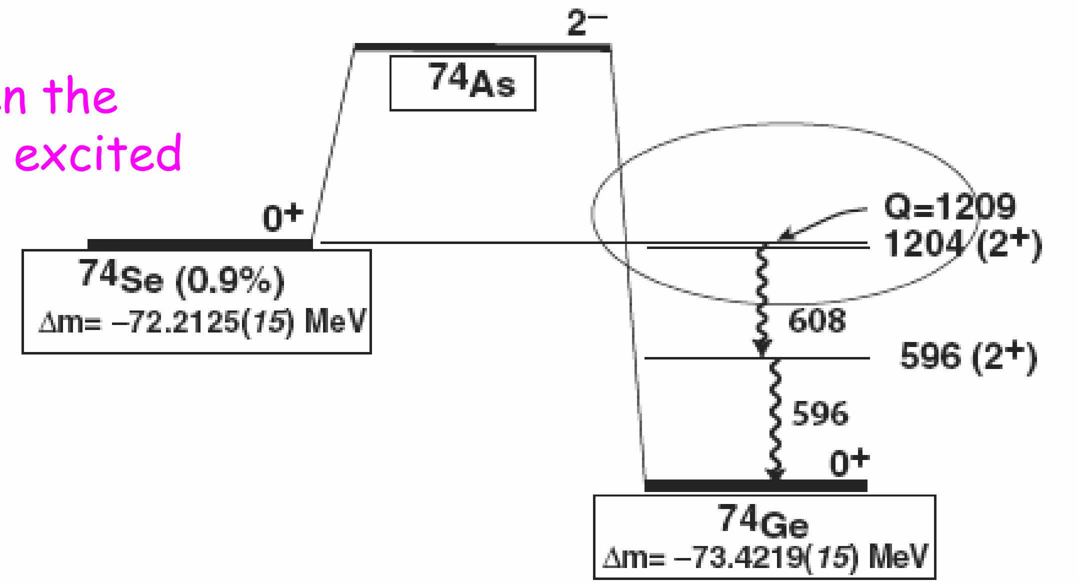
A comment on the two electron capture process:

The neutrinoless process obviously cannot go by itself, it would violate energy conservation. It could go with emission of a photon, with low energy preferred $\sim 1/\omega$.

But what if there is a resonance, an energy degeneracy?

From Frekers hep-ex/0506002

Near degeneracy between the ^{74}Se ground state and an excited 2^+ state in ^{74}Ge



Resonance condition: $Q \approx E^* + B_e(1) + B_e(2) = E$

The decay rate is then

$$1/\tau = (\Delta M)^2 \times \Gamma/[(Q - E)^2 + \Gamma^2/4] ,$$

where Γ is the final state width and ΔM is the weak interaction coupling matrix element. At resonance the rate goes like $4/\Gamma$ while off resonance it goes like $\Gamma/(Q-E)^2$, large enhancement.

However, $\Gamma \sim O(10eV)$ is dominated by the electron vacancies. It is rather unlikely that one could find such a perfect match.

Prehistory of ββ decay

(slides by John Wilkerson)

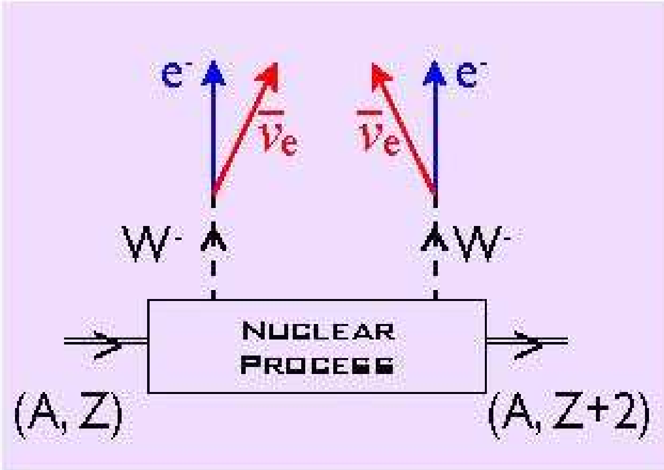
2ν double-beta decay (2νββ): Nucleus (A, Z) → Nucleus (A, Z+2) + e^- + $\overline{\nu}_e$ + e^- + $\overline{\nu}_e$

Allowed second-order weak process

Maria Goeppert-Mayer (1935)

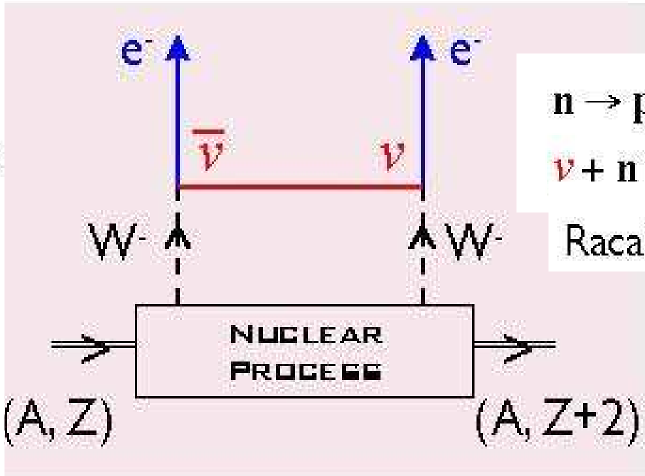
2νββ observed for

^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr ^{100}Mo, ^{116}Cd ^{128}Te, ^{130}Te, ^{150}Nd



0ν double-beta decay (0νββ): Nucleus (A, Z) → Nucleus (A, Z+2) + e^- + e^-

Ettore Majorana (1937) realized symmetry properties of Dirac's theory allowed the possibility for electrically neutral spin-1/2 fermions to be their own anti-particle



$n \rightarrow p + e^- + \overline{\nu}$

$\nu + n \rightarrow p + e^-$

Racah (1937),

Early Estimates of ββ Decay Rates

2ν double-beta decay (2νββ)	**0ν double-beta decay (0νββ)**

Maria Goeppert-Mayer (1935)
using Fermi Theory

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \propto \text{Phase Space (4-body)} \propto Q^{11}$$

$$T_{1/2}^{2\nu\beta\beta} \approx 10^{22} \text{ years}$$

Furry (1939), assuming Parity
conserved, so no preferential handedness

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} \propto \text{Phase Space (2-body)} \propto Q^{5}$$

$$T_{1/2}^{0\nu\beta\beta} \approx 10^{16} \text{ years}$$

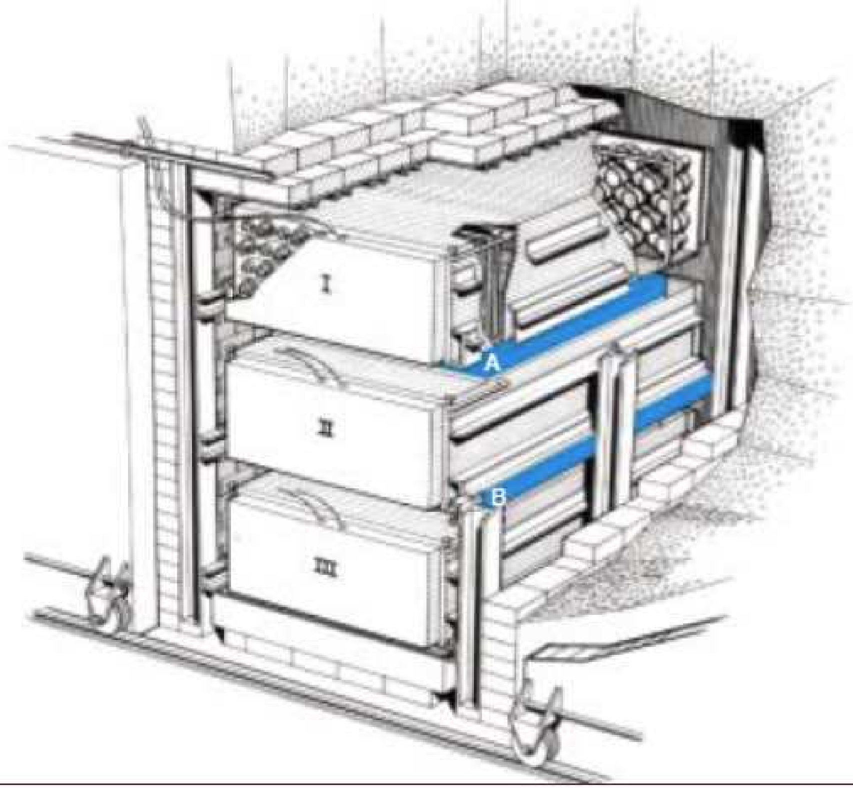
0νββ mode highly favored over 2νββ

So, at that time it looked as that if the ββ decay is observed with $T_{1/2} << 10^{20}$y neutrinos are Majorana particles but if it is observed with $T_{1/2} > 10^{20}$y than neutrinos are Dirac particles.

Unfortunately, real life is not that simple.

1956 The $\bar{\nu}$ is first observed (in SC)

Reines, Cowan, Harrison, McGuire, and Kruse
Science, July 1956



PHYSICAL REVIEW VOLUME 117, NUMBER 1 JANUARY 1, 1960

Detection of the Free Antineutrino*

F. REINES,† C. L. COWAN, JR.,‡ F. B. HARRISON, A. D. MCGUIRE, AND H. W. KRUSE
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

Weak Interaction maximally violates parity

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. Lee, *Columbia University, New York, New York*

AND

C. N. Yang,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

Experimental Test of Parity Conservation in Beta Decay*

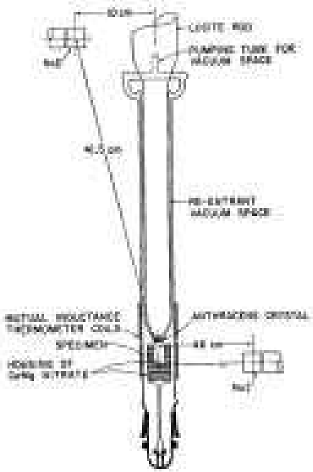
C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

In a recent paper[1] on the question of parity in weak interactions, Lee and Yang critically surveyed the experimental information concerning this question and reached the conclusion that there is no existing evidence either to support or to refute parity conservation in weak interactions. They proposed a number of experiments on beta decays and hyperon and meson decays which would provide the necessary evidence for parity conservation

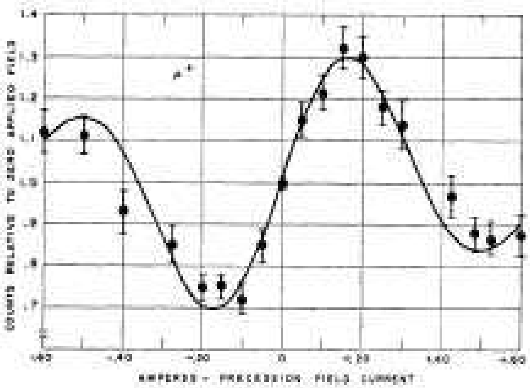


Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

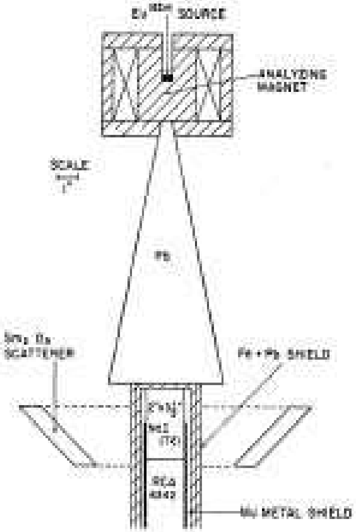
Richard L. Garwin,† Leon M. Lederman, and Marcel Weinrich

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

(Received January 15, 1957)



1958 Goldhaber-Grodzins-Sunyar

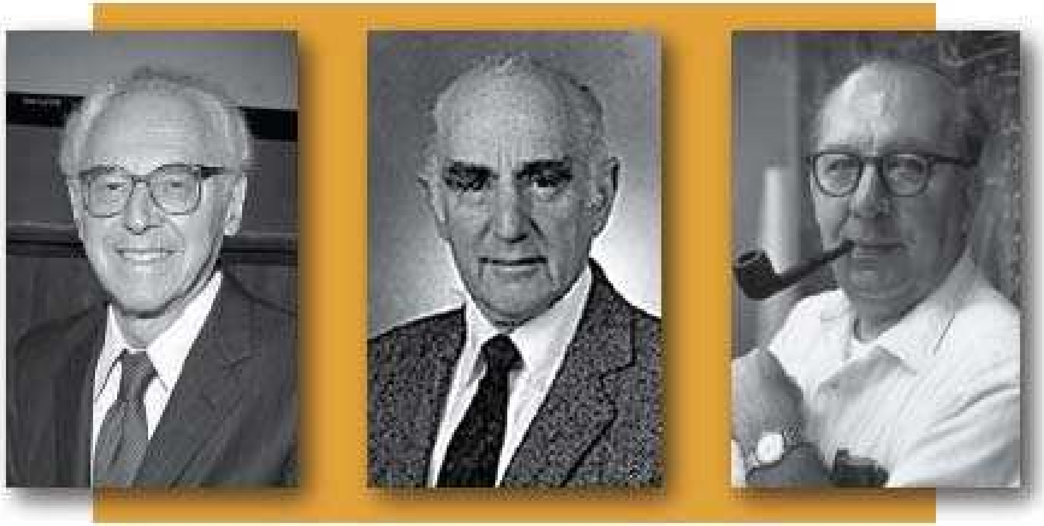


Helicity of Neutrinos*

M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR
Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

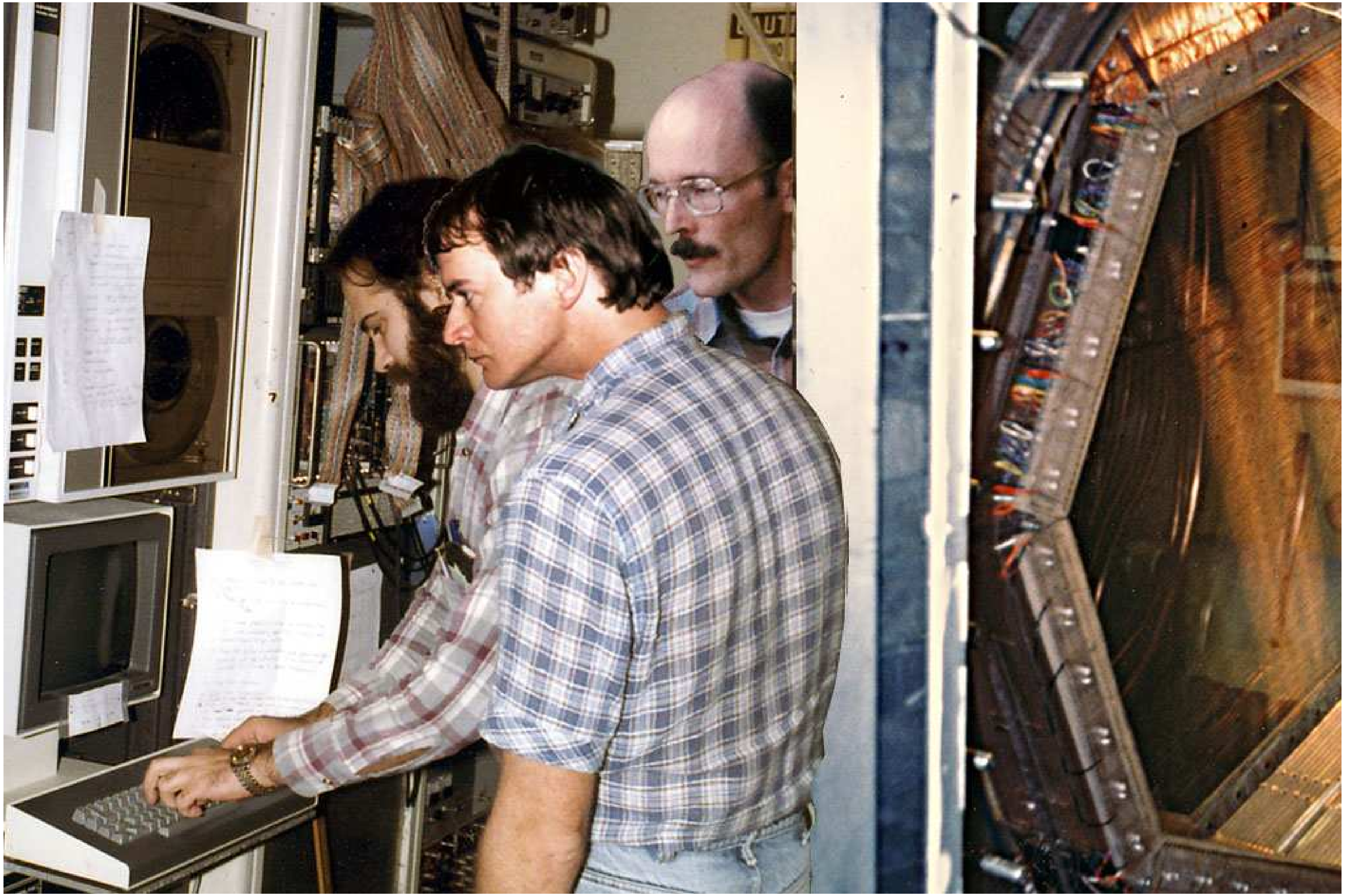
A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\sigma_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).



- Weak Interaction
  - maximally violates parity
  - V-A nature
  - neutrinos emitted in beta-decay have intrinsic handedness

1987 First observation of 2νββ decay in a `live' experiment

Elliott, Hahn, and Moe (this is a composite picture by Mike Moe of a real situation)



That was the size of a ββ experimental group then, here is the size of another ββ experimental group now (EXO collaboration, May 2008)

$2\nu\beta\beta$ decay rate, spectrum, etc.

Here $qR \ll 1$ hence long wavelength approximation is valid, only GT operators $\sigma\tau$ need to be considered.
The phase space is simply:

$$) \int_{m_e}^{E_0 - m_e} F(Z, E_{e1}) p_{e1} E_{e1} dE_{e1} \int_{m_e}^{E_0 - E_1} F(Z, E_{e2}) p_{e2} E_{e2} dE_{e2} (E_0 - E_{e1} - E_{e2})^5 / 30$$

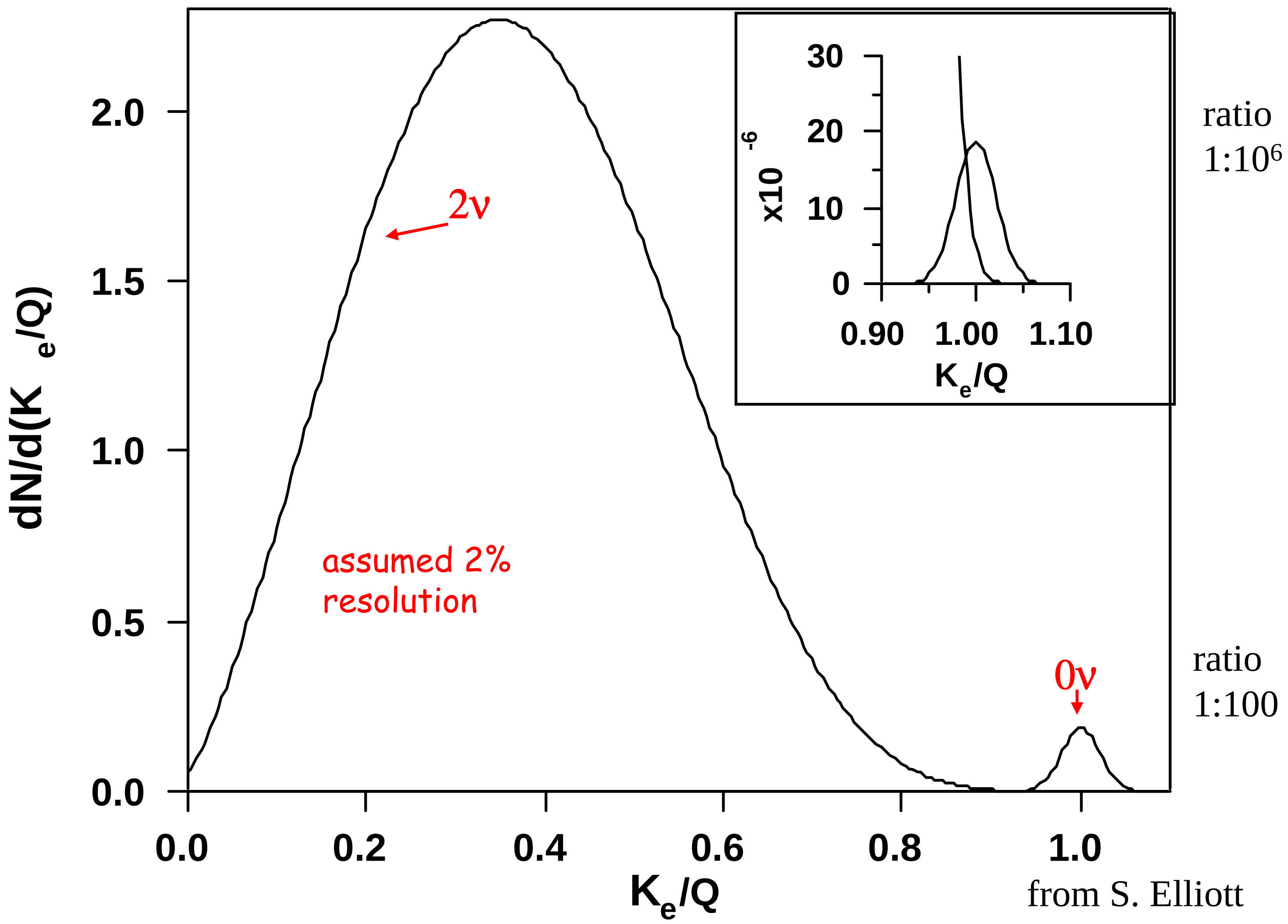
In a simple approximation (Primakoff-Rosen)

$$\frac{dN}{dK} \sim K(T_0 - K)^5 \left(1 + 2K + \frac{4K^2}{3} + \frac{K^3}{3} + \frac{K^4}{30}\right)$$

Here K is the sum kinetic energy of the electrons, hence total rate $1/\tau \sim T_0^{11}$, and near T_0 the spectrum goes like $(T_0 - K)^5$
The fraction of events near the endpoint, in the dimensionless interval $\delta = \Delta E/Q$ is $F \sim 7Q\,\delta^6/m_e$. Thus signal/background is

$S/B \sim m_e/\ 7Q\,\delta^6\, T^{2\nu}_{1/2}/\ T^{0\nu}_{1/2}$, inversely proportional to ΔE^6.

One can distinguish the two modes by measuring the sum electron energy. Ultimately, though, the 2ν decay is an unavoidable background to the 0νββ.



The nuclear matrix elements are

$$M^{2\nu} = \Sigma_m \frac{\langle 0_f^+ | \vec{\sigma}_i \tau_i^+ | m \rangle \langle m | \vec{\sigma}_k \tau_k^+ | 0_i^+ \rangle}{E_m - (M_i + M_f)/2}$$

And the decay rate is $1/T_{1/2} = G^{2\nu}(Q,Z)\,(M^{2\nu})^2$. Knowing the half-life we can extract $M^{2\nu}$ easily.

Isotope	$T^{2\nu}_{1/2}$ (y)	$M^{2\nu}_{GT}$ (MeV^{-1})
^{48}Ca	$(3.9 \pm 0.7 \pm 0.6) \times 10^{19}$	0.05 ± 0.01
^{76}Ge	$(1.7 \pm 0.2) \times 10^{21}$	0.13 ± 0.01
^{82}Se	$(9.6 \pm 0.3 \pm 1.0) \times 10^{19}$	0.10 ± 0.01
^{96}Zr	$(2.0 \pm 0.3 \pm 0.2) \times 10^{19}$	0.12 ± 0.02
^{100}Mo	$(7.11 \pm 0.02 \pm 0.54) \times 10^{18}$	0.23 ± 0.01
^{116}Cd	$(2.8 \pm 0.1 \pm 0.3) \times 10^{19}$	0.13 ± 0.01
$^{128}Te^{(1)}$	$(2.0 \pm 0.1) \times 10^{24}$	0.05 ± 0.005
^{130}Te	$(7.6 \pm 1.5 \pm 0.8) \times 10^{20}$	0.032 ± 0.003
^{136}Xe	$> 1.0 \times 10^{22}$ (90% CL)	<0.01
^{150}Nd	$(9.2 \pm 0.25 \pm 0.73) \times 10^{18}$	0.06 ± 0.003
$^{238}U^{(2)}$	$(2.0 \pm 0.6) \times 10^{21}$	0.05 ± 0.01

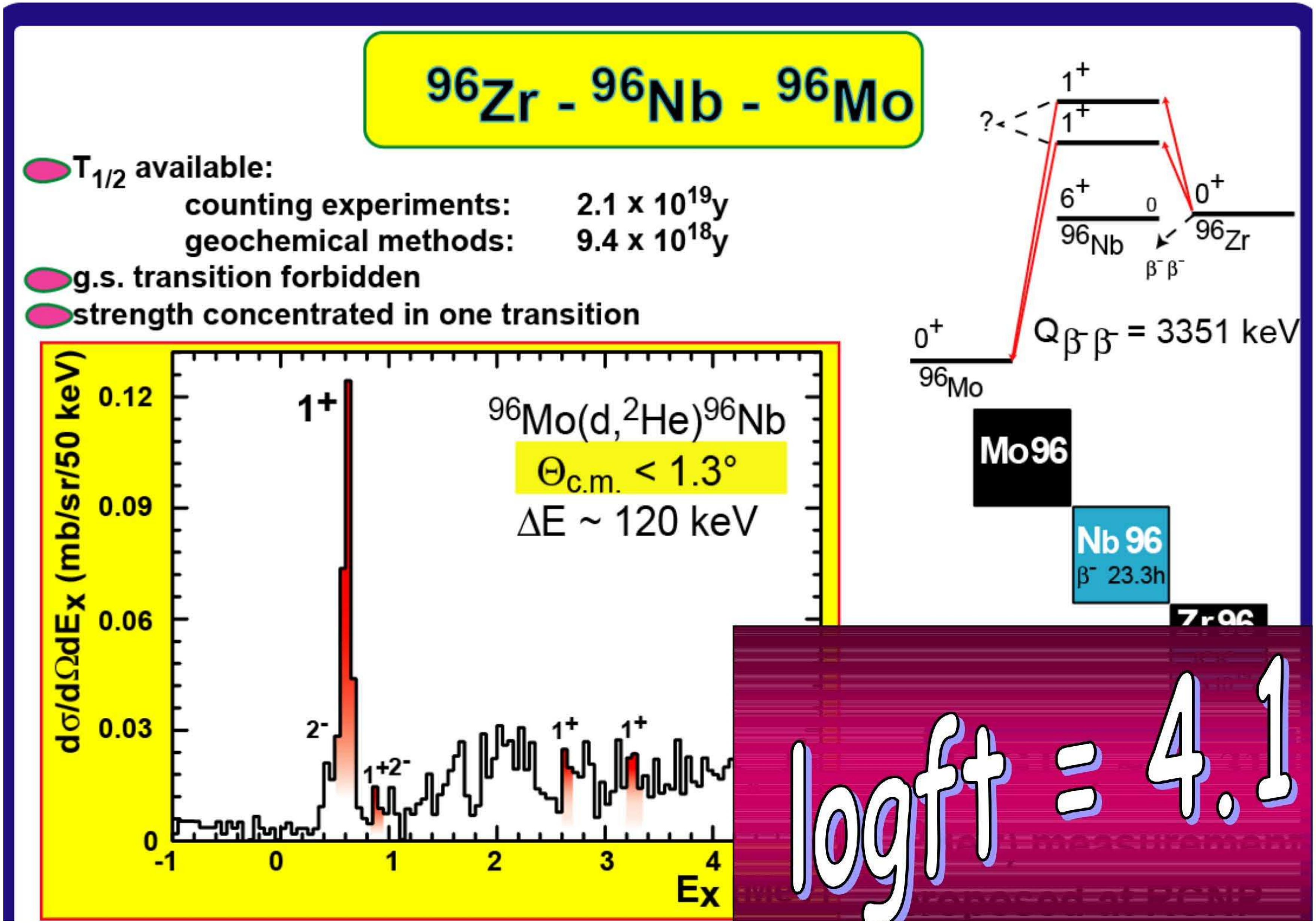
This is a summary of present $2\nu\beta\beta$ measurements, mostly from NEMO.

The amplitudes $\langle m|\vec{\sigma}_k \tau_k^+|0_i^+\rangle$

and $\langle 0_f^+|\vec{\sigma}_i \tau_i^+|m\rangle$

represent the β^- and β^+ strength of the initial and Final nucleus respectively.
They can be (at least for the few lowest states) determined by the appropriate charge exchange reactions (p,n), (^{3}He,t) and (n,p),(d,^{2}He), etc.
A number of experiments along these lines is currently conducted.

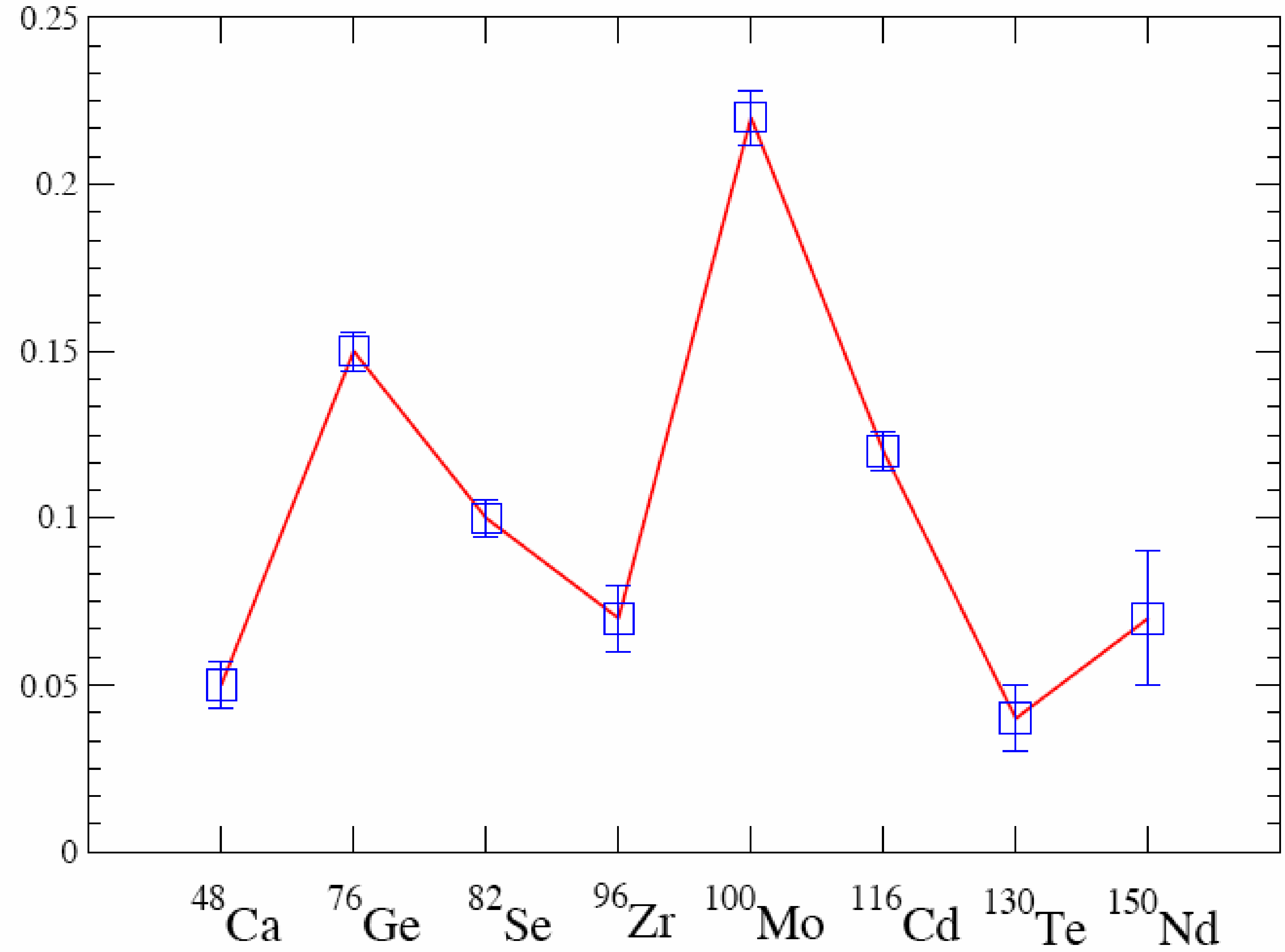
An example of such data (D. Frekers, Blaubeuren conf. 2007)



Nuclear matrix elements for the 2ν decay deduced from measured halflives.
Note the pronounced shell dependence.

$$1/T_{1/2} = G(E,Z)\,(M_{GT}^{2\nu})^2$$

easily calculable phase space factor



0νββ decay rate formulae:

$$\omega_{0\nu} = 2\pi \Sigma_{spin} |R_{0\nu}|^2 \delta(E_{e1} + E_{e2} + E_f - M_i) d^3 p_{e1} d^3 p_{e2}$$

The transition amplitude $R_{0\nu}$ includes leptons and nucleons, the lepton part is a product of two left-handed currents

$$\bar{e}(x)\gamma_\rho \frac{1}{2}(1-\gamma_5)\nu_j(x)\bar{e}(y)\gamma_\sigma \frac{1}{2}(1-\gamma_5)\nu_k(y)$$

The implied contraction over the two neutrino operators is possible only for Majorana neutrinos. After the substitution for neutrino propagator the lepton part becomes

$$-i\delta_{jk} \int \frac{d^4 q}{(2\pi)^4} \frac{e^{-iq(x-y)}}{q^2 - m_j^2} \bar{e}(x)\gamma_\rho \frac{1}{2}(1-\gamma_5)(q^\mu \gamma_\mu + m_j)\frac{1}{2}(1-\gamma_5)\gamma_\sigma e^C(y)$$

and

$$\frac{1}{2}(1-\gamma_5)(q^\mu \gamma_\mu + m_j)\frac{1}{2}(1-\gamma_5) = m_j \frac{1}{2}(1-\gamma_5)$$

Thus the amplitude is proportional to m_j, the Majorana neutrino mass.

After integrating over dq^0 , and taking into account the energy denominator of the second order perturbation we arrive at

$$H(r, E_m) = \frac{R}{2\pi^2 g_A^2} \int \frac{d\vec{q}}{\omega} \frac{1}{\omega + A_m} e^{i\vec{q}\cdot\vec{r}} = \frac{2R}{\pi r g_A^2} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + A_m)}$$

This is a `neutrino potential' (A_m is the total energy in the virtual nuclear state with respect to $(M_i + M_f)/2$). The constants are added for future convenience. r is the distance between the two neutrons that are transformed into protons.

This H(r, E), together with spin and isospin operators will appear in the nuclear matrix elements, In compact form

$H(r) = R/r\ \Phi(\omega r)$ where $\Phi(\omega r) \leq 1$ is a slowly varying

function. Since $r < R$ the potential is ≥ 1 (but less than 5-10).

The phase space integral is very simple

$$G^{0\nu}(Q,Z) \sim \int F(Z,E_{e1})F(Z,E_{e2})p_{e1}p_{e2}E_{e1}E_{e2}\delta(E_0 - E_{e1} - E_{e2})dE_{e1}dE_{e2} \ .$$

with the proportionality constant

$$(G_F \cos\theta_C g_A)^4 \left(\frac{\hbar c}{R}\right)^2 \frac{1}{\hbar}\frac{1}{\ln(2)32\pi^5} \ ,$$

The single electron spectrum (Primakoff-Rosen approximation) is

$$\frac{dN}{dT_e} \sim (T_e + 1)^2 (T_0 - T_e + 1)^2 \ ,$$

peaked, naturally, in the middle at $T_e = T_0/2$

In an exact expression we will have the transition operator

$$\Omega = \tau^+\tau^+ \frac{(-h_F + h_{GT}\sigma_{12} - h_T S_{12})}{q(q + E_m - (M_i + M_f)/2)} \ , \ \sigma_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \ , \ S_{12} = 3\vec{\sigma}_1 \cdot \hat{q}\vec{\sigma}_2 \cdot \hat{q} - \sigma_{12} \ .$$

With a bit more complicated q dependence:

$$h_{GT} = g_A^2 \left[1 - \frac{2}{3}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m_\pi^2}\right)^2 \right], \ h_T = g_A^2 \left[\frac{2}{3}\frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m_\pi^2}\right)^2 \right].$$

This comes from the induced pseudoscalar current and the use of the Goldberger-Treiman relation.
In the integral over dq the sin(qr)/qr is replaced by **j_0(qr)** for F and GT and **$-j_2$(qr)** for T

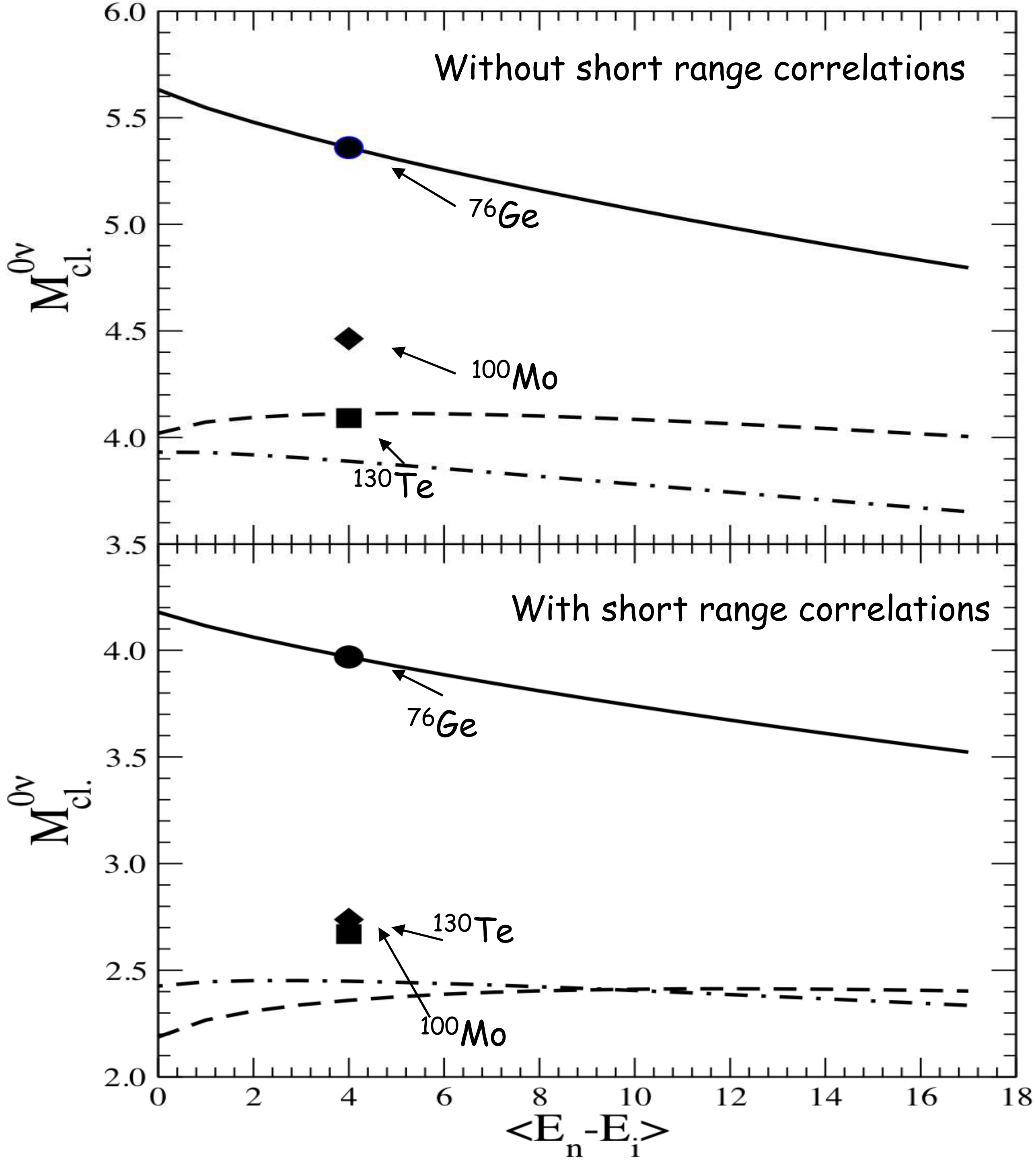
The matrix element that we need to evaluate is then

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \langle f | \frac{M^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu} | i \rangle \ ,$$

with the constant chosen such that we could modify g_A if needed

We see that the `neutrino potentials' depend on the nuclear excitation energy E_n. Thus, formally, one needs to use a slightly different potential for every state, and integrate over dq. How important is that dependence?

If it is not very important, e.g. if $q \gg E_n$, we might do the dq integrals first and not worry about the intermediate states at all. This is the **closure approximation** used in the nuclear shell model.



How good is the closure approximation?

Comparison between the QRPA $M^{0\nu}$ with the proper energies of the virtual intermediate states (symbols with arrows) and the closure approximation (lines) with different $\langle E_n - E_i \rangle$.

Note the mild dependence on $\langle E_n - E_i \rangle$ and the fact that the exact results are reasonably close to the closure approximation results for $\langle E_n - E_i \rangle < 20$ MeV.

Graph by F. Simkovic

A few words about the QM of Majorana particles

A bit of simple theory: Weyl, Dirac and Majorana fermions:

Free fermions obey the Dirac equation $(\not{p} - m)\Psi = 0$

The four component Dirac equation can be rewritten as two coupled two component equations

$$-m\psi_- + (E - \vec{\sigma} \cdot \vec{p})\psi_+ = 0$$
$$(E + \vec{\sigma} \cdot \vec{p})\psi_- - m\psi_+ = 0$$

Here $\psi_\pm$ are the chiral projections and the representation is used where

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

In the limit $m \to 0$ these equations decouple and we obtain the states with a definite helicity and chirality $\vec{\sigma} \cdot \hat{p}\psi_\pm = \pm\psi_\pm$.

Thus massless fermions obey the two-component Weyl equations:

$$(E - \vec{\sigma} \cdot \vec{p})\psi_+ = 0$$
$$(E + \vec{\sigma} \cdot \vec{p})\psi_- = 0$$

The two states, $\psi_+ = \psi_R$, and $\psi_- = \psi_L$ are so-called van der Waerden spinors that transform independently under the two nonequivalent simplest two dimensional representation of the Lorentz group.

Massive fermions have two possibilities:

They can obey the four component Dirac equation, which can be recast as shown before as two coupled two-component equations, or rewritten in the four-component form $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$

Alternatively, Ettore Majorana pointed out that there is a two-component alternative:

$(E - \vec{\sigma} \cdot \vec{p})\psi_R - m\epsilon\psi_R^* = 0$

and an independent second equation with a different mass value m'

$(E + \vec{\sigma} \cdot \vec{p})\psi_L + m'\epsilon\psi_L^* = 0$

where $\epsilon = i\sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

The Majorana fields can be also formally expressed in the four-component form (but only two components are independent)

$$\Psi_L(x) = \begin{pmatrix} -\epsilon\psi_L^* \\ \psi_L \end{pmatrix} \qquad \Psi_R(x) = \begin{pmatrix} \psi_R \\ \epsilon\psi_R^* \end{pmatrix}$$

$-\epsilon\psi_L^*$ **also represents the solution of Majorana eq. with the same mass** m

To show it, substitute it into the eq. for ψ_R**:**

$(E - \vec{\sigma}\cdot\vec{p})(-\epsilon\psi_L^*) - m\epsilon(-\epsilon\psi_L) = 0$

Then multiply by ϵ **from left and use that**

$\epsilon\vec{\sigma}\epsilon = \vec{\sigma}^*$ **and** $(E \pm \vec{\sigma}^*\cdot\vec{p})^* = -(E \pm \vec{\sigma}\cdot\vec{p})$

Thus we obtain $(E + \vec{\sigma}\cdot\vec{p})\psi_L + m\epsilon\psi_L^* = 0$ **which is what we wanted.**

Comparing the Majorana eq. for ψ_R with the corresponding part of the coupled Dirac equations:

M: $(E - \vec{\sigma} \cdot \vec{p})\psi_R - m\epsilon\psi_R^* = 0$

D: $(E - \vec{\sigma} \cdot \vec{p})\psi_R - m\psi_L = 0$

one can see that they become identical if $\epsilon\psi_R^* = \psi_L$. The same is true for the other pair.

The four-component Dirac field is therefore equivalent to two degenerate ($m = m'$) Majorana fields, with the corresponding relation between ψ_L and $\epsilon\psi_R^*$.

It is also easy to see that the four-component form of the Majorana field obeys formally the Dirac equation provided we use the relations $\epsilon\psi_R^* = \psi_L$ etc.

Charge conjugation matrix C in our (Weyl) representation is

$$C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix}.$$

Dirac bispinor $\Psi = \begin{pmatrix} \chi \\ \epsilon\xi* \end{pmatrix}$ **transforms as** $\Psi_D^c = C\gamma^0\Psi_D^* = \begin{pmatrix} \xi \\ \epsilon\chi* \end{pmatrix}$

In contrast the Majorana bispinor $\Psi_M = \begin{pmatrix} \chi \\ \epsilon\chi^* \end{pmatrix}$ **transforms as**

$$\Psi_M^c = C\gamma^0\Psi^* = \Psi_M$$

This is called the Majorana condition.

Mass term in the Lagrangian must be Lorentz invariant and hermitian. There are two possible types of mass terms:

Dirac: $\bar{\psi}\psi$ and $\bar{\psi}^c\psi^c$

Majorana: $\bar{\psi}\psi^c + \bar{\psi}^c\psi$

The Dirac mass term is invariant under a global phase transformation $\psi \to e^{i\theta}\psi$; $\psi^c \to e^{-i\theta}\psi$ and the Majorana mass term is obviously not. Thus, the Dirac mass term can be associated with a conserved quantum number ('lepton number'). The Majorana mass term violates conservation of the lepton number by two units.

The most general mass term is therefore

$$-2L_m = \frac{1}{2}[\bar{\psi} m_D \psi + \bar{\psi}^c m_D \psi^c + \bar{\psi} m_M \psi^c + \bar{\psi}^c m_m^* \psi].$$

This Lagrangiam is hermitian and Lorentz invariant. It can be rewritten in a matrix form:

$$\frac{1}{2}\,(\bar{\psi}, \bar{\psi}^c) \begin{pmatrix} m_D & m_M \\ m_M^* & m_D \end{pmatrix} \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}$$

If we use, more realistically, the chiral projections then
Dirac mass: $\bar{\psi}_L \psi_R$ or $\bar{\psi}_R \psi_L$
Majorana: $\bar{\psi}_L (\psi^c)_R$ etc.

With chiral projections the dimension of the mass matrix doubles; only two eigenvalues and eigenvectors are independent. The eigenvectors are Majorana fields.

See-saw:

Lets do it more carefully:

The mass term is
$m_D[\bar{\psi}_L \psi_R + h.c.] + m_L/2[(\bar{\psi^c})_R \psi_L + h.c] + m_R/2[(\bar{\psi^c})_L \psi_R + h.c]$

We can rewrite it in terms of the charge-conjugation eigenstates:
$f = [\psi_L + (\psi)^c)_R]/\sqrt{2}$
$F = [\psi_R + (\psi)^c)_L]/\sqrt{2}$

The mass term is then
$m_D(\bar{f}F + \bar{F}f) + m_L \bar{f}f + m_R \bar{F}F$

$$(\bar{f}, \bar{F}) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} f \\ F \end{pmatrix}$$

This is the same matrix we had before.

See-saw, cont.:

Lets consider now special cases:

1) $m_L = m_R = 0$.
The eigenvalues are $\pm m_D$ and we use the γ_5 trick to make them degenerate. As expected, we recover the Dirac case.

2) $m_L, m_R \ll m_D$
This is the quasi-Dirac case. A pair of close lying Majorana states with opposite CP eigenvalues.

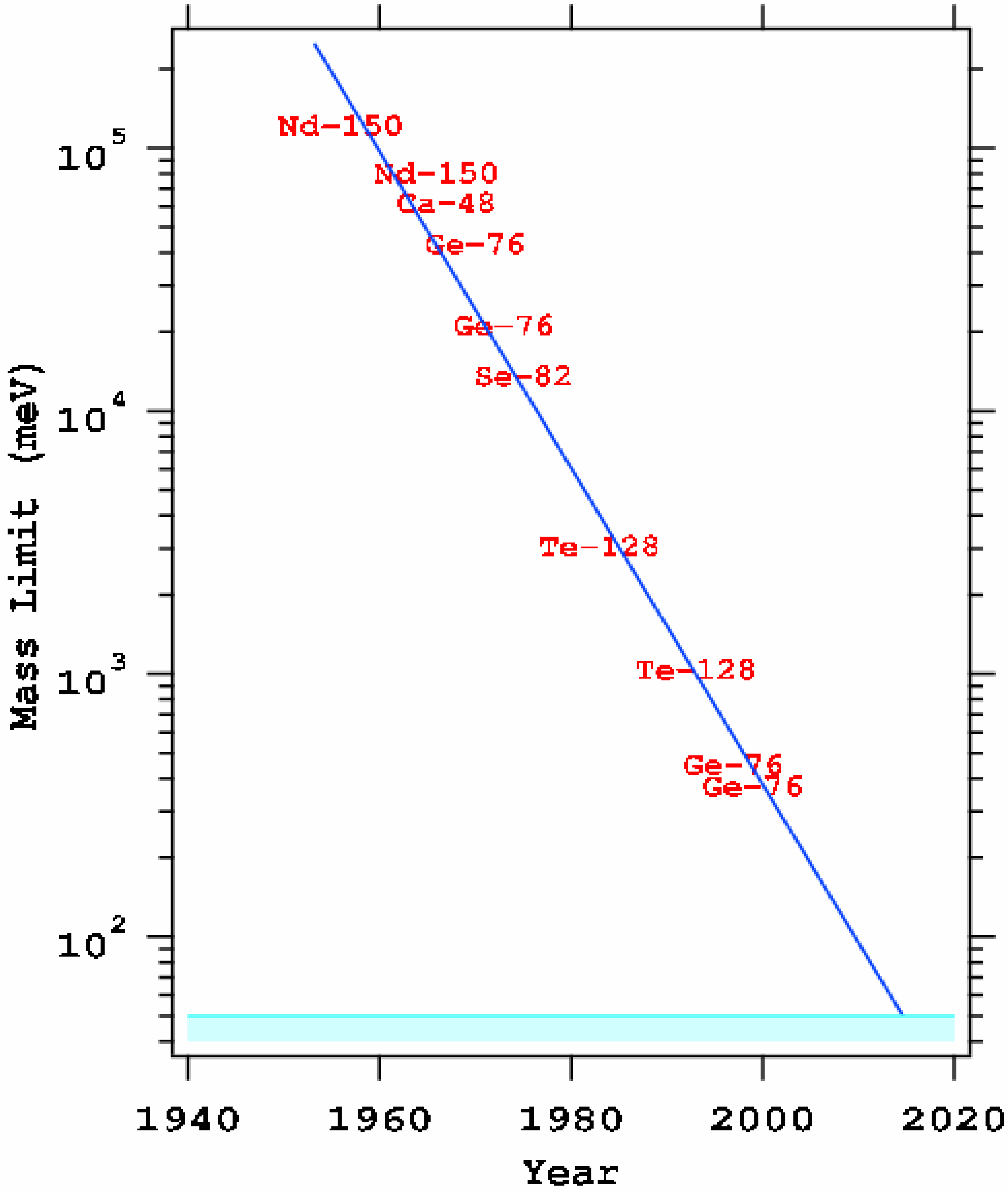
3) Finally, the most interesting case when $m_L \sim 0$ and $m_R \gg m_D$. The eigenvalues are

$m_\nu = \frac{m_D^2}{m_R}$ and $m_N = m_R + \frac{m_D^2}{m_R} \sim m_R$.

Given all of that we have the machinery (except the nuclear matrix elements, to be discussed next) to determine $\langle m_{\beta\beta} \rangle$ if we could observe the $0\nu\beta\beta$ decay.

So, where are we in that quest?

Moore's law of 0νββ decay:



There is a steady progress in the sensitivity of the searches for 0νββ decay. Several experiments that are funded and almost ready to go will reach sensitivity to ~0.1 eV. There is one (**so far unconfirmed**) claim that the 0νββ decay of ^{76}Ge was actually observed. The deduced mass $\langle m_{\beta\beta} \rangle$ would be then 0.3-0.7 eV.

spares

See-saw:

Lets write the general mass term with chiral projections:

$$(\bar{\psi}_L, \bar{\psi}^c_L) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \psi^c_R \\ \psi_R \end{pmatrix}$$

where $m_{R,L} = m_1 \pm |m_2$ using the previous notation.

Since only ψ_L and ψ^c_R are interacting, we can use a different notation for them, and for their sterile partners and write the Langrangian as

$$(\bar{\nu}_L, \bar{N}^c_L) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ N_R \end{pmatrix}$$

The eigenvalues are

$$\lambda_\pm = \tfrac{1}{2}[(m_L + m_R) \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}]$$

The eigenstates are $\phi_{1,2}$. They are eigenstates of CP with eigenvalues $\epsilon_{1,2}$. The mass term is now diagonal

$\lambda_+ \bar{\phi}_1 \phi_1 + \lambda_- \bar{\phi}_2 \phi_2$.

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably weak isospin invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

Since $I_W(\nu_R) = 0$, *Right-Handed Majorana mass terms* $m_R \overline{\nu_R^c} \nu_R$ are allowed by the SM symmetries.

Then quite likely *Majorana masses* occur in nature too.

slide by B.Kayser

Possible Majorana mass terms:

$$H_{SM} H_{SM} \overline{\nu_L^c} \nu_L, \quad H_{I_W=1} \overline{\nu_L^c} \nu_L, \quad m_R \overline{\nu_R^c} \nu_R$$

Not renormalizable

This Higgs not in SM

No Higgs

slide by B.Kayser

Why Majorana Masses ⟹ Majorana Neutrinos

The objects ν_L and $\nu_L{}^c$ in $m_L\overline{\nu_L}\,\nu_L{}^c$ are not the mass eigenstates, but just the neutrinos in terms of which the model is constructed.

$m_L\overline{\nu_L}\,\nu_L{}^c$ induces $\nu_L \leftrightarrow \nu_L{}^c$ mixing.

As a result of $K^0 \leftrightarrow \overline{K^0}$ mixing, the neutral K mass eigenstates are —

$$K_{S,L} \cong (K^0 \pm \overline{K^0})/\sqrt{2}\,. \qquad \overline{K_{S,L}} = K_{S,L}\,.$$

As a result of $\nu_L \leftrightarrow \nu_L{}^c$ mixing, the neutrino mass eigenstate is —

slide by B.Kayser

$$\nu_i = \nu_L + \nu_L{}^c = \text{“}\,\nu + \overline{\nu}\,\text{”}. \quad \overline{\nu_i} = \nu_i\,.$$

The mass eigenstates are explicitly charge conjugation eigenstates. They do not have fixed chirality.