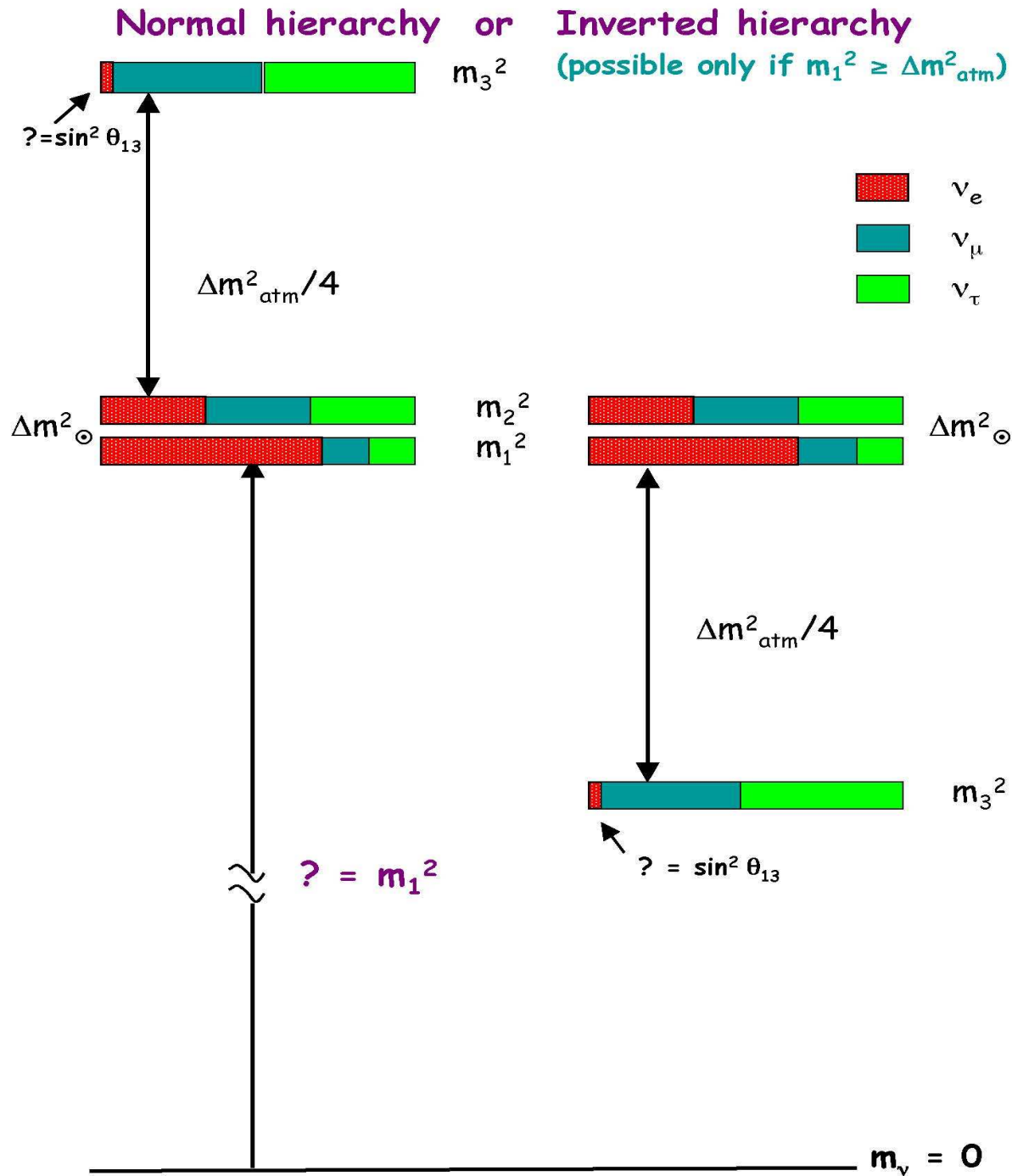


Lecture #1

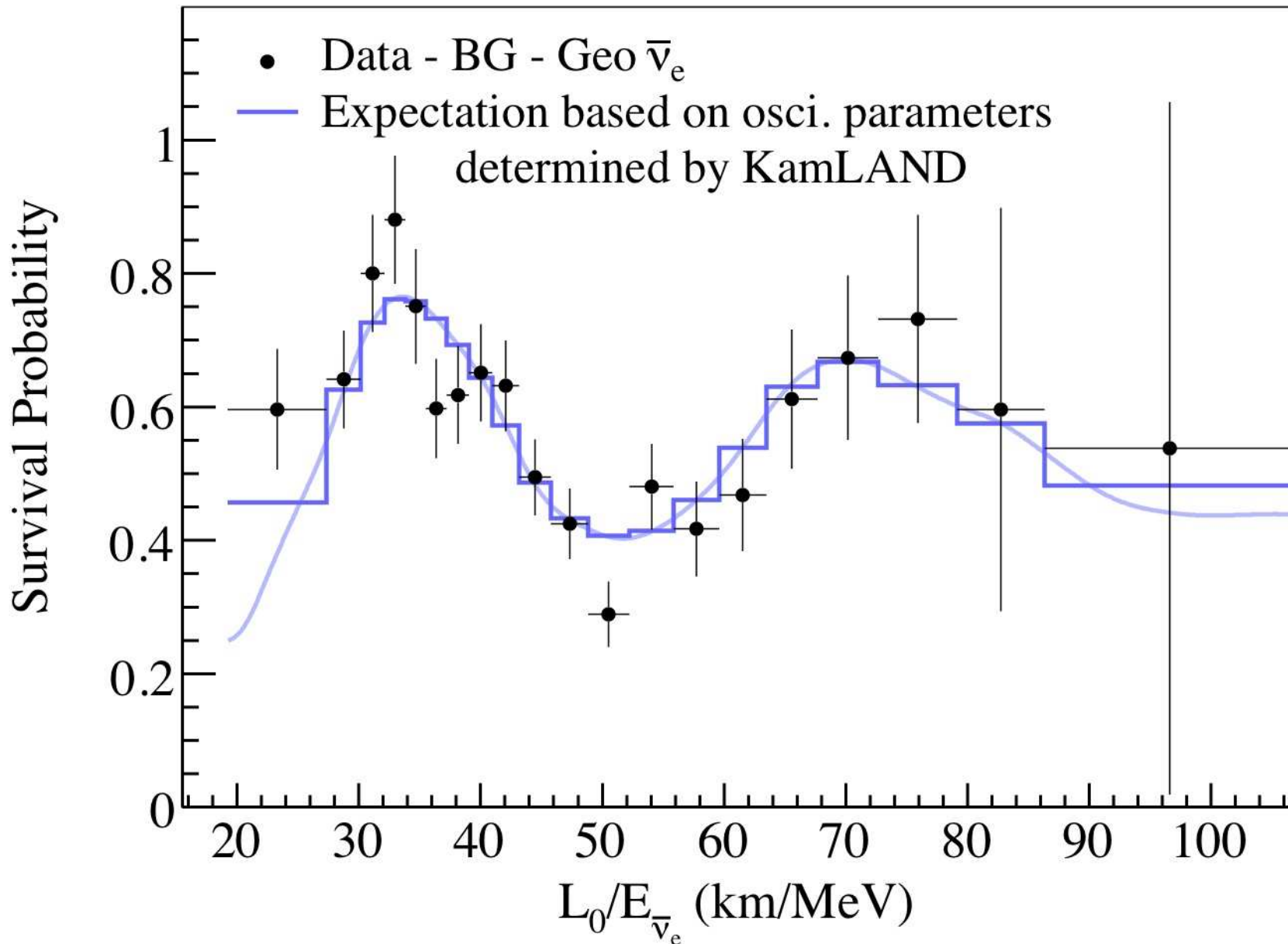
- a) Brief review of oscillation results
- b) Motivation for the search of $0\nu\beta\beta$ decay
- c) Mechanism of $0\nu\beta\beta$ decay
- d) Role of $0\nu\beta\beta$ decay in determination of neutrino mass



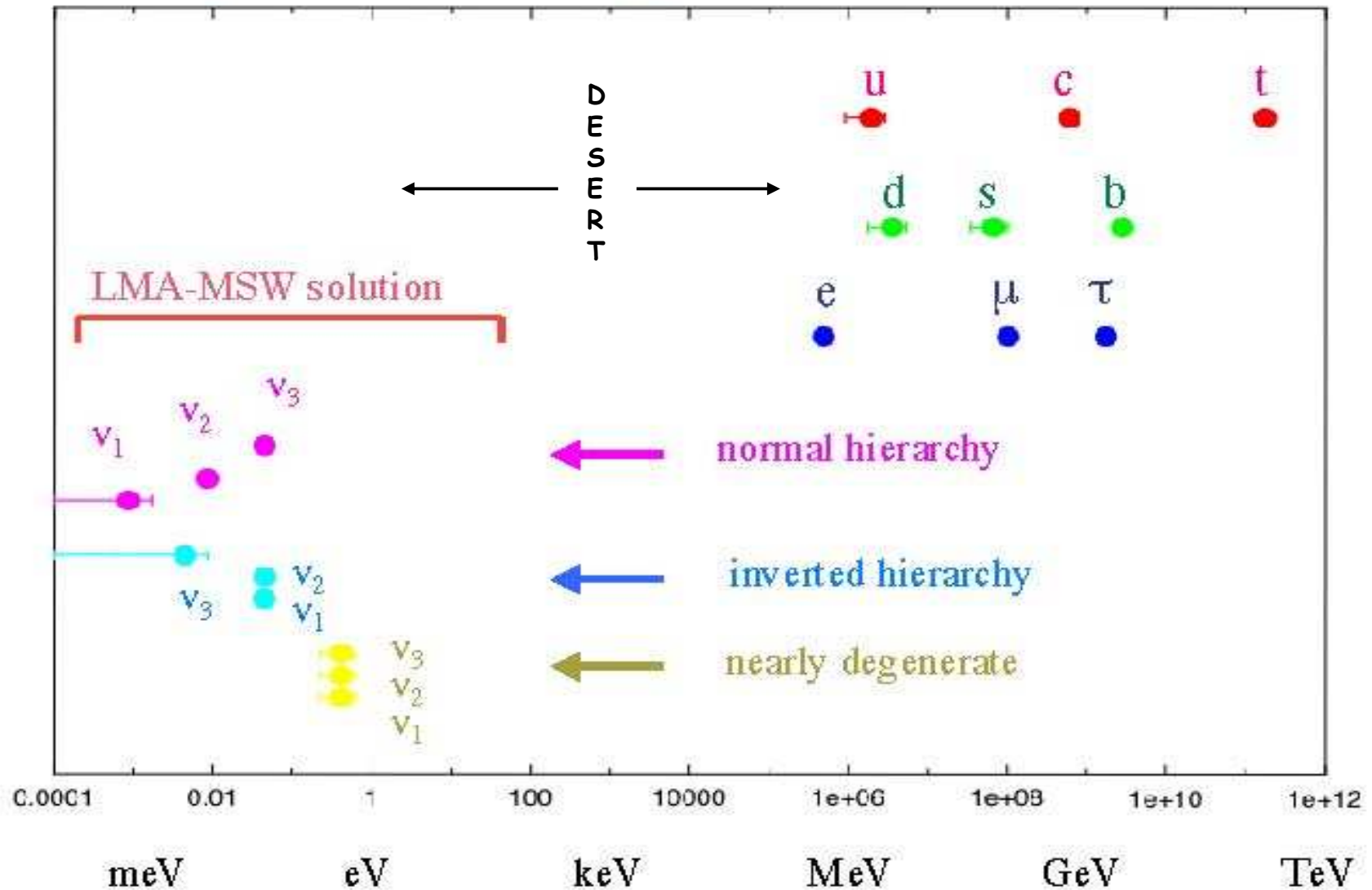
The status of the present knowledge of the neutrino oscillation phenomena is schematically depicted in this slide. Three quantities are unknown at present:

- The mass m_1
- The angle θ_{13}
- Whether the normal or inverted hierarchy is realized.

Reactor $\bar{\nu}_e$ survival probability is really an oscillating function of L/E_ν
(from KamLAND preprint arXiv:0801.4589)



However, ν masses are much smaller than the masses of other fermions



Is that a possible "Hint of" a new mass-generating mechanism?

To solve the dilemma of 'unnaturally' small neutrino mass we can give up on renormalizability and add operators of dimension $d > 4$ that are suppressed by inverse powers of some scale Λ but are consistent with the SM symmetries.

Weinberg already in 1979 (PLR 43, 1566) showed that there is **only one** dimension $d=5$ gauge-invariant operator given the particle content of the standard model:

$$L^{(5)} = C^{(5)}/\Lambda (\bar{L}^c \varepsilon H)(H^T \varepsilon L) + \text{h.c.}$$

Here $\bar{L}^c = L^T C$, where C is charge conjugation and $\varepsilon = -i\tau_2$. This operator clearly violates the lepton number by two units and represents neutrino Majorana mass

$$L^{(M)} = C^{(5)}/\Lambda v^2/2 (\underline{\nu}_L^c \nu_L) + \text{h.c.}$$

If Λ is larger than v , the Higgs vacuum expectation value, the neutrinos will be 'naturally' lighter than the charged fermions.

The energy scale Λ is more or less the energy above which the effective operator expressions above are no longer valid.

In order to estimate the magnitude of Λ suppose that $C^{(5)} \sim O(1)$ and neutrino mass ~ 0.1 eV. Then

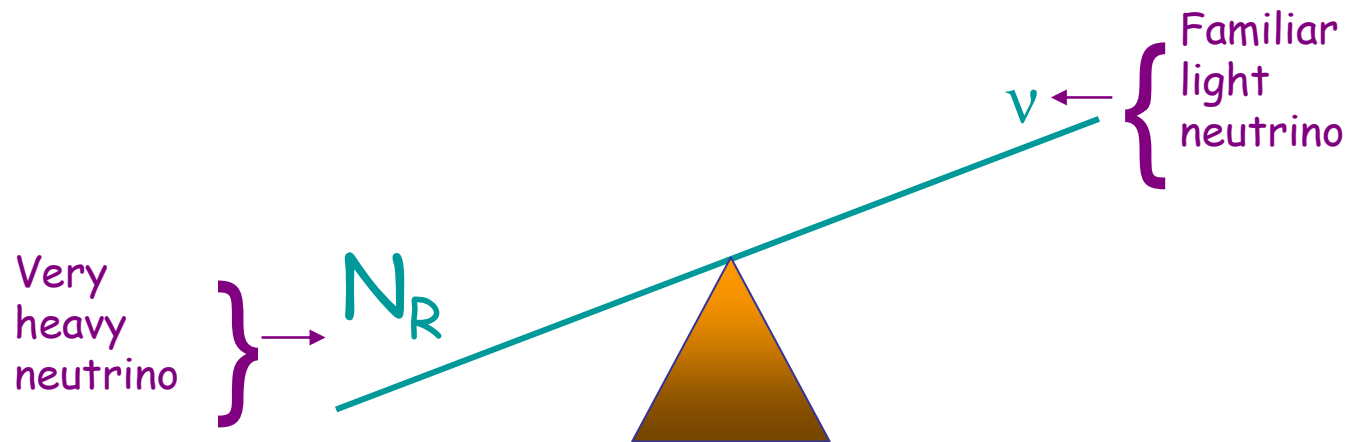
$$\Lambda \sim v^2/m_\nu \sim 10^{15} \text{ GeV}$$

It is remarkable, but perhaps a coincidence, that this scale Λ is quite near the scale at which the running gauge coupling constants meet, $M_{\text{GUT}} \sim 10^{15-16} \text{ GeV}$.

The most popular theory of why neutrinos are so light is the —


See-Saw Mechanism

(Gell-Mann, Ramond, Slansky (1979), Yanagida(1979), Mohapatra, Senjanovic(1980))



It assumes that the very heavy neutrinos N_R exist. Their mass plays an analogous role as the scale Λ of Weinberg, i.e., $m_\nu \sim v^2/M_N$. Both the light and heavy neutrinos are Majorana fermions.

Current experimental goals in neutrino physics

- Measure mixing parameters (esp. unknowns θ_{13} and δ_{CP})
 - Resolve the mass 'hierarchy'
 - Determine magnitude of at least one m_ν
 - **Demonstrate Majorana or Dirac hypothesis** These lectures
 - Use neutrinos as astrophysical probes
 - Look for the unknown
- 

How can we tell whether the total lepton number is conserved?

A partial list of processes where the lepton number would be violated:

Neutrinoless $\beta\beta$ decay: $(Z,A) \rightarrow (Z\pm 2,A) + 2e^{(\pm)}$, $T_{1/2} > \sim 10^{25}$ y

Muon conversion: $\mu^- + (Z,A) \rightarrow e^- + (Z-2,A)$, BR $< 10^{-12}$

Anomalous kaon decays: $K^+ \rightarrow \pi^- \mu^+ \mu^+$, BR $< 10^{-9}$

Flux of ν_e from the Sun: BR $< 10^{-4}$

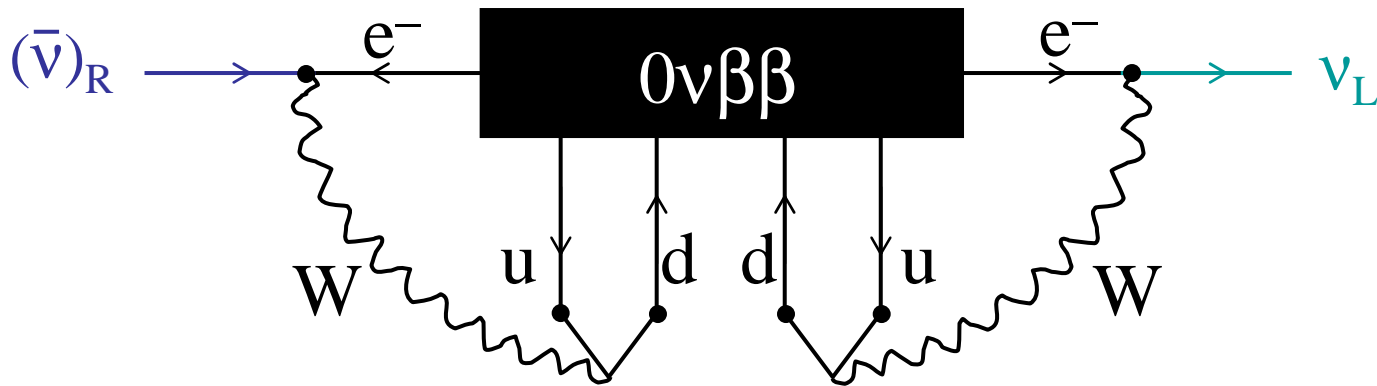
Flux of $\bar{\nu}_e$ from a nuclear reactor: BR $< ?$

Observing any of these processes would mean that the lepton number is not conserved, and that neutrinos are massive Majorana particles.

It turns out that the study of the $0\nu\beta\beta$ decay is by far the most sensitive test of the total lepton number conservation, so we restrict further discussion to this process.

Whatever processes cause $0\nu\beta\beta$, its observation would imply the existence of a **Majorana mass term**:

Schechter and Valle,82



By adding only Standard model interactions we obtain

$$(\bar{\nu})_R \rightarrow (\nu)_L \text{ **Majorana mass term**}$$

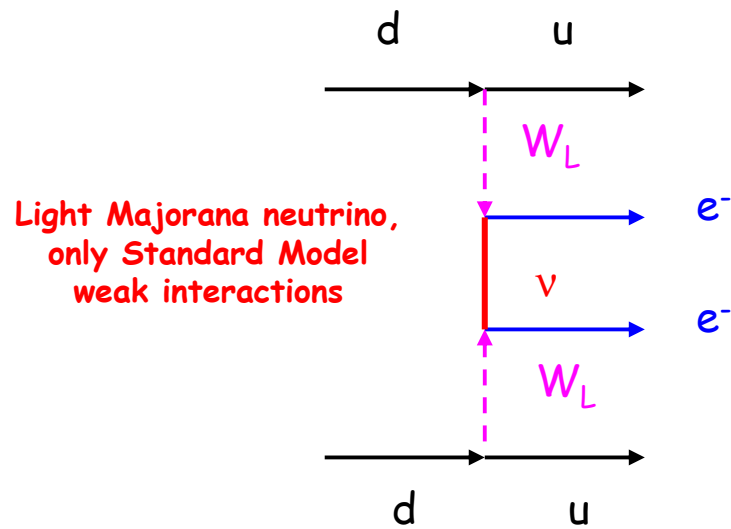
Hence observing the $0\nu\beta\beta$ decay guaranties that ν are massive Majorana particles.

If (or when) the $0\nu\beta\beta$ decay is observed two problems must be resolved:

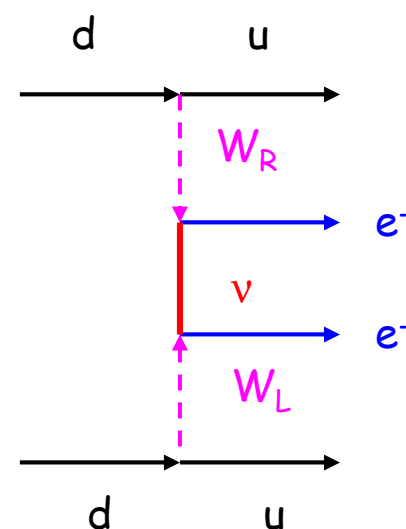
- a) What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (or quarks)?
- b) How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix element?

What is the nature of the 'black box'? In other words, what is the mechanism of the $0\nu\beta\beta$ decay?

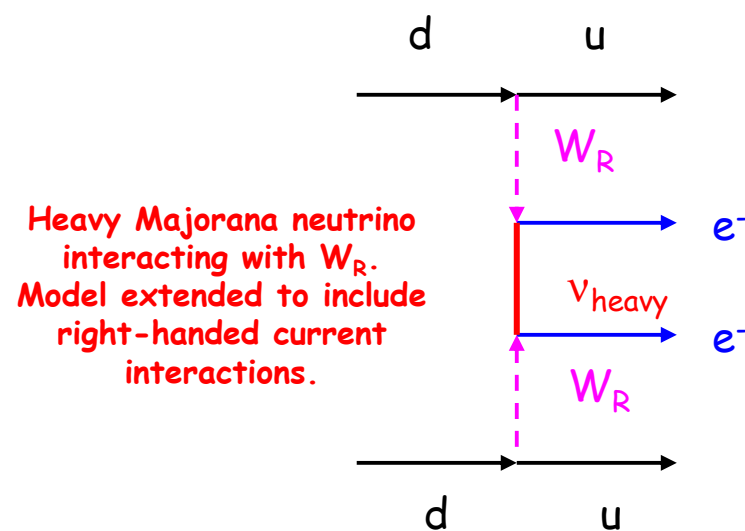
All these diagrams can contribute to the $0\nu\beta\beta$ decay amplitude



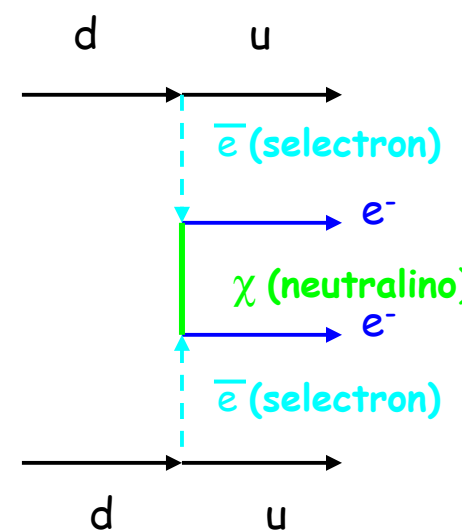
Light Majorana neutrino, only Standard Model weak interactions



Light or heavy Majorana neutrino. Model extended to include right-handed W_R . Mixing extended between the left and right-handed neutrinos.



Heavy Majorana neutrino interacting with W_R . Model extended to include right-handed current interactions.



Supersymmetry with R-parity violation. Many new particles invoked. Light Majorana neutrinos exist also.

The relative size of heavy (A_H) vs. light particle (A_L) exchange to the decay amplitude is (a crude estimate):

$$A_L \sim G_F^2 m_{\beta\beta} / \langle k^2 \rangle, \quad A_H \sim G_F^2 M_W^4 / \Lambda^5,$$

where Λ is the heavy scale and $k \sim 100$ MeV is the virtual neutrino momentum.

For $\Lambda \sim 1$ TeV and $m_{\beta\beta} \sim 0.1 - 0.5$ eV $A_L/A_H \sim 1$, hence both mechanism contribute equally. If $\Lambda \gg 1$ TeV, the heavy particle exchange results in unobservably small $0\nu\beta\beta$ rate.

From the observation of the $0\nu\beta\beta$ decay it is, in general, impossible to decide which of the possible graphs is relevant.

A diagnostic tool in deciding which mechanism dominates is in linking LNV to LFV violation.

$$A_L/A_H \sim m_{\beta\beta} \Lambda^5 / \langle k^2 \rangle M_W^4$$

Thus for $m_{\beta\beta} = 0.2 \text{ eV}$, $\langle k^2 \rangle = 50^2 \text{ MeV}^2$, and $A_L/A_H \sim 1$
 $\Lambda^5 \sim 50^2 \times 10^{12} \times 80^4 \times 10^{36} / 0.2 \text{ eV} \sim 5 \times 10^{59} \text{ eV}$
 $\Rightarrow \Lambda \sim 10^{12} \text{ eV} = 1 \text{ TeV}$

Clearly, the heavy particle mechanism could compete with the light Majorana neutrino exchange only if the heavy scale Λ is between about 1 - 5 TeV. Smaller Λ are already excluded and larger ones will be unobservable due to the fast Λ^5 scale dependence.

Observing the $0\nu\beta\beta$ decay will not (in general) make it possible to draw conclusion about the 'mechanism' of the process. We need additional information.

We shall discuss how the study of lepton flavor violation (LFV) can help us to decide what mechanism is responsible for the $0\nu\beta\beta$ decay if it is observed in a foreseeable future.

This is based on "Lepton number violation without supersymmetry"

Phys.Rev.D 70 (2004) 075007

V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

and on "Neutrinoless double beta decay and lepton flavor violation" Phys. Rev. Lett. 93 (2004) 231802

V. Cirigliano, A. Kurylov, M.J.Ramsey-Musolf, and P.V.

In the standard model lepton flavor conservation is a consequence of vanishing neutrino masses. However, the observation of neutrino oscillations shows that neutrinos are massive and that the flavor is not conserved. Hence a more general theory must contain LFV of charged leptons generated probably at some high scale.

There is a long history of searches for LFV with charged leptons, like $\mu \rightarrow e + \gamma$, muon conversion $\mu^- + (Z, A) \rightarrow e^- + (Z, A)$, or $\mu^+ \rightarrow e^+ + e^+ + e^-$.

Impressive limits for the branching ratios have been established:

$$B_{\mu \rightarrow e \gamma} \equiv \frac{\Gamma(\mu^+ \rightarrow e^+ \gamma)}{\Gamma(\mu^+ \rightarrow e^+ \nu \bar{\nu})} < 1.2 \times 10^{-11} \quad 90\% \text{C.L.},$$

$$B_{\mu \rightarrow e}^A \equiv \frac{\Gamma[\mu^- + A(N, Z) \rightarrow e^- + A(N, Z)]}{\Gamma[\mu^- + A(Z, N) \rightarrow \nu_\mu + A(Z - 1, N + 1)]} < 8 \times 10^{-13} \quad 90\% \text{C.L.}$$

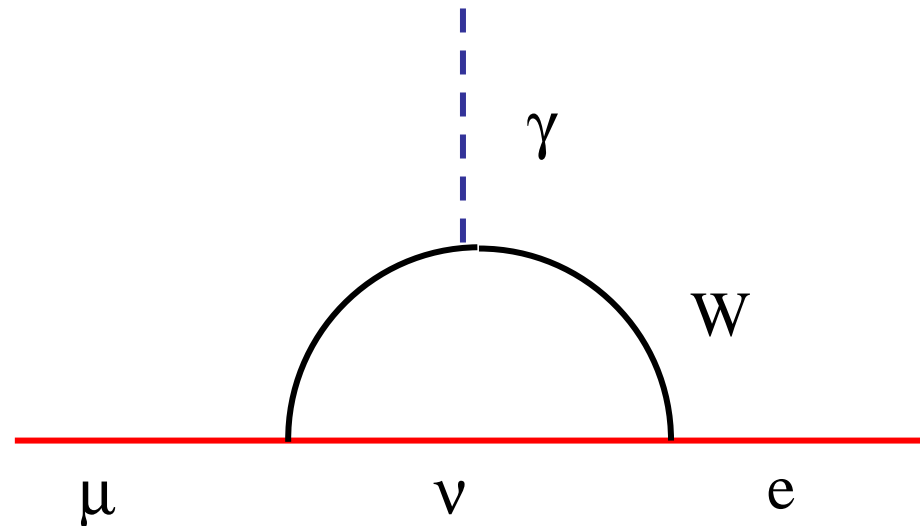
There are ambitious new proposals with much better sensitivities:

MECO (now unfortunately cancelled): $B_{\mu \rightarrow e} < 5 \times 10^{-17}$ on Al

MEG (now beginning to run at PSI): $B_{\mu \rightarrow e+\gamma} < 4 \times 10^{-14}$

i.e. improvement by a factor of $\sim 1000 - 10000$.

The direct effect of neutrino mass is "GIM suppressed" by a factor of $(\Delta m_n^2 / M_W^2)^2 \sim 10^{-50}$ hence unobservable.



So, why are people even looking for LFV?

Because most particle physics models of 'physics beyond the Standard Model' contain LFV originating at some high mass scale. Many of them also contain LNV and, naturally, all realistic models should include light and mixed neutrinos, known to exist.

If the scales of **both** LFV and LNV are well above the weak scale, then $\Gamma_{0\nu\beta\beta} \sim \langle m_{\beta\beta} \rangle^2$ and $\langle m_{\beta\beta} \rangle$ can be derived from the $0\nu\beta\beta$ decay rate. However, the 'dangerous' case is when **both** LFV and LNV scales are low (\sim TeV). In that case there might be an ambiguity in interpreting the results of $0\nu\beta\beta$ decay experiments.

In the popular SUSY-GUT scenario (for SU(5) GUT) one has the branching ratios (*Barbieri and Hall, 94*)

$$B_{\mu \rightarrow e \gamma} = 2.4 \times 10^{-12} \left(\frac{|V_{ts}|}{0.04} \frac{|V_{td}|}{0.01} \right)^2 \left(\frac{100 \text{ GeV}}{m_{\tilde{\mu}}} \right)^4$$

$$B_{\mu \rightarrow e}^{\text{Ti}} = 5.8 \times 10^{-12} \alpha \left(\frac{|V_{ts}|}{0.04} \frac{|V_{td}|}{0.01} \right)^2 \left(\frac{100 \text{ GeV}}{m_{\tilde{\mu}}} \right)^4$$

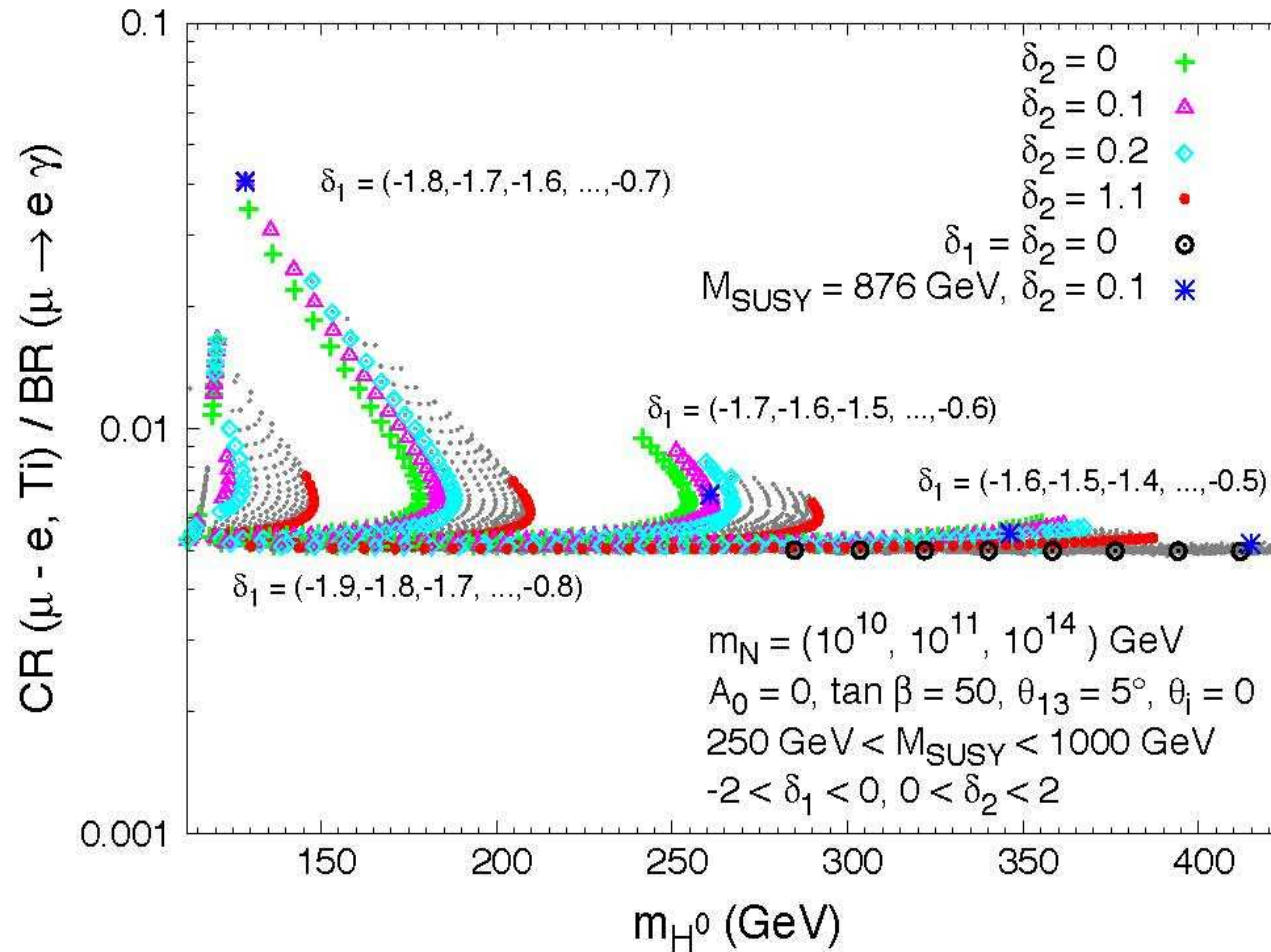
Thus a) MEG and MECO should see an effect, and
b) $\mu \rightarrow e + \gamma$ is enhanced by a factor $\sim 1/\alpha$
compared to $\mu \rightarrow e$ conversion.

The feature b) is generic for theories with high scale LNV

$\mu - e$ conversion in nuclei within the CMSSM seesaw:
 universality versus non-universality

arXiv:0707.2955

E. Arganda^a, M. J. Herrero^a and A. M. Teixeira^b



Ratio of the branching ratios for μ conversion to $\mu \rightarrow e + \gamma$ as a function of the Higgs mass. Note the typical value of $\sim 1/200$.

However, there are exceptions. Albright and Chen in arXiv:0802.4228 find the ratio as large as 0.3 in SUSY SO(10) but with high scale LNV

Linking LNV to LFV Summary:

$$B_{\mu \rightarrow e \gamma} \equiv \frac{\Gamma(\mu^+ \rightarrow e^+ \gamma)}{\Gamma(\mu^+ \rightarrow e^+ \nu \bar{\nu})} \quad B_{\mu \rightarrow e}^A \equiv \frac{\Gamma[\mu^- + A(N, Z) \rightarrow e^- + A(N, Z)]}{\Gamma[\mu^- + A(Z, N) \rightarrow \nu_\mu + A(Z - 1, N + 1)]}$$

- SM extensions with **low (" TeV) scale LNV** \Downarrow^{**}

** In absence of fine-tuning or hierarchies
in flavor couplings. Important caveat!

$$\mathcal{R} = B_{\mu \rightarrow e} / B_{\mu \rightarrow e \gamma} \gg 10^{-2}$$

- SM extensions with **high (GUT) scale LNV** [$\Gamma_{0\nu\beta\beta} \sim \langle m_{\beta\beta} \rangle^2$] \Downarrow

$$\mathcal{R} \sim \mathcal{O}(\alpha/\pi) \sim 10^{-3} - 10^{-2}$$

Thus the ratio \mathcal{R} can be used as a 'diagnostic tool' for
low scale LNV

Effective theory description

$$\mathcal{L}_{0\nu\beta\beta} = \sum_i \frac{\tilde{c}_i}{\Lambda^5} \tilde{O}_i$$

Operators (omitting L ★ R)

$$\tilde{O}_i = \bar{q}\Gamma_1 q \bar{q}\Gamma_2 q \bar{e}\Gamma_3 e^c$$

$$\mathcal{L}_{\text{LFV}} = \sum_i \frac{c_i}{\Lambda^2} O_i$$

$$O_{\sigma L} = \frac{e}{(4\pi)^2} \bar{\ell}_{iL} \sigma_{\mu\nu} i \not{D} \ell_{jL} F^{\mu\nu} + \text{h.c.}$$

$$O_{\ell L} = \bar{\ell}_{iL} \ell_{jL}^c \bar{\ell}_{kL}^c \ell_{mL}$$

$$O_{\ell q} = \bar{\ell}_i \Gamma \ell_j \bar{q} \Gamma q$$

- $O_{\sigma L}$ arises at loop level, hence $1/(4\pi)^2$ explicitly included
- $O_{\ell L}$, $O_{\ell q}$ may arise at tree level
- Leading pieces in c_i are nominally of order (Yukawa)²

The ratio R can be expressed in terms of the constants c_i as follows

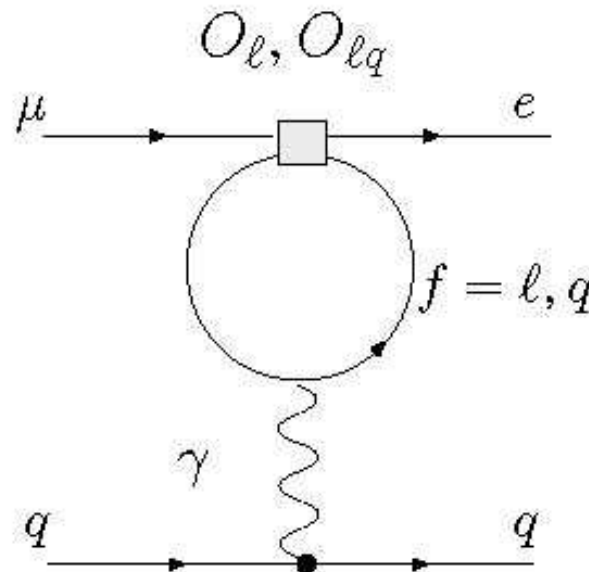
$$\mathcal{R} = \frac{\Phi}{48\pi^2} \frac{\left| e^2 \eta_1 c_{\sigma L} + e^2 (\eta_2 c_{\ell L} + \eta_3 c_{\ell q}) \log \frac{\Lambda^2}{m_\mu^2} + \eta_4 (4\pi)^2 c_{\ell q} + \dots \right|^2}{e^2 \left(|c_{\sigma L}|^2 + |c_{\sigma R}|^2 \right)}$$

- Phase space + overlap integrals: $\Phi = \frac{Z F_p^2(m_\mu^2)}{g_V^2 + 3g_A^2} \sim O(1)$ for light nuclei

- η_n are coefficients of $O(1)$

- **Origin of large logs:**
one loop operator mixing

[Raidal-Santamaria '97]



Thus from the expression for \mathcal{R} it follows:

$$\mathcal{R} = \frac{\Phi}{48\pi^2} \frac{\left| e^2 \eta_1 c_{\sigma L} + e^2 (\eta_2 c_{\ell L} + \eta_3 c_{\ell q}) \log \frac{\Lambda^2}{m_\mu^2} + \eta_4 (4\pi)^2 c_{\ell q} + \dots \right|^2}{e^2 \left(|c_{\sigma L}|^2 + |c_{\sigma R}|^2 \right)}$$

(i) No tree level $c_{\ell L}$, $c_{\ell q}$ \Downarrow $\mathcal{R} \sim \frac{\Phi \eta_1^2 \alpha}{12\pi} \sim 10^{-3} - 10^{-2}$

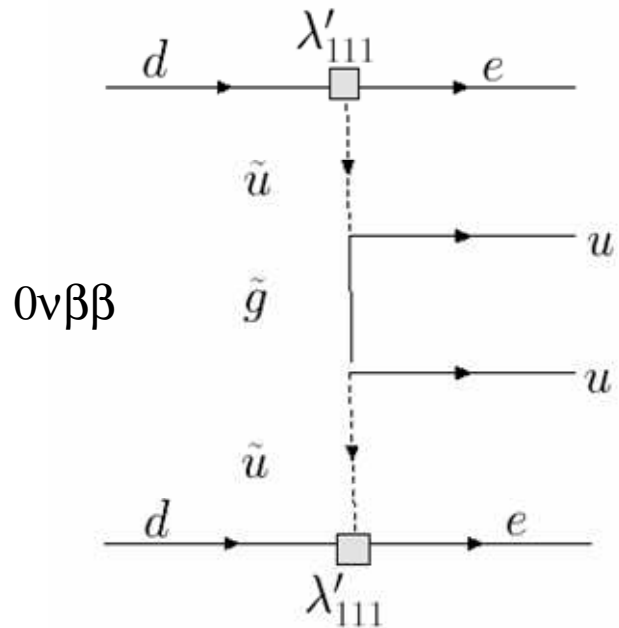
(ii) Tree level $c_{\ell L}$, $c_{\ell q}$ \Downarrow log enhancement and $\mathcal{R} \sim O(1)$

(iii) Tree level $c_{\ell q}$ \Downarrow $\mathcal{R} \gg 1$

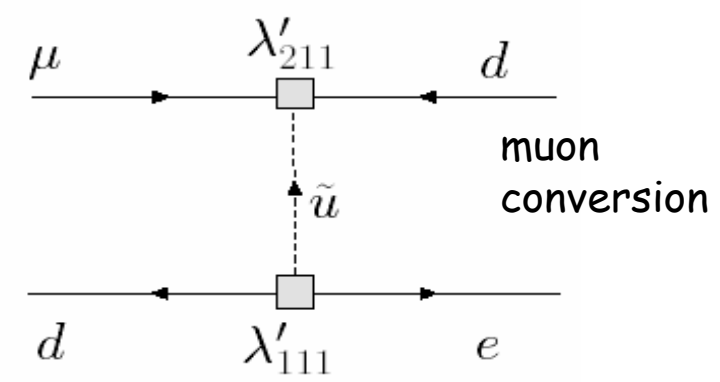
We need to show that in models with low scale LNV O_l and/or O_{lq} are generated at tree level. We offer no general proof, but two illustrations.

Illustration I: RPV SUSY [R = (-1)^{3(B-L) + 2s}]

$$W_{\text{RPV}} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i L_i H_w$$



$$\frac{\tilde{c}_i}{\Lambda^5} \sim \frac{\pi\alpha_s}{m_{\tilde{g}}} \frac{\lambda_{111}^2}{m_{\tilde{f}}^4}, \quad \frac{\pi\alpha_2}{m_\chi} \frac{\lambda_{111}^2}{m_{\tilde{f}}^4}$$



$$\frac{c_\ell}{\Lambda^2} \sim \frac{\lambda_{i11} \lambda_{i21}^*}{m_{\tilde{\nu}_i}^2}, \quad \frac{\lambda_{i11}^* \lambda_{i12}}{m_{\tilde{\nu}_i}^2}$$

$$\frac{c_{\ell q}}{\Lambda^2} \sim \frac{\lambda_{11i}^* \lambda'_{21i}}{m_{\tilde{d}_i}^2}, \quad \frac{\lambda_{1i1}^* \lambda'_{2i1}}{m_{\tilde{u}_i}^2}$$

$$\frac{c_\sigma}{\Lambda^2} \sim \frac{\lambda \lambda^*}{m_{\tilde{\ell}}^2}, \quad \frac{\lambda' \lambda'^*}{m_{\tilde{q}}^2}$$

Clearly, the way to avoid the connection between LFV and LNV is if $\lambda'_{111} \gg \lambda'_{211}$, etc. That is if λ' is nearly flavor diagonal. Note that empirically both λ_{ijk} and λ'_{ijk} are small $\ll 1$.

For the discussion of neutrino masses in the R-parity violating supersymmetric models see Y. Grossman and S. Rakshit, hep-ph/0311310

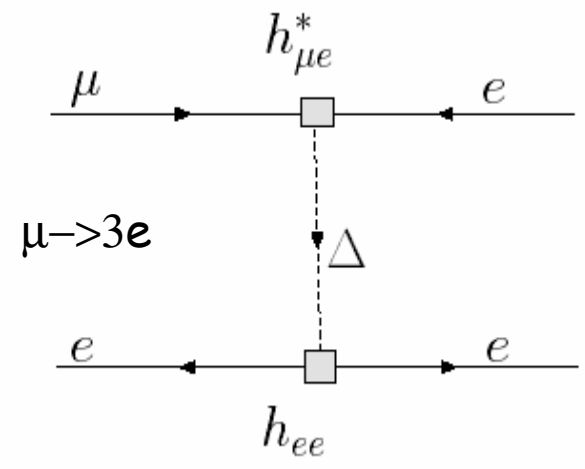
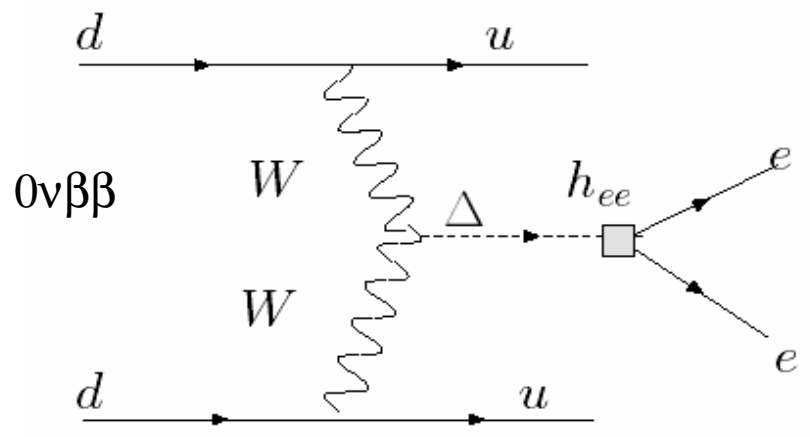
Generally, a hierarchical neutrino spectrum is predicted, but small neutrino masses require some fine tuning. Note also that R-parity violation excludes LSP as a dark matter candidate. Discovering it would exclude R-parity violation.

Illustration II: Left-Right Symmetric Model

$$SU(2)_L \rightsquigarrow SU(2)_R \rightsquigarrow U(1)_{B-L} \quad \Downarrow \quad SU(2)_L \rightsquigarrow U(1)_Y \quad \Downarrow \quad U(1)_{EM}$$

$$\mathcal{L}_Y^{\text{lept}} = -\bar{L}_L^i (y_D^{ij} \Phi + \tilde{y}_D^{ij} \tilde{\Phi}) L - (\bar{L}_L)^{ci} y_M^{ij} \tilde{\Delta}_L L_L^j - (\bar{L}_R)^{ci} y_M^{ij} \tilde{\Delta}_R L_R^j.$$

$\Delta_{L,R}$ - lepton interaction $\mathcal{L} \sim \Delta_{L,R}^{\pm\pm} \bar{\ell}_i^c h_{ij} (1 \pm \gamma_5) \ell_j$ (h_{ij} are related to the heavy neutrino mixing)



$$\frac{\tilde{c}_i}{\Lambda^5} \sim \frac{g_2^4}{M_{W_R}^4} \frac{1}{M_{\nu_R}};$$

$$\frac{g_2^3}{M_{W_R}^3} \frac{h_{ee}}{M_\Delta^2}$$

$$\frac{c_\ell}{\Lambda^2} \sim \frac{h_{\mu i} h_{ie}^*}{m_\Delta^2}$$

$$\frac{c_\sigma}{\Lambda^2} \sim \frac{(h^\dagger h)_{e\mu}}{M_{W_R}^2}$$

h_{ij} are coupling constants of leptons and the doubly charged Higgs

They are related to the mixing matrix K_R of the heavy neutrinos

$$h_{ij} = \sum_{n=\text{heavy}} (K_R)_{ni} (K_R)_{nj} \sqrt{x_n}, \quad x_n = \left(\frac{M_n}{M_{W_2}} \right)^2$$

$$(h^\dagger h)_{e\mu} = (\tilde{h}^\dagger \tilde{h})_{e\mu} = \sum_{n=\text{heavy}} x_n (K_R^\dagger)_{en} (K_R)_{n\mu} \equiv g_{\text{lfv}}$$

Note that g_{lfv} vanishes for degenerate heavy neutrinos, but h_{ij} need not.
In addition g_{lfv} also vanishes for unmixed heavy neutrinos.

Within LRSM the LFV branching ratios depend only on g_{lfv} to a good approximation.

$$B_{\mu \rightarrow e \gamma} \approx 10^{-7} \times |g_{\text{lfv}}|^2 \left(\frac{1 \text{ TeV}}{M_{W_R}} \right)^4$$

$$B_{\mu \rightarrow e}^{\text{Al}} \approx 10^{-7} \times \alpha |g_{\text{lfv}}|^2 \left(\frac{1 \text{ TeV}}{M_{\delta_R^{++}}} \right)^4 \left(\log \frac{M_{\delta_R^{++}}^2}{m_\mu^2} \right)^2$$

Thus the present limits suggest that either the scale is $\gg 1 \text{ TeV}$, or that g_{lfv} is very small, i.e. that the heavy neutrino spectrum is degenerate or has very little mixing.

As long as only a limit on the $0\nu\beta\beta$ decay rate exists, we can constrain all parameters entering the decay amplitudes (light and heavy neutrino masses, strength of the right-handed current, SUSY R-parity violating amplitude, etc.).

However, once the decay rate is convincingly measured, we need to determine which of the possible mechanism is responsible for the observation.

Let us in the following assume that the three light active neutrinos, ν_1, ν_2, ν_3 , are Majorana particles. The $0\nu\beta\beta$ decay exists then for sure, and we will concentrate on the corresponding rate.

What is the relation of the deduced fundamental parameters and the neutrino mixing matrix? Or, in other words, what is the relation between the $0\nu\beta\beta$ decay rate and the absolute neutrino mass?

As long as the mass eigenstates ν_i that are the components of the flavor neutrinos ν_e , ν_μ , and ν_τ are Majorana neutrinos, the $0\nu\beta\beta$ decay will occur, with the rate

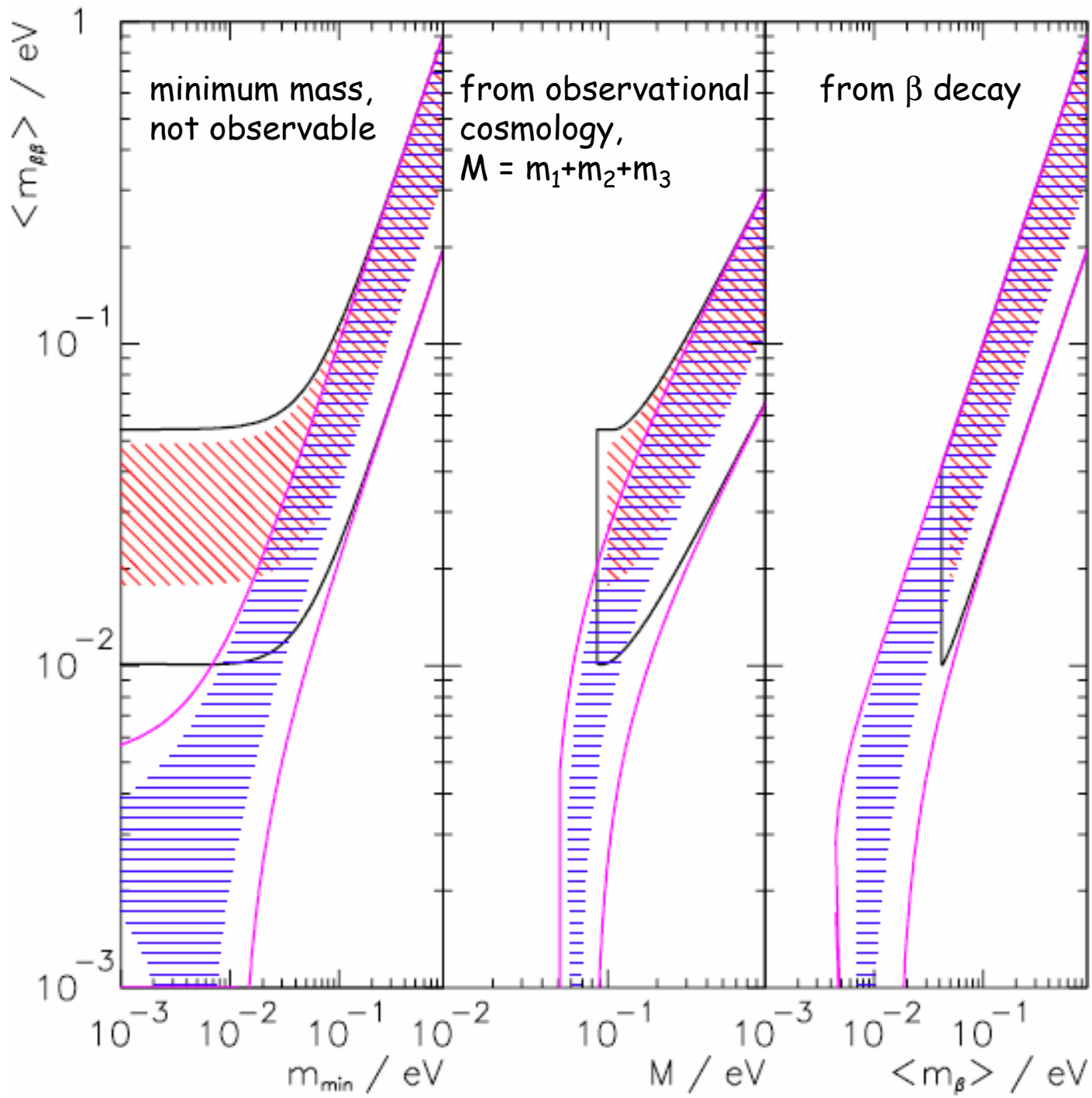
$$1/T_{1/2} = G(E_{\text{tot}}, Z) (M^{0\nu})^2 \langle m_{\beta\beta} \rangle^2,$$

where $G(E_{\text{tot}}, Z)$ is easily calculable phase space factor, $M^{0\nu}$ is the nuclear matrix element, calculable with difficulties (and discussed later), and

$$\langle m_{\beta\beta} \rangle = \left| \sum_i |U_{ei}|^2 \exp(i\alpha_i) m_i \right|,$$

where α_i are unknown Majorana phases (only two of them are relevant).

Using the formula above we can relate $\langle m_{\beta\beta} \rangle$ to other observables related to the absolute neutrino mass.



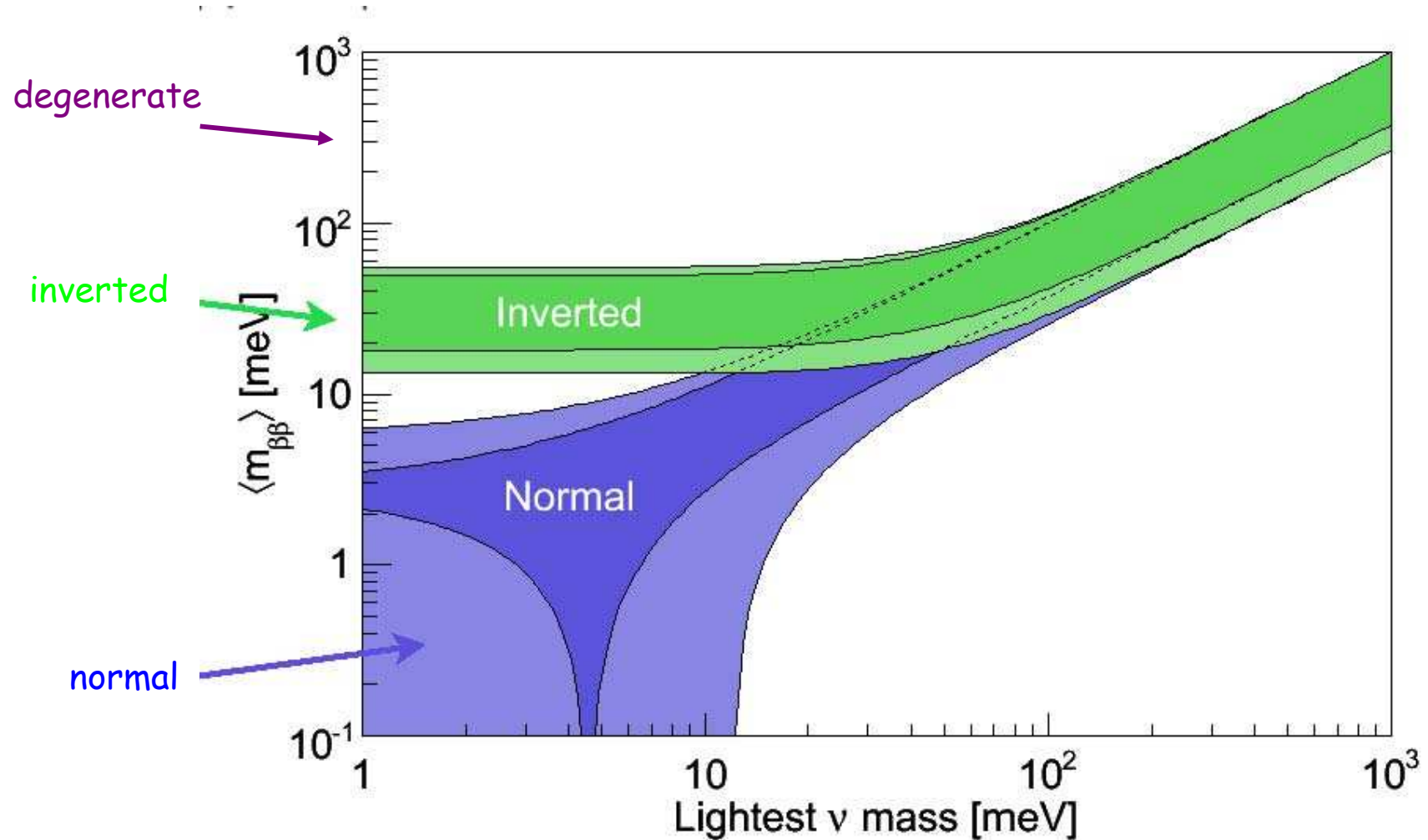
$\langle m_{\beta\beta} \rangle$ vs. the absolute mass scales

blue shading: normal hierarchy, $\Delta m^2_{31} > 0$.
 red shading: inverted hierarchy $\Delta m^2_{31} < 0$

shading: best fit parameters, lines 95% CL errors.

Thanks to A. Piepke

Alternate representation of the first panel. Shows that the $\langle m_{\beta\beta} \rangle$ axis can be divided into three distinct regions. However, it creates the impression (false) that determining $\langle m_{\beta\beta} \rangle$ would help to decide between the two competing hierarchies.



Summary of methods of neutrino mass determination and (optimistic) sensitivities:

Neutrino oscillations: θ_{12} (U_{12}), $m_1^2 - m_2^2$, etc.

observed $\sim 10^{-5} \text{ eV}^2$ (only mass square differences, independent of Dirac vs. Majorana)

Single beta decay:

0.2 eV (independent of Dirac vs. Majorana)

$$\langle m_\beta \rangle^2 = \sum m_i^2 |U_{ei}|^2$$

Double beta decay:

0.01 eV (only for Majorana)

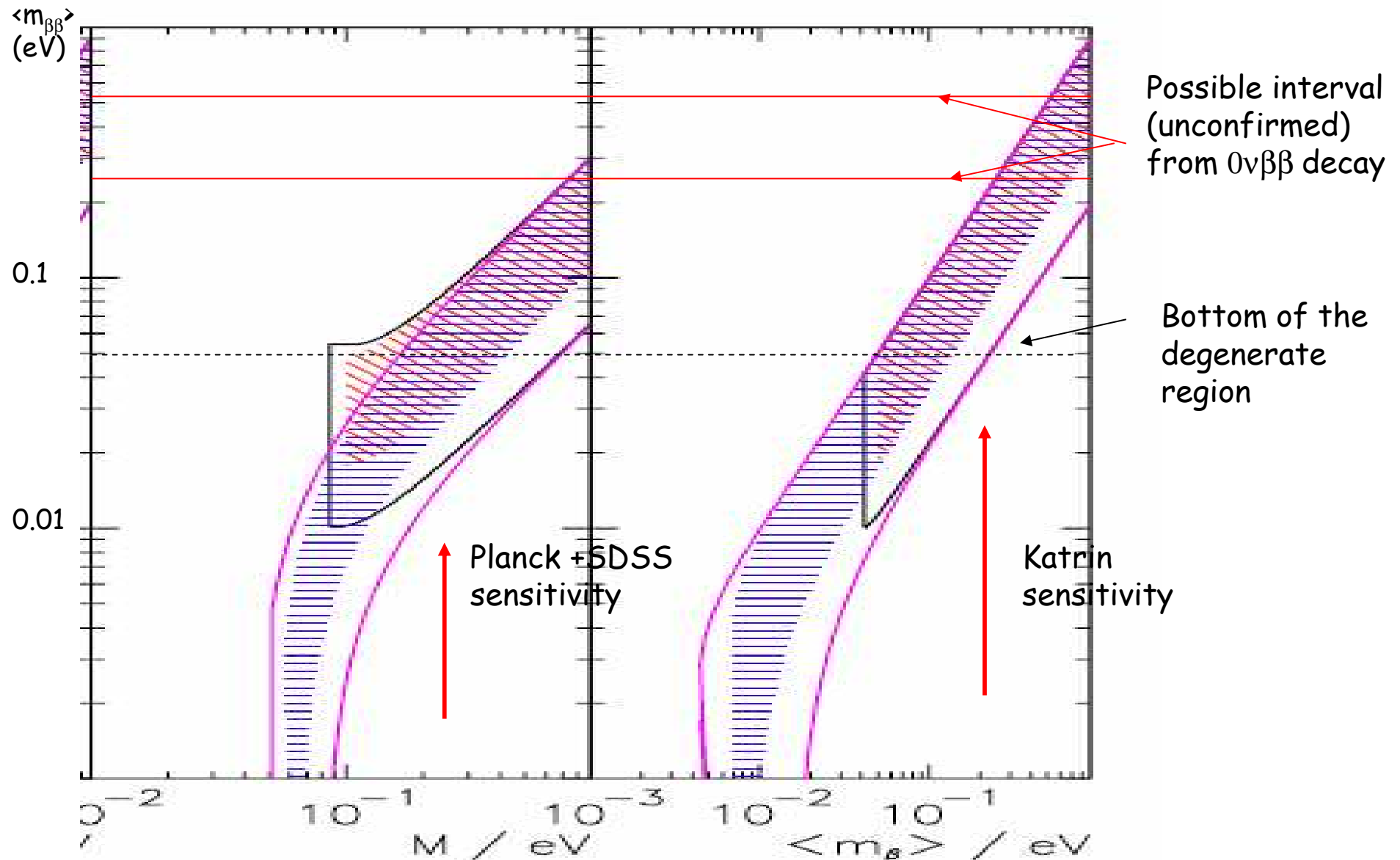
$$\langle m_{\beta\beta} \rangle = \left| \sum m_i |U_{ei}|^2 \epsilon_i \right|$$

(Majorana phases)

Observational cosmology: $M = \sum m_i$

0.1 eV (independent of Dirac vs. Majorana)

The degenerate mass region will be explored by the next generation of $0\nu\beta\beta$ experiments and also probed by ways independent on Majorana nature of neutrinos.

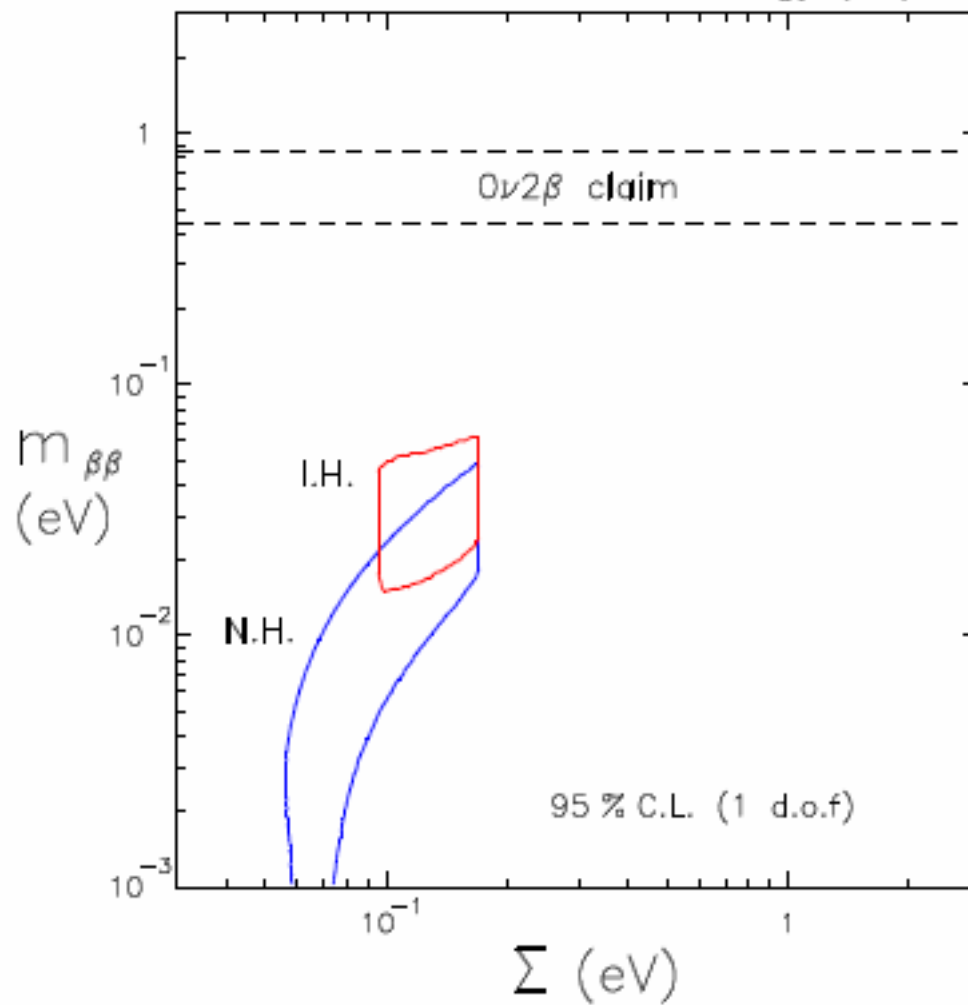


Three regions of $\langle m_{\beta\beta} \rangle$ of interest:

- i) Degenerate mass region where all $m_i \gg \Delta m_{31}^2$. There $\langle m_{\beta\beta} \rangle > 0.05$ eV. $T_{1/2}$ for $0\nu\beta\beta$ decay $< 10^{26-27}$ y in this region. This region will be explored during the next 3-5 years with $0\nu\beta\beta$ decay experiments using ~ 100 kg sources. Moreover, most if not all of that mass region will be explored also by study of ordinary β decay and by the 'observational cosmology'. These latter techniques are independent of whether neutrinos are Majorana or Dirac particles.
- ii) Inverted hierarchy region where m_3 could be $< \Delta m_{31}^2$. However, quasidenegerate normal hierarchy is also possible for $\langle m_{\beta\beta} \rangle \sim 20-100$ meV. $T_{1/2}$ for $0\nu\beta\beta$ decay is 10^{27-28} years here, and could be explored with \sim ton size experiments. Proposals for such experiments, with timeline ~ 10 years, exist.
- iii) Normal mass hierarchy, $\langle m_{\beta\beta} \rangle < 20$ meV. It would be necessary to use ~ 100 ton experiments. There are no realistic ideas how to do it.

spares

Combined results of the claimed ^{76}Ge $0\nu\beta\beta$ discovery and the most restrictive observational cosmology constraint. There is a clear conflict in this case.



From Fogli et al, hep-ph/0608060

Leaving aside the all important question whether the $0\nu\beta\beta$ experimental evidence will withstand further scrutiny and whether the cosmological constraint is reliable and model independent, lets discuss various possible scenarios suggested by this test of consistency.

Possibility #1: Both neutrino mass determination give a positive and consistent result (the results intersect on the expected 'band' and both suggest a degenerate mass pattern.

(Everybody is happy, even though somewhat surprised since the degenerate scenario is a bit unexpected.)

Possibility #2: $0\nu\beta\beta$ will not find a positive evidence (the present claim will be shown to be incorrect) but observational cosmology will give a positive evidence for a degenerate mass scenario, i.e., a situation opposite to the previous slide. (This will also be reluctantly accepted as an evidence that neutrinos are not Majorana but Dirac.)

Possibility #3: The situation on the previous slide is confirmed. The positive evidence stemming from $0\nu\beta\beta$ decay is confronted with a lack of evidence from observational cosmology. **What now? Is there a possible scenario that would accommodate such a possibility?**

The answer is **yes** and deserves a more detailed explanation. Actually, this can happen for two reasons:

- 1) The $0\nu\beta\beta$ decay is not caused by the exchange of the light Majorana neutrinos, but by some other mechanism. The obvious question then is **how can we tell which mechanism is responsible for the $0\nu\beta\beta$ decay.**
- 2) Even though the $0\nu\beta\beta$ decay is caused by the exchange of the light Majorana neutrinos the relation between the decay rate and $\langle m_{\beta\beta} \rangle$ is rather different than what we thought, i.e. the nuclear matrix elements we used are incorrect. The obvious question then is **how uncertain the nuclear matrix elements really are.**

In conclusion of this first lecture lets add few general remarks regarding the neutrino mass determination.

The two-body decays, like $\pi^+ \rightarrow \mu^+ + \nu_\mu$ are very simple conceptually: Consider pion decay in its rest frame, there

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu ,$$

but the sensitivity is only to $m_\nu \sim 170$ keV with little hope of a substantial improvement.

Another conceptually simple methods of neutrino mass determination, like TOF, are not sensitive enough either

The time delay, with respect to massless particle, is

$$\Delta t(E) = 0.514 (m_\nu/E_\nu)^2 D,$$

where m is in eV, E in MeV, D in 10 kpc, and Δt in sec.

But there are no massless particles emitted by SN at the same time as neutrinos. Alternatively, we might look for a time delay between the charged current signal (i.e. ν_e) and the neutral current signal (dominated by ν_x). In addition, one might look for a broadening of the signal, and rearrangement according to the neutrino energy.

Note as a curiosity:

$\langle m_{\beta\beta} \rangle$ may vanish even though all m_i are nonvanishing and all ν_i are Majorana neutrinos.

What can we do in that case?

In principle, although probably not in practice, we can look for the lepton number violation involving muons.

Numerical example: take $\theta_{13} = 0$, and Majorana phase $\alpha_2 - \alpha_1 = \pi$ (only for this choice of phases can $\langle m_{\beta\beta} \rangle$ vanish when $\theta_{13} = 0$).

$\langle m_{\beta\beta} \rangle = 0$ if $m_1/m_2 = \tan^2\theta_{12}$, with $m_2 = (m_1^2 + \Delta m_{sol}^2)^{1/2}$.

That happens for $m_1 = 4.58$ meV and $m_2 = 10$ meV

(this is, therefore, fine tuning).

But then $\langle m_{\mu e} \rangle = \sin 2\theta_{12} \cos \theta_{23} / 2 \times (m_1 + m_2) = 4.78$ meV,

Which is, at least in principle, observable using

$\mu^- + (Z, A) \rightarrow e^+ + (Z-2, A)$.

LRSM Matter fields:

$$L_{iL} = \begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_L : (1/2:0:-1) \quad L_{iR} = \begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_R : (0:1/2:-1)$$

$$Q_{iL} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_L : (1/2:0:1/3) \quad Q_{iR} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_R : (0:1/2:1/3)$$

Higgs sector

bi-doublet $\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \langle \phi \rangle = \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2/\sqrt{2} \end{pmatrix}$

2 triplets $\Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix} \langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}$