Neutrino masses and mixings and...

cosmology, astrophysics, LHC

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Present

Two direct evidences for violation of lepton flavour.

Anomaly	Solar	Atmospheric
first hint	1968	1986
confirmed	2002	1998
evidence	13σ	20σ
for	$\nu_e \to \nu_{\mu,\tau}$	$\nu_\mu \to \nu_\tau$
seen by	Cl,2Ga,SK,SNO,KL,Bo	SK,Macro, K2K,NuMi
disappearance	seen	seen
appearance	seen	partly seen
oscillations	seen	$\approx$ seen
$\sin^2 2\theta$	0.88 ± 0.03	1.02 ± 0.04
Δm^2	$(7.58 \pm 0.21) 10^{-5}\,\mathrm{eV}^2$	$(2.40 \pm 0.15) 10^{-3}\,\mathrm{eV}^2$

Theory

Neutrino oscillations

Ultrarelativistic neutrinos with 3×3 mass matrix:

$$m_\nu = V^* \,\mathrm{diag}(m_1 e^{-2i\beta}, m_2 e^{-2i\alpha}, m_3) V^\dagger$$

where

$$V = R_{23}(\theta_{23}) \cdot R_{13}(\theta_{13}) \cdot \mathrm{diag}\,(1, e^{i\phi}, 1) \cdot R_{12}(\theta_{12})$$

is the neutrino mixing matrix, oscillate in normal matter as dictated by

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = H \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \qquad \text{where} \qquad H = \frac{m_\nu^\dagger m_\nu}{2E} + \sqrt{2} G_\mathrm{F} N_e \,\mathrm{diag}(1,0,0)$$

Main facts can be understood in terms of 2ν vacuum oscillations.

2ν vacuum oscillations

(Derivation as simple as the well-known $e^{iE_i t}$ hand-waving, and correct)
Oscillations from interference between states with different mass and same E
Often stationary fluxes. Always energy resolution $\Delta E \gg 1/\Delta t$: $\langle e^{i\Delta E\cdot t}\rangle = 0$

At the production region $x \approx 0$

$$|\nu(x\approx 0)\rangle = |\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

At a generic x

$$|\nu(x)\rangle = e^{ip_1 x}\cos\theta|\nu_1\rangle + e^{ip_2 x}\sin\theta|\nu_2\rangle.$$

Since $p_i^2 = \sqrt{E^2 + m_i^2} \simeq E - m_i^2/2E$ at the detection region $x \approx L$

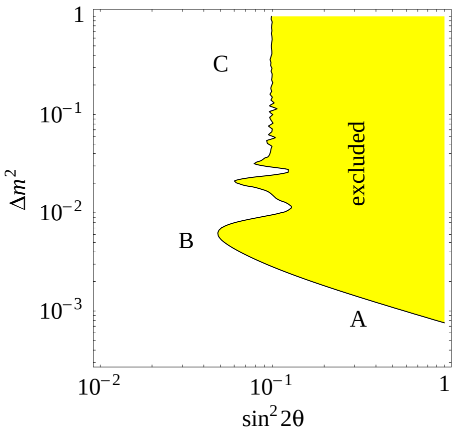
$$P(\nu_\mu \to \nu_\mu) = |\langle\nu_\mu|\nu(L)\rangle|^2 \simeq 1 - S_{12}\sin^2 2\theta$$

$$S_{ij} \equiv \sin^2 \frac{c^3}{\hbar}\frac{\Delta m_{ij}^2 L}{4E} = \sin^2 1.27\frac{\Delta m_{ij}^2}{\text{eV}^2}\frac{L}{\text{Km}}\frac{\text{GeV}}{E}.$$

Need low E and big L to see this macroscopic quantum phenomenon

Limiting cases

A **Oscillations with short base-line**: $S \ll 1$, reduces to perturbation theory $P(\nu_e \to \nu_\mu) \propto L^2$: enough to fix factor-2 ambiguity!



C **ΔE, ΔL averaged oscillations**: $\langle S \rangle = 1/2$

$$\begin{aligned} P(\nu_e \to \nu_e) &= 1 - \tfrac{1}{2}\sin^2 2\theta \\ &= \sin^4\theta + \cos^4\theta = \end{aligned}$$

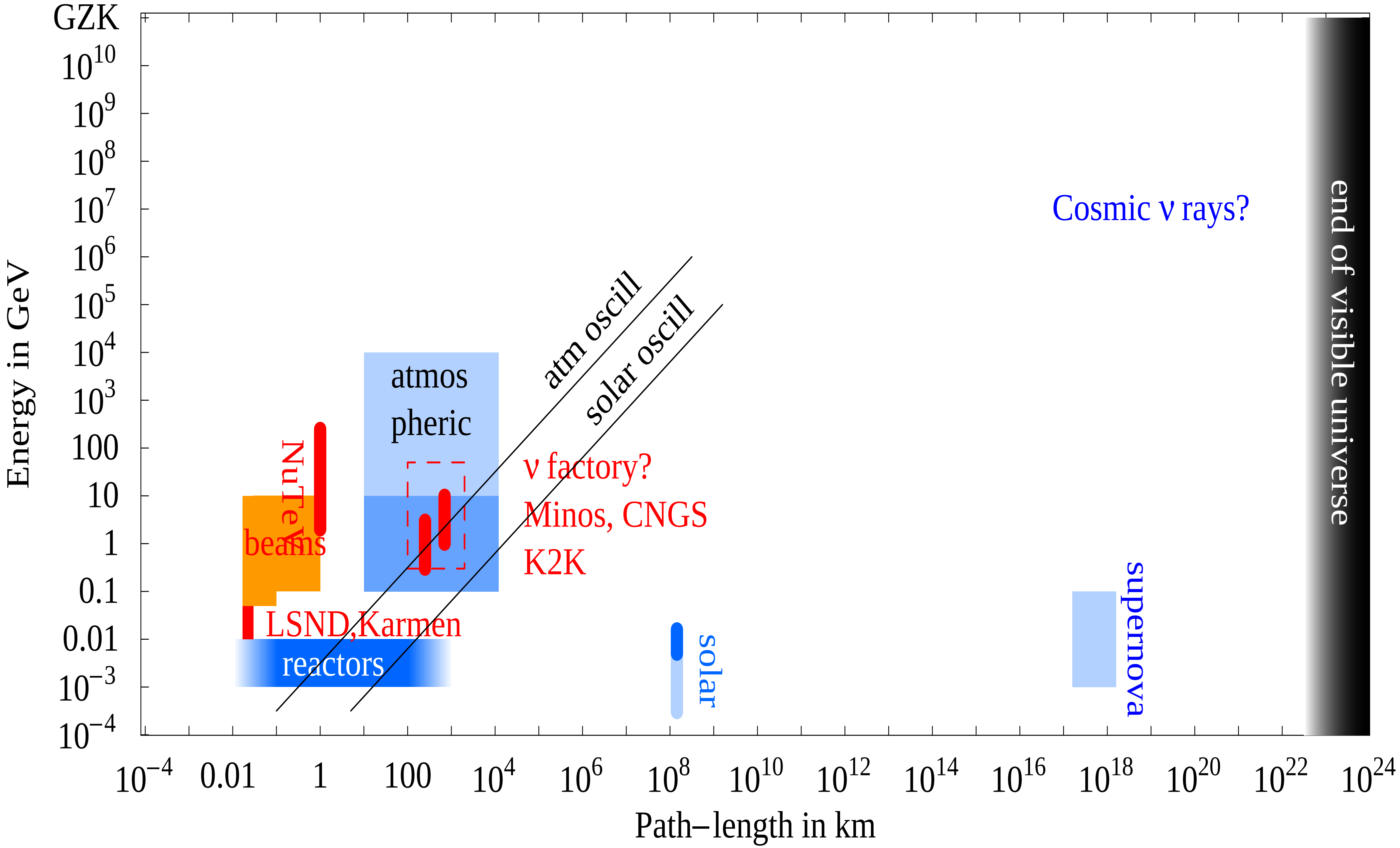
$$\begin{tikzcd} & \nu_2 \arrow[dr, "\sin^2\theta"] & \\ \nu_e \arrow[ur, "\sin^2\theta"] \arrow[dr, "\cos^2\theta"'] & & \nu_e \\ & \nu_1 \arrow[ur, "\cos^2\theta"'] & \end{tikzcd}$$

like

$$\begin{tikzcd} & d\ (\pi) \arrow[dr, "\sin^2\theta_C"] & \\ c\ (\text{e.g. } \Lambda) \arrow[ur, "\sin^2\theta_C"] \arrow[dr, "\cos^2\theta_C"'] & & c \\ & s\ (K) \arrow[ur, "\cos^2\theta_C"'] & \end{tikzcd}$$

The information on the phase is lost: combine probabilities, not amplitudes

B **The intermediate region**. Coherence is lost when neutrinos with different E have too different oscillation phases $\phi \sim \Delta m^2 L/E$, i.e. when $\Delta\phi \approx n\phi \gtrsim 1$. With energy resolution ΔE one can see $n \sim E/\Delta E$ oscillations.



Atmospheric and solar discoveries based on careful study of natural ν sources

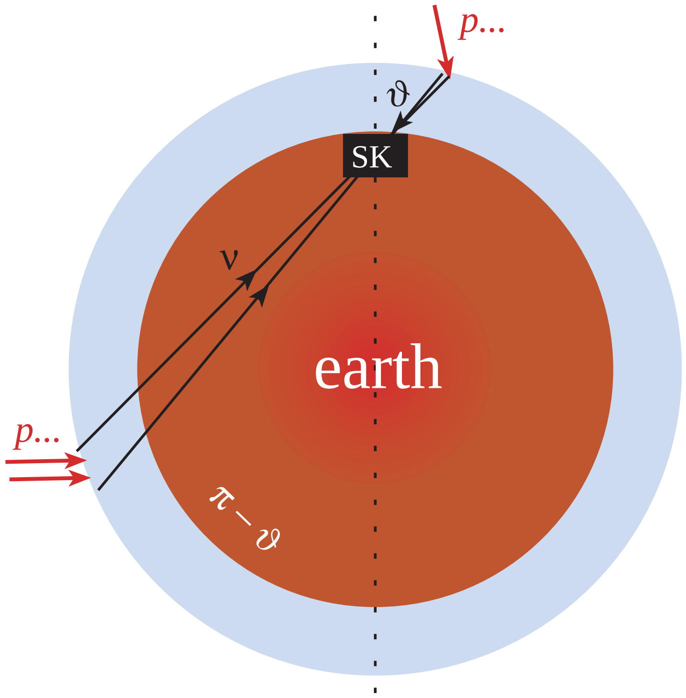
The atmospheric anomaly

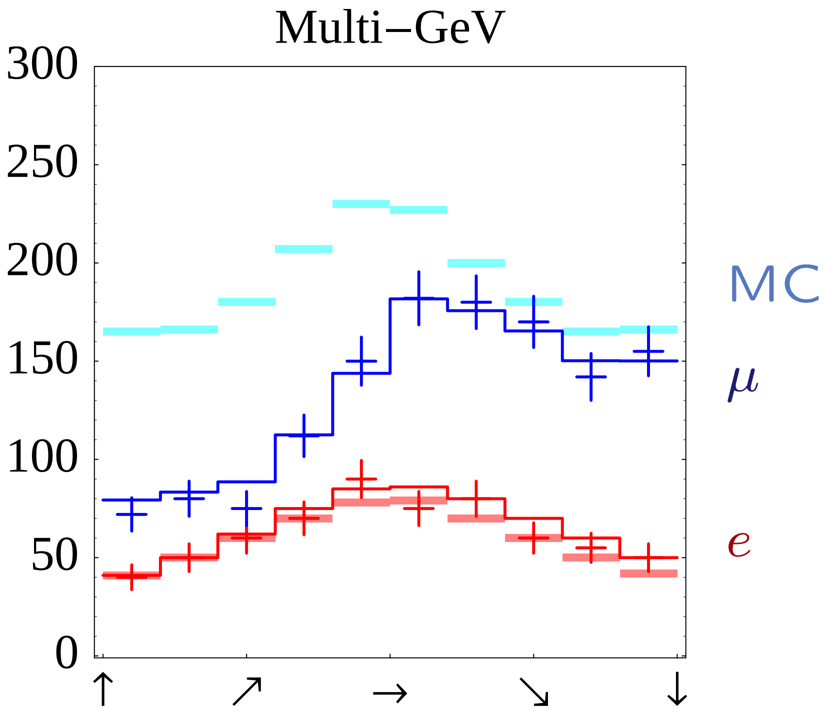
The atmospheric anomaly

SK detects $\nu_\ell N \to \ell N$ distinguishing μ from e. In the multi-GeV sample

$$E_\ell \lesssim E_\nu \sim 3\,\text{GeV}, \qquad \vartheta_\ell \sim \vartheta_\nu \pm 10^\circ$$

Without oscillations $N(\cos\vartheta_{\text{zenith}})$ is up/down symmetric





No doubt that there is an anomaly

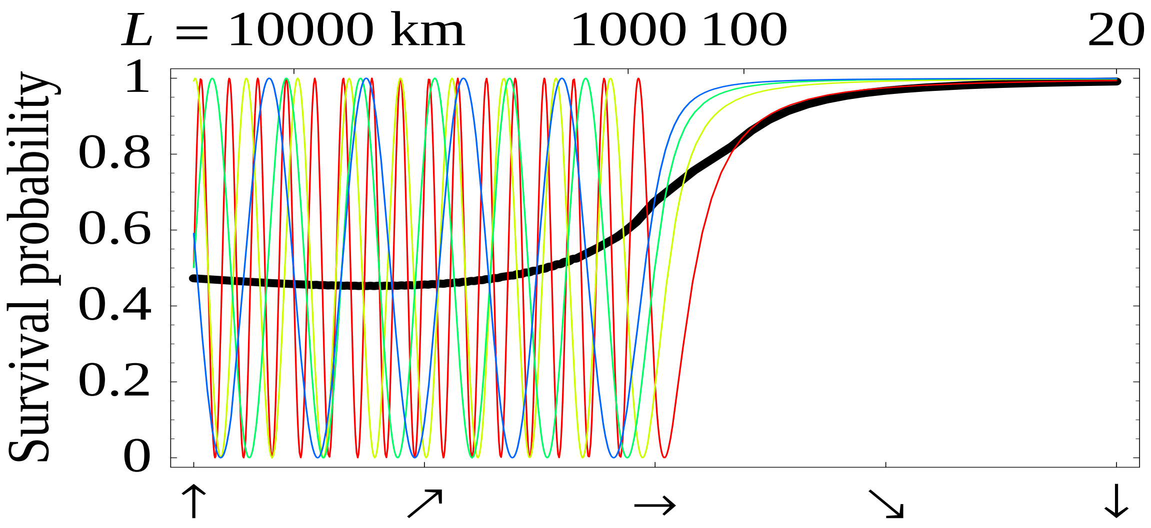
Atmospheric oscillations?

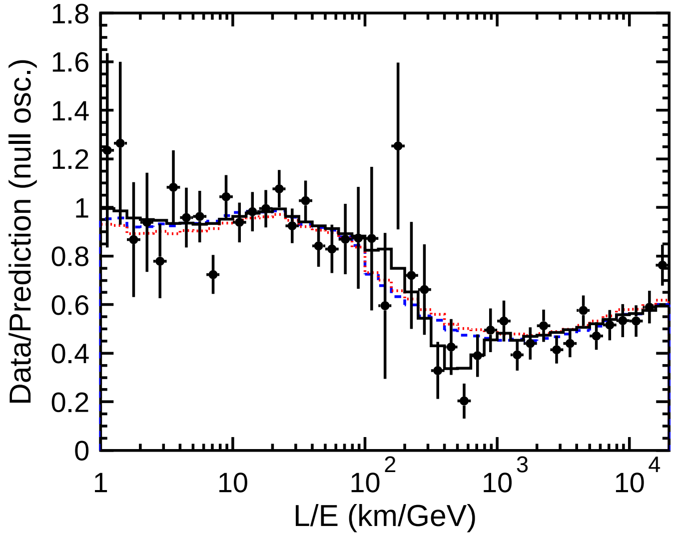
$$P_{ee} = 1 \qquad P_{e\mu} = 0 \qquad P_{\mu\mu} = 1 - \sin^2 2\theta_{\rm atm}\ \sin^2 \frac{\Delta m^2_{\rm atm} L}{4E_\nu}$$

- $\sin^2 2\theta_{\rm atm} = 2 - 2\frac{N_\uparrow}{N_\downarrow} = 1 \pm 0.1$ i.e. $\theta_{\rm atm} \sim 45$
- oscillatations start 'horizontal', $L \sim 1000\,\text{km}$: $\Delta m^2_{\rm atm} \sim \frac{E_\nu}{L} \sim 3\ 10^{-3}\,\text{eV}^2$

$P_{\mu\mu}(E_\nu)$: the anomaly disappears at high energy, as predicted by oscillatons.

$P_{\mu\mu}(L)$: at SK $\sigma_{E_\nu} \sim E_\nu$: **oscillation dip** averaged out: SK sees a hint



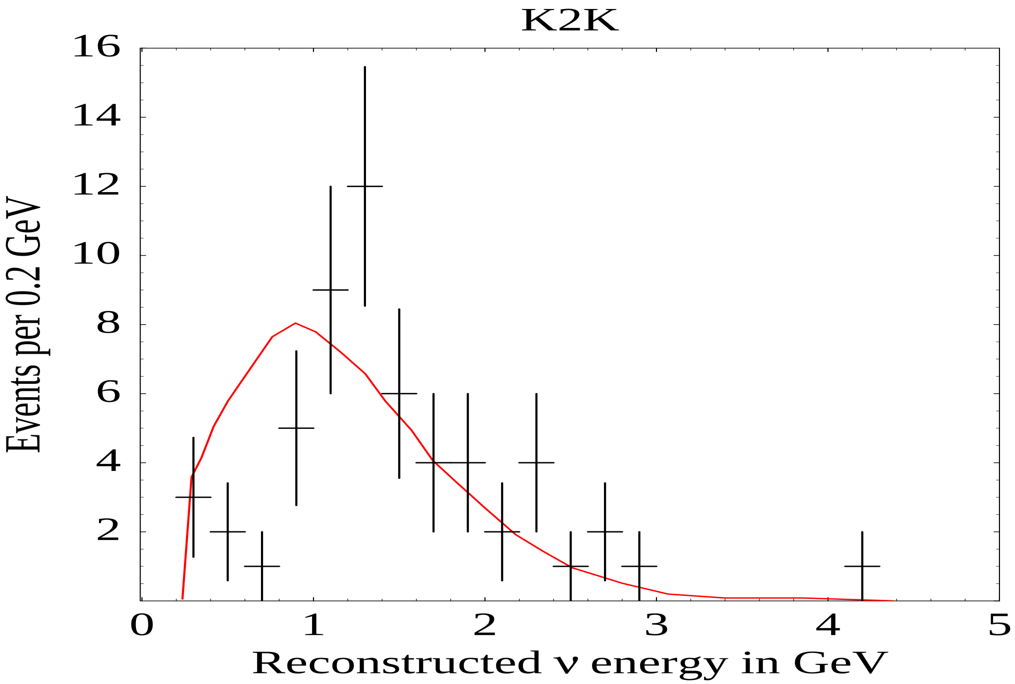


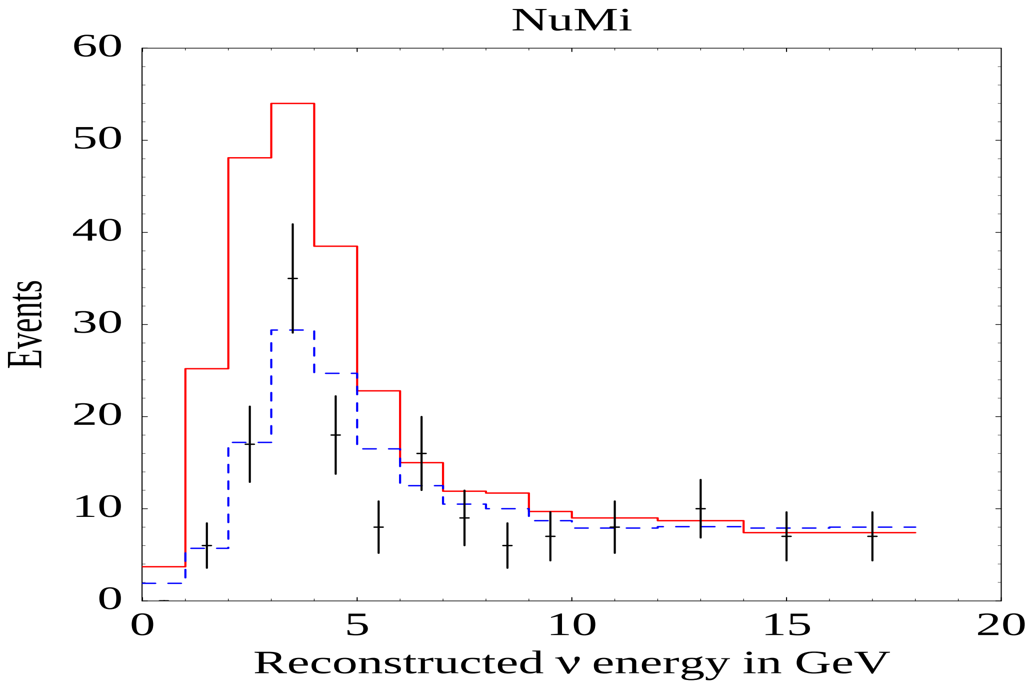
K2K and NuMi

ν_μ beam. Gosplan:

- Energy $E_\nu \sim$ few GeV $\sim m_p$ chosen such that $\Delta\theta_\mu \sim 1$.
- $\boldsymbol{E_\nu}$ **reconstructed** from $E_\mu, \Delta\theta_\mu$ since ν source known.
- Distances $L = 250\,\text{km}, 735\,\text{km}$ chosen such that $\Delta m^2_{\text{atm}} L/E_\nu \sim 1$.

$4\sigma \oplus 6\sigma$ deficit, with (?) spectral distortion: good sensitivity to Δm^2_{atm}.

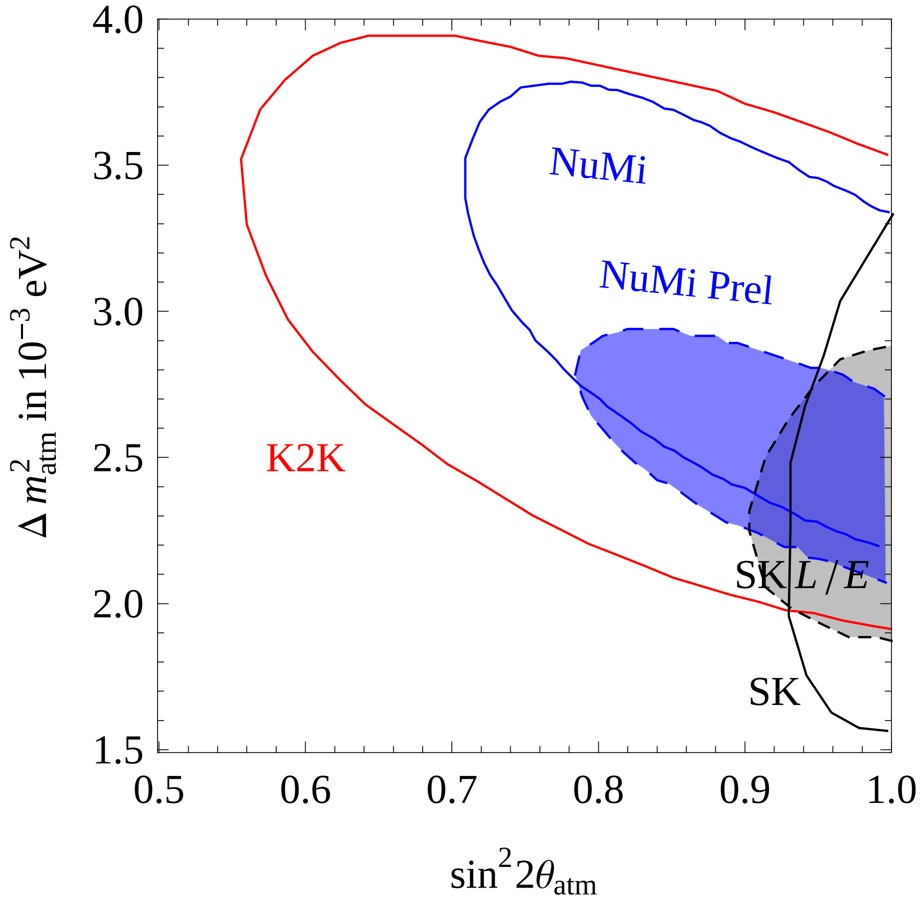




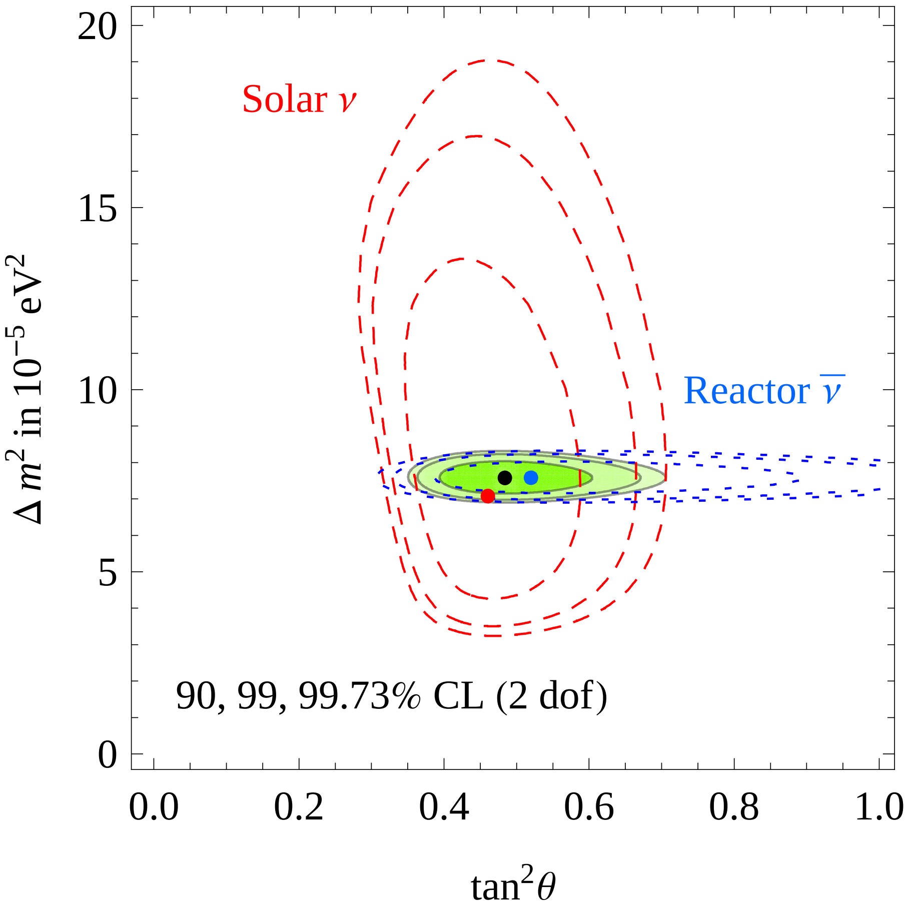
... and CNGS is running at higher energy to see a few τ events from $\nu_\mu \to \nu_\tau$

Global fits

atmospheric



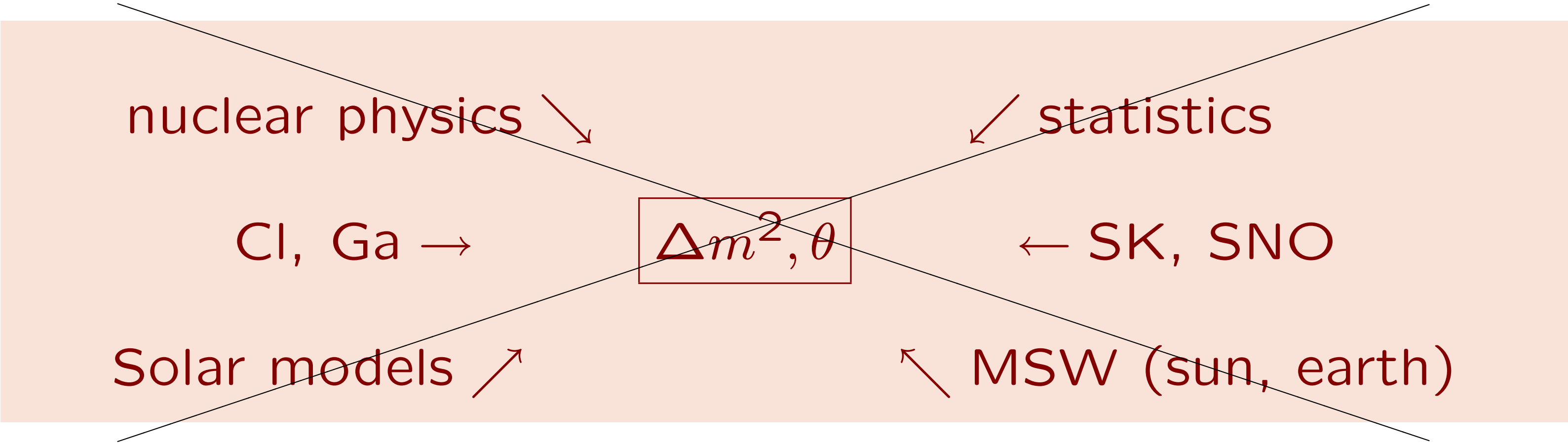
solar



The solar anomaly

The solar ν anomaly

Previously based on global fits of many ingredients:



nuclear physics ↘ ↙ statistics

Cl, Ga → $\boxed{\Delta m^2, \theta}$ ← SK, SNO

Solar models ↗ ↖ MSW (sun, earth)

Today we can choose best and simpler pieces of data

KamLAND confirms the solar anomaly with reactor $\bar{\nu}_e$.

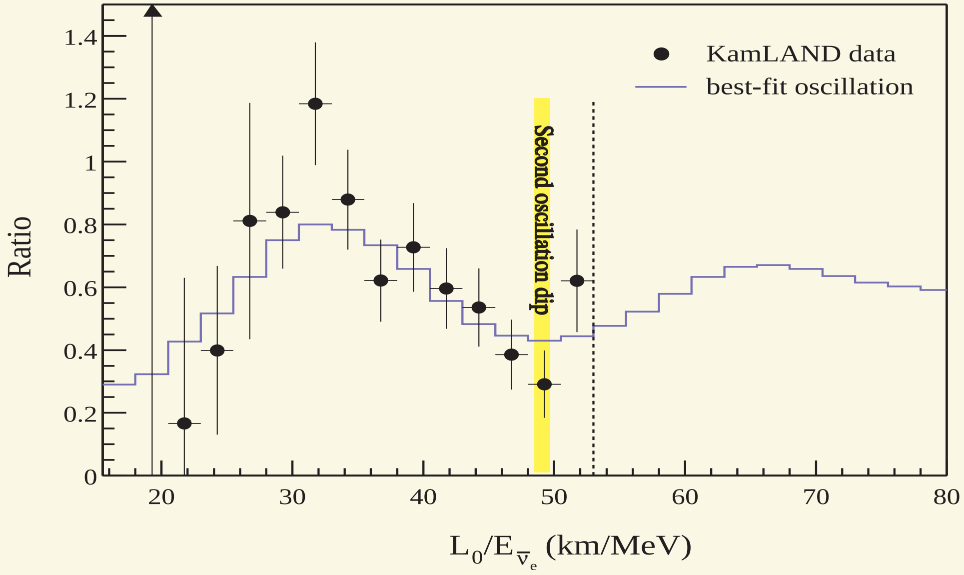
SNO measures ν_e and $\nu_{\mu,\tau}$ solar rates at $E_\nu \sim 10$ MeV.

Simple arguments allow to extract results quantitatively.

Fit without fit

Solar mass splitting

Data dominated by KamLAND:



Theory: II dip of vacuum oscillations:

$$\Delta m^2 = 6\pi \left.\frac{E}{L}\right|_{\text{dip}} = (8.0 \pm 0.3)10^{-5}\,\text{eV}^2$$

Solar mixing angle

Data dominated by SNO:

$$\langle P(\nu_e \to \nu_e)\rangle = 0.357 \pm 0.030.$$

Theory: at largest energies

$$P(\nu_e \to \nu_e) \simeq |\langle \nu_2|\nu_e\rangle|^2 = \sin^2\theta.$$

Small correction due to

$$\nu_e(\text{center of sun}) \neq \nu_2 :$$

$$\langle P(\nu_e \to \nu_e)\rangle \approx 1.15 \sin^2\theta$$

So:

$$\tan^2\theta = 0.45 \pm 0.05$$

Global fits needed to check if all the rest is consistent... and for movies

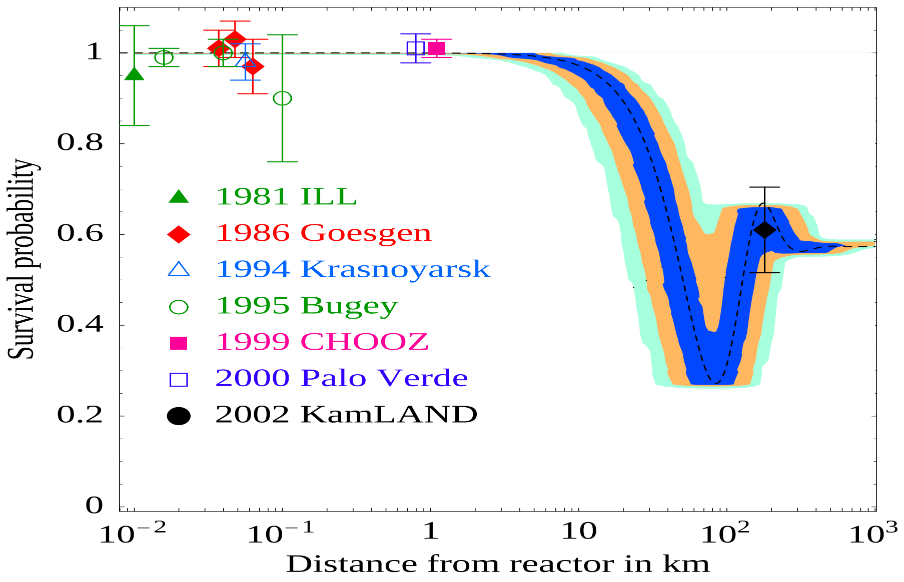
KamLAND 素晴らしい 結果

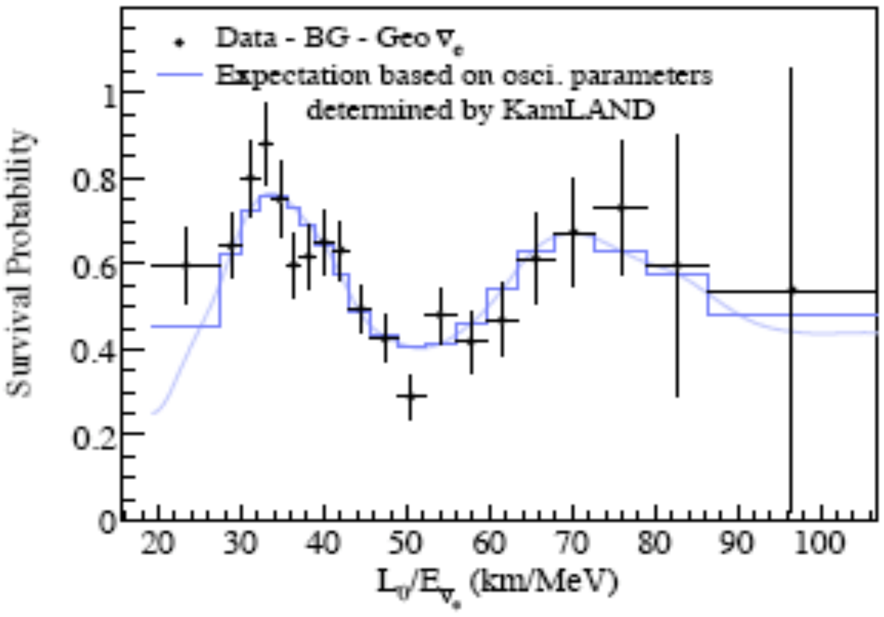
Čerenkov scintillator that detects $\bar{\nu}_e$ from terrestrial (japanese) reactors using $\bar{\nu}_e p \to \bar{e} n$

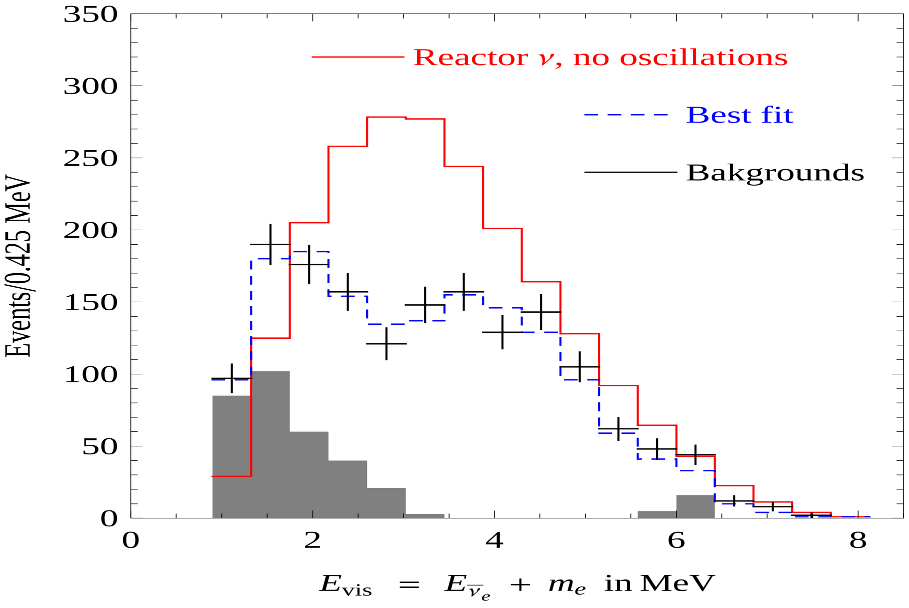
- Delayed $\bar{e}n$ coincidence: $\sim$ no bck (geo$\bar{\nu}_e$ background at $E_{\text{vis}} < 2.6\,\text{MeV}$)
- 1609 events seen, 2179 ± 89 expected
- Most reactors at $L \sim 180$ km.

  $E_{\bar{\nu}} \ll m_p$: $E_{\bar{\nu}} \approx E_e + m_n - m_p$:

  L/E distortion seen at 5σ

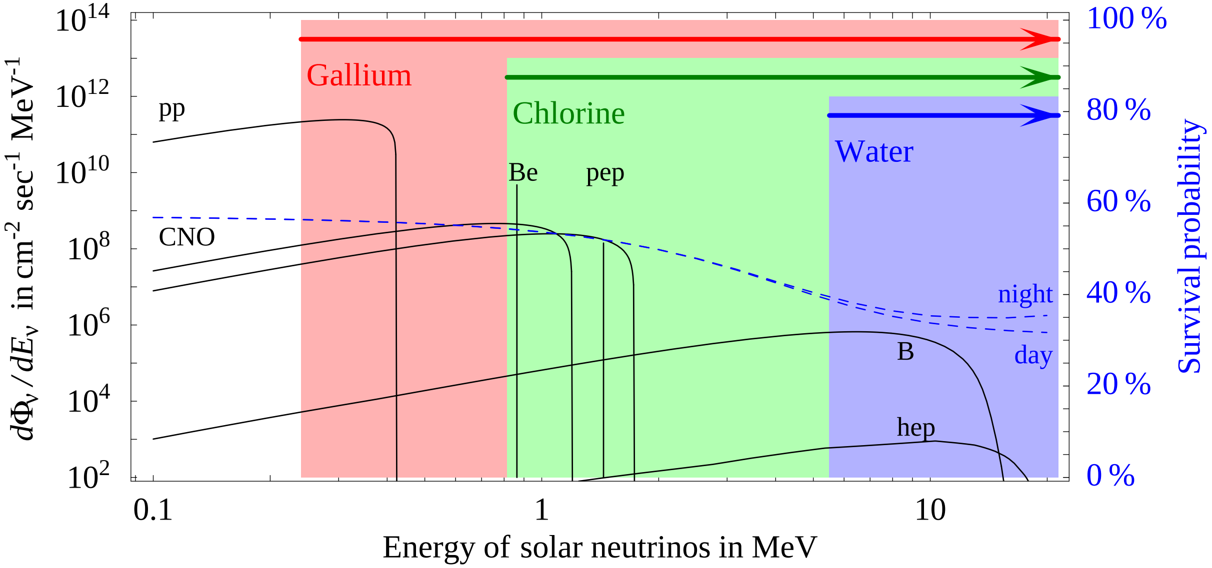


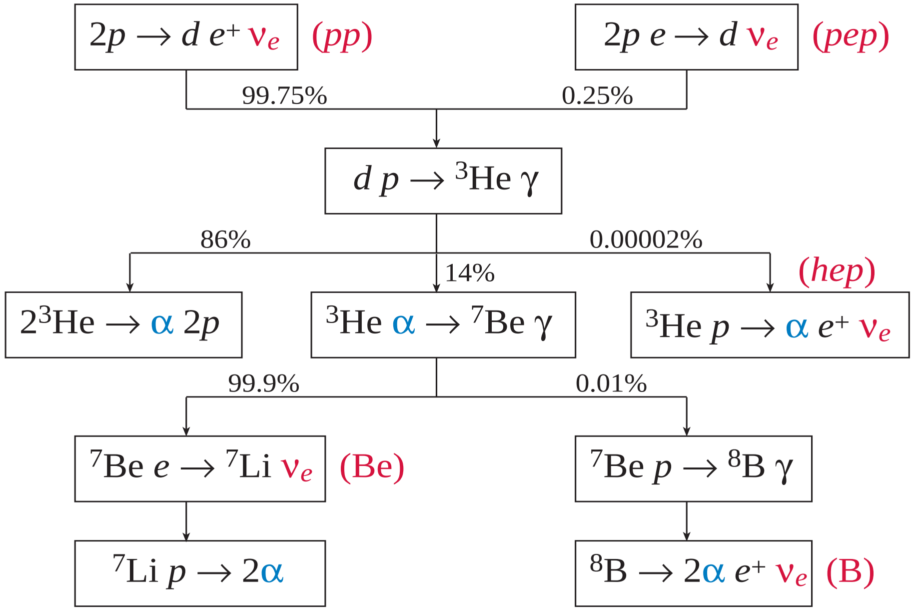




Solar ν fluxes

The sun shines as $4p + 2e \to {}^4\mathrm{He} + 2\nu_e$ ($Q = 26.7\,\mathrm{MeV}$).
Proceeds in steps giving a complex ν spectrum





$2p \to d\, e^+ \nu_e$ (*pp*)

$2p\, e \to d\, \nu_e$ (*pep*)

99.75%

0.25%

$d\, p \to {}^3\mathrm{He}\,\gamma$

86%

14%

0.00002%

$2\,{}^3\mathrm{He} \to \alpha\, 2p$

${}^3\mathrm{He}\,\alpha \to {}^7\mathrm{Be}\,\gamma$

(*hep*) ${}^3\mathrm{He}\, p \to \alpha\, e^+ \nu_e$

99.9%

0.01%

${}^7\mathrm{Be}\, e \to {}^7\mathrm{Li}\,\nu_e$ (Be)

${}^7\mathrm{Be}\, p \to {}^8\mathrm{B}\,\gamma$

${}^7\mathrm{Li}\, p \to 2\alpha$

${}^8\mathrm{B} \to 2\alpha\, e^+ \nu_e$ (B)

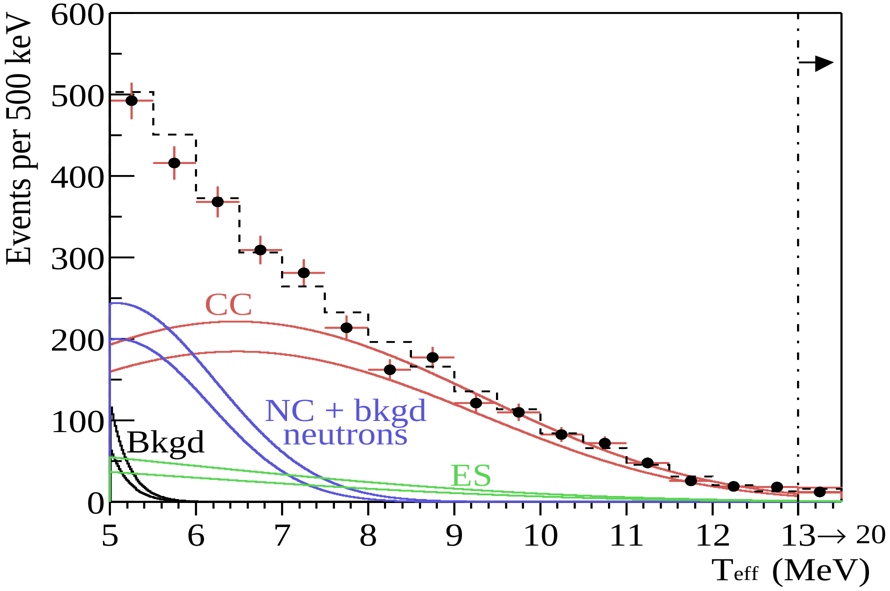
- pp: energy $< 0.42\,\mathrm{MeV} \sim 2m_p - m_d - m_e$: too small for most expreriments.
  Precisly known flux $\Phi \sim 2K_\odot/Q \sim 6.5 \cdot 10^{10}/\mathrm{cm}^2\mathrm{s}$.
  Vacuum oscillations: $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$.

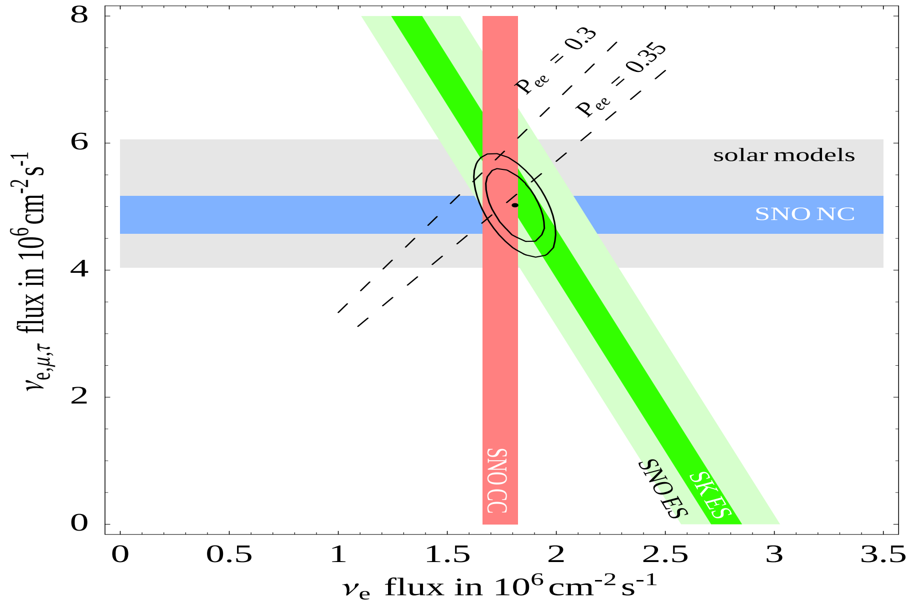
- B: highest energy, small flux predicted to $\pm 20\%$.
  Adiabatic MSW resonance: $P(\nu_e \to \nu_e) = \sin^2\theta$.

SNO

Čerenkov detector similar to SK (smaller, cleaner) with $H_2O \to D_2O$

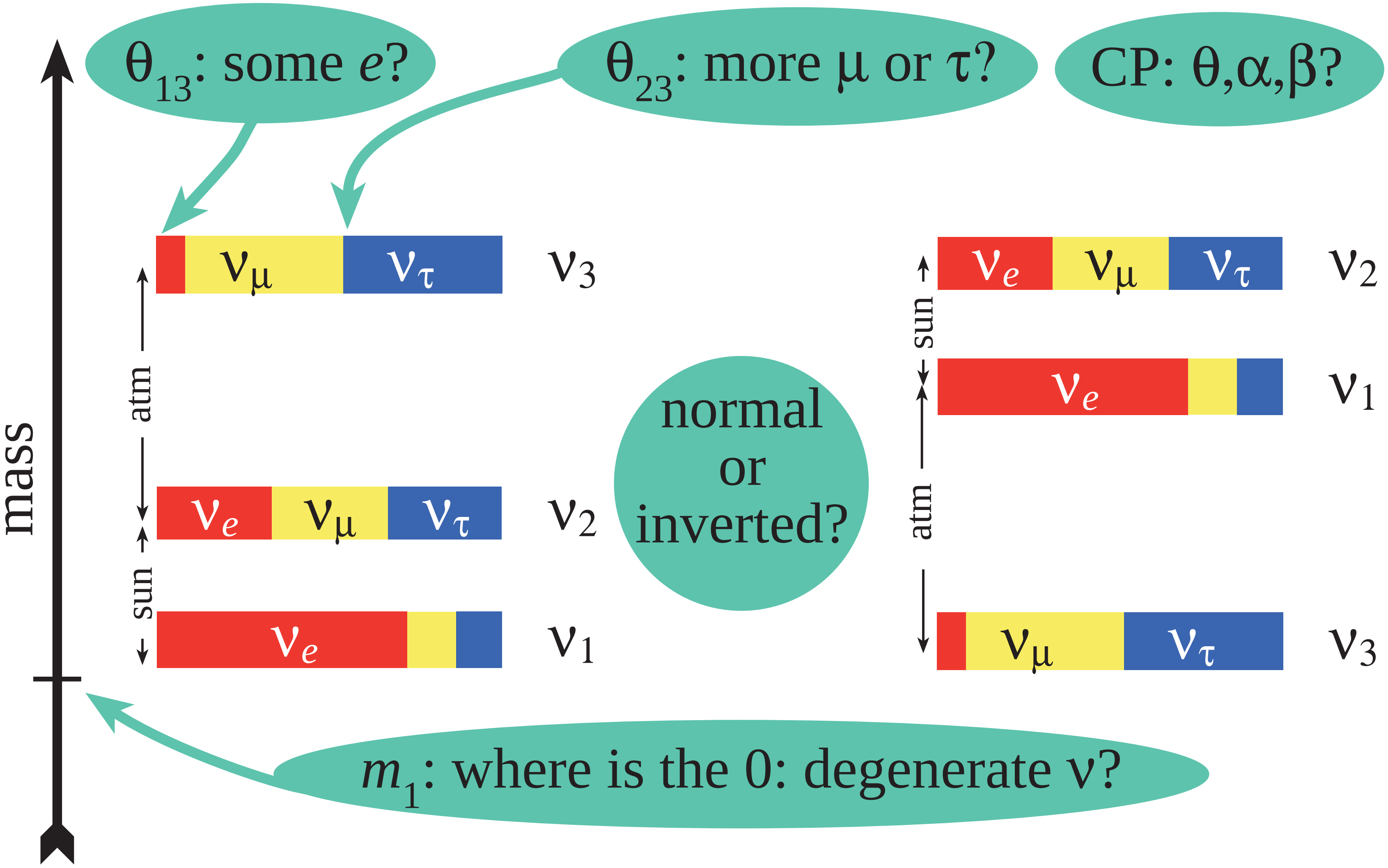
$\mathrm{CC} + \frac{1}{6}\mathrm{NC}:\ \nu e \longrightarrow \nu e \qquad \mathrm{CC}:\ \nu_e d \longrightarrow ppe \qquad \mathrm{NC}:\ \nu d \longrightarrow \nu pn$





More oscillations?

Known unknowns



θ_{13}: some e?

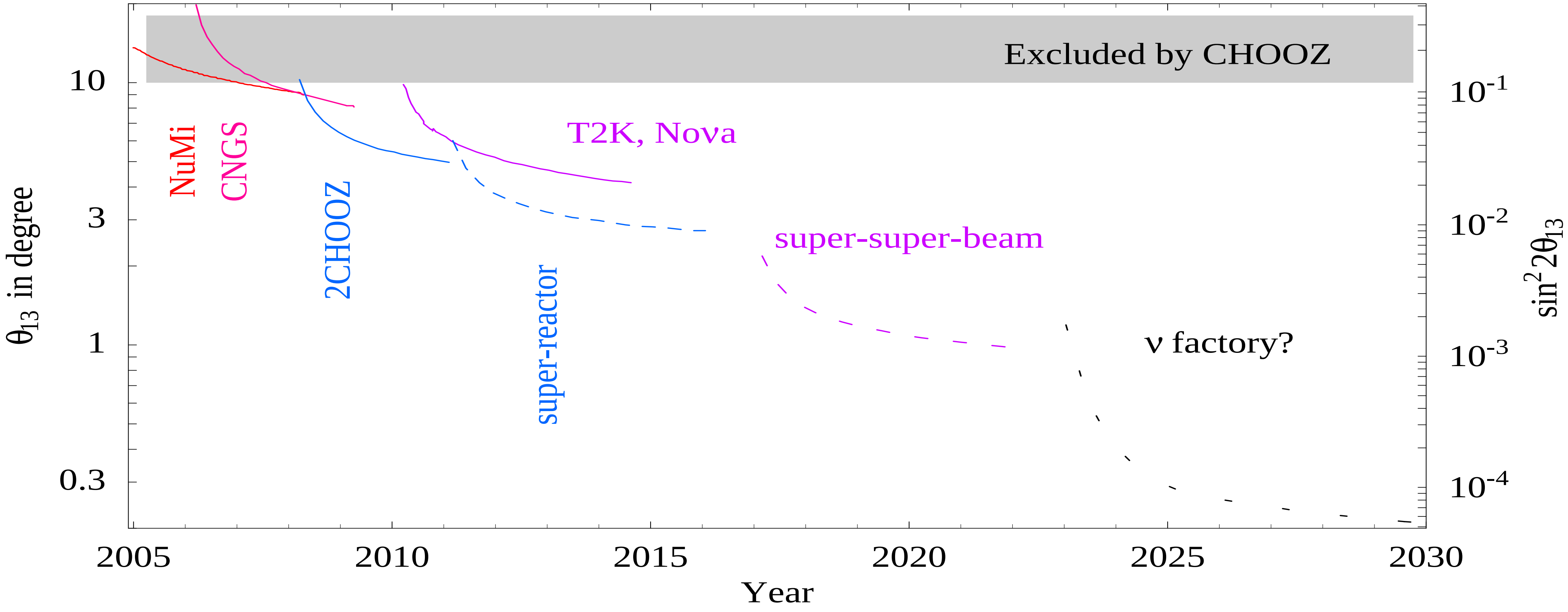
θ_{23}: more μ or τ?

CP: θ,α,β?

normal or inverted?

m_1: where is the 0: degenerate ν?

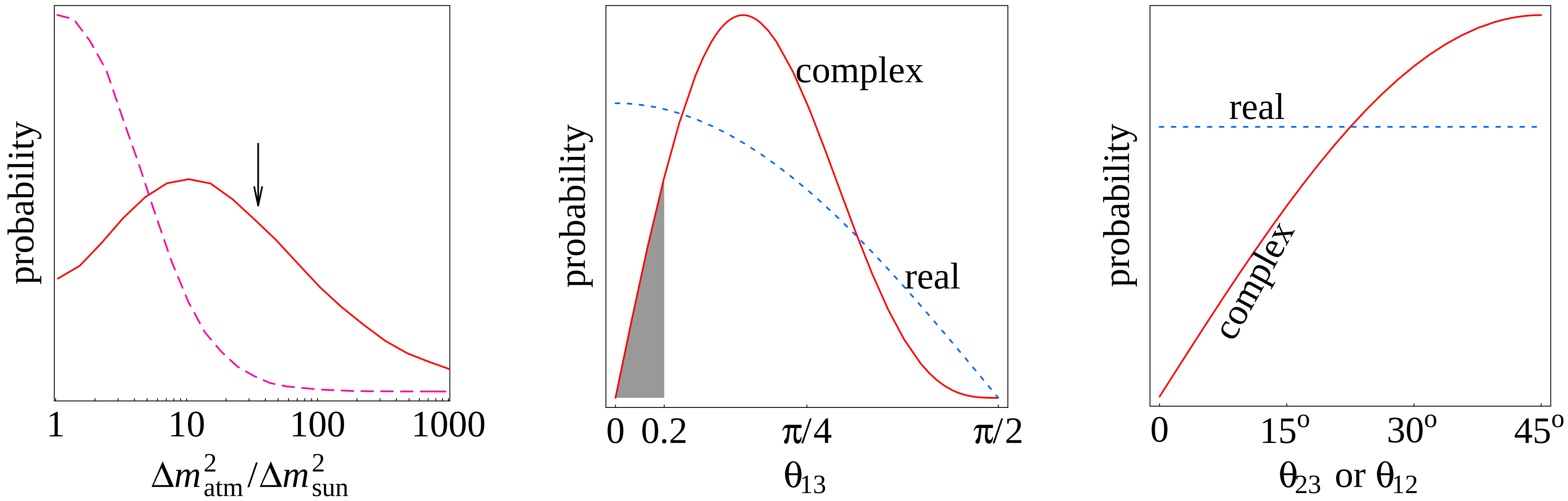
Sensitivity to θ_{13}



Experiment	from	to	baseline	beam	ν energy in GeV	off-axis	start
K2K	KEK	Kamioka	250 km	ν_μ	$\sim (0.5 \div 2)$	0	1999
NuMi	FermiLab	Soudan	735 km	ν_μ	$\sim (2 \div 10)$	0	2005
CNGS	CERN	Gran Sasso	730 km	ν_μ	$\sim (5 \div 30)$	0	2006
T2K	JPARC	Kamioka	295 km	$\overset{(-)}{\nu}_\mu$	$\approx (0.3 \div 1.3)$	2°	2008
NOνA	FermiLab	Ash river	810 km	$\overset{(-)}{\nu}_\mu$	$\approx (1 \div 3)$	0.8°	2010
Supernova	Milky Way	Earth	10 kpc	all	≈ 0.03	–	20??

Predictions for θ_{13}

- If all is random $\mathcal{O}(1)$, $P(\theta_{13}) \propto \theta_{13}^2$ (like latitude, unlike longitude)



- GUT: $\theta_{13} \gtrsim \sqrt{\frac{m_e}{2m_\mu}} \approx 3^\circ$.
- Commonsense: $\theta_{13} \gtrsim \theta_{\text{sun}} \sqrt{\frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}}} \sim 6^\circ$.
- Zerology: $\theta_{13} \simeq \frac{1}{2}\sqrt{\frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}}} \sin 2\theta_{12} \tan\theta_{23} = 4.8^\circ(1 \pm 0.1)$, etc, etc.

Oscillations for 2CHOOZ

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{13} L}{4 E_\nu}$$

Oscillations for K2K, NuMi, CNGS

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \sin^2 \delta$$

$$P(\nu_\mu \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2((1-r)\delta)}{(1-r)^2}$$

atmospheric phase $\delta = \frac{\Delta m^2_{13} L}{4 E_\nu} \sim 1$

Earth matter effects $r \equiv \frac{2\sqrt{2} G_\mathrm{F} N_e E_\nu}{\Delta m^2_{13}} \sim \frac{E_\nu}{10\ \mathrm{GeV}}$

Oscillations for long baseline exps

Compute $P_{ij} = |\exp(-itH_{\text{eff}})|^2_{ij}$ treating Δm^2_{12} and θ_{13} as perturbation

$$\varepsilon \equiv \frac{\Delta m^2_{12}}{\Delta m^2_{13}} \approx \pm 0.04 \qquad\qquad \varepsilon' \equiv \sin 2\theta_{13} \lesssim 0.2.$$

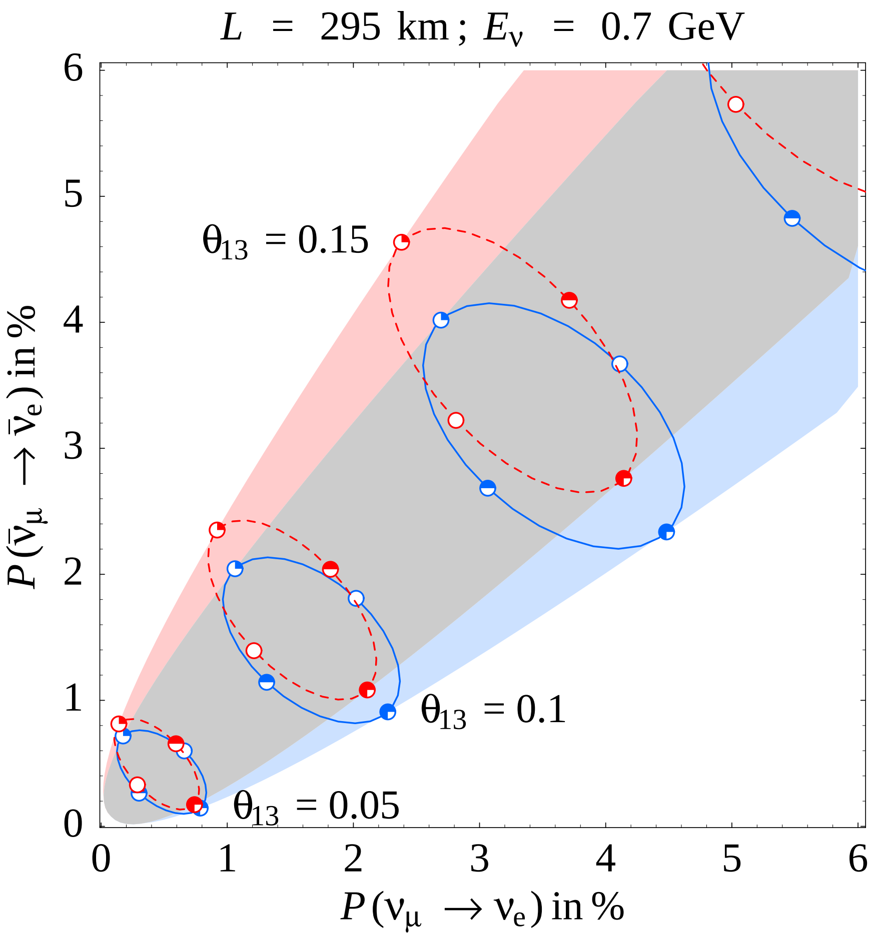
over the large atmospheric $\mu \leftrightarrow \tau$ and earth matter effects.

$$\begin{aligned} P(\nu_\mu \to \nu_e) &\simeq \varepsilon^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(r\delta)}{r^2} + && \text{sun} \\ &+ \varepsilon'^2 \sin^2\theta_{23} \frac{\sin^2((1-r)\delta)}{(1-r)^2} + && \text{atm} \\ &+ \varepsilon\varepsilon' \sin(2\theta_{12}) \sin(2\theta_{23}) \frac{\sin(r\delta)\sin((1-r)\delta)}{r(1-r)} \cos(\delta + \phi) && \text{inteference} \end{aligned}$$

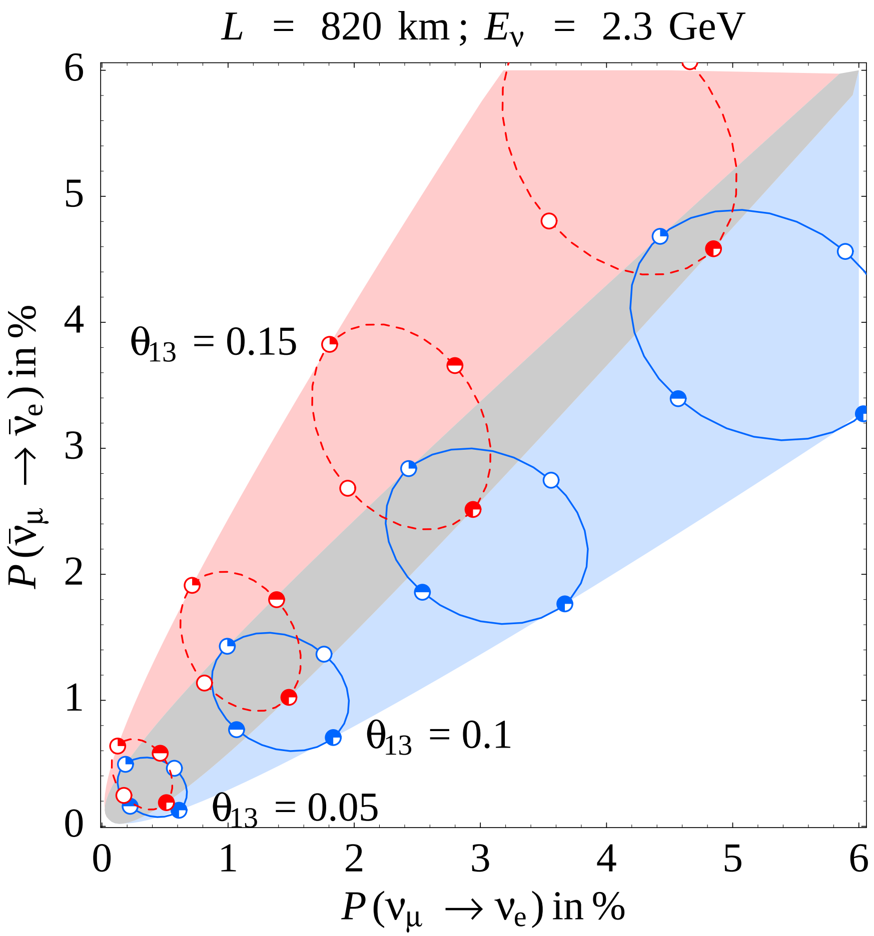
Swap $r \to -r$ for $P(\bar\nu_\mu \to \bar\nu_e)$. Swap $\delta \to -\delta$ for $P(\nu_e \to \nu_\mu)$. Both for $P(\bar\nu_e \to \bar\nu_\mu)$.

- Matter effects give a fake CP asymmetry. (No fake T for constant matter).
- $\sin(r\delta) = 0$ i.e. only atm at the 'magic baseline' $L = \sqrt{2}\pi/G_{\mathrm{F}}N_e \approx 7400\,\text{km}$.

T2K



NOνA



Observables as function of θ_{13} and ϕ for normal and inverted hierarchy.

T2K could kaizen into a MW p-driver and Mton (p-decay, LFV, d_μ, atm, sun..)

Why off-axis?

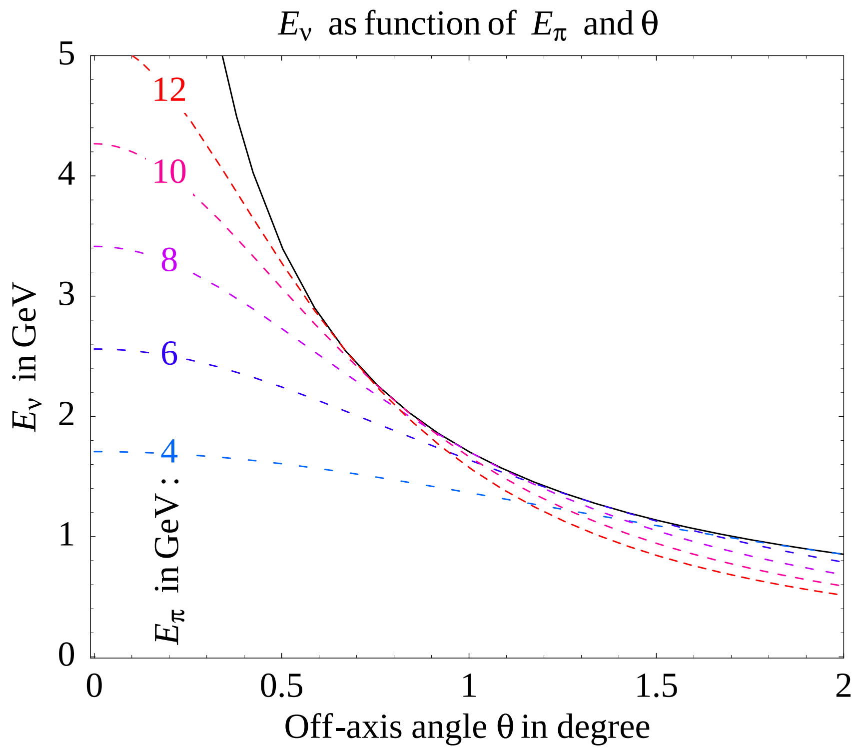
A π at rest decays as $\pi \to \mu\nu_\mu$ so $E_\nu^{\text{CM}} = 30\,\text{MeV}$. Boosting, at $E_\pi \gg m_\pi$:

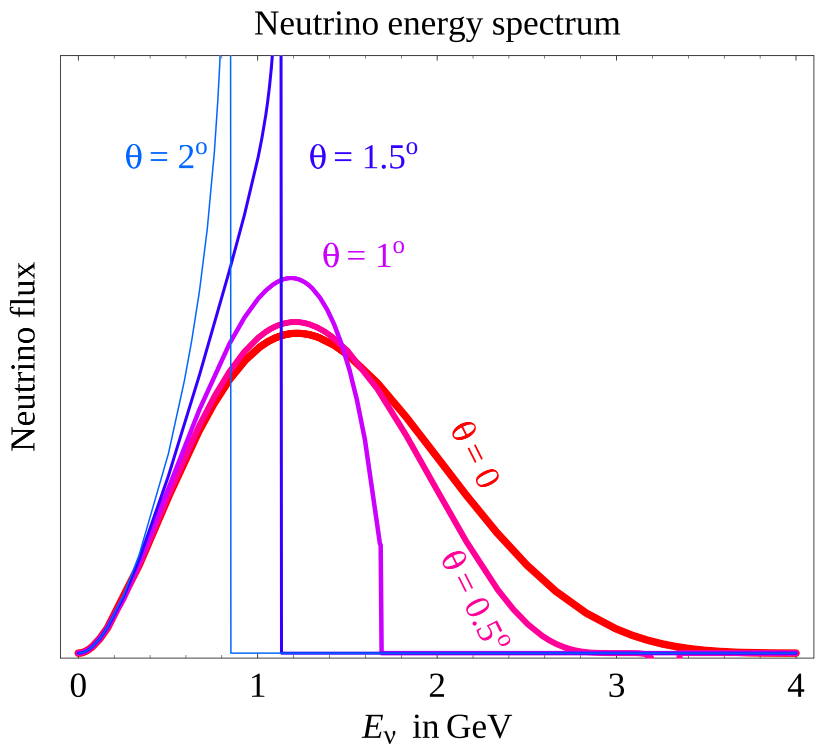
$$E_\nu = \frac{2E_\nu^{\text{CM}} E_\pi m_\pi}{m_\pi^2 + E_\pi^2 \tan^2\theta} \le \frac{30\,\text{MeV}}{\tan\theta} \equiv E_{\text{max}}$$

Indeed

$$E_\nu \simeq p_\nu^{\parallel} = \frac{p_\nu^{\perp}}{\tan\theta} = \frac{E_\nu^{\text{CM}} \sin\theta^{\text{CM}}}{\tan\theta} \le \frac{30\,\text{MeV}}{\tan\theta}$$

So going off-axis by θ one has a roughly monochromatic beam at $E_\nu \approx E_{\text{max}}$.


E_ν as function of E_π and θ


Neutrino energy spectrum

Neutrino masses

How to detect $m_\nu \gtrsim \sqrt{\Delta m^2_{\rm atm}} \approx 0.05$ eV?

Some techniques are almost there; improvements are not easy

	Astrophysics	**Cosmology**	**β decay**	**$0\nu 2\beta$**
Signal	Time delay from supernova	LSS and CMB: reduced $P(k)$	End-point spectrum	Electrons with $E_{ee} = Q$-value
Needs	—	Simple cosmology	—	Majorana
Measures	Δm_ν	$\sum m_\nu$	$(m^\dagger m)^{1/2}_{ee}$	m_{ee}
Today	$< 20\,$eV	$< 0.3\,$eV	$< 2\,$eV	$< 0.4h\,$eV
From	SN1987A	MAP,SDSS,Ly	Mainz, Troitsk	HM,Igex,Cuoric
Implies	$m_\nu < 20\,$eV	$m_\nu \lesssim 0.1\,$eV	$m_\nu \lesssim 2\,$eV	$m_\nu/h \lesssim 1$ eV
Future	eV	$0.03\,$eV	$0.2\,$eV	$0.05\,$eV
If normal	too small	$(51 \div 66)\,$meV	$(4.6 \div 10)\,$meV	$(1.1 \div 4.5)\,$meV
If inverted	too small	$(83 \div 114)\,$meV	$(42 \div 57)\,$meV	$(12 \div 57)\,$meV

Constraints and predictions at 99% C.L.

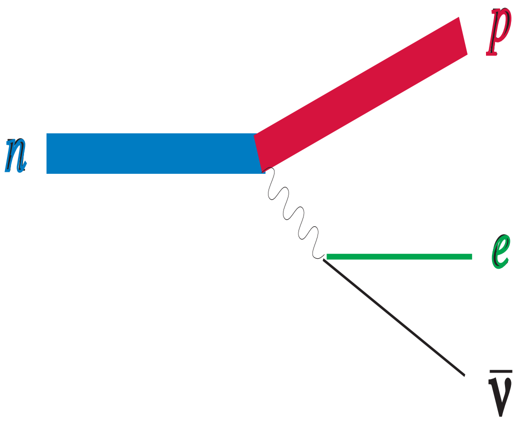
β decay

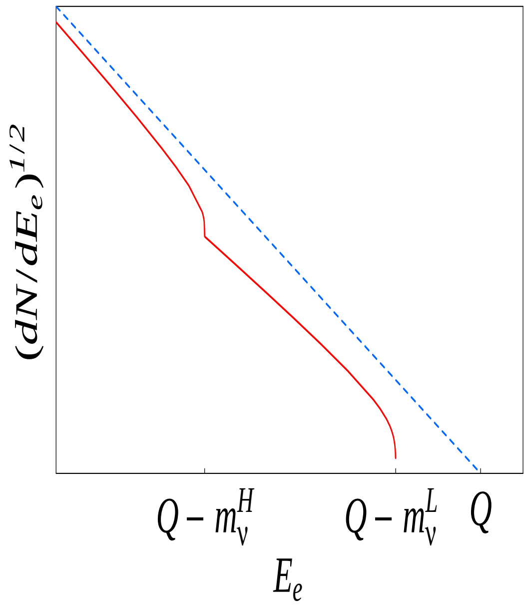
Normal β decay: m_ν affects end-point of

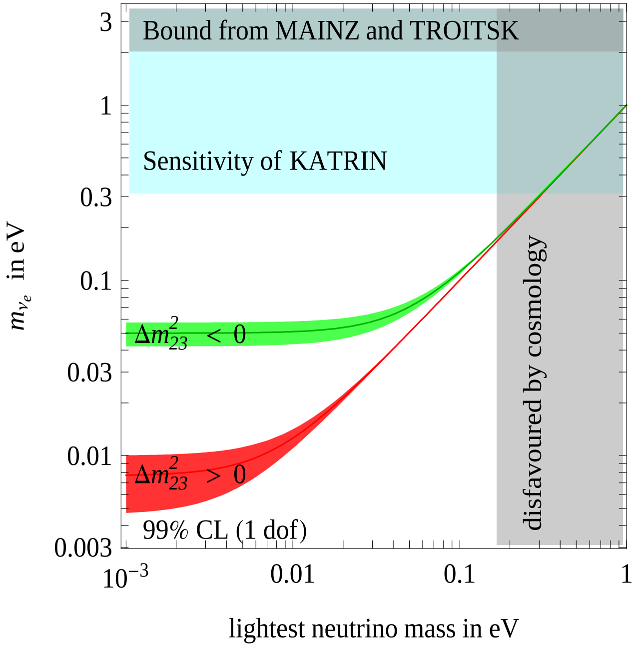
$$^3\text{H} \to {}^3\text{He}\; e\; \bar{\nu}_e \qquad (\text{small } Q = 18.6\,\text{keV})$$

by reducing the phase space $\propto E_\nu p_\nu$:

$$\frac{dN_e}{dE_e} = F \cdot \sum_i |V_{ei}|^2 (Q - E_e)\sqrt{(Q-E_e)^2 - m_i^2} \;\approx\; F(Q-E_e)\sqrt{(Q-E_e)^2 - m_{\nu_e}^2}$$





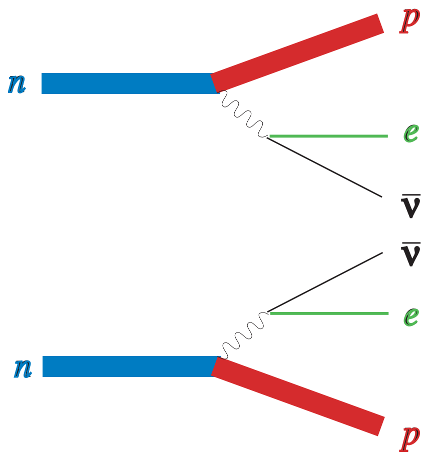


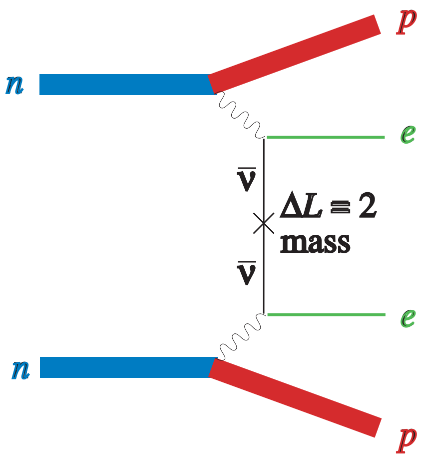
$2\nu2\beta$ and $0\nu2\beta$ decay

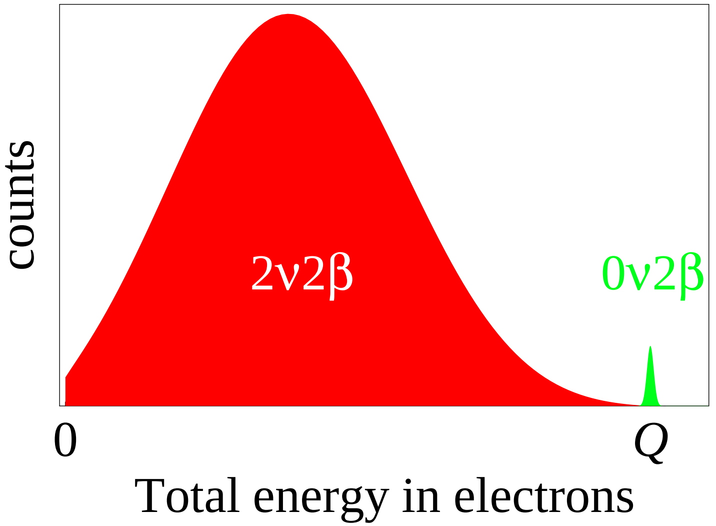
Double β decay: ${}^{76}_{32}$Ge cannot β-decay to ${}^{76}_{33}$As that is heavier, so it $\beta\beta$ decays

$$\tau({}^{76}_{32}\text{Ge} \to {}^{76}_{34}\text{Se}\ e\ e\ \bar\nu_e\ \bar\nu_e) \sim 10^{21}\,\text{yr} \qquad \text{with } Q = 2038.6\,\text{keV}$$

Neutrino-less double β decay: rate $= |m_{ee}|^2 \times$ uncertain nuclear physics.

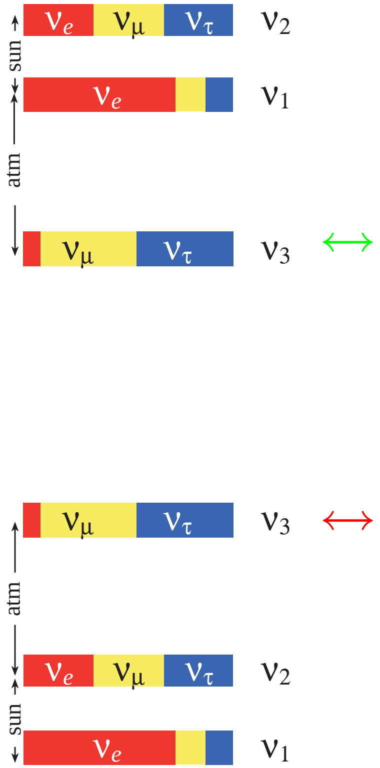


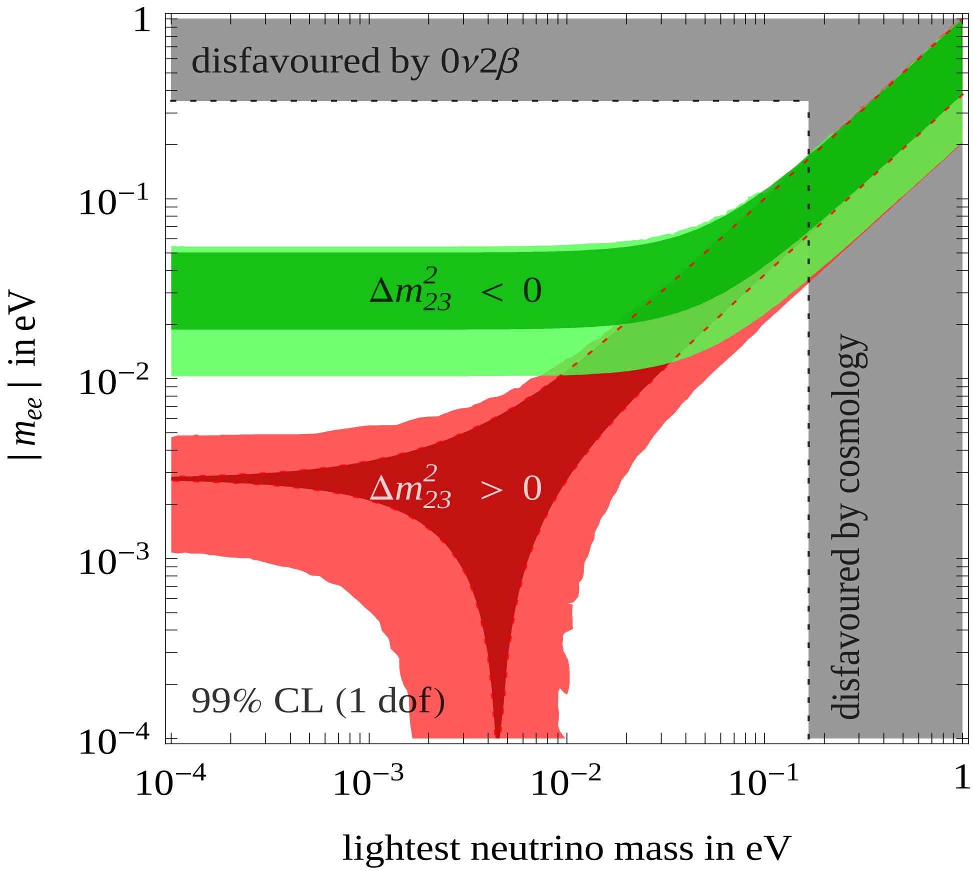




Predictions for $0\nu 2\beta$

$$|m_{ee}| = |\sum_i V_{ei}^2\, m_i| = |\cos^2\theta_{13}(m_1\cos^2\theta_{12} + m_2 e^{i\alpha}\sin^2\theta_{12}) + m_3 e^{i\beta}\sin^2\theta_{13}|$$





The $|m_{ee}|$ range restricts to the darker regions if we assume present best-fit values of $\Delta m^2, \theta$ with zero errors ($\theta_{13} = 0$).

Future $0\nu 2\beta$ experiments should test degenerate and inverted neutrinos.

Cosmology

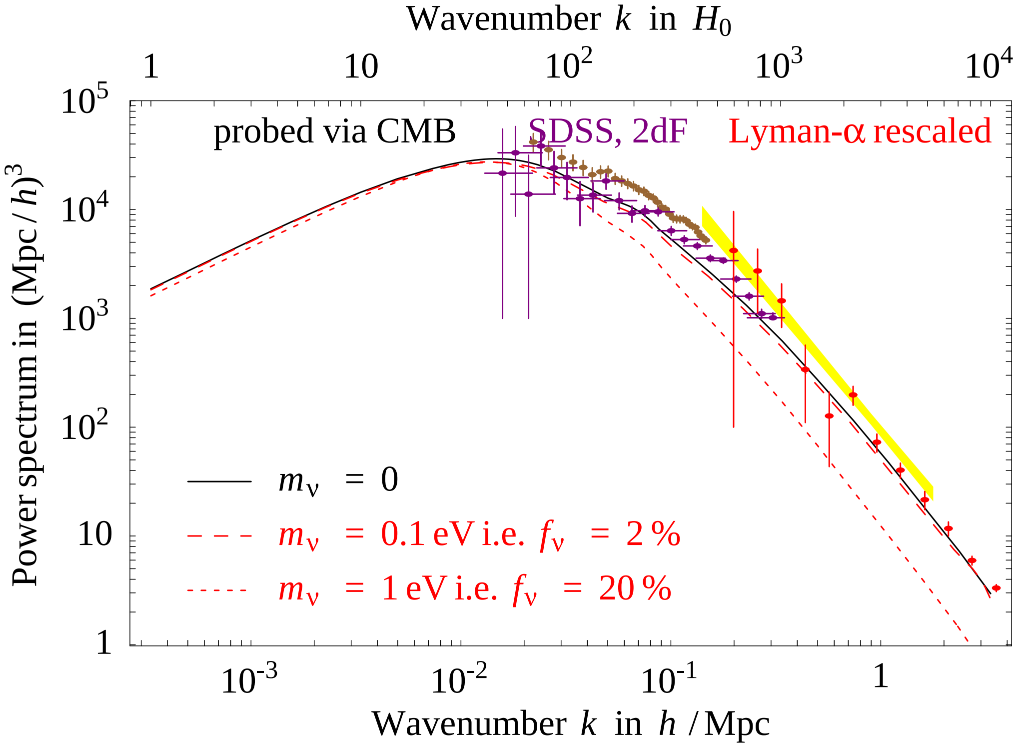
Neutrinos suppress clustering in way which depends on m_ν because:

- Heavier neutrinos contribute more
- Lighter neutrinos travel more.

Growth of $\delta_{\rm DM} \equiv \delta\rho_{\rm DM}/\rho_{\rm DM}$ during matter domination: Newton equation

$$\ddot\delta_{\rm DM} + 2H\dot\delta_{\rm DM} = 4\pi G_{\rm N}\,\delta\rho_{\rm tot} \qquad \delta\rho_{\rm tot} \simeq \delta\rho_{\rm DM} = (1-f)\rho_{\rm tot}\cdot\delta_{\rm DM}$$

neutrino $f$raction $f = \rho_\nu/\rho_{\rm DM} = (\sum m_\nu/94\,{\rm eV})/0.23$. Solution:



$$\frac{\delta_{\rm DM}({\rm now})}{\delta_{\rm DM}(T\approx m_\nu)} = \left(\frac{m_\nu}{3\,{\rm K}^\circ}\right)^p \qquad p = 1-\frac{3}{5}f$$

Power spectrum:

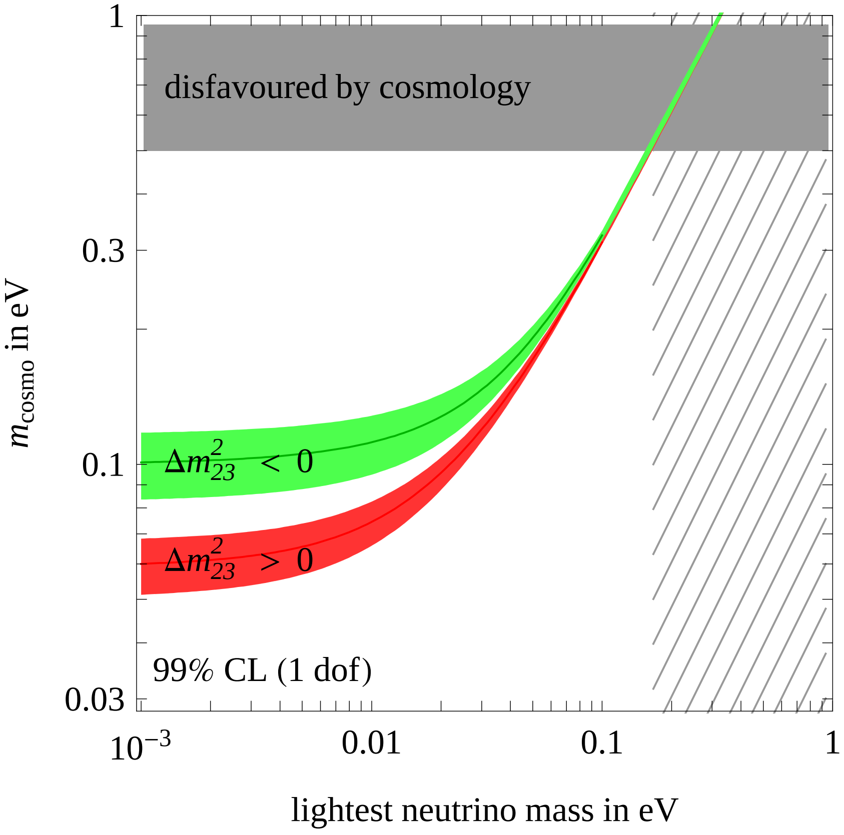
$$\frac{P(m_\nu,k)}{P(0,k)} \approx \begin{cases} 1 & k \lesssim k_{\rm NR} \\ (k_{\rm NR}/k)^{4-4p} & k_{\rm NR} \lesssim k \lesssim k_0 \\ (k_{\rm NR}/k_0)^{4-4p} & k \gtrsim k_0 \end{cases}$$

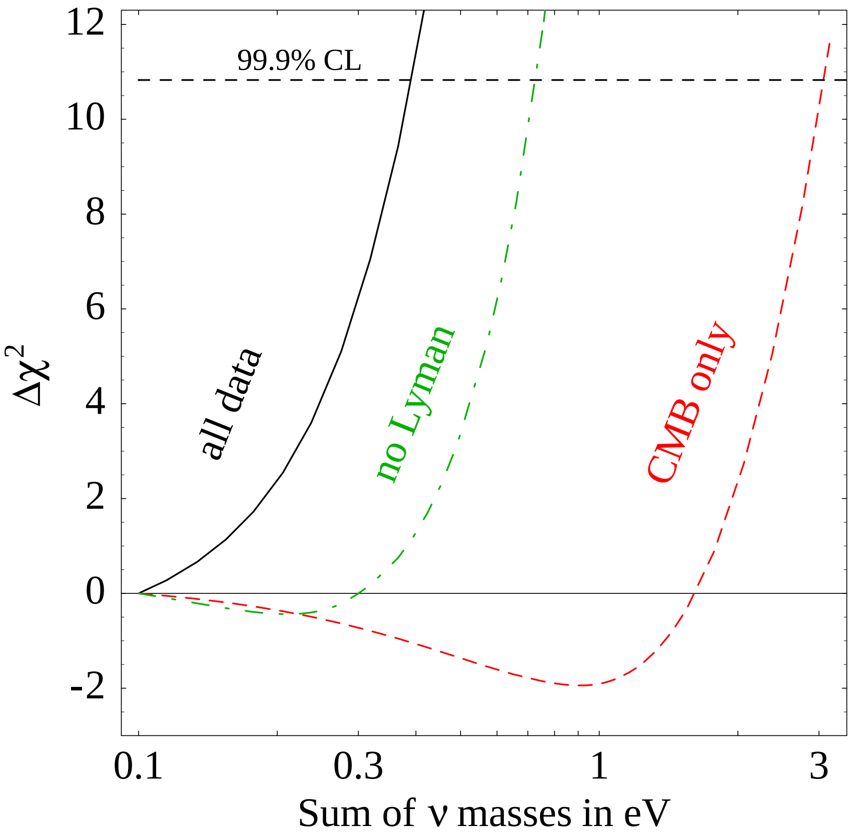
$$k_{\rm NR} = k_{\rm Jeans}(T = m_\nu) \approx 60 H_0\sqrt{m_\nu/\,{\rm eV}}$$

$$k_0 = k_{\rm Jeans}(T = 3\,{\rm K}^\circ) \approx 5000 H_0\,(m_\nu/\,{\rm eV})$$

Present status

CMB demands $N_\nu = 4.4 \pm 1.5$ freely streaming neutrinos, with mass $m_\nu \lesssim 0.5\,\text{eV}$
Large scale structures suggest $m_\nu \lesssim 0.2\,\text{eV}$ (Lyman-α? bias? Primordial tilt?)





Future: **weak lensing** (of CMB or of 10^7 galaxies) should allow to reach $0.05\,\text{eV}$, to directly measure δ_{DM} and its growth. Precise and safe!

Testing origin of ν masses

Behind neutrinos

Surely we saw violation of **lepton flavour** (absent in SM),
likely due to **oscillations** induced by **neutrino masses** (absent in SM),
presumably of **Majorana** type ($\Delta L = 2$: $\mathscr{L} = \mathscr{L}_{\rm SM} + (LH)^2/\Lambda_L$),
maybe induced by new physics **around 10^{14} GeV** (see-saw?)...

first manifestation of a new scale in nature, $\Lambda_L \sim 10^{14}$ GeV?

History: operators suppressed by the EW scale $\mathscr{L} = \mathscr{L}_{\rm QED} + (\bar{e}\nu)(\bar{p}n)/\Lambda^2_{\rm EW}$ first seen as β radioactivity by Rutherford in 1896. The SM, guessed in 1968, predicts operators in terms of 2 parameters, directly probed now at LEP, LHC.

Back to neutrinos: in next few × 10 yrs the 1st mostly experimental stage might be completed, seeing all 9 $(L_i H)(L_j H)$ operators accessible at low energy.

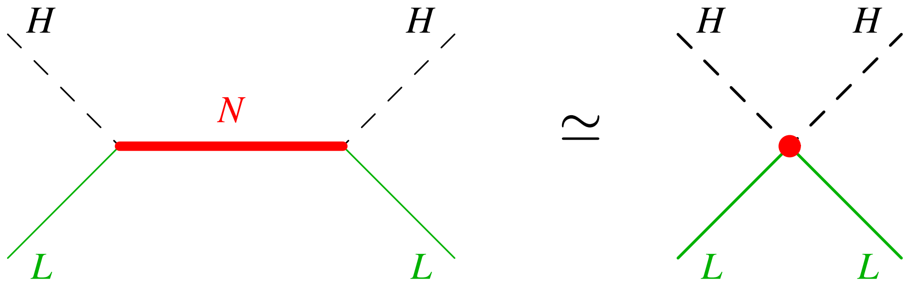
See-saw 'predicts' 9 Majorana ν parameters in terms of 18 parameters. bad
The physics behind m_ν seems either too heavy or too weakly coupled. worse
Leptogenesis or $\mu \to e\gamma$ in SUSY-see-saw might give extra hints? hope...

See-saw

Add neutral ‘right-handed neutrinos’ N. The generic Lagrangian becomes

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \bar{N}\not{\partial}N + M\frac{N^2}{2} + \lambda\, H L N$$

Exchange of heavy N gives the dimension-5 neutrino mass operator:



$$= \frac{\lambda^2}{M}\frac{(LH)^2}{2} \to \frac{(\lambda v)^2}{M}\frac{\nu^2}{2}$$

More explicit: the neutrino mass matrix is

$$\begin{array}{c c} & \begin{array}{cc} \nu & N \end{array} \\ \begin{array}{c} \nu \\ N \end{array} & \begin{pmatrix} 0 & \lambda v \\ \lambda v & M \end{pmatrix} \end{array}$$

for $M \gg \lambda v$ the eigenvalues are $\simeq M$ and $m_\nu \simeq (\lambda v)^2/M$.

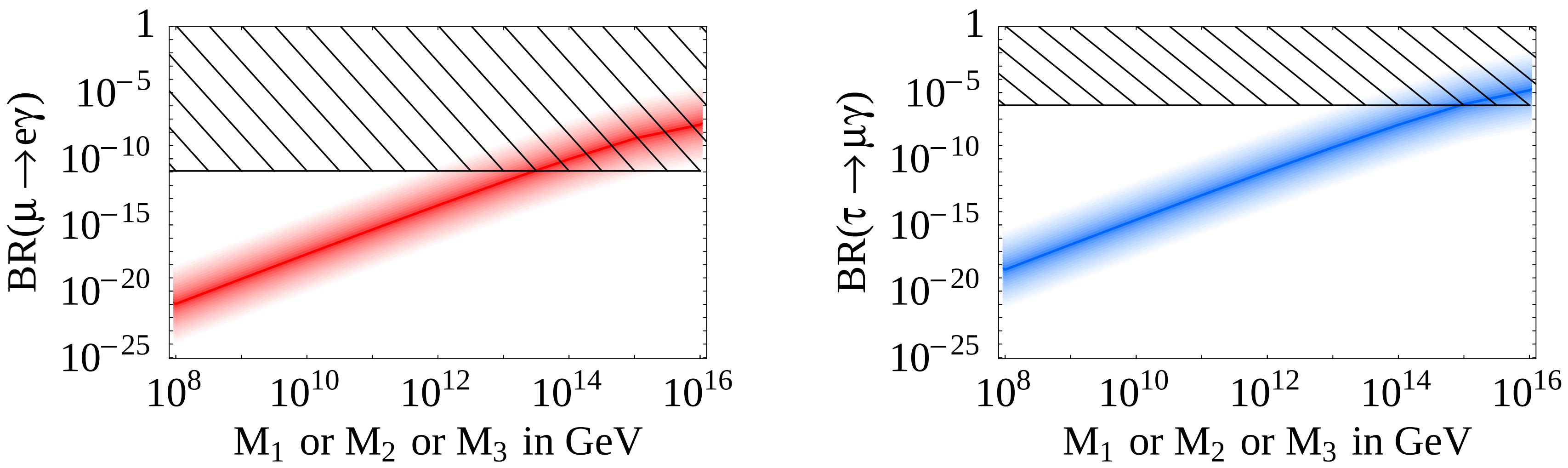
[P. Minkowski, Phys. Lett. B 67 (1977) 421]

$\mu \to e\gamma$ from SUSY λ_ν

In the SM BR($\mu \to e\gamma$) $\sim (m_\mu/\Lambda_L)^2 \sim 10^{-40}$. **In SUSY see-saw quantum effects imprint LFV in slepton masses.** Starting from universal m_0^2 at $M_{\rm GUT}$

$$\boldsymbol{m}^2_{\tilde{L}} = m_0^2 \mathbb{1} - \frac{3m_0^2}{(4\pi)^2}\boldsymbol{\lambda}_\nu^\dagger \ln(\frac{M_{\rm GUT}^2}{\boldsymbol{M}\boldsymbol{M}^\dagger})\boldsymbol{\lambda}_\nu + \cdots$$

Even assuming large ν mixings also in λ_ν one gets loose predictions



because BR($\mu \to e\gamma$) $\sim 10^{-8}\lambda_\nu^4$ while $m_\nu = \lambda_\nu^2 v^2/M$ is measured.

Baryogenesis

The universe contains γ, e, baryons (p, Helium, Deuterium, ...), likely ν.
We understand why $n_e = n_p$, why $n_{^4\mathrm{He}}/n_p \approx 0.25$, why $n_\mathrm{D}/n_p \approx 3\ 10^{-5}$, ...
We do not understand $n_B/n_\gamma \sim 6\ 10^{-10}$ i.e. why at $T \approx m_p$ we survived as

$$1000000001\ \frac{\text{protons}}{\text{pico-m}^3} \qquad - \qquad 1000000000\ \frac{\text{anti-protons}}{\text{pico-m}^3}$$

Might be the initial condition, but suspiciously small or large (in inflation).

Can a $p/\bar{p}$ asymmetry can be generated dynamically from nothing?

Yes, if 3 trivial Sacharov conditions are satisfied
(his big achivement was realizing that it is an interesting question).

1. Baryon number B is violated
2. C and CP are violated
   (otherwise p and $\bar{p}$ behave in the same way)
3. At some epoch the universe went out of equilibrium
   (CPT implies $m_p = m_{\bar{p}}$ so that in thermal equilibrium $n_p = n_{\bar{p}}$)

Leptogenesis

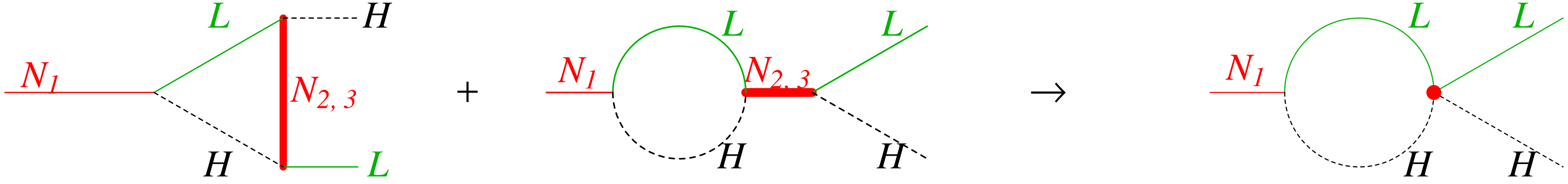
The trivial ν_R produce not only m_ν but also n_B.

See-saw with ν_R: $N_{1,2,3}$ with Yukawa $\lambda_{1,2,3}$ and masses $M_1 < M_2 < M_3$.
$m_1 < m_2 < m_3$: ν_L masses. $\tilde{m}_i \equiv \lambda_i^2 v^2 / M_i =$ 'N_i contribution to ν_L masses'.
Maybe $\tilde{m}_1 = m_{\text{atm}}$ or $\gtrsim m_{\text{sun}}$ or $< m_{\text{sun}}$ or anywhere between 0 and ∞.

$N_1 \to HL$ decays violate CP (ε) and proceed out of equilibrium (η) generating

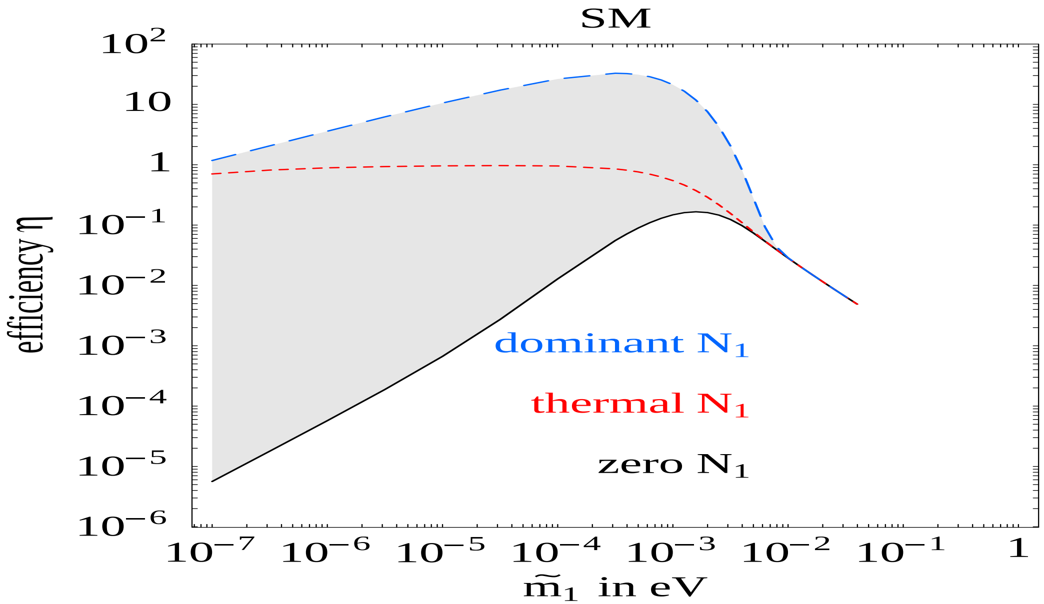
$$(6.15 \pm 0.25)\ 10^{-10} = \frac{n_B}{n_\gamma} \approx \frac{\varepsilon\eta}{100}$$

$$\varepsilon \simeq \frac{3}{16\pi}\frac{\tilde{m}_{2,3} M_1}{v^2}\sin\delta = 10^{-6}\frac{\tilde{m}_{2,3}}{0.05\,\text{eV}}\frac{M_1}{10^{10}\,\text{GeV}}\sin\delta \qquad M_{2,3} \gg M_1$$



$$\eta \text{ related to } \frac{H}{\Gamma_N} \sim \frac{m^*}{\tilde{m}_1} \text{ where } m^* \equiv \frac{256\sqrt{g_*}v^2}{3M_{\text{Pl}}} = 2.2\ 10^{-3}\,\text{eV}$$

Leptogenesis



Result: ‘optimal’ at $M_1 \sim 10^{10}$ GeV (gravitino over-production in SUSY?)

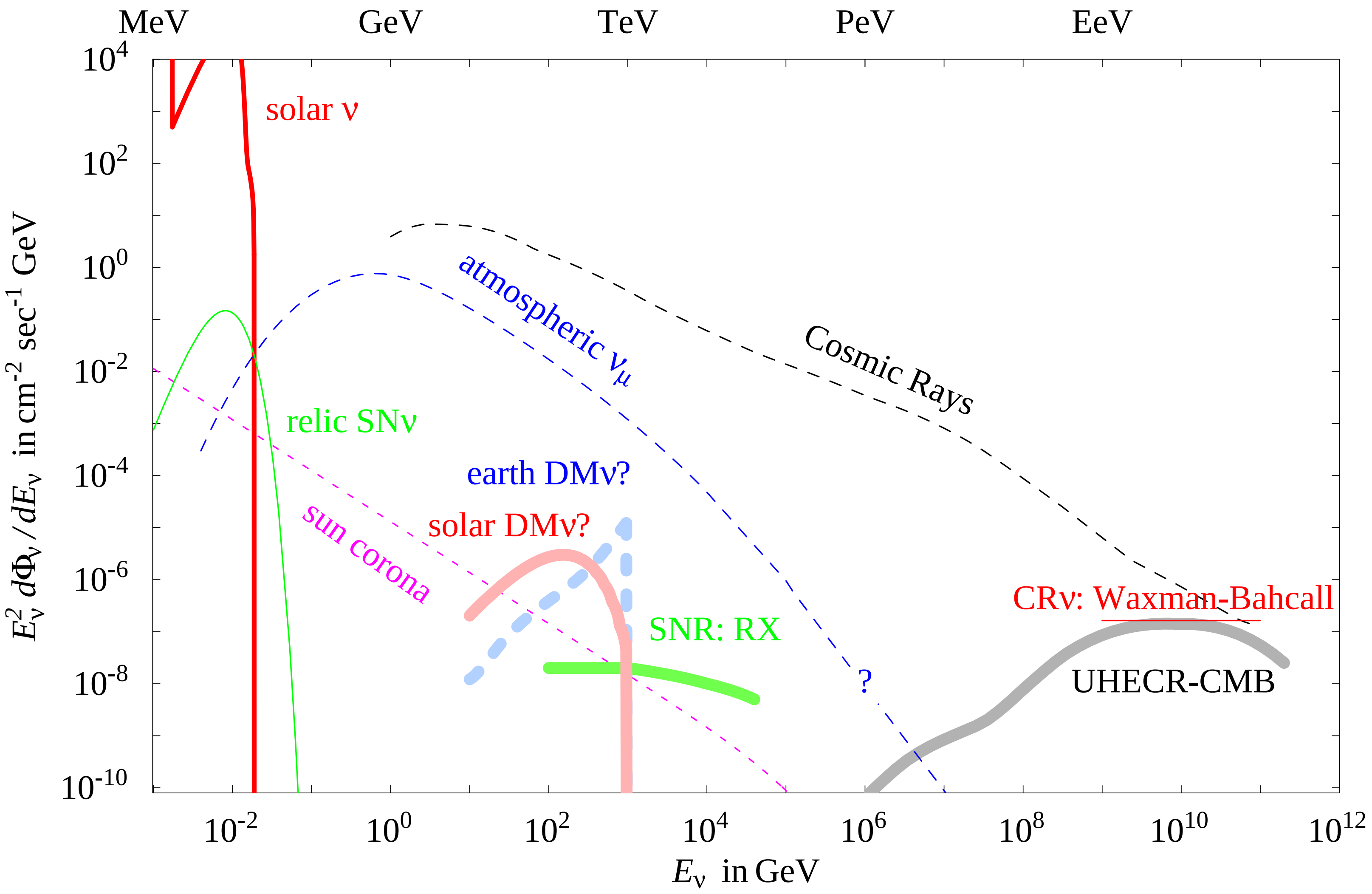
But no real bound or prediction. Not even in models with a single CP phase.

Too many flavour parameters. Hard to proceed without understanding it.

UHE

Astrophysical ν sources

Plausible optimistic predictions, expectations, guesses, prejudices



Present bounds not shown are (of course!) somewhat above all unseen sources.

Measuring $\sigma_{\nu N}$ at $\sqrt{s} = 1\div 100$ TeV

If UHEν are seen, their zenith-angle distribution tells σ

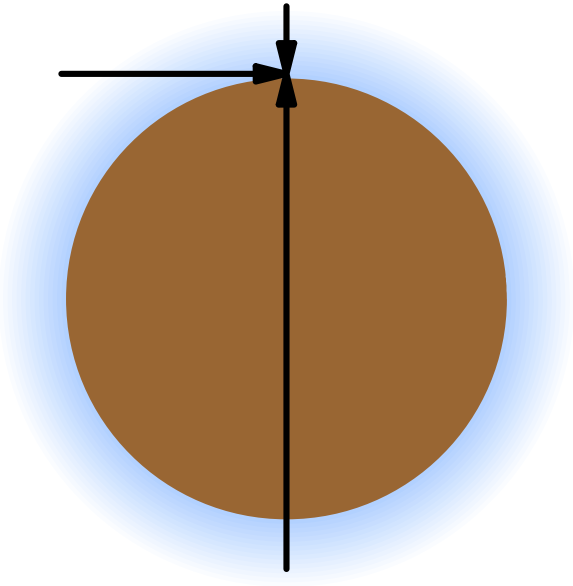
Atmospheric ν guarantee $\sqrt{s} \lesssim$ TeV.

The thickness of the Earth is:

$$L_\downarrow = 0.01033\,\text{kmwe} = 1\,\text{Pascal}$$
$$L_\rightarrow = 0.36\,\text{kmwe}$$
$$L_\uparrow = 1.1\ 10^5\,\text{kmwe}$$

Compare with the interaction length

$$L = 1.7\ 10^4\,\text{kmwe}\frac{\text{nb}}{\sigma}$$



- CR interact in the upper atmosphere, so they have hadronic $\sigma \approx$ barn
- UHE neutrinos have $\sigma \approx 10\,\text{nb}$ in the SM, so $L_\rightarrow \ll L \ll L_\uparrow$ and

$$N_\downarrow \ll N_\rightarrow \propto \sigma \qquad N_\uparrow \propto 1/\sigma$$

Neutrino cross sections at $E_\nu \gg$ 10 TeV

The SM predicts, at transferred momentum $t \sim m_N E_\nu \gg M^2_{W,Z}$:

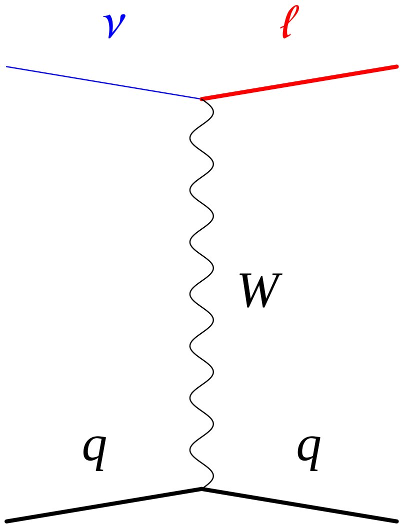
$$\sigma_{\text{CC}}(\nu N) \simeq \sigma_{\text{CC}}(\bar\nu N) \approx 28\,\text{nb}\,(\frac{E_\nu}{10^{10}\,\text{GeV}})^{0.40}$$

Why σ grows with E_ν, while unitarity tells that it should decrease?

- t-channel W exchange gives a Coulomb-like force, $e^{-M_W r}/r$, and thus a constant partonic

$$\hat\sigma_{\text{CC}}(\nu q) \sim g_2^4/32\pi M_W^2.$$

- Multiply $\hat\sigma$ by the number N of partons with $t \gg M^2_{W,Z}$ i.e. those that carry a fraction $x \gtrsim M^2_{W,Z}/m_N E_\nu$ of the nucleon momentum. HERA and BFKL tell that N diverges as $x \cdot q(x) \propto x^{-\beta}$ with $\beta \approx 0.5$, giving $\sigma \propto E_\nu^\beta$.

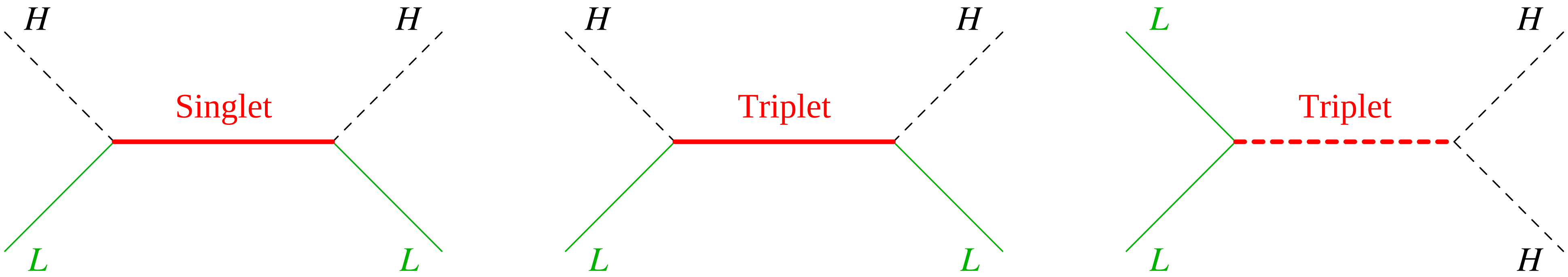


Do not trust $\sigma^{\text{SM}}_{\nu N}$ above 10^8 GeV

Neutrino masses and LHC?

See-saw: type I, II and III

3 kinds of particles with mass M can mediate neutrino masses at tree level:



LHC needs $M \lesssim$ TeV. No motivation for that (anthropic leptogenesis?).
For the singlet, the rate is anyhow 0, unless there are extra couplings.

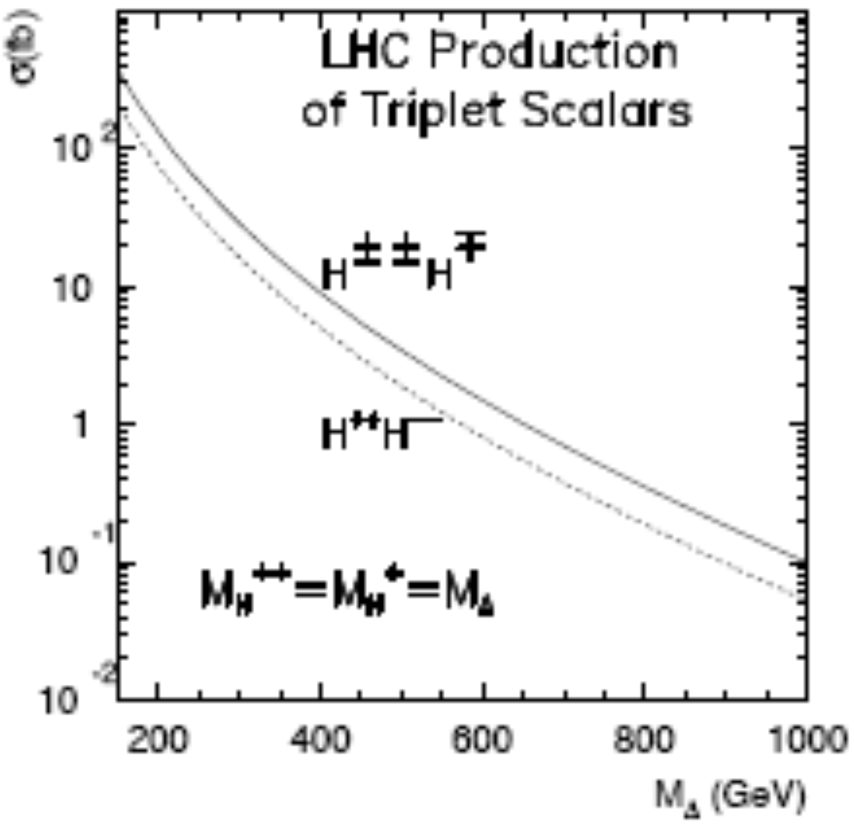
Scalar triplets at LHC

For the scalar triplet $T = \{T^0, T^+, T^{++}\}$ $M \ll M_{\text{Pl}}$ opens a hierarchy problem.

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + |D_\mu T|^2 - M^2|T|^2 + \frac{1}{2}(\lambda_L LLT + M\lambda_H HHT^* + \text{h.c.})$$

$m_\nu = \lambda_L \lambda_H v^2/M$. Production via gauge interactions, $\sigma \sim \beta^3 g^4/4\pi s$:

$$q\bar{q} \to W^\pm \to T^{\pm\pm}T^\mp, \qquad q\bar{q} \to \gamma, Z \to T^+$$



Decays into predicted flavors:

$$\Gamma(T^{++} \to W^+W^+) \approx \lambda_H^2 M/4\pi$$

$$\Gamma(T^{++} \to \ell_1^+\ell_2^+) \approx \lambda_L^2 M/4\pi$$

Similar for $T^+ \to W^+Z$, $\ell^+\nu$ and

$$T^0 \to ZZ \ , \ \nu\nu$$

Lepton-number is violated only by both:

$$pp \to T^{++}T^{--} \to \ell_1\ell_2 W^+W^+$$

Fermion triplets at LHC

Majorana N^0 with Dirac $N^\pm$

$$\tilde{m}_1 = \lambda^2 v^2/M$$

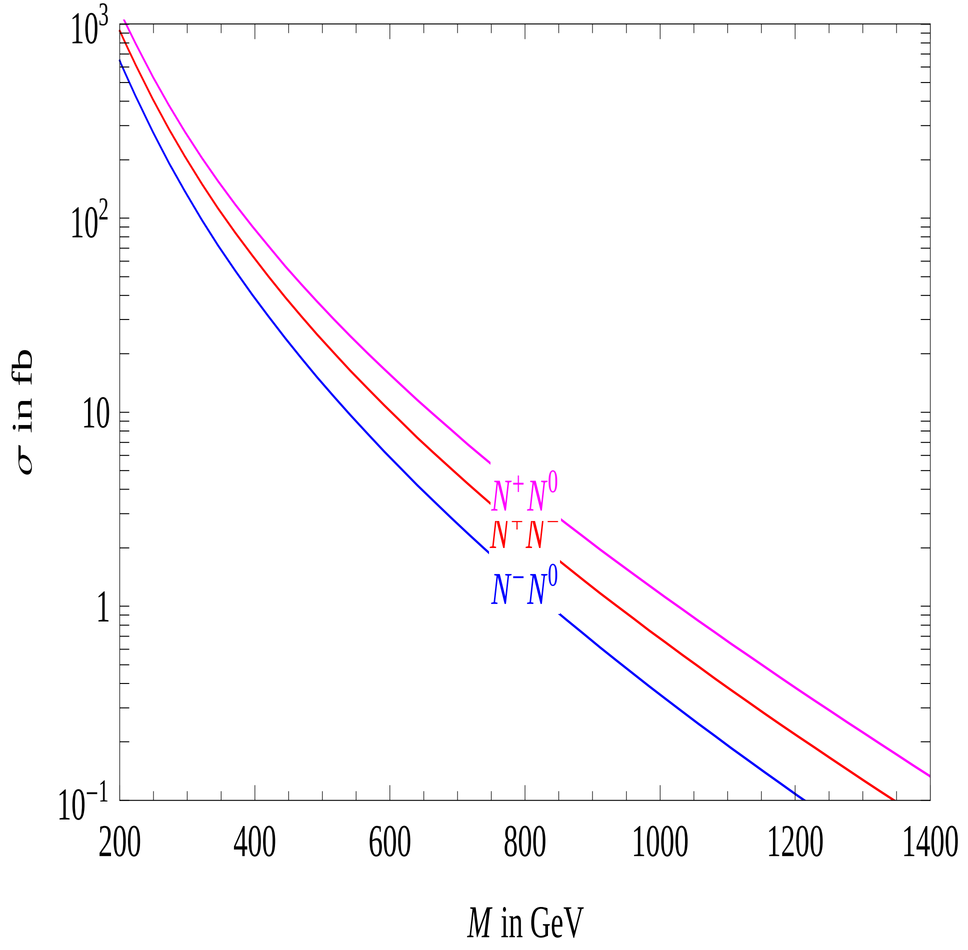
Production via gauge interactions: $\sigma \sim \beta g^4/4\pi s$, peaked at $\beta \sim 0.7$, is 10× bigger for scalars.

Decay via neutrino Yukawas:

$$\tau_{N_0} \overset{M\gg v}{\simeq} \tau_{N_\pm} \overset{M\gg v}{\simeq} \frac{8\pi v^2}{\tilde{m}_1 M^2} =$$

$$= 1.5\,\text{cm}\frac{\text{meV}}{\tilde{m}_1}\left(\frac{100\text{ GeV}}{M}\right)^2$$

detectably displaced if $\tau \gtrsim 0.1\,\text{mm}$.



Lepton flavor and lepton-number-violating decays (Z is Z or h):

$$pp \to \ell_1\bar{\ell}_2 ZW^+, \qquad \ell_1\bar{\ell}_2 ZZ, \qquad pp \to \ell_1\ell_2 ZW^+.$$

$M_{N_\pm} - M_{N_0} \approx 166\,\text{MeV} > m_\pi$, so $\Gamma(N^\pm \to N^0\pi^\pm) \sim 1/\,\text{cm}$ can also give

$$pp \to \ell_1\ell_2\pi^\pm W^+W^+, \qquad \ell_1\ell_2\pi^+W^+Z, \qquad \ell_1\ell_2\pi^+\pi^-W^+W^+.$$

experiment	status	name	start	cost in 億¥ ≈ M$ ≈ M€
WČ (3 kton)	terminated	Kamiokande	1983	5
WČ (50 kton)	running	SuperKamiokande	1996	100
WČ (1000 kton)	proposals	HyperK, UNO?	2015?	500?
Solar B	running	SNO	2001	$100 + 500$ (target)
Solar Be	construction	Borexino	2004?	25
Solar pp	running	Gallex ≈ SAGE/2	1991	$1 + 15$ (target)
Solar pp	proposals	many or none	2010??	100??
Reactor	terminated	CHOOZ	1997	1.5
Reactor	running	KamLAND	2002	20
Long baseline	construction	CNGS	2006	50 (beam) + 80
Long baseline	construction	NuMI	2005	110 (beam) + 60
Long baseline	approved	T2K	2008?	130
Long baseline	discussions	ν factory	2020??	2000??
β decay at 0.2 eV	approved	Katrin	2008	25
$0\nu2\beta$ at 0.01 eV	proposals		2010??	$20 \div 100$
$e\bar{e}$ collider (0.2 TeV)	terminated	LEP	1989	1200
$e\bar{e}$ collider (0.5 TeV)	proposals	ILC	2020??	5000?
pp collider (7 TeV)	construction	LHC	2007?	3000?
pp collider (20 TeV)	not approved	SSC		11000
Satellite	flying	WMAP	2003	150
Space Station	flying	ISS		50000?

Conclusions

Solar and atmospheric anomalies **established**, oscillations almost seen.
Future experiments will give redundancy, testing minimal theory.
Progress driven by 300M€ of experiments, simple theory, nice phenomenology.

“a piece of 20th century physics that fell by chance into the 21th century”

Unexplained fundamental parameters increased from 17 to 21.
Probably bigger experiments will access a few more in next years.
Probably a window to physics at 10^{14} GeV: how to reconstruct it?

2008: KamLAND, Borexino, AUGER

2009: LHC, IceCUBE, MEG

2010: 2CHOOZ, Katrin

2011: T2K, NOνA