Neutrinos and electrons in dense matter: a new approach

Alexander Studenikin

Moscow State University

Padua,
10/03/08

R.N.Mohapatra, A.Y.Smirnov,

Neutrino mass and New Physics, Ann.Rev.Nucl.Part.Phys. 56 (2006)

“Recent discovery of
flavour conversion of
solar, atmospheric, reactor and accelerator
neutrinos have conclusively established that
neutrinos have nonzero mass
and they mix among themselves
much like quarks, providing the first evidence of
new physics beyond the standard model.”

Crucial role of neutrino

ν is a “tiny” particle :

- very light $m_{\nu_f} \ll m_f, \quad f = e, \mu, \tau$
- electrically neutral $q_\nu = 0$ $q_\nu < 4 \times 10^{-17} e$
- with very small magnetic moment μ_ν ?
- weak interactions are indeed weak

$\sigma_{\nu_e N} \sim 10^{-39}\ cm^2$ ν-N scattering

$\sigma_{\bar{\nu}_e p} \sim 10^{-40}\ cm^2$ inverse β-decay

$\sigma_{\nu_e e} \sim 10^{-43}\ cm^2$ ν-e scattering

at the final stages of development of particular elementary particle physics framework

ν manifests itself most vividly under the influence of extreme external conditions:

- dense background matter

and

- strong external (electromagnetic …) fields

Particle interactions under the influence of external conditions

- **strong electromagnetic fields**
- **dense matter**

3 problems:

1. Spin light of ν in matter $\boxed{SL\nu}$
2. Spin light of electron in matter $\boxed{SLe}$
3. ν energy quantization in rotating matter

ν and e quantum states in matter

New approach to particles in matter

… Consistent approach to $SL\nu$

A.S., “Neutrinos and electrons in background matter: a new approach”, **Ann.Fond. de Broglie 31** (2006) 289

- We present quite **powerful method** for description of **neutrinos** (and **electrons**) motion in **background matter** which implies the use of **modified Dirac equations** with effective matter potentials being included.

ν and e in matter being treated within the method of exact solutions of quantum wave equations -

«method of exact solutions »

A.Studenikin, A.Ternov,
Phys.Lett.B 608 (2005) 107;

hep-ph/0410297,
"Neutrino quantum states in matter";

hep-ph/0410296,
"Generalized Dirac-Pauli equation and neutrino quantum states in matter"

A.Grigoriev, A.Studenikin, A.Ternov,
Phys.Lett.B 608 622 (2005) 199

A.Studenikin,

J.Phys.A: Math.Gen.39 (2006) 6769;

Ann. Fond. de Broglie 31 (2006) 289,
"Neutrinos and electrons in background matter: a new approach"

Outline (in addition to 3 mentioned above main problems)

- electromagnetic properties of ν
  - ν magnetic moment (th. & exp.)
- direct influence of e.m. fields on ν
  - spin (spin-flavour) oscillations in $B_\perp$
  - spin oscillations in arbitrary (e.m.) external fields
- indirect influence of e.m. fields on
  - beta-decay of neutron in $B_\perp$
  - spin-flavour ν oscillations in magnetized matter

Interaction of particles in external electromagnetic fields

(**Furry representation** in quantum electrodynamics)

$$e \xrightarrow{\mathrm{B}_\perp} e + \gamma$$

synchrotron radiation

Potential of electromagnetic field

$$\boxed{A_\mu(x) = A^q_\mu(x) + A^{ext}_\mu(x)},$$

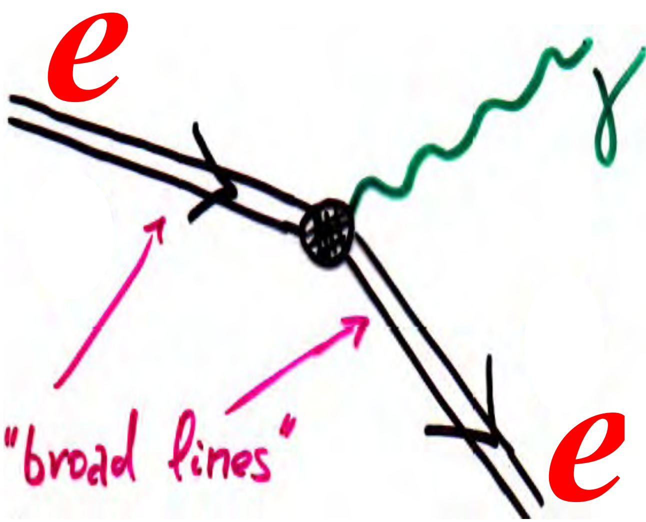
quantized part of potential

evolution **operator**

$$\boxed{U_F(t_1,t_2) = Texp\Big[- i \int\limits_{t_1}^{t_2} j^\mu(x) A^q_\mu(x) dx \Big]},$$

charged particles **current**

$$\boxed{j_\mu(x) = \frac{e}{2}\left[\overline{\Psi}_F \gamma_\mu, \Psi_F\right]},$$



Dirac equation in **external classical (non-quantized) field** $A^{ext}_\mu(x)$

$$\boxed{\Big\{ \gamma^\mu \Big(i\partial_\mu - eA^{ext}_\mu(x) \Big) - m_e \Big\} \Psi_F(x) = 0}$$

…remark on

How can ν be affected by $\vec{B}$?

1) *"direct influence"*

non-trivial electromagnetic properties

- ✱ $\mu_\nu \neq 0 \; (m_\nu \neq 0)$
- ✱ spin and
- ✱ spin-flavour oscillations in B

!

2) "indirect influence" of B on interacting with ν particles

- ✱ $n \xrightarrow{B} p^+ + e^- + \tilde{\nu}_e$
- ✱ $\nu + n \xrightarrow{B} p^+ + e^-$
- ✱ flavour and spin ν oscillations in polarized (by B) matter $(e, n, p, \ldots)$

3) *"direct-indirect influence"*

Spin light of ν in matter and e.m.fields

Direct influence of B on ν

ν magnetic moment interaction

electromagnetic field

$$\Delta\mathcal{L} = \mu_\nu \bar{\Psi}_\nu \sigma_{\lambda\rho} \Psi'_\nu F^{\lambda\rho}, \quad \sigma_{\lambda\rho} = \frac{1}{2}(\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda),$$

$$\bar{\Psi}\sigma_{\lambda\rho}\Psi' = \dots = \bar{\Psi}_L \sigma_{\lambda\rho}\Psi'_R + \bar{\Psi}_R\sigma_{\lambda\rho}\Psi'_L,$$

$$\Psi_{L,R} = \frac{1}{2}(1\mp\gamma_5)\Psi.$$

{Neutrino spin $\nu_L \xleftrightarrow{B_\perp} \nu_R$ oscillations

- (mixing due to $\frac{\Delta m_\nu^2}{2E_\nu}\sin 2\theta_{vac} \rightarrow \boxed{2\mu B_\perp}$)

*

$$\boxed{\begin{aligned} P(\nu_L\leftrightarrow\nu_R) &= \sin^2 2\theta_{eff}\,\sin^2\frac{\pi x}{L_{eff}}, \\ \sin^2 2\theta_{eff} &= \frac{(2\mu B_\perp)^2}{(2\mu B_\perp)^2+\underline{\Omega^2}}, \end{aligned}}$$

$$* \quad \Omega = \frac{\Delta m_\nu^2}{2E_\nu}A(\theta_{vac}) - \sqrt{2}\,G_F\, n_{eff},$$

particle number density

$$L_{eff} = \frac{2\pi}{\sqrt{\underline{\Omega^2} + (2\mu B_\perp)^2}}$$

E. Akhmedov
C.-S. Lim, W. Marciano

(1988)

$\Omega^2 \rightarrow 0$: resonance in $\nu_L \leftrightarrow \nu_R$ neutrino spin oscillations

$\Downarrow$

- Spin and spin-flavour oscillations for $\nu_\odot$ and ν_{SN}

Neutrino conversions and oscillations in magnetic field

⊛ $\nu_\odot$ problem

$$\boxed{\nu_L \overset{B}{\leftrightarrow} \nu_R}$$

Cisneros, 1971

* { Voloshin, Vysotsky, Okun, 1986
Barbieri, Fiorentini, 1988

$\odot$ twisting B { Smirnov, 1991
Akhmedov, Petcov, Smirnov 1993

⊛ Supernova $\nu_L \overset{B}{\leftrightarrow} \nu_R$

Dar, 1987

Fujikawa, Shrock, 1988

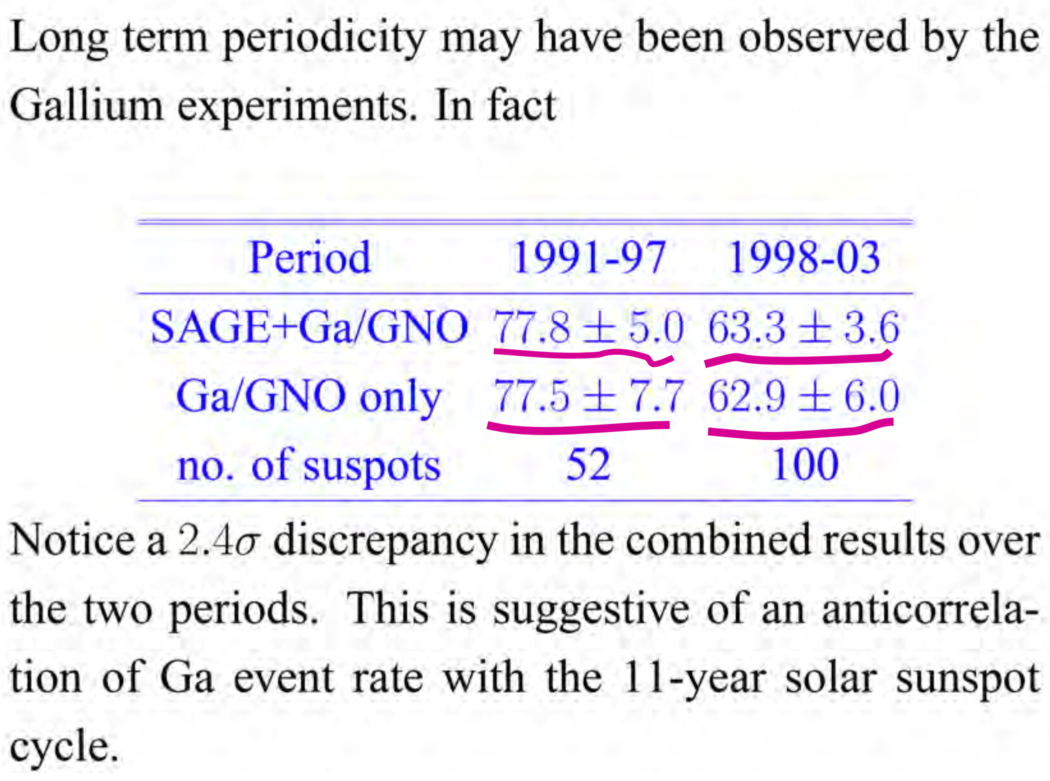
Voloshin, 1988

$\nu_{\odot}$

SPIN FLAVOUR PRECESSION AND LMA

João M. Pulido
CFTP - Instituto Superior Técnico, Lisbon

2005



Long term periodicity may have been observed by the Gallium experiments. In fact

Period	1991-97	1998-03
SAGE+Ga/GNO	77.8 ± 5.0	63.3 ± 3.6
Ga/GNO only	77.5 ± 7.7	62.9 ± 6.0
no. of suspots	52	100

Notice a 2.4σ discrepancy in the combined results over the two periods. This is suggestive of an anticorrelation of Ga event rate with the 11-year solar sunspot cycle.

Periodicity of the active solar neutrino flux is probably the most important issue to be investigated after LMA has been ascertained as the dominant solution to the $\odot$ ν problem. If confirmed it will imply the existence of a sizable neutrino magnetic moment μ_ν and hence a wealth of new physics.

Idea was introduced in 1986 by Russian physicists Voloshin, Vysotsky and Okun

$$Strong\ B_\odot \rightarrow large\ \mu_\nu B_\odot \rightarrow large\ conversion$$

…from J.Pulido

Indirect influence of B on ν

β-decay of neutron in magnetic field

{Birth of ν astrophysics in B}

$$n \xrightarrow{B} p + e + \tilde{\nu}_e$$

- L. Korovina, "β-decay of polarized neutron in magnetic field", Sov.Phys.J., # 6 (1964) 86
- I. Ternov, B. Lysov, L. Korovina, Mosc.Univ.Bull.,Phys.,Astron., #5 (1965) 58 "On the theory of neutron β-decay in external magnetic field",
- J. Matese, R. O'Connell, "Neutron beta decay in a uniform magnetic field", Phys.Rev.180 (1969) 1289
- L. Fassio-Canuto, "Neutron beta decay in a strong magnetic field" Phys.Rev.187 (1969) 2141
- G. Greenstein, Nature 223 (1969) 938

⊛ Asymmetry in $\tilde{\nu}$ emission

$$\frac{W(B)}{W_0} = \frac{1}{2}\int \sin\theta_\nu \, d\theta_\nu \left\{ 1 + \frac{2(\alpha^2+\alpha)}{1+3\alpha^2} S_n \underline{\underline{\cos\theta_\nu}} - 4.9\,\frac{eB}{\Delta^2}\left(\frac{\alpha^2-1}{1+3\alpha^2} \underline{\underline{\cos\theta_\nu}} + \frac{2(\alpha^2-\alpha)}{1+3\alpha^2} S_n \right) \right\}$$

neutrino momentum

astrophysical applications

Weak reaction rates in B

Inter-conversion between n and p through

inverse β-decay

$$* \; n + \nu \overset{B}{\longleftrightarrow} p + e^-,$$

$$* \; n + e^+ \overset{B}{\longleftrightarrow} p + \bar{\nu},$$

$$* \; n \overset{B}{\longleftrightarrow} p + e^- + \bar{\nu},$$

n/p ratio in various astrophysical processes such as Greenstein, 1969; Matese, O'Connell, 1970, 1969;

I Big-Bang Nucleosynthesis Cheng, Schramm, Truran, 1993;

* "Magnetic Fields in the Early Universe", Phys. Rep. 348, 2001, 163. } ← Grasso, Rubinstein, 1995, 1996.

Recent studies of β-processes in B

(neutron star cooling and kick velocities)

increasing interest

Vilenkin, 1995

Goyl, 1997;

Roulet, 1997;

Leinson, Perez, 1998;

Lai, Qian, 1998;

Arras, Lai, 1999;

Bhattacharya, Pal, 2003;

Duan, Qian, 2004;

Kauts, Savochkin, Studenikin, 2004.

Gvozdev, Ognev, 1999;

Bisnovatyi-Kogan, 1993.

S. Shinkevich, A.S., **Pramana, 65 (2005) 215-244**

Relativistic theory of inverse β-decay of polarized neutron

$$\boxed{\nu_e + n \to p + e^-}$$

in strong B

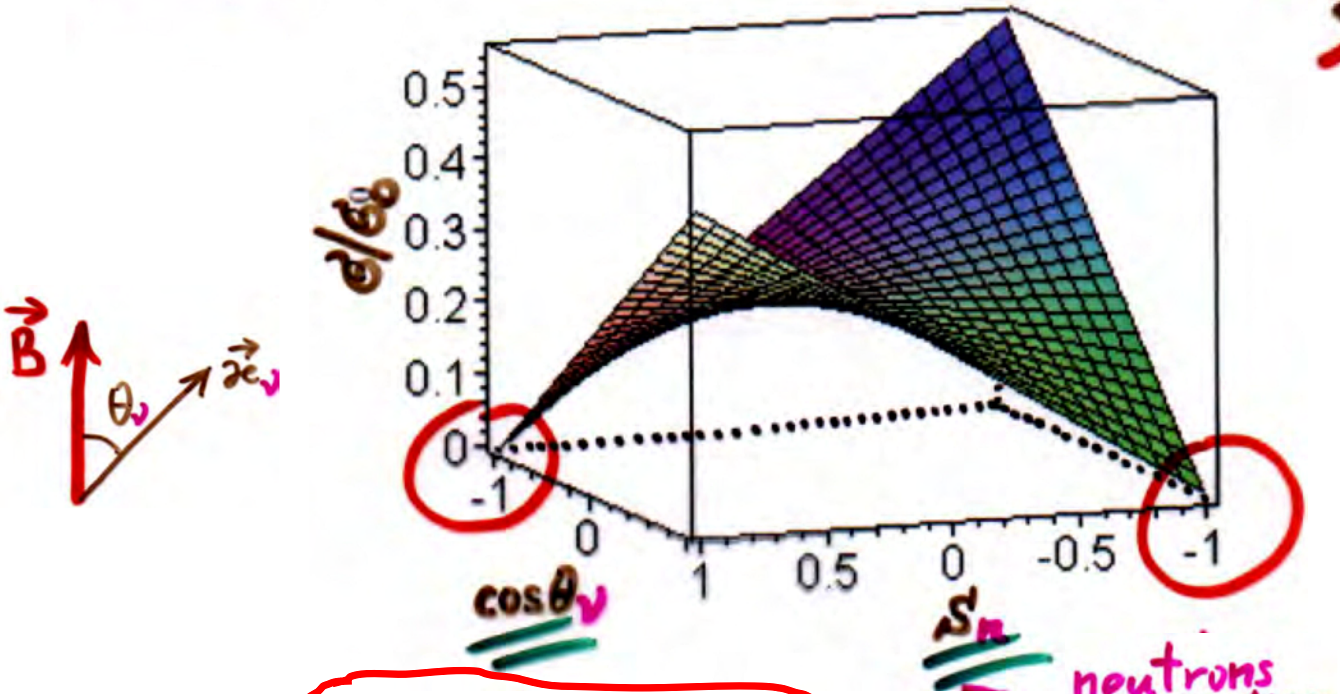
⇒ effects of proton momentum quantization and proton recoil motion are included.

⊛ neutron matter is transparent for ν if

$\longrightarrow \nu \quad S_n \longleftarrow n$ in strong and super-strong B.

Cross section σ in strong $B=B_{cr}=1.1\cdot10^{16}\,G$

$\varkappa = 10\,MeV$



S.Shinkevich, A.Studenikin, Pramana, 65 (2005) 215-244

$\alpha = \frac{g_A}{g_V}$

$\alpha = 1.26$

if $S_n \cos\theta_\nu = -1$,

$$\sigma\Big|_{\cos\theta_\nu=-1,\,S_n=1} = 0, \quad \sigma\Big|_{\cos\theta_\nu=+1,\,S_n=-1} \sim (1-\alpha)^2 < 0.1$$

neutron matter is transparent for ν...

⊙ **K.Kouzakov, A.S.**
Phys.Rev.C 72 (2005) 015502

$$n \xrightarrow{B} pe + \tilde{\nu}_e$$

“Bound-state beta-decay of neutron in strong magnetic field”

Usual (continuum-state) β decay $\quad n \to p + e^- + \bar{\nu}_e$

"Rare" (bound-state) β decay $\quad n \to (pe^-) + \bar{\nu}_e$

R. Daudel, M. Jean, and M. Lecoin, J. Phys. Radium **8**, 238 (1947)

$$\frac{w_b}{w_c} \cong 4.2 \times 10^{-6}$$

$$\tau_c \sim 15 \text{ min}$$

$$\tau_b \sim 7 \text{ years}$$

J.N. Bahcall, Phys. Rev. **124**, 495 (1961) [Dirac equation]

L.L. Nemenov, Sov. J. Nucl. Phys. **15**, 582 (1972) [Schrödinger equation]

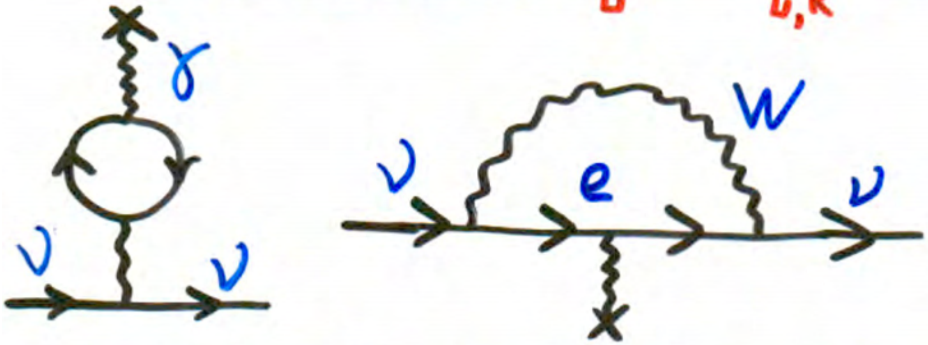
X. Song, J. Phys. G: Nucl. Phys. **13**, 1023 (1987) [Bethe-Salpeter equation]

… another example of

Indirect influence of B *on* ν

Indirect influence of external $F_{\mu\nu}$ on oscillations $\nu_L \leftrightarrow \nu_{L,R}$ in matter

J. D'Olivo, J. Nieves, P. Pal, 1989,
S. Esposito, G. Capone, 1996,
Elmfors, D. Grasso, G. Raffelt, 1996,
V. Semikoz, J. Valle, 1994; 1997,



J. D'Olivo, J. Nieves, 1996,
H. Nunokawa, V. Semikoz, A. Smirnov, J. Valle
1997

one-loop finite-density contribution to energy of ν in magnetized matter

⇔ matter polarization effects in $\vec{B}$

⇙

Extra term in ν effective potential

✓ $$V_{\nu_e} = \sqrt{2}\, G_F N_e - \frac{e G_F}{\sqrt{2}} \left(\frac{3N_e}{\pi^4}\right)^{1/3} \boxed{\frac{\vec{p}_\nu \vec{B}}{E_\nu}} \sim B_\parallel$$

(degenerate electron gas)

$N_e = n_e - n_{\bar{e}}$

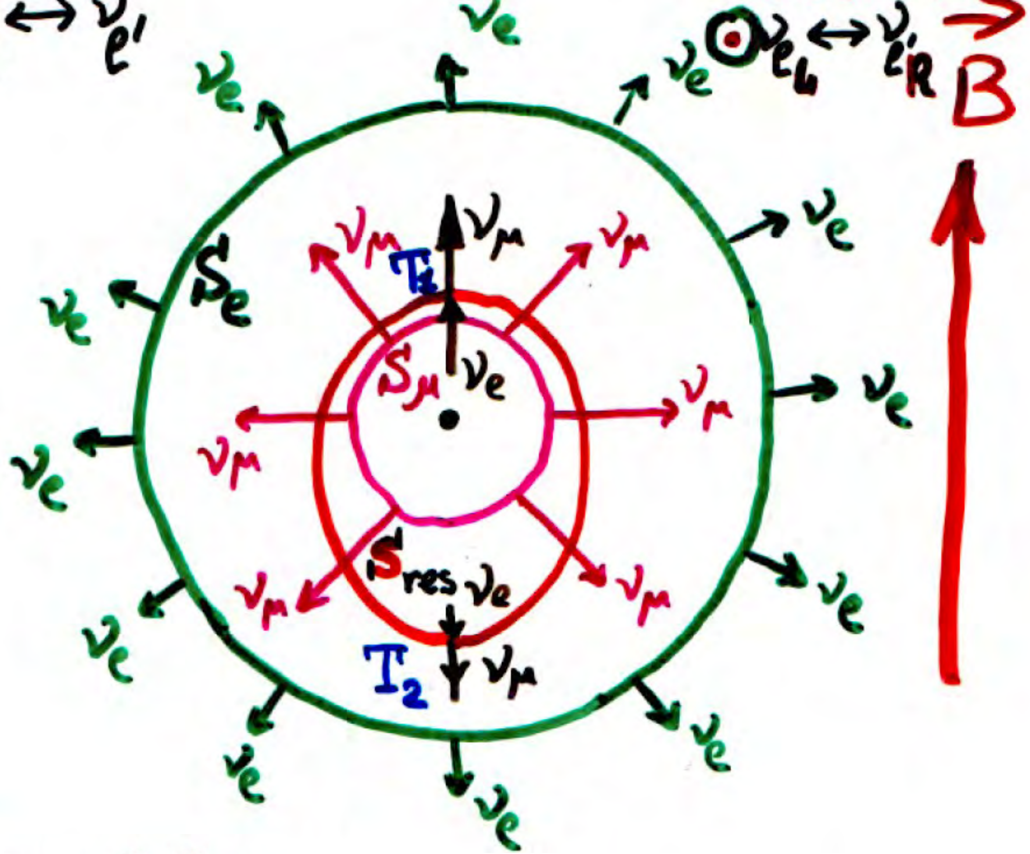
Pulsar kick and ν oscillations in B

Kusenko, Serge, 1996

$\odot\, \nu_\ell \leftrightarrow \nu_{\ell'}$

Akhmedov, Lanza, Sciama, 1997

$\odot\, \nu_{e_L} \leftrightarrow \nu_{e_R}$ → B



S_μ , S_e , S_{res} are muon, electron neutrino-spheres

and $\nu_e \leftrightarrow \nu_\mu$ resonance surface (ellipsoid)

In directions where RS is close (far) to center → larger(smaller) ν momentum, since $T_1 > T_2$.

…back to main stream of discussion…

Method of exact solutions

Modified **Dirac equations** for e and ν
(containing the correspondent effective matter potentials)

+

exact solutions (particles wave functions)

⇓

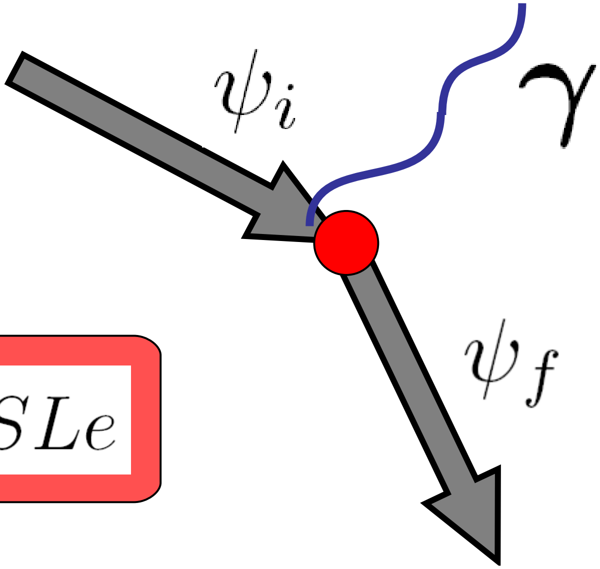
a basis for investigation of different phenomena which can proceed when **neutrinos** and **electrons** move in dense media
(**astrophysical** and **cosmological** **environments**).

2003-2004-2005

Spin light of neutrino in matter $\boxed{SL\nu}$

2005-2008

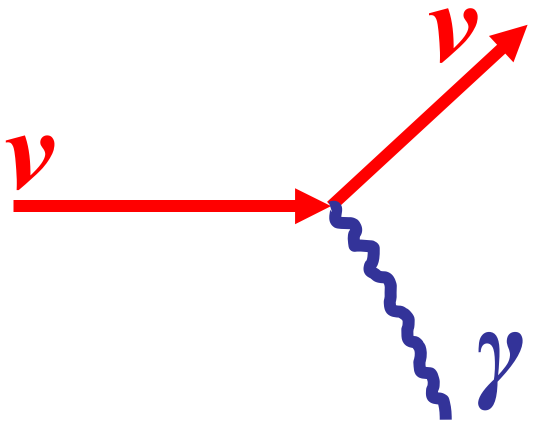
Spin light of electron in matter $\boxed{SLe}$



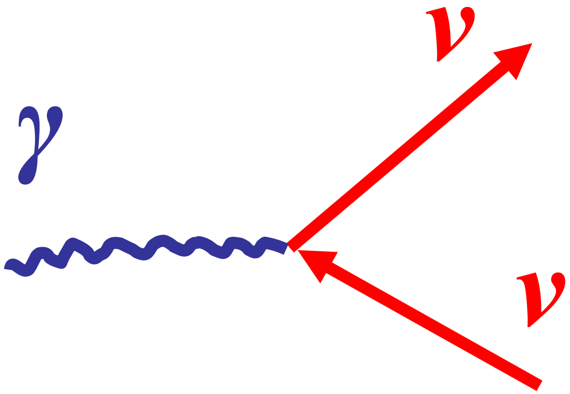
- We predict the existence of a **new mechanism** of the electromagnetic process stimulated by the presence of matter, in which a neutrino or electron due to **spin precession** can emit light.

New mechanism of electromagnetic radiation

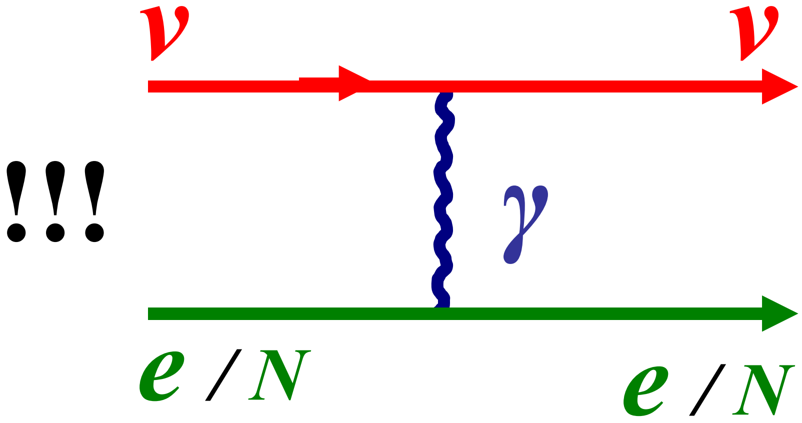
Neutrino – photon couplings (I)



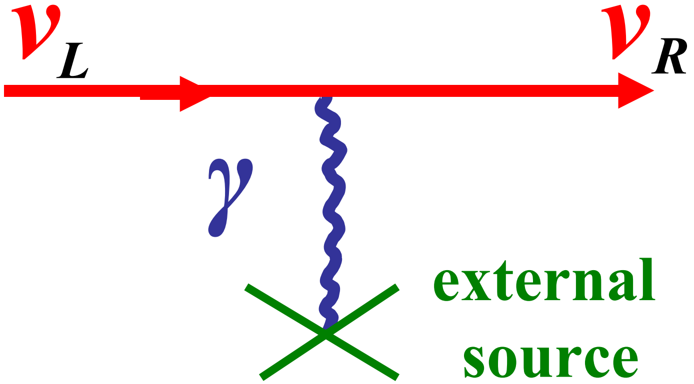
ν decay, Cherenkov radiation



γ decay in plasma

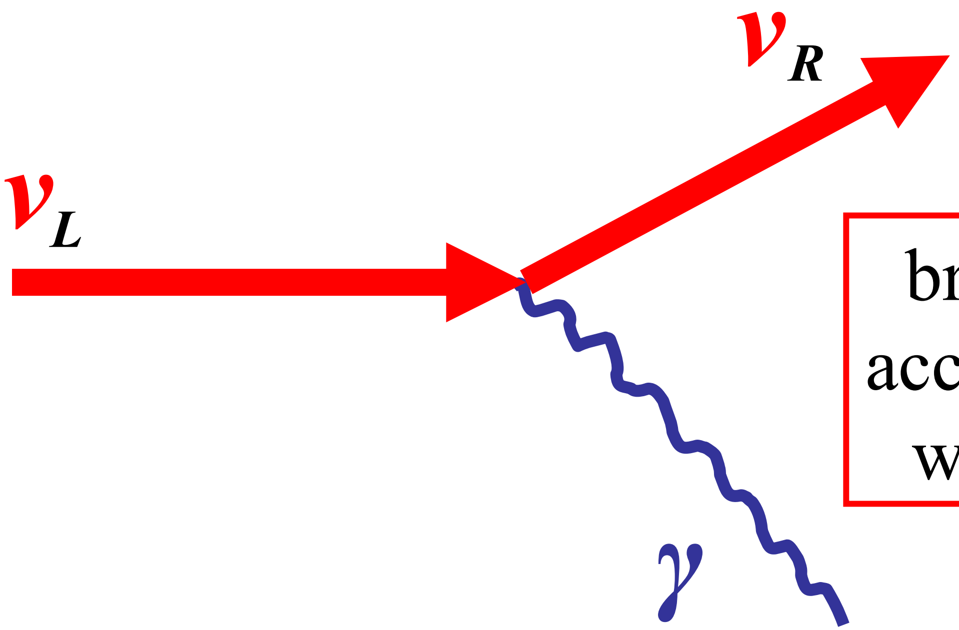


Scattering



Spin precession

Neutrino – photon couplings (II)



broad neutrino lines account for interaction with environment

"Spin light of neutrino in matter"

…within the quantum treatment…

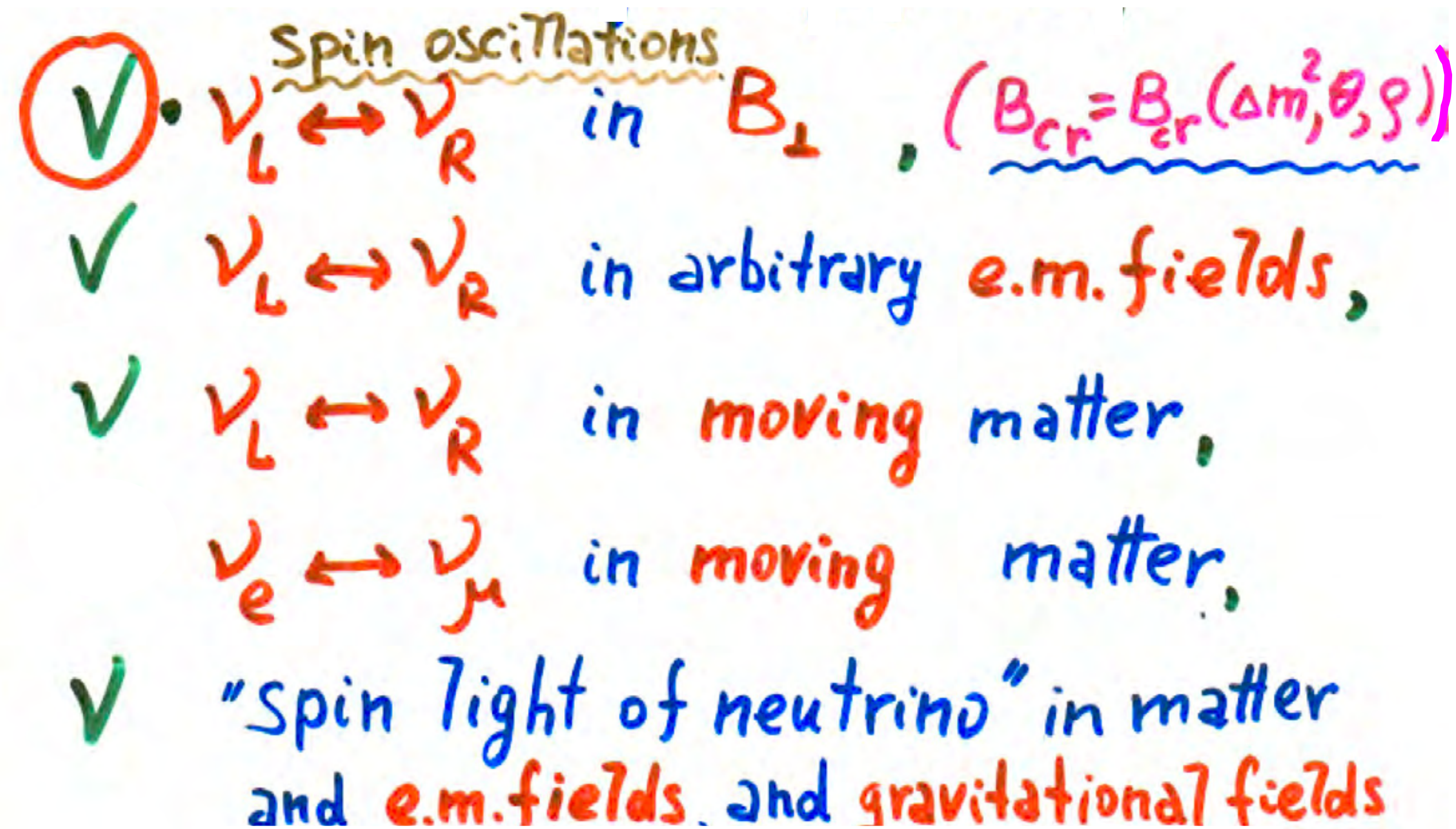
… evaluation of the **method**

● **- within a project of research on ν**
in dense matter and external fields
- stimulated by a need to obtain a consistent theory of "spin light of neutrino (electron) in matter"

A.S.,
"Neutrinos and electrons in background matter: a new approach",
Ann.Fond. de Broglie 31 (2006) 289;

"Method of wave equations exact solutions in studies of neutrinos and electron interactions in dense matter",
J.Phys.A: Math.Theor. (2008) *accept. for publ.*

Main results of our previous studies



1994-1997 ✓ · $\overbrace{\nu_L \leftrightarrow \nu_R}^{\text{Spin oscillations}}$ in $B_\perp$, $(\underline{B_{cr}=B_{cr}(\Delta m^2,\theta,\rho)})$

1998-2000 ✓ $\nu_L \leftrightarrow \nu_R$ in arbitrary e.m. fields,

2000-2002 ✓ $\nu_L \leftrightarrow \nu_R$ in moving matter,

1995-2002 $\nu_e \leftrightarrow \nu_\mu$ in moving matter,

2003-2005 ✓ "spin light of neutrino" in matter and e.m. fields and gravitational fields

2004-2006… ✓ quantum theory of neutrino motion in background matter

NB ! These studies are performed within the Standard Model of interaction

A.Studenikin, **J.Phys.A: Math.Theor.** (2008) *acc. for publ.*
A.Studenikin, **J.Phys.A: Math.Gen. 39** (2006) 6769; **Ann.Fond. de Broglie 31** (2006) 289
A.Studenikin, **Phys.Atom.Nucl. 70** (2007) 1275; *ibid* **67** (2004)1014
A.Grigoriev, A.Savochkin, A.Studenikin, **Russ.Phys. J. 50** (2007) 845
A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, I.Trofimov, **Russ.Phys. J. 50** (2007) 596
A.Studenikin, A.Ternov, **Phys.Lett.B 608** (2005) 107; **Grav. & Cosm. 14** (2008)
A.Grigoriev, A.Studenikin, A.Ternov, **Phys.Lett.B 622** (2005) 199
Grav. & Cosm. 11 (2005) 132 ; **Phys.Atom.Nucl. 6 9** (2006)1940
K.Kouzakov, A.Studenikin, **Phys.Rev.C 72** (2005) 015502
M.Dvornikov, A.Grigoriev, A.Studenikin, **Int.J Mod.Phys.D 14** (2005) 309
S.Shinkevich, A.Studenikin, **Pramana 64** (2005) 124
A.Studenikin, **Nucl.Phys.B** (Proc.Suppl.) **143** (2005) 570
M.Dvornikov, A.Studenikin, **Phys.Rev.D 69** (2004) 073001
Phys.Atom.Nucl. 64 (2001) 1624
Phys.Atom.Nucl. **67** (2004) 719
JETP 99 (2004) 254; **JHEP 09** (2002) 016
A.Lobanov, A.Studenikin, **Phys.Lett.B 601** (2004) 171
Phys.Lett.B 564 (2003) 27
Phys.Lett.B 515 (2001) 94
A.Grigoriev, A.Lobanov, A.Studenikin, **Phys.Lett.B 535** (2002) 187
A.Egorov, A.Lobanov, A.Studenikin, **Phys.Lett.B 491** (2000) 137

Electromagnetic properties of ν

ν **magnetic moment ？**

Neutrino mass

$$\Rightarrow \quad m_\nu \neq 0 \; !$$

ν ⊙

Neutrino magnetic moment

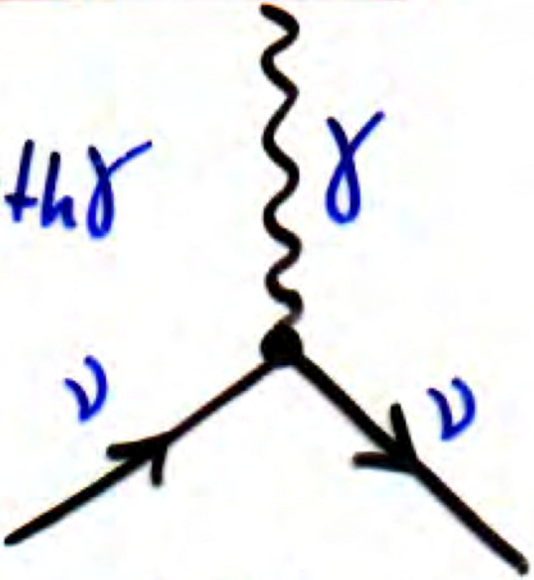
$$\mu_\nu \neq 0 \; ? \; (!)$$

⊛ Lee, Shrock 1977

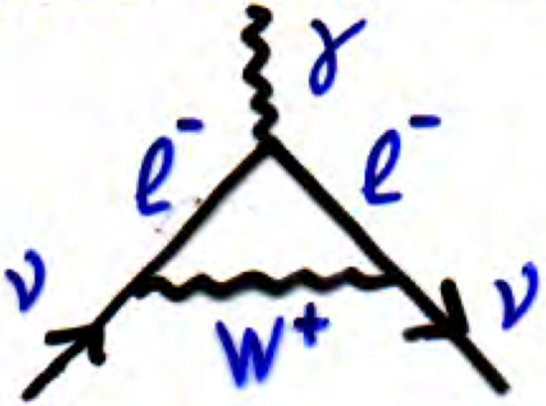
Shrock, Fujikawa 1980

...Massive neutrino electromagnetic properties...

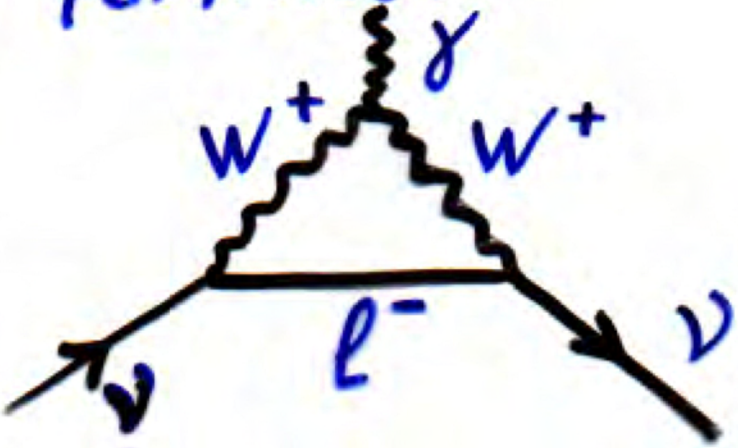
② Electromagnetic ν properties



$\underline{Q_\nu = 0}$ => interaction with γ entirely from loop effects through weak interactions with charged particles:



,



Theory (Standard Model with ν_R)

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3\cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1eV}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee
Shrock, 1977; Fujikawa
Shrok, 1980

In the Standard Model : $m_\nu = 0$,
there is no ν_R $\Longrightarrow$
ν magnetic moment $\mu_\nu = 0$.

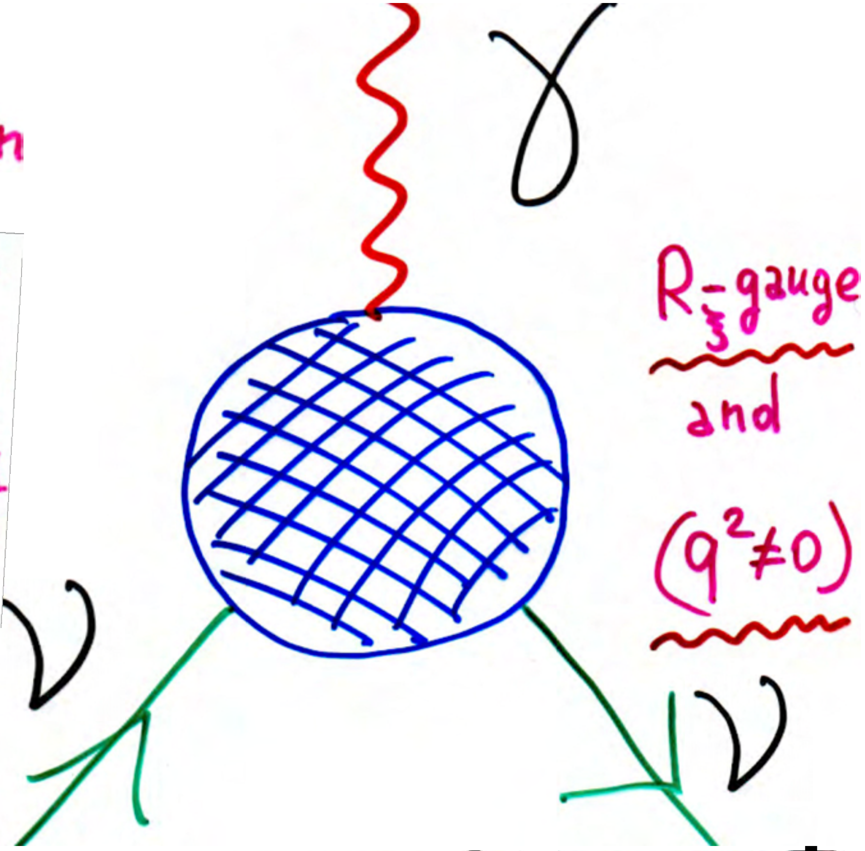
Thus, $\mu_\nu \neq 0$ ← beyond the SM.

The most general study of the **massive neutrino** vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R accounting for masses of particles in polarization loops

⇘

M.Dvornikov, A.Studenikin

- Phys.Rev.D 63, 073001, 2004, "Electric charge and magnetic moment of massive neutrino";
- JETP 126 (2004), N8, 1 "Electromagnetic form factors of a massiv neutrino."



R_ξ-gauge and $(q^2\neq 0)$

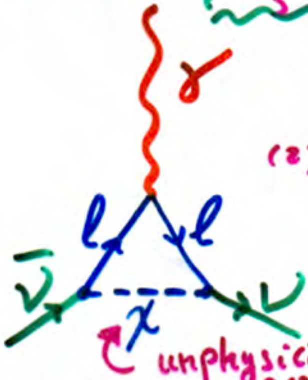
$$\begin{aligned}\Delta_\mu(q) = {} & \overset{\text{charge}}{\underline{f_Q(q^2)}}\,\gamma_\mu + \overset{\text{magnetic moment}}{\underline{f_M}}(q^2)\, i\sigma_{\mu\nu} q^\nu - \\ & - \underset{\text{electric moment}}{\underline{f_E}}(q^2)\, i\sigma_{\mu\nu} q^\nu \gamma_5 - \underset{\text{anapole moment}}{\underline{f_A}}(q^2)\,(q^2\gamma_\mu - q_\mu \not{q})\,\gamma_5\end{aligned}$$

$a=\left(\frac{m_\ell}{m_W}\right)^2,$

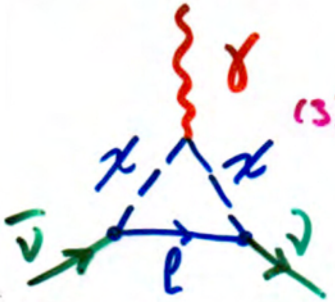
$B=\left(\frac{m_\nu}{m_W}\right)^2$

Proper vertices $\boxed{R_\xi\text{-gauge}}$

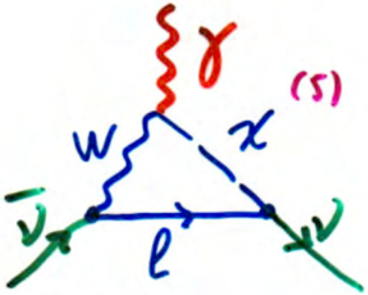

(1)

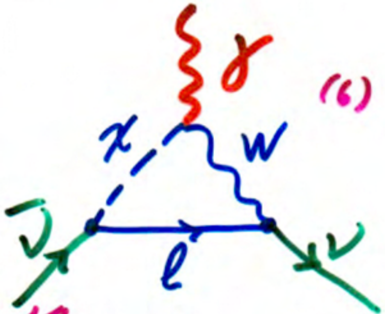

(2)

unphysical charged scalar boson


(3)


(4)


(5)

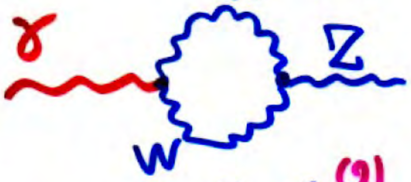

(6)

$$\Lambda_\mu(q) = \sum_{i=1}^{14} \Lambda_\mu^i(q)$$

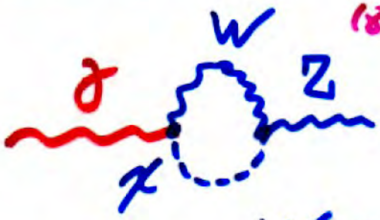
$$\Lambda^{j}_{\mu}(q) = \frac{g}{2\cos\theta_W}\,\Pi^{(j)}_{\mu\nu}(q)\,\frac{1}{q^2 - M_Z^2}$$

$$\times \left\{ g^{\nu\alpha} - (1-\alpha_Z)\frac{q^\nu q^\alpha}{q^2 - \alpha_Z M_Z^2} \right\} \gamma^4_\alpha\,, \; j=7,\dots,14$$

γ-Z self-energy diagrams

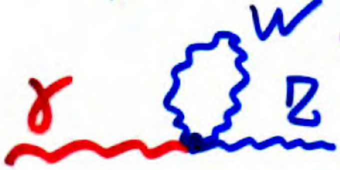


(7)



(8)

(9)

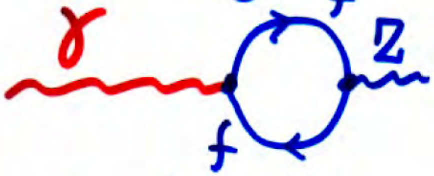


(10)

(11)

(12)

(13)



(14)

f = u, c, t, d, s, b
quarks

Calculation of ν magnetic moment
(massive ν, arbitrary R_ξ- gauge)

Dvornikov, Studenikin, PRD 2004

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)\underline{i\sigma_{\mu\nu}q^\nu}-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 +f_A(q^2)(q^2\gamma_\mu-q_\mu \not{q})\gamma_5$$

magnetic moment

ν

$$\boxed{\mu(a,b,\alpha)=f_M(q^2=0)}$$

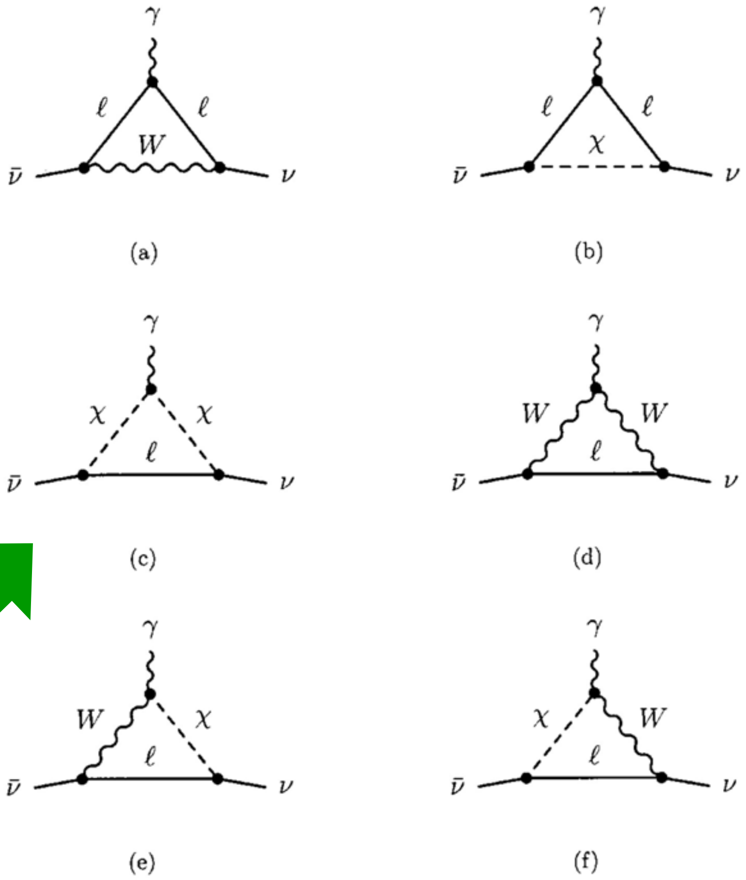
two mass parameters $a=\left(\frac{m_\ell}{M_W}\right)^2$ $b=\left(\frac{m_\nu}{M_W}\right)^2$

and gauge-fixing parameter $\alpha=\frac{1}{\xi}$

$\xi=0$ **- unitary gauge,** $\xi=1$ **- 't Hooft-Feynman gauge**

$$\boxed{\mu(a,b,\alpha)=\sum_{i=1}^{6}\mu^{(i)}(a,b,\alpha)}$$

Proper vertices



(a) (b) (c) (d) (e) (f)

Contributions of proper vertices diagrams

(dimensional-regularization scheme)

- $$\Lambda_\mu^{(1)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\left[g^{\kappa\lambda}-(1-\alpha)\frac{k^\kappa k^\lambda}{k^2-\alpha M_W^2}\right]\times\frac{\gamma_\kappa^L(\not{p}\,'-\not{k}+m_\ell)\gamma_\mu(\not{p}-\not{k}+m_\ell)\gamma_\lambda^L}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-M_W^2]},$$

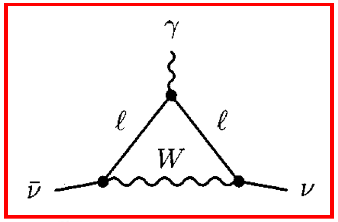
- $$\Lambda_\mu^{(2)}=i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}\frac{(m_\nu P_L-m_\ell P_R)(\not{p}\,'-\not{k}+m_\ell)\gamma_\mu(\not{p}-\not{k}+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-\alpha M_W^2]},$$

- $$\Lambda_\mu^{(3)}=i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}(2k-p-p')_\mu\frac{(m_\nu P_L-m_\ell P_R)(\not{k}+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-\alpha M_W^2][k^2-m_\ell^2]},$$

- $$\Lambda_\mu^{(4)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\gamma_\kappa^L(\not{k}+m_\ell)\gamma_\lambda^L\left[\delta_\beta^\kappa-(1-\alpha)\frac{(p'-k)^\kappa(p'-k)_\beta}{(p'-k)^2-\alpha M_W^2}\right]\left[\delta_\gamma^\lambda-(1-\alpha)\frac{(p-k)^\lambda(p-k)_\gamma}{(p-k)^2-\alpha M_W^2}\right]$$
$$\times\frac{\delta_\mu^\beta(2p'-p-k)^\gamma+g^{\beta\gamma}(2k-p-p')_\mu+\delta_\mu^\gamma(2p-p'-k)^\beta}{[(p'-k)^2-M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]},$$

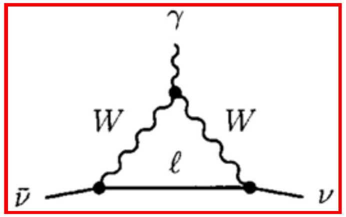
- $$\Lambda_\mu^{(5)+(6)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}$$
$$\times\left\{\frac{\gamma_\beta^L(\not{k}-m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-M_W^2][(p-k)^2-\alpha M_W^2][k^2m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p'-k)^\beta(p'-k)_\mu}{(p'-k)^2-\alpha M_W^2}\right]\right.$$
$$\left.-\frac{(m_\nu P_L-m_\ell P_R)(\not{k}-m_\ell)\gamma_\beta^L}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p-k)^\beta(p-k)_\mu}{(p-k)^2-\alpha M_W^2}\right]\right\}$$

… after loop integrals calculations (e.g., for diagrams (a) and (d) contributing in unitary gauge)



$$\mu^{(1)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\int_0^1 dz\, z(1-z^2)\frac{1}{D}-\frac{1}{2}\int_0^1 dz(1-z)^3(a-bz)\left[\frac{1}{D_\alpha}-\frac{1}{D}\right]\right.$$

$$\left.-\frac{1}{2}\int_0^1 dz(1-z)(1-3z)[\ln D_\alpha-\ln D]\right\},$$



$$\mu^{(4)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\frac{1}{2}\int_0^1 dz\, z^2(1+2z)\frac{1}{D}\right.$$

$$+\frac{b}{2}\int_0^1 dz\int_0^z dy(1-z)^2[z(1-z)-2y]\left[\frac{1}{D_\alpha+y(1-\alpha)}-\frac{1}{D}\right]$$

$$\left.+\frac{1}{2}\int_0^1 dz\int_0^z dz(-2+9z-4z^2-6y)\{\ln[D_\alpha+y(1-\alpha)]-\ln D\}\right\},$$

where $D_\alpha=a+(\alpha-a)z-bz(1-z)$ **and** $D=D_{\alpha=1}$

$$\boxed{a=\left(\frac{m_\ell}{M_W}\right)^2} \qquad \boxed{\alpha=\frac{1}{\xi}}$$

Dvornikov, Studenikin,
PRD 2004, JETP 2004

… within exact calculations it is possible to expand over mass parameter $b=\left(\frac{m_\nu}{M_W}\right)^2$

$$\mu(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\sum_{i=1}^{6}\{\underline{\overline{\mu}_0^{(i)}}(a,\alpha)+b\,\underline{\overline{\mu}_1^{(i)}}(a,\alpha)+\mathcal{O}(b^2)\}$$

Cabral-Rosetti, Bernabéu, Vidal, Zepeda, EPJ 2000

$$\mu_0(a,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\frac{3}{4(1-a)^3}(2-7a+6a^2-2a^2\ln a-a^3)+\mathcal{O}(a^2)$$

$a=\left(\frac{m_\ell}{M_W}\right)^2$

$$\overline{\mu}_1(a,\alpha)=\sum_{i=1}^{6}\overline{\mu}_1^{(i)}(a,\alpha)=\frac{1}{12(1-a)^5}(5-26a+6a\ln a-36a^2-60a^2\ln a+58a^3-18a^3\ln a-a^4)$$

… μ_ν **gauge independent and finite value…**

ν magnetic moment

(heavy massive neutrino)

- **LEP data** →

only 3 light νs coupled to Z°,

for any additional neutrino

$$\boxed{m_\nu \geq 45\ Gev}$$

- $m_\nu \ll m_e \ll M_W$ **light** ν

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$

$$\boxed{\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}\left(2 - 7a + 6a^2 - 2a^2 \ln a - a^3\right)} , \quad a = \left(\frac{m_e}{M_W}\right)^2$$

*Gabral-Rosetti, Bernabeu, Vidal, Zepeda, Eur.Phys.J **C 12** (2000) 633*

*Dvornikov, Studenikin, Phys.Rev.**D 69** (2004) 073001; JETP **99** (2004) 254*

- $m_e \ll m_\nu \ll M_W$ **intermediate** ν

$$\boxed{\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1 + \frac{5}{18} b\right\}} , \quad b = \left(\frac{m_\nu}{M_W}\right)^2$$

- $m_e \ll M_W \ll m_\nu$ **heavy** ν

$$\boxed{\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu}$$

Status of Experiments on the Neutrino Magnetic Moment Measurement

…stolen from the talk of
Alexander Starostin (*ITEP*)

given at the
13th Lomonosov Conference on
Elementary Particle Physics
(Moscow, August 23-29, 2007)

Moscow State University

Faculty of Physics

THIRTEENTH
LOMONOSOV
CONFERENCE
ON
ELEMENTARY
PARTICLE
PHYSICS

Moscow, August 23-29, 2007

Mikhail Lomonosov
1711-1765

Electroweak Theory
Tests of Standard Model & Beyond
Developments in QCD (Perturbative and Non-Perturbative Effects)
Heavy Quark Physics
Neutrino Physics
Astroparticle Physics
Gravitation and Cosmology
Physics at the Future Accelerators

SEVENTH
INTERNATIONAL
MEETING
ON
PROBLEMS
OF INTELLIGENTSIA

August 29, 2007

Rights and Responsibility
of the Intelligentsia

V.Sadovnichy (Rector of MSU) - Chairman

International Advisory Committee

- E.Akhmedov (KTH, Stokholm & Kurchatov Inst.)
- S.Belayev (Kurchatov Inst., Moscow)
- V.Belokurov (MSU)
- V.Berezinsky (LNGS, Gran Sasso)
- S.Bilenky (JINR, Dubna)
- J.Bleimaier (Princeton)
- M.Danilov (ITEP, Moscow)
- G.Diambrini-Palazzi (Univ. of Rome)
- A.Dolgov (INFN, Ferrara & ITEP, Moscow)
- V.Kadyshevsky (JINR, Dubna)
- A.Logunov (IHEP, Protvino)
- V.Matveev (INR, Moscow)
- L.Okun (ITEP, Moscow)
- M.Panasyuk (MSU)
- V.Rubakov (INR, Moscow)
- D.Shirkov (JINR,Dubna)
- J.Silk (Univ. of Oxford)
- A.Sissakian (JINR, Dubna)
- A.Skrinsky (INP, Novosibirsk)
- A.Slavnov (MSU & Steklov Math. Inst.)
- A.Smirnov (ICTP, Trieste & INR, Moscow)
- P.Spillantini (INFN, Florence)
- A.Starobinsky (Landau Inst., Moscow)
- V.Trukhin (MSU)

Organizing Committee

- V.Bagrov (Tomsk Univ.)
- V.Braginsky (MSU)
- A.Egorov (ICAS)
- D.Galtsov (MSU)
- A.Grigoriev (MSU)
- S.Kapitza (EAPS)
- P.Kashkarov (MSU)
- A.Kataev (INR, Moscow)
- O.Khrustalev (MSU)
- V.Mikhailin (MSU & ICAS)
- A.Mourao (IST/CENTRA, Lisbon)
- N.Narozhny (MEPHI, Moscow)
- N.Nikiforova (MSU)
- A.Nikishov (Lebedev Phys. Inst., Moscow)
- V.Ritus (Lebedev Phys. Inst., Moscow)
- Yu.Popov (MSU)
- V.Savrin (MSU)
- A.Studenikin (MSU & ICAS)

Organizers and sponsors

Faculty of Physics, Moscow State University
Skobeltsyn Institute of Nuclear Physics, Moscow State University
Interregional Centre for Advanced Studies
Joint Institute for Nuclear Research (Dubna)
Institute of Theoretical and Experimental Physics (Moscow)
Institute for Nuclear Research (Moscow)
Institute for High Energy Physics (Protvino)
Budker Institute of Nuclear Physics (Novosibirsk)
Russian Foundation for Basic Research
Ministry for Education and Science of Russia
Russian Academy of Sciences
Russian Agency for Atomic Energy

INTERREGIONAL CENTRE
FOR ADVANCED STUDIES

For contact: Alexander Studenikin, Chairman, studenik@srd.sinp.msu.ru; Alexander Grigoriev, Scientific Secretary, ane@srd.sinp.msu.ru

Department of Theoretical Physics,
Moscow State University, 119992 Moscow, Russia
Phone (007-495) 939-50-33 Fax (007-495) 932-88-20 http://www.icas.ru

www.icas.ru

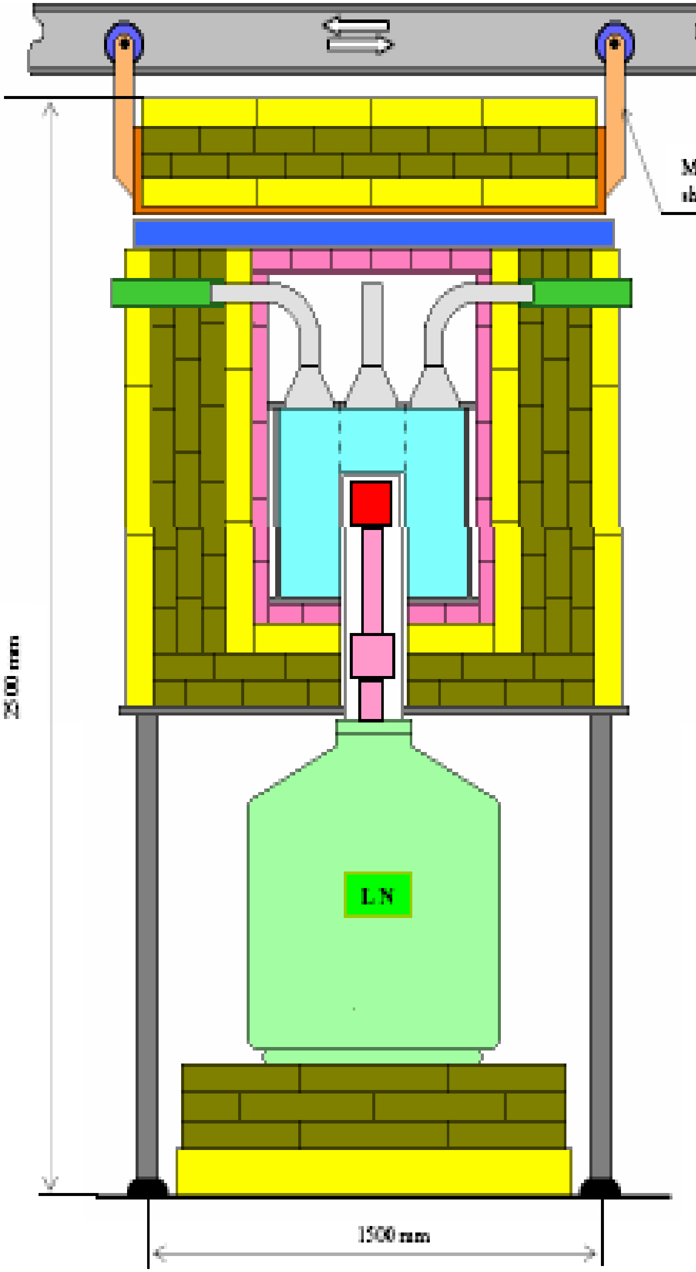
14th Lomonosov Conference
August, 2009

Experiment GEMMA

(**G**ermanium **E**xperiment for measurement of **M**agnetic **M**oment of **A**ntineutrino)

- ITEP – LNP JINR Dubna

[Phys. of At.Nucl.,67,№11(2004)1948]



- Spectrometer includes a HPGe detector of 1.5 kg installed within NaI active shielding.

- HPGe + NaI are surrounded with multi-layer passive shielding – electrolytic copper, borated polyethylene and lead.
- Circuit noises were discriminated by means method of frequency analysis of signals.

Studies of ν-e scattering - most sensitive method of experimental investigation of μ_ν

Cross-section:

- $$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\text{SM}} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu},$$

where the Standard Model contribution

- $$\left(\frac{d\sigma}{dT}\right)_{\text{SM}} = \frac{G_{\text{F}}^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T}{E_\nu^2}\right],$$

T is the electron recoil energy and

$$g_V = \begin{cases} 2\sin^2\theta_W + \frac{1}{2} & \text{for } \nu_e\,, \\ 2\sin^2\theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau\,, \end{cases} \qquad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e\,, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases}$$

for anti-neutrinos $g_A \to -g_A$,

to incorporate charge radius: $g_V \to g_V + \frac{2}{3}M_W^2\langle r^2\rangle\sin^2\theta_W$.

ν-γ coupling

- $$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1 - T/E_\nu}{T}\right]\mu_\nu^2$$

with change of helicity, contrary to SM

Effective Lagrangian for the spin component of ν vertex

$$\boxed{L = \frac{1}{2}\bar{\nu}_j \sigma_{\eta\xi}(\beta_{ij} + \varepsilon_{ij}\gamma_5)\nu_i F^{\eta\xi} + \text{h.c.}\,,}$$

e.m. field tensor

magnetic and **electric** moments
which couple together mass eigenstates
$(\nu_i)_L$ and $(\nu_j)_R$ ⇒ change of the helicity states

- $\nu_i = \nu_j$ ⇒ diagonal moments
- $\nu_i \neq \nu_j$ ⇒ transitional moments
- $\varepsilon_{ii} = \beta_{ii} = 0$ for Majorana ν

E.M. properties
↳ a way to distinguish Dirac and Majorana ν

Effective ν_e magnetic moment measured in ν-e scattering experiments ? $\boxed{\mu_e^2}$

Two steps:

1) consider ν_e as superposition of mass eigenstates ($i=1,2,3$) at some distance L, and then sum up magnetic moment contributions to ν-e scattering amplitude of each of mass components induced by their magnetic moments

$$\boxed{A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji}}$$

2) amplitudes combine incoherently in total cross section

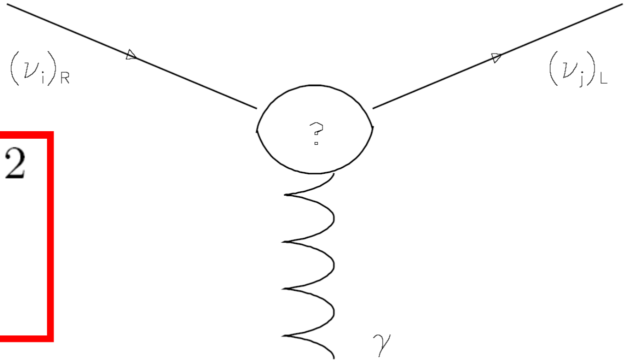
$$\boxed{\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2}$$

J.Beacom, P.Vogel, 1999

NB! Summation over $j=1,2,3$ is outside the square because of **incoherence** of different final mass states contributions to cross section.

ν magnetic moment in experiments

(for neutrino produced as ν_l with energy E_ν
and after traveling a distance L)



$$\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_iL} \mu_{ji} \right|^2$$

where neutrino mixing matrix $\mu_{ij} \equiv |\beta_{ij} - \varepsilon_{ij}|$

Observable μ_ν is an effective parameter that depends on neutrino flavour composition at the detector.

H.Wong,
H.-B.Li, 2005

Implications of μ_ν limits from different experiments (reactor , solar ^{8}B and ^{7}Be) are different.

$$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\rm SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$

ν-γ coupling

- $$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1-T/E_\nu}{T}\right]\mu_\nu^2$$

with change of helicity,
contrary to SM

T is the electron recoil energy: $$0 \leq T \leq \frac{2E_\nu^2}{2E_\nu + m_e}$$

If neutrino has electric dipole moment,
or electric or magnetic transition moments,
these quantities would also contribute to scattering cross section

$$\mu_\nu^2 = \sum_{j=\,\nu_e,\ \nu_\mu,\ \nu_\tau} |\,\mu_{ij} - \epsilon_{ij}\,|^2 \,,$$ ***i*** *refers to initial neutrino flavour*

Possibility of ***distractive interference*** between **magnetic** and **electric** transition moments of **Dirac** neutrino
(**Majorana** neutrino has only magnetic or electric transition moments, but not both if CP is conserved)

Magnetic moment contribution is dominated at low electron recoil energies

and $\boxed{\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}}$ when $\boxed{\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4} \mu_\nu^2}$

… the lower the smallest measurable electron recoil energy is,

the smaller values of μ_ν^2 can be probed in scattering experiments:

Savannah River (1976), ***first observation of*** ν ***- e***

- $\boxed{\mu_\nu \leq 2 \div 4 \times 10^{-10} \mu_B}$

Vogel, Engel, 1989

Kurchatov, Krasnoyarsk (1992),

Rovno (1993) reactors

SuperKamiokande (2004)

- $\boxed{\mu_\nu \leq \ 1.1 \times 10^{-10} \ \mu_B}$
- $\boxed{\mu_\nu \leq \ 9 \times 10^{-11} \ \mu_B}$

MUNU (Bugey reactor, 2005)

- $\boxed{\mu_\nu \leq \ few \times 10^{-11} \mu_B}$

Beta-beams

McLaughlin, Volpe, 2004

GEMMA (2007) result of the 1st year

- (anti)neutrino magnetic moment:

$$! \quad \mu_\nu \leq 5.8 \cdot 10^{-11}\ \mu_B\ \mathit{(90\%\ CL)} \quad !$$

- *Available as hep-ex/0705.4576*

- Compared with the TEXONO experiment

$$\mu_\nu \leq 7.2 \cdot 10^{-11}\ \mu_B\ \mathit{(90\%\ CL)}$$

Alexander Starostin, **talk given at**
13th Lomonosov Conference on Elementary Particle Physics,
Moscow State University, August 24, 2007

Astrophysics bounds on μ_ν

$$\boxed{\mu_\nu(\text{astro}) < 10^{-10}\text{–}10^{-12}\ \mu_B}$$

Mostly derived from consequences of **helicity-state change** in astrophysical medium:

- available degrees of freedom in BBN,
- stellar cooling via plasmon decay,
- cooling of SN1987a.

Red Giant lumin.
$\mu_\nu \leqslant 3 \cdot 10^{-12} \mu_B$
G. Raffelt, D. Dearborn,
J. Silk, 1989.

The bounds depend on

- modeling of the astrophysical systems,
- on assumptions on the neutrino properties.

Generic assumption:

- absence of other nonstandard interactions except for μ_ν .

A global treatment would be desirable, incorporating **oscillation** and **matter effects** as well as the complications due to interference and competitions among various channels.

MUNU experiment at Bugey reactor (2005)

$$\boxed{\mu_{\nu} \leq 9 \times 10^{-11} \mu_B}$$

TEXONO collaboration at Kuo-Sheng power plant (2006)

$$\boxed{\mu_{\nu} \leq 7 \times 10^{-11} \mu_B}$$

GEMMA (2007) $\boxed{\mu_\nu \leq \ 5.8 \times 10^{-11} \mu_B}$

μ_{ν} is presently known to be in the range

$$\boxed{10^{-20} \mu_B \leq \ \mu_{\nu} \ \leq 10^{-10} \mu_B}$$

μ_{ν} provides a tool for exploration possible physics beyond the Standard Model.

large magnetic moment

$$\mu_\nu = \mu_\nu(m_\nu, m_{B^+}, m_{e^-})$$
$$\Uparrow$$

- In the L-R symmetric models $(SU(2)_L \times SU(2)_R \times U(1))$

Kim, 1976
Beg, Marciano, Ruderman, 1978

- **M.Voloshin (ITEP),**

"On compatibility of small m_ν with lar μ_ν of neutrino",
Sov.J.Nucl.Phys. 48 (1988) 512

… there may be $SU(2)_\nu$ symmetry that forbids m_ν but not μ_ν

- supersymmetry
- extra dimensions

considerable enhancement of μ_ν to experimentally relevant range

ν e.m. form factors are affected by matter and B

⊛ magnetic moment $\mu_\nu = \mu_\nu(B)$

Egorov, Studenikin 1997

Borisov, Zhukovsky, Kurilin, Ternov, 1985

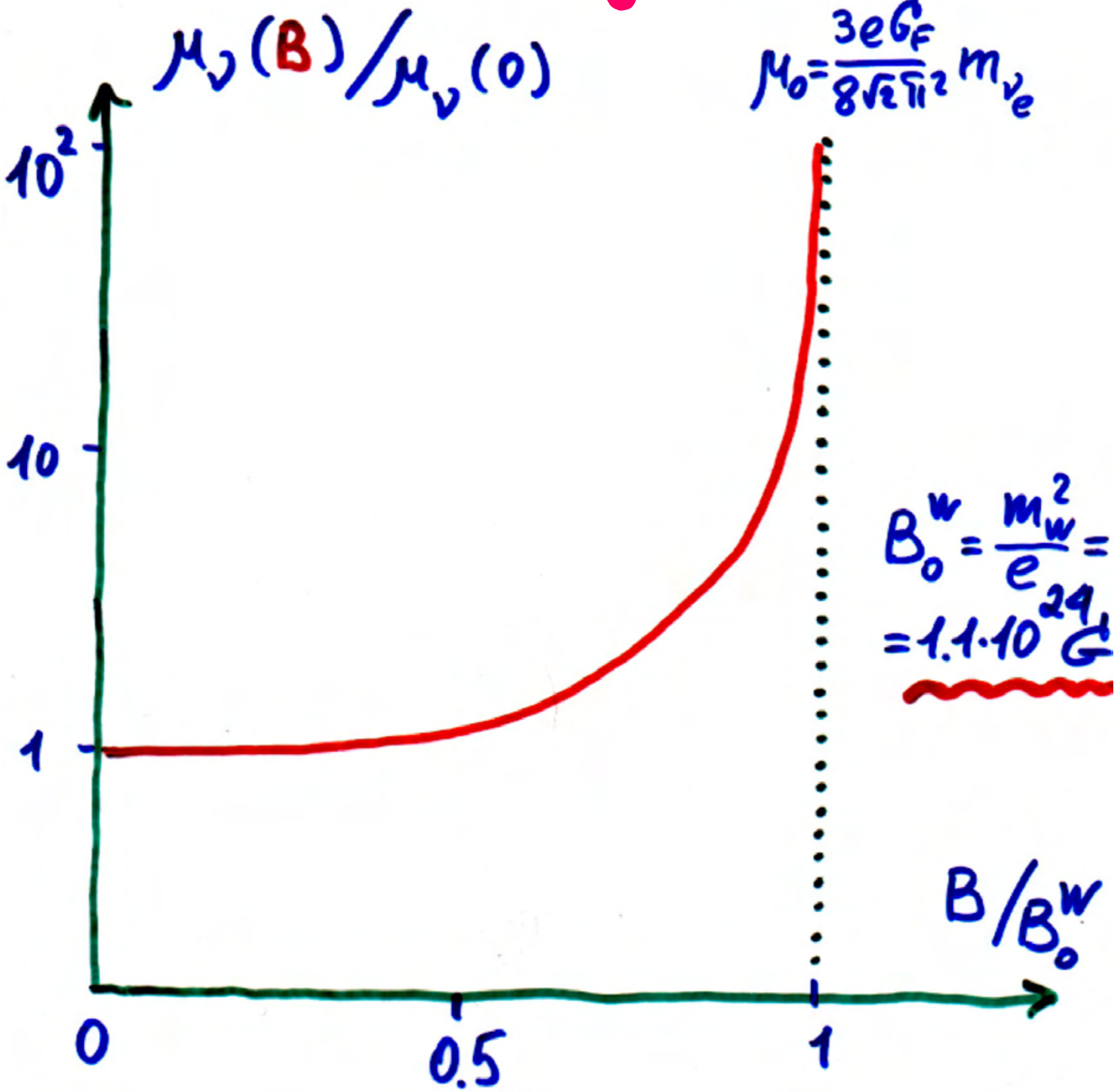
⊛ induced electric charge of ν in magnetized matter

⊛ Oraevsky, Semikoz, Smorodinsky, 1986

Bhattacharaya, Ganguly, Konar, 2002

Nieves, 2003

Neutrino magnetic moment



$$\mu_0 = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$

$$B_0^W = \frac{m_W^2}{e} = 1.1 \cdot 10^{24}\, G$$

Borisov,
Zhukovs'ky,
Kurilin,
Ternov, 1985;

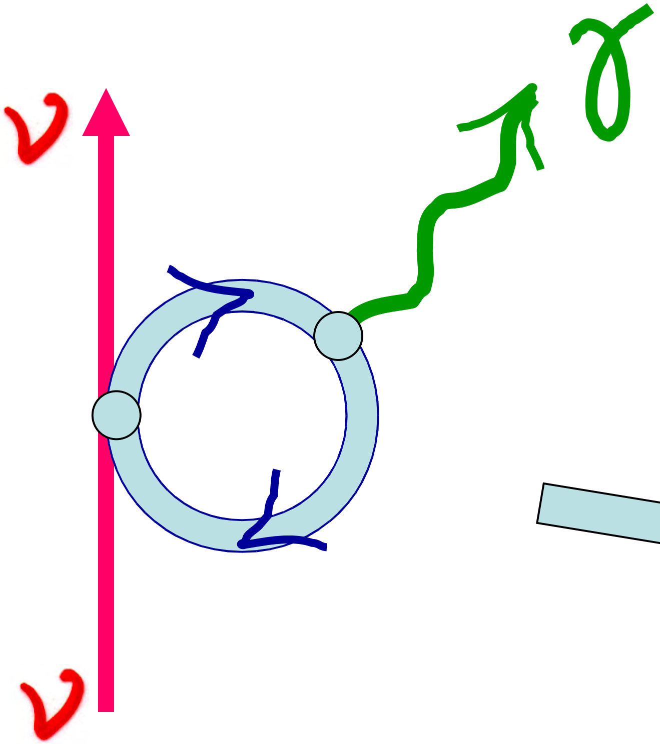
Masood,
Perez Rojas,
Gaitan,
Rodrigues-Romo,
1999

ν "effective electric charge" in magnetized plasma

- ● νs do not couple with γs in vacuum,

  . . . however, when

- ○ ν in thermal medium (e^- and e^+)



ν

V.Oraevsky, V.Semikoz, Ya.Smorodinsky, JETP Lett. 43 (1986) 709;
J.Nieves, P.Pal, Phys.Rev.D 49 (1994) 1398;
T.Altherr, P.Salati, Nucl.Phys.B421 (1994) 662;
K.Bhattacharya, A.Ganguly, 2002

…different νγ interactions in astrophysical and cosmological media

…more about

Indirect influence of external $F_{\mu\nu}$

- $\nu\gamma$ interactions

$$\nu \rightarrow \nu + \gamma$$

$$\gamma \rightarrow \nu + \nu$$

$$\gamma\gamma \rightarrow \nu\bar{\nu} \ \ldots$$

DeRaad, Milton, Hari Dass
Galtsov, Nikitina, Skobelev
Chistakov, Gvozdev, Mikheev, Vasilevskaya
Ionnisian, Raffelt
Dicus, Repko, Shaisultanov
Borisov, Zhukovsky, A.Ternov, Eminov
Radomski, Grimus, Sakuda
Mohanty, Samal
Nieves, Pal . . .

- νe interactions

$$e \rightarrow e\nu\nu$$

$$\nu e \rightarrow \nu e \ \ldots$$

Landstreet, Baier, Katkov, Strakhovenko
Ritus, Nikishov
Loskutov, Zakhartsov
I.Ternov, Rodionov, Studenikin
Borisov, Kurilin
Narynskaya . . .

… astrophysical applications …

ν ***magnetic moment evolution in matter and external fields***

⊛ New effects: #1, #2, #3, #4

hep-ph/0407010,
A. Studenikin: Neutrino in electromagnetic fields and moving matter,
Phys. Atom. Nucl. 67 (N5) 1024, 2004.

"The four new effects in neutrino scillations",
Nucl.Phys.B (Proc.Suppl.) **143** (2005) 570

"Neutrinos in matter and external fields",
Phys.Atom.Nucl. 70 (2007) 1275

#1 Lorentz invariant approach to ν spin evolution in arbitrary e.m. field $F_{\mu\nu}$ (only $B_\perp$ ⊛ was considered before)

⇒

predictions for new resonances in $\nu_L \leftrightarrow \nu_R$ in various configurations of e.m. fields (e.m. wave etc...)

① $\nu_e \xleftrightarrow{vac} \bar{\nu}_e$, B. Pontecorvo, 1957

② $\nu_e \xleftrightarrow{vac} \nu_\mu$, Z. Maki, M. Nakagava, S. Sakata, 1962

③ $\nu_e \xleftrightarrow{matter,\ \rho = const} \nu_\mu$, L. Wolfenstein, 1978

④ $\nu_e \xleftrightarrow{matter,\ \rho \neq const} \nu_\mu$, S. Mikheev, A. Smirnov, 1985

- resonances in ν flavour oscillations ⇒ MSW-effect, solution for $\nu_\odot$-problem

⑤ $\nu_{e_L} \xleftrightarrow{B_\perp} \nu_{e_R}$, A. Cisneros, 1971
M. Voloshin, M. Vysotsky, L. Okun, 1986, $\nu_\odot$

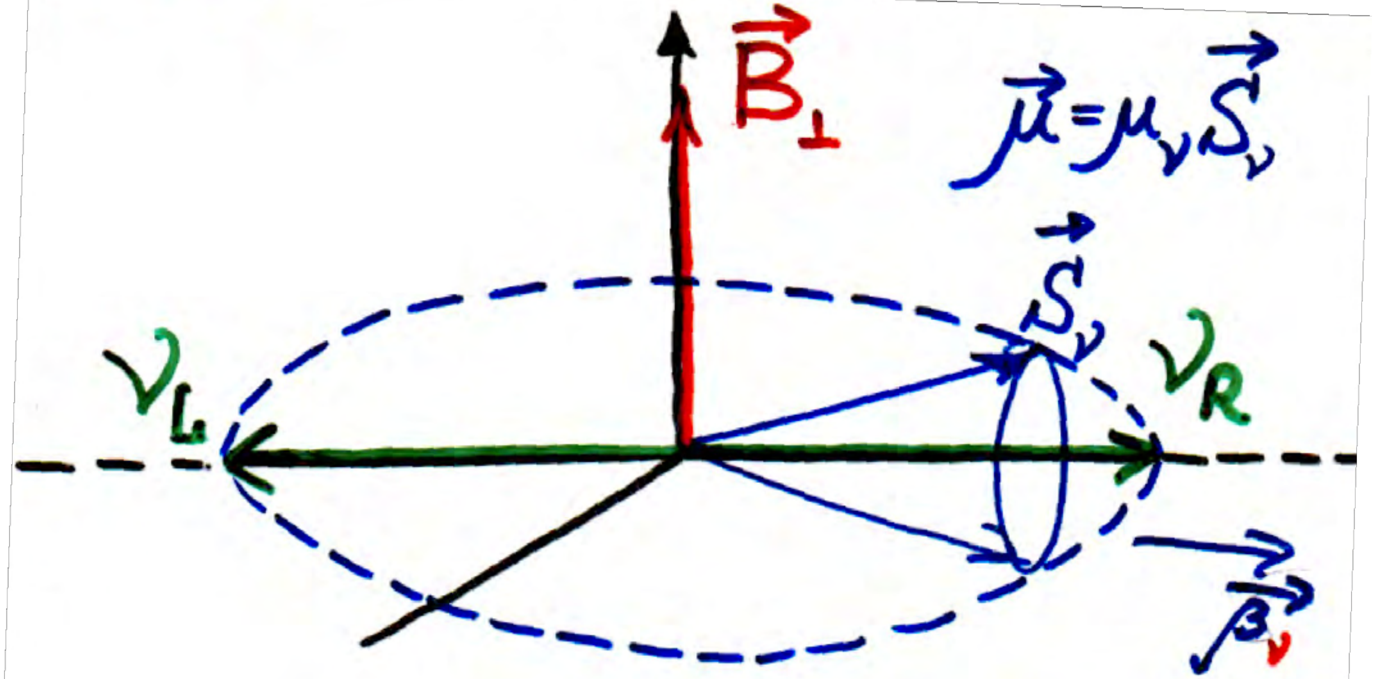
⑥ $\nu_{e_L} \xleftrightarrow{B_\perp} \nu_{e_R}, \nu_{\mu_R}$, E. Akhmedov, 1988
C.-S. Lim & W. Marciano, 1988

- resonances in ν spin (spin-flavour) oscillations in matter

only in $B_\perp$ and matter at rest

#2 ... matter effect included ...

ν spin precession can be stimulated not only by e.m. interactions with e.m. field $F_{\mu\nu}$ but also by ν weak interactions with matter!



$$\frac{d\vec{S}_\nu}{dt} = 2\mu_\nu[\vec{S}_\nu \times \underline{\underline{\vec{B}}}\,] + \boxed{2\mu_\nu[\vec{S}_\nu \times \vec{G}\,]}$$

electromagnetic interaction with e.m. field

weak interaction with matter

#3 $\nu_L \leftrightarrow \nu_R$ and $\nu_\ell \leftrightarrow \nu_{\ell'}$, $\ell \neq \ell'$

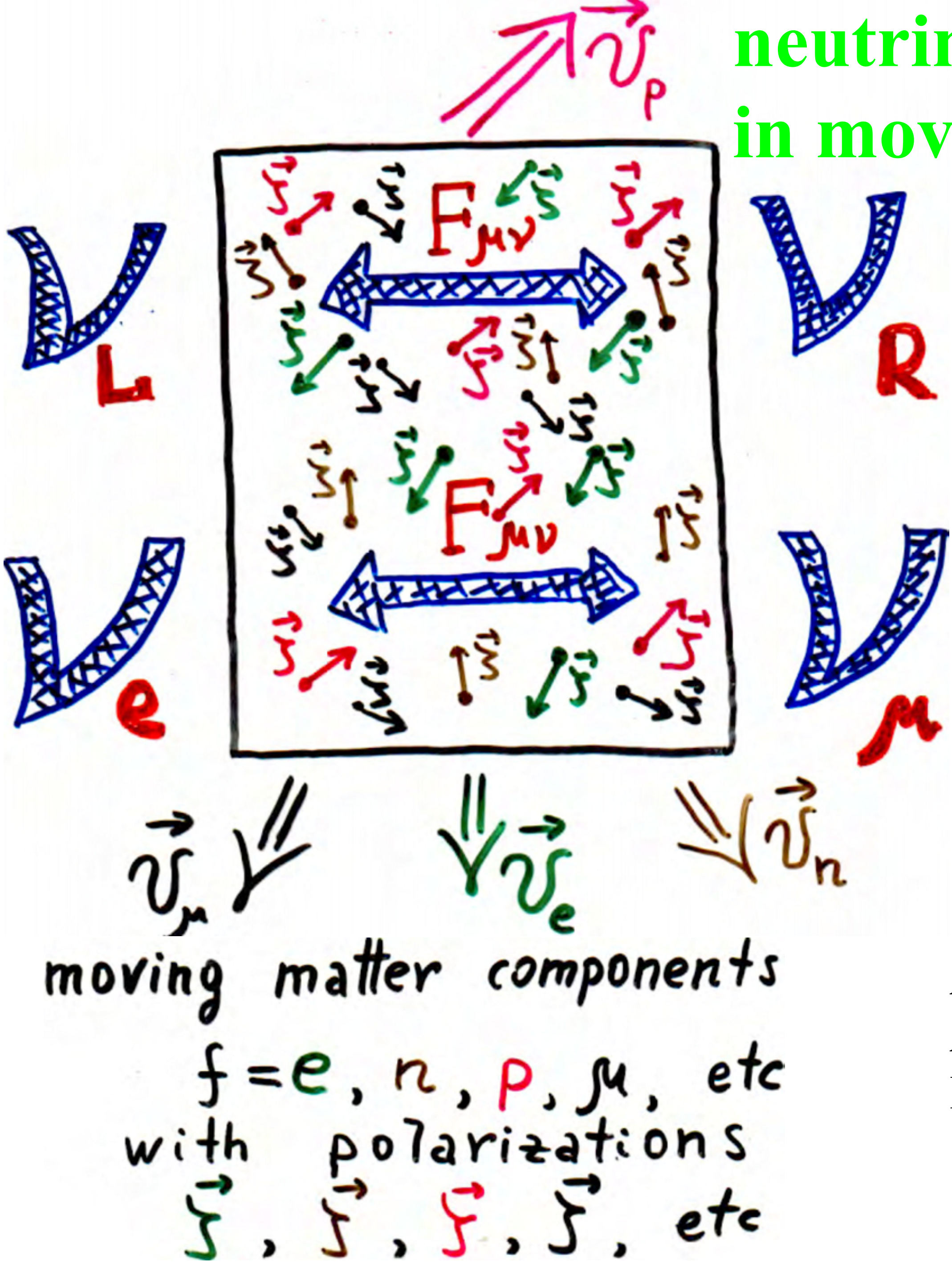
(neutrino spin and flavour oscillations)

in moving and polarized matter

⇒

matter motion can significantly change the neutrino oscillation pattern

neutrino spin oscillations in moving matter



G.Likhachev,
A.Studenikin,
1995 (unpublished)

A.Egorov, A.Lobanov,
A.Studenikin,
Phys.Lett.B 491 (2000) 137

A.Lobanov, A.Studenikin,
Phys.Lett.B 515 (2001) 94

A.Lobanov, A.Grigoriev,
A.Studenikin,
Phys.Lett.B 535 (2002) 187

Neutrino spin evolution in arbitrary electromagnetic field $F_{\mu\nu}$ and moving and polarized matter

Bargmann-Michel-Telegdi equation for spin vector $\mathcal{S}_\mu$ of neutral particle:

$$\frac{d\mathcal{S}^\mu}{d\tau} = 2\mu\left[F^{\mu\nu}\mathcal{S}_\nu - u^\mu(u_\nu F^{\nu\lambda}\mathcal{S}_\lambda)\right] + \underbrace{2\epsilon\left[\tilde{F}^{\mu\nu}\mathcal{S}_\nu - u^\mu(u_\nu \tilde{F}^{\nu\lambda}\mathcal{S}_\lambda)\right]}_{\cancel{\text{T-invariance}}}$$

magnetic

dipole moments

electric

- direct interaction of ~~ν~~ spin with $F_{\mu\nu}$
- $\mathcal{L}$ invariant theory

arbitrary e.m. field

Neutrino spin evolution equation

for ν general interactions

(e.g., ~~P-in~~variant weak interactions)

with moving and polarized matter

neutrino speed

$u_\mu = (\gamma, \gamma\vec{\beta})$, $\gamma = (1-\beta^2)^{-1/2}$, $\vec{\beta} = \text{const.}$, $S^2 = -1$, $u_\mu S^\mu = 0$

Lorentz invariant generalization of BMT eq. :

$$F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}$$

ν interactions with moving and polarized matter

⊛Evaluation of $G_{\mu\nu}$:

- ν evolution eq. has to be linear over S_μ , $F_{\mu\nu}$ and characteristics of matter

$$j_f^\mu = (n_f , n_f \vec{v}_f), \qquad \boxed{f = e, n, p, \mu,} \ldots$$

fermions currents

$$\left.\lambda_f^\mu = \left(n_f \vec{\zeta}_f \vec{v}_f , n_f \vec{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}} \right) \right\} \text{in the laboratory frame of reference}$$

polarizations

n_f → number density of background f

$\vec{v}_f$ → speed of reference frame in which mean momentum of fermions f is zer

$\vec{\zeta}_f$ → mean value of polarization vectors of f in above mentioned ref. frame

Thus, in general case of ν interaction with different background fermions f matter effects are described by antisymmetric tensor

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g^{(1)}_{\rho} \underset{\nu\ \text{speed}}{u_{\lambda}} - \left(g^{(2)\mu} u^{\nu} - u^{\mu} g^{(2)\nu}\right),$$

where

matter current and polarization

$$g^{(1)\mu} = \sum_{f} \rho_f^{(1)} j_f^{\mu} + \rho_f^{(2)} \lambda_f^{\mu},$$

$$g^{(2)\mu} = \sum_{f} \xi_f^{(1)} j_f^{\mu} + \xi_f^{(2)} \lambda_f^{\mu},$$

- summation is performed over fermions f,
- coefficients $\rho_f^{(1),(2)}$, $\xi_f^{(1),(2)}$ are determined by ν interaction model.

In the usual notations

$$\underset{\substack{\uparrow \\ \text{e.m. field}}}{F_{\mu\nu}} = (\vec{E}, \vec{B}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\boxed{G_{\mu\nu} = (-\vec{P}, \vec{M})},$$

where

$$g_\mu^{(1,2)} \equiv \left(g_0^{(1,2)}, \vec{g}^{(1,2)}\right)$$

$$\vec{M} = \gamma\left\{ g_0^{(1)}\vec{\beta} - \vec{g}^{(1)} - [\vec{\beta} \times \vec{g}^{(2)}] \right\},$$

$$\vec{P} = -\gamma\left\{ g_0^{(2)}\vec{\beta} - \vec{g}^{(2)} + [\vec{\beta} \times \vec{g}^{(1)}] \right\}.$$

Substitution $\boxed{F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}}$

implies:

✓

$$\boxed{\begin{aligned} \vec{B} &\longrightarrow \vec{B} + \vec{M} \\ \vec{E} &\longrightarrow \vec{E} - \vec{P} \end{aligned}}$$

effects of interaction with moving and polarized matter ✓

Finally:

three-dimentional ν spin vector

$$\boxed{\frac{d\vec{S}}{d\underset{\sim}{t}} = \frac{2\mu}{\gamma_\nu}\left[\vec{S}\times(\vec{B}_0+\vec{M}_0)\right] + \frac{2\epsilon}{\gamma}\left[\vec{S}\times(\vec{E}_0-\vec{P}_0)\right],}$$

effects of matter

laboratory frame

$\boxed{\vec{B}_0, \vec{E}_0, \vec{M}_0, \vec{P}_0}$

in the rest frame of ν are expressed in terms of quantities determined in laboratory frame

$$\vec{B}_0 = \gamma_\nu\left(\vec{B}_\perp + \frac{1}{\gamma_\nu}\vec{B}_\parallel + \sqrt{1-\frac{1}{\gamma_\nu^2}}\,[\vec{E}_\perp\times\vec{n}]\right), \quad \gamma_\nu = \frac{E_\nu}{m_\nu}$$

energy

mass of ν

$$\vec{E}_0 = \gamma_\nu\left(\vec{E}_\perp + \frac{1}{\gamma_\nu}\vec{E}_\parallel - \sqrt{1-\frac{1}{\gamma_\nu^2}}\,[\vec{B}_\perp\times\vec{n}]\right),$$

in rest frame of ν

$\boxed{\vec{n} = \vec{\beta}/\beta}$

A. Egorov,
A. Lobanov,
A.S.
PLB491(200
p.137

$$\vec{M}_0 = \gamma_\nu\vec{\beta}\left(g_0^{(1)} - \frac{\vec{\beta}\vec{g}^{(1)}}{1+\gamma_\nu^{-1}}\right) - \vec{g}^{(1)},$$

$$\vec{P}_0 = -\gamma_\nu\vec{\beta}\left(g_0^{(2)} - \frac{\vec{\beta}\vec{g}^{(2)}}{1+\gamma_\nu^{-1}}\right) + \vec{g}^{(2)}.$$

$g = g(j_f, \lambda_f)$

weak interaction of neutrino with matter

For SM+SU(2)-singlet ν_R and matter $f=e$

$$\boxed{\frac{d\vec{S}_\nu}{dt} = \frac{2\mu_\nu}{\gamma_\nu}\left[\vec{S}_\nu \times (\vec{B}_0 + \vec{M}_0)\right]},$$

in rest frame of neutrino

interaction of neutrino with an electromagnetic field

$$\vec{B}_0 = \gamma_\nu\left(\vec{B}_\perp + \frac{1}{\gamma_\nu}\vec{B}_\parallel + \sqrt{1-\frac{1}{\gamma_\nu^2}}\left[\vec{E}_\perp \times \vec{n}\right]\right),$$

interaction of neutrino with matter

$$\vec{M}_0 = \gamma_\nu \rho n_e \left(\vec{\beta}_\nu\left(1-\vec{\beta}_\nu \vec{v}_e\right) - \frac{1}{\gamma_\nu}\vec{v}_{e\perp}\right),$$

$$\gamma_\nu = \frac{E_\nu}{m_\nu},$$

matter density

$\parallel$

$\perp$

ν *spin procession and oscillations in arbitrary electromagnetic field*

Now we know:

#1 how to treat ν spin oscillations in arbitrary e.m. fields within Lorentz invariant approach ⇨ new resonances in $\nu_L \leftrightarrow \nu_R$ in various e.m. fields (e.m. wave etc...)

? complete and exact solution for ν spin precession in an arbitrary $F_{\mu\nu}$, in particular:

A.Egorov, A.Lobanov, A.Studenikin, Phys.Lett.B 49 (2000) 137

1) $\vec{B}_{\parallel} \neq 0$, $\xcancel{\vec{P}_{\nu} \perp \vec{B}}$

2) electromagnetic wave...

etc... $\boxed{F \neq B_{\perp}}$

$\Rightarrow$

the probability amplitude gets its max. value ($\sin^2 2\theta_{eff} = 1$) for any strength of the electromagnetic wave field **B** when the **resonance condition** is fulfilled:

$$\boxed{\frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{g \enclose{circle}{\omega}}{2}\left(1 - \frac{\beta}{\beta_0}\cos\varphi\right) = 0} \, .$$

$A = A(\theta_{vac})$

e.m. wave speed, $\beta_0 < 1$

Prediction for new type of ν resonances $\nu_L \leftrightarrow \nu_R$ in electromagnetic field of the wave.

… once again…

For SM+SU(2)-singlet ν_R and matter $f=e$

$$\boxed{\frac{d\vec{S}_\nu}{dt} = \frac{2\mu_\nu}{\gamma_\nu}\left[\vec{S}_\nu \times \left(\vec{B}_0 + \vec{M}_0\right)\right]},$$

in rest frame of neutrino

$$\vec{B}_0 = \gamma_\nu\left(\underline{\vec{B}_\perp} + \frac{1}{\gamma_\nu}\underline{\vec{B}_\parallel} + \sqrt{1-\frac{1}{\gamma_\nu^2}}\left[\underline{\vec{E}_\perp} \times \vec{n}\right]\right),$$

$$\vec{M}_0 = \gamma_\nu \rho\, n_e \left(\underline{\vec{\beta}_\nu}\left(1 - \underline{\vec{\beta}_\nu}\,\underline{\vec{v}_e}\right) - \frac{1}{\gamma_\nu}\underline{\vec{v}_{e\perp}}\right),$$

$$\gamma_\nu = \frac{E_\nu}{m_\nu},$$

matter density

$\parallel$

$\perp$

interaction of neutrino with an electromagnetic field

interaction of neutrino with matter

→ ν *spin procession in matter !!!*

without any electromagnetic field

ν spin evolution in presence of general external fields

M.Dvornikov, A.Studenikin,
JHEP 09 (2002) 016

General types non-derivative interaction with external fields

$$\boxed{\begin{array}{c} -\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p\pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \\ +\frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g'_t}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu, \end{array}}$$

scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor fields:

$$s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}),$$

$$T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$$

Relativistic equation (quasiclassical) for ν spin vector:

$$\begin{aligned} \dot{\vec{\zeta}}_\nu = 2g_a\Big\{A^0[\vec{\zeta}_\nu\times\vec{\beta}] - \frac{m_\nu}{E_\nu}[\vec{\zeta}_\nu\times\vec{A}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{A}\vec{\beta})[\vec{\zeta}_\nu\times\vec{\beta}]\Big\} \\ +2g_t\Big\{[\vec{\zeta}_\nu\times\vec{b}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{b})[\vec{\zeta}_\nu\times\vec{\beta}] + [\vec{\zeta}_\nu\times[\vec{a}\times\vec{\beta}]]\Big\} + \\ +2ig'_t\Big\{[\vec{\zeta}_\nu\times\vec{c}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{c})[\vec{\zeta}_\nu\times\vec{\beta}] - [\vec{\zeta}_\nu\times[\vec{d}\times\vec{\beta}]]\Big\}. \end{aligned}$$

- *Neither* S *nor* π *nor* V *contributes to spin evolution*

- **Electromagnetic interaction**

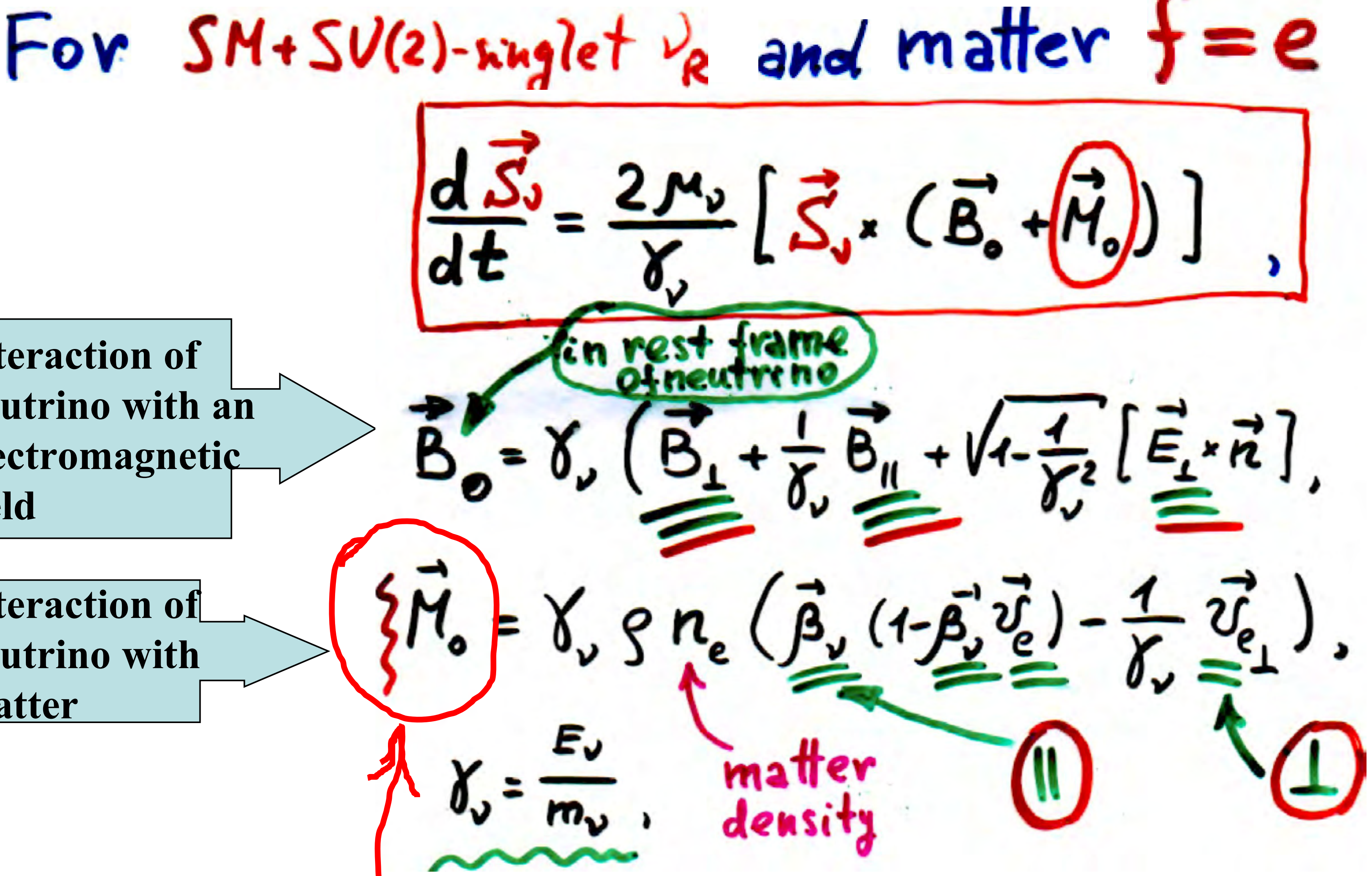
$$T_{\mu\nu} = F_{\mu\nu} = (\vec{E}, \vec{B})$$

- **SM weak interaction**

$$G_{\mu\nu} = (-\vec{P}, \vec{M})$$

$$\vec{M} = \gamma(A^0\vec{\beta} - \vec{A})$$
$$\vec{P} = -\gamma[\vec{\beta}\times\vec{A}],$$

… once more…



For SM+SU(2)-singlet ν_R and matter $f=e$

$$\boxed{\frac{d\vec{S}_\nu}{dt} = \frac{2\mu_\nu}{\gamma_\nu}\left[\vec{S}_\nu \times \left(\vec{B}_0 + \vec{M}_0\right)\right],}$$

in rest frame of neutrino

$$\vec{B}_0 = \gamma_\nu\left(\underline{\vec{B}_\perp} + \frac{1}{\gamma_\nu}\underline{\vec{B}_\parallel} + \sqrt{1-\frac{1}{\gamma_\nu^2}}\left[\underline{\vec{E}_\perp}\times\vec{n}\right]\right),$$

$$\vec{M}_0 = \gamma_\nu \rho n_e \left(\underline{\vec{\beta}_\nu}\left(1-\underline{\vec{\beta}_\nu}\,\underline{\vec{v}_e}\right) - \frac{1}{\gamma_\nu}\underline{\vec{v}_{e\perp}}\right),$$

$$\gamma_\nu = \frac{E_\nu}{m_\nu},$$

matter density

$\parallel$

$\perp$

interaction of neutrino with an electromagnetic field

interaction of neutrino with matter

ν *spin procession in moving matter !!!*

Neutrino ν_e spin evolution in relativistic flux of electrons ($f \equiv e$)

Effects of moving and polarized matter

$$\boxed{\vec{M}_0} = n_e \gamma \vec{\beta} \Big\{ \Big[\rho^{(1)} + \rho^{(2)} \vec{\zeta} \vec{v}_e \Big] (1 - \vec{\beta} \vec{v}_e) + \rho^{(2)} \sqrt{1 - v_e^2} \Big[\frac{\vec{\zeta} \vec{v}_e \, \vec{\beta} \vec{v}_e}{1 + \sqrt{1 - v_e^2}} - \vec{\zeta} \vec{\beta} \Big] + O(\gamma^{-1}) \Big\}$$

- slowly moving matter, $v_e \ll 1$:

$$\vec{M}_0 = \underbrace{n_e \gamma \vec{\beta} \Big(\rho^{(1)}}_{\text{Wolfenstein term (1978)}} \underbrace{- \rho^{(2)} \vec{\zeta} \vec{\beta} \Big)}_{\text{H. Nunokawa, V. Semikoz, A. Smirnov, J.W.F. Valle (1997)}} . \qquad \gamma = \frac{E_\nu}{m_\nu}$$

Wolfenstein term
(1978)

H. Nunokawa,
V. Semikoz,
A. Smirnov,
J.W.F. Valle (1997)

$$\boxed{\vec{\zeta} \equiv \vec{\zeta}_e}$$

mean value of polarization vector of electrons

$$\rho^{(1)} = \frac{G_F}{2\mu\sqrt{2}} \left(1 + 4\sin^2\theta_W\right), \quad \rho^{(2)} = -\frac{G_F}{2\mu\sqrt{2}}$$

for SM + SU(2)-singlet ν_R

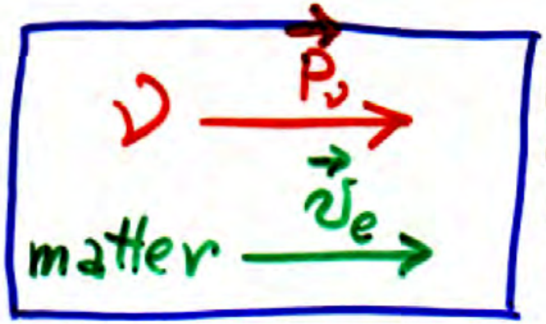
- relativistic flux of e, $v_e \sim 1$:

$$\vec{M}_0 = n_e \gamma \vec{\beta} \left(\rho^{(1)} + \rho^{(2)} \vec{\xi}_e \vec{v}_e\right) \boxed{\left(1 - \vec{\beta}_\nu \vec{v}_e\right)}$$

!

$c=1$ speed of light

In case of

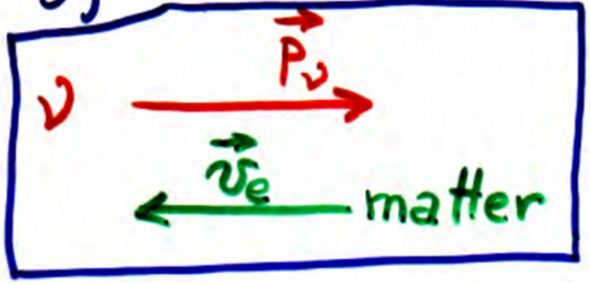


matter effect contribution to ν spin evolution equation is suppressed !

invariant electron number density

$$\boxed{n_e = \frac{n_0}{\sqrt{1 - v_e^2}}}$$

In case of



ν and relativistic matter ($v_e \sim 1$) motion in opposite directions matter term gets its maximum

$$\vec{M}_0^{max} = \boxed{\frac{2}{\sqrt{1-v_e^2}}} \vec{M}_0^{(v_e \ll 1)}$$

$\gg 1$ if $v_e \sim 1$

$\Rightarrow$ substantial increase of matter effects in ν oscillations !

flavour oscillations

!

Unpolarized but moving matter

$(\vec{\zeta}_e = 0,\ v_e \neq 0)$

MSW effect

Resonance condition:

- $$\boxed{\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta = \sqrt{2}\, G_F\, n_e^{(0)} \frac{1-\vec{\beta}_\nu \vec{v}_e}{\sqrt{1-v_e^2}}}$$

$n_e^{(0)}$ – invariant matter density in r.f.

A.Lobanov,
A.Grigoriev,
A.Studenikin,
Phys.Lett.B 535 (2002) 187

If $\xleftrightarrow{\quad}\vec{\beta}_\nu$, $\xleftrightarrow{\quad}\vec{v}_e$:

- $$\left.\frac{1-\vec{\beta}_\nu \vec{v}_e}{\sqrt{1-v_e^2}}\right|_{\beta_\nu\approx 1} = \begin{cases} \left.\sqrt{\dfrac{1-v_e}{1+v_e}}\right|_{v_e\approx 1} \approx \dfrac{\sqrt{1-v_e}}{\sqrt{2}} & \nu \xrightarrow{\qquad} \vec{\beta}_\nu,\quad \text{matter} \xrightarrow{\qquad} \vec{v}_e \\ \left.\sqrt{\dfrac{1+v_e}{1-v_e}}\right|_{v_e\approx 1} \approx \dfrac{\sqrt{2}}{\sqrt{1-v_e}} & \nu \xrightarrow{\qquad} \vec{\beta}_\nu,\quad \text{matter} \xleftarrow{\qquad} \vec{v}_e \end{cases}$$

Relativistic motion of matter along (against) neutrino propagation could provide resonance in $\nu \leftrightarrow \nu$ if matter density $n_e^{(0)}$ is too high (low) for resonance appearance in non-moving matter.

{in restframe of matter}

★ Resonance condition in Lorentz invariant form :

A.S., Phys.Atom.Nucl.2004

$$\boxed{\Delta \cos 2\theta = \sqrt{2} G_F n_0 p_\mu u^\mu} \quad , \qquad \Delta = \delta m_\nu^2 / 2E \quad ,$$

$$p_\mu = m\gamma(1, \boldsymbol{\beta}) \quad , \quad u_\mu = \gamma_e(1, \mathbf{v_e}) \quad , \quad \gamma_e = (1 - v_e^2)^{1/2}$$

#3 $\nu_L \leftrightarrow \nu_R$ and $\nu_\ell \leftrightarrow \nu_{\ell'}$, $\ell \neq \ell'$

(neutrino spin and flavour oscillations)

in moving and polarized matter

⇒ **Now we know**

matter motion can significantly change the neutrino oscillation pattern

#4

New mechanism of e.m. radiation by ν in matter and e.m. fields, and gravitational fields

$$\boxed{\nu \xrightarrow[\text{environment}]{\text{background}} \nu + \gamma,}$$

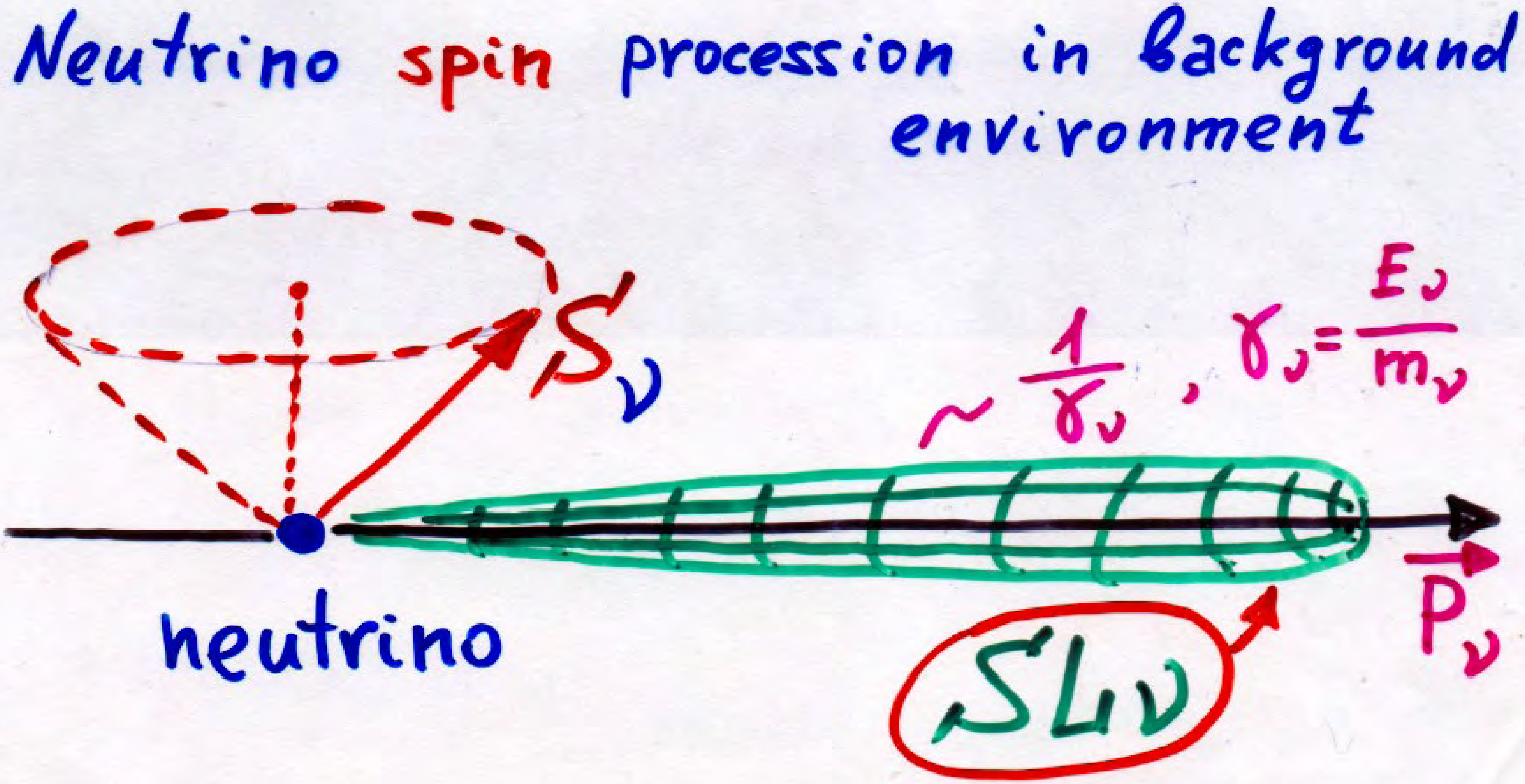
‖ "Spin Light of Neutrino": "SLν"

A.Lobanov, A.Studenikin,
Phys.Lett.B 564 (2003) 27

Quasi-classical theory of spin light of neutrino in matter and gravitational field

$SL\nu$

A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,
Phys.Lett. B 601 (2004) 171;
M.Dvornikov, A.Grigoriev, A.Studenikin, Int.J.Mod.Phys. D 14 (2005) 309



Neutrino spin $\vec{S}$ precession
is described by the **generalized Bargmann-Michel-Telegdi equation**:

A.Egorov, A.Lobanov, A.Studenikin
PLB 491 (2000) 137, PLB 515 (2001) 94

$$\boxed{\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma}\left[\vec{S}\times(\vec{B}_0+\vec{M}_0)\right]}$$

$$\vec{B}_0 = \gamma(\vec{B}_\perp + \tfrac{1}{\gamma}\vec{B}_\parallel + \sqrt{1-\tfrac{1}{\gamma^2}}[\vec{E}_\perp\times\vec{n}]),$$

$\vec{n} = \vec{\beta}/\beta$,

speed of neutrino

$\vec{F}_\perp$ and $\vec{F}_\parallel$ are transversal and longitudinal e.m. fields $\vec{F} = \vec{B}, \vec{E}$ in laboratory frame.

$$F_{\mu\nu} \longrightarrow E_{\mu\nu} = F_{\mu\nu} + G_{\mu\nu}$$

interaction with electro-magnetic field

weak interaction with matter

A.Lobanov, A.Studenikin,
Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

SLν radiation power:

$$\boxed{I = \frac{16}{3}\mu_\nu^4\left[\left(2\mu_\nu(\vec{B}_0+\vec{M}_0)^2\right)^2 + (\dot{\vec{B}}_0+\dot{\vec{M}}_0)^2\right],}$$

$$\vec{M}_0 = \vec{M}_{0\parallel} + \vec{M}_{0\perp}, \qquad \gamma_\nu = \frac{p_0^\nu}{m_\nu}$$

matter density

$$\vec{M}_{0\parallel} = \gamma_\nu\vec{\beta}_\nu\frac{n_0}{\sqrt{1-v_e^2}}\left\{\rho^{(1)}\left(1-\frac{\vec{v}_e\cdot\vec{\beta}_\nu}{1-\gamma_\nu^{-2}}\right) - \rho^{(2)}\left(\vec{\xi}_e\cdot\vec{\beta}_\nu\sqrt{1-v_e^2} + \frac{\vec{\xi}_e\cdot\vec{v}_e\,\vec{\beta}_\nu\cdot\vec{v}_e}{1+\sqrt{1-v_e^2}}\right)\frac{1}{1-\gamma_\nu^{-2}}\right\},$$

$$\vec{M}_{0\perp} = -\frac{n_0}{\sqrt{1-v_e^2}}\left\{\vec{v}_{e\perp}\left(\rho^{(1)} + \rho^{(2)}\frac{\vec{\xi}_e\vec{\beta}_\nu}{1+\sqrt{1-v_e^2}}\right) + \vec{\xi}_{e\perp}\rho^{(2)}\sqrt{1-v_e^2}\right\}.$$

Now we know:

#4 new mechanism of e.m. radiation by ν in matter (with or without e.m. field being superimposed)

— spin light of neutrino —

that must be important for dense astrophysical (SN,... gamma-ray Bursts) cosmological (the early Universe) environments.

...however !!!

Quantum treatment of neutrino in matter

A.Studenikin, J.Phys.A: Math.Gen 39 (2006) 6769

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Atom.Nucl. 69 (2006) 1940

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199

Grav. & Cosm. 11 (2005) 132

I.Pivovarov, A.Studenikin, PoS (HEP2005) 191

Standard model electroweak interaction of a flavour neutrino in matter (*f* = *e*)

Interaction Lagrangian (it is supposed that **matter contains only electrons**)

$$\boxed{\begin{aligned} L_{int} = &-\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1+\gamma_5)\nu_e - \bar{e}\gamma^\mu(1-4\sin^2\theta_W+\gamma_5)e\right]Z_\mu \\ &-\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu(1+\gamma_5)eW^+_\mu - \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu_e W^-_\mu \end{aligned}}$$

⇨ **Charged current** interactions contribution to neutrino potential in matter

$$\Delta L^{CC}_{eff} = \sqrt{2}\,G_F\left\langle \bar{e}\gamma^\mu(1+\gamma_5)e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

⇨ **Neutral current** interactions contribution to neutrino potential in matter

$$\Delta L^{NC}_{eff} = -\frac{G_F}{\sqrt{2}}\left\langle \bar{e}\gamma^\mu[(1-4\sin\theta^2_W)+\gamma_5]e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

Matter current and polarization

When the **electron field bilinear**

$$\left\langle \bar{e}\gamma^{\mu}(1+\gamma_5)e \right\rangle$$

is **averaged** over the background

$$\langle \bar{e}\gamma_0 e\rangle \sim density\,,$$

$$\langle \bar{e}\gamma_i e\rangle \sim velocity\,, \qquad \boldsymbol{i = 1, 2, 3}$$

$$\langle \bar{e}\gamma_\mu\gamma_5 e\rangle \sim spin\,,$$

it can be replaced by the **matter** (electrons) **current**

☆ $$\boxed{j^{\mu} = (n, n\mathbf{v}),}$$

invariant number density

speed of matter

and **polarization**

☆ $$\boxed{\lambda^{\mu} = \left(n(\boldsymbol{\zeta}\mathbf{v}), n\boldsymbol{\zeta}\sqrt{1-v^2} + \frac{n\mathbf{v}(\boldsymbol{\zeta}\mathbf{v})}{1+\sqrt{1-v^2}} \right)}$$

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L^{CC}_{eff} + \Delta L^{NC}_{eff} = -f^{\mu}\Big(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\Big)$$

where

$$f^{\mu} = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^{2}\theta_W)j^{\mu} - \lambda^{\mu}\Big)$$

matter current

matter polarization

$$\boxed{\Big\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\Big\}\Psi(x) = 0}$$

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,’88; J.Panteleone,’91; K.Kiers, N.Weiss, M.Tytgat,’97-’98; P.Manheim,’88; D.Nötzold, G.Raffelt,’88; J.Nieves,’89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang’91; M.Kachelriess,’98; A.Kusenko, M.Postma,’02.

A.Studenikin, A.Ternov, hep-ph/0410297; ***Phys.Lett.B*** **608 (2005) 107**

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Neutrino wave function and energy spectrum in matter (I)

In the **rest frame** of **unpolarized matter**

$$\boxed{\left\{i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m\right\}\Psi(x) = 0}$$

$$f^\mu = \frac{1}{2\sqrt{2}}\tilde{G}_F(n,0,0,0), \quad \tilde{G}_F = G_F(1+4\sin^2\theta_W)$$

The Hamiltonian form of the equation:

$$\boxed{i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}_{matt}\Psi(\mathbf{r},t)},$$

where

$$\boxed{\hat{H}_{matt} = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta}m + \hat{V}_{matt},}$$

$$\boxed{\hat{V}_{matt} = \frac{1}{2\sqrt{2}}(1+\gamma_5)\tilde{G}_F n}$$

number density of background matter (electrons)

$$\hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_0\boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0,$$

$$\tilde{G}_F = G_F(1+4\sin^2\theta_W)$$

The form of the Hamiltonian implies that the **operators of the momentum, $\hat{\mathbf{p}}$**, and **longitudinal polarization**, $\hat{\Sigma}\mathbf{p}/p$, are the **integrals of motion:**

$$\boxed{\frac{\hat{\Sigma}\mathbf{p}}{p}\Psi(\mathbf{r},t) = s\Psi(\mathbf{r},t),} \quad \hat{\Sigma} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix}, \quad \boxed{s = \pm 1}$$

positive-helicity

negative-helicity

In the relativistic limit the **negative-helicity** neutrino state is dominated by the **left-handed chiral** state: $\nu_- \approx \nu_L$, $\nu_+ \approx \nu_R$.

Stationary states

$$\Psi(\mathbf{r},t)=e^{-i(E_\varepsilon t-\mathbf{pr})}u(\mathbf{p},E_\varepsilon),$$

neutrino wave function in matter

and

$$E_\varepsilon=\varepsilon\sqrt{\mathbf{p}^2\Big(1-s\alpha\frac{m}{p}\Big)^2+m^2}+\alpha m$$

neutrino energy spectrum in matter

$s\ =\ \pm1$ for two **helicity** states ,

where the **matter density parameter** $\alpha=\dfrac{1}{2\sqrt{2}}\tilde{G}_F\dfrac{n}{m}$,

J.Panteleone, 1991
(if NC interaction
were left out)

$\frac{1}{2\sqrt{2}}\tilde{G}_F n\sim 1\ eV$ for $n=10^{37}\ cm^{-3}$ ← density of matter in a neutron star

Neutrino energy in the background matter depends on the state of the neutrino **longitudinal** polarization (helicity), i.e. in the relativistic case the left-handed **and right**-handed neutrinos with equal momenta have different energies.

Neutrino wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon t-\mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;e^{i\delta}\\ s\varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;e^{i\delta}\end{pmatrix}}$$

A.Studenikin, A.Ternov, hep-ph/0410297;

***Phys.Lett.B* 608 (2005) 107;**

$$\eta=\text{sign}\left(1-s\alpha\frac{m}{p}\right),\ \delta=\arctan\left(p_2/p_1\right)$$

$$\boxed{E_\varepsilon-\alpha m=\varepsilon\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}}$$

A.Grigoriev, A.Studenikin, A.Ternov,
***Phys.Lett.B* 622 (2005) 199**

The quantity $\boxed{\varepsilon=\pm1}$ splits the solutions into the two branches that in the limit of **vanishing matter density,** $\boxed{\alpha\to0,}$ reproduce the **positive** and **negative-frequency** solutions, respectively.

Neutrino flavour oscillations in matter

Consider the two flavour neutrinos, ν_e and ν_μ , propagating in electrically neutral matter of **electrons** , **protons** and **neutrons** : $n_e = n_p$.

The matter density parameters are

$$\boxed{\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(1+4\sin^2\theta_W)+n_p(1-4\sin^2\theta_W)-n_n\Big)}$$

and

$$\boxed{\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(4\sin^2\theta_W-1)+n_p(1-4\sin^2\theta_W)-n_n\Big)}$$

, respectively.

The energies of the **relativistic** **active** neutrinos are

$$\boxed{E^{s=-1}_{\nu_e,\nu_\mu} \approx E_0 + 2\alpha_{\nu_e,\nu_\mu} m_{\nu_e,\nu_\mu}} ,$$

and the energy difference

$$\boxed{\Delta E \equiv E^{s=-1}_{\nu_e} - E^{s=-1}_{\nu_\mu} = \sqrt{2}G_F n_e}$$

⇨ **MSW effect**

Neutrino processes in matter

- **Neutrino reflection from interface between vacuum and matter**
- **Neutrino trapping in matter**
- **Neutrino-antineutrino pair annihilation at interface between vacuum and matter**
- **Spontaneous neutrino-antineutrino pair creation in matter**

L.Chang, R.Zia,’88
A.Loeb,’90
J.Panteleone,’91
K.Kiers, N.Weiss, M.Tytgat,’97-’98
M.Kachelriess,’98
A.Kusenko, M.Postma,’02 H.Koers,’04
A.Studenikin, A.Ternov,’04
A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, ’05
I.Pivovarov, A.Studenikin,’05
A.Ivanov, A.Studenikin, ‘05

Neutrino reflection from interface between vacuum and matter

$$1 < \alpha_1 < 2$$

If the neutrino energy in **vacuum** E

is less than the neutrino minimal energy in **medium** $\alpha_1 m + m$

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}$$

matter density parameter

$$\boxed{m \leq E < \alpha_1 m + m}$$

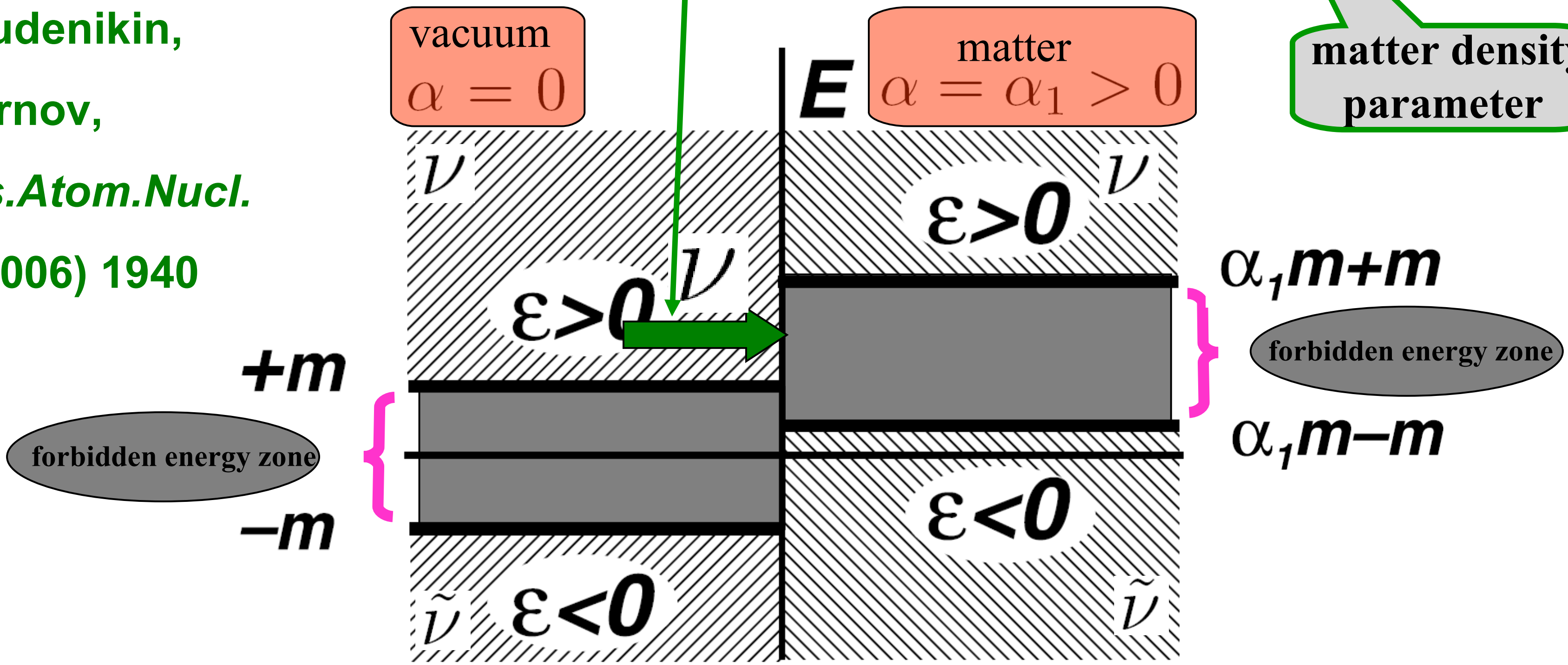
A.Grigoriev,

A.Studenikin,

A.Ternov,

Phys.Atom.Nucl.

69 (2006) 1940



then the appropriate energy level inside the medium is **not accessible** for neutrino → neutrino **is reflected** from the interface.

Neutrino propagation in matter

I.Pivovarov, A.Studenikin, PoS(HEP2005) 2006, 191

Equation for neutrino **Green function** in matter

$$\boxed{\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu \left(1+\gamma_5\right) f^\mu - m \right\} G\left(x\right) = -\delta(x)}\ , \quad f^\mu = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^2\theta_W)j^\mu - \lambda^\mu\Big),$$

matter current and polarization

in the **momentum representation**

$$\boxed{\left(\hat{p} - m - \hat{f} P_L\right) G(p) = -1}\ , \qquad P_L = \frac{1+\gamma_5}{2}, \quad P_R = \frac{1-\gamma_5}{2} \ .$$

Neutrino Green function in matter

$$\boxed{G_{matt}(p) = \frac{-\left(p^2-m^2\right)\left(\hat{p}+m\right) + \hat{f}\left(\hat{p}-m\right) P_L \left(\hat{p}+m\right) - f^2 \hat{p} P_L + 2\left(fp\right) P_R \left(\hat{p}+m\right)}{\left(p^2-m^2\right)^2 - 2\left(fp\right)\left(p^2-m^2\right) + f^2 p^2}}$$

Spin Light

of Neutrino in matter

Quantum theory of $\boxed{SL\nu}$

- 🔴 A.Studenikin, A.Ternov, *Phys. Lett.**B*** **608** (2005) 107;
- 🟡 A.Grigoriev, A.Studenikin, A.Ternov, *Phys. Lett.**B*** **622** (2005) 199, hep-ph/0502231, hep-ph/0507200;
- 🟢 A.Grigoriev, A.Studenikin, A.Ternov, *Grav. & Cosm.* **11** (2005) 132;

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Atom.Nucl. 69 (2006) 1940,
hep-ph/0502210, hep-ph/0511311,
hep-ph/0511330;

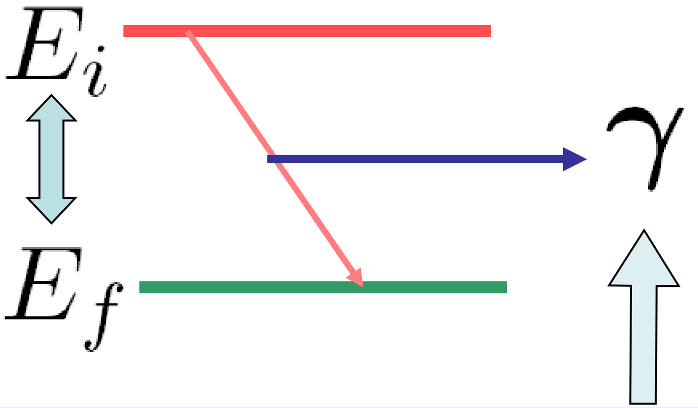
A.Studenikin, A.Ternov, hep-ph/0410296, hep-ph/0410297

Quantum theory of spin light of neutrino (I)

$SL\nu$

Quantum treatment of *spin light of neutrino* in matter shown**s** that this process originates from the **two subdivided phenomena:**

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,



$$E=\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}+\alpha m$$

$$s=\pm 1$$

the radiation of the photon in the process of the neutrino transition from the **“excited” helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

hep-ph/0507200, hep-ph/0502210,

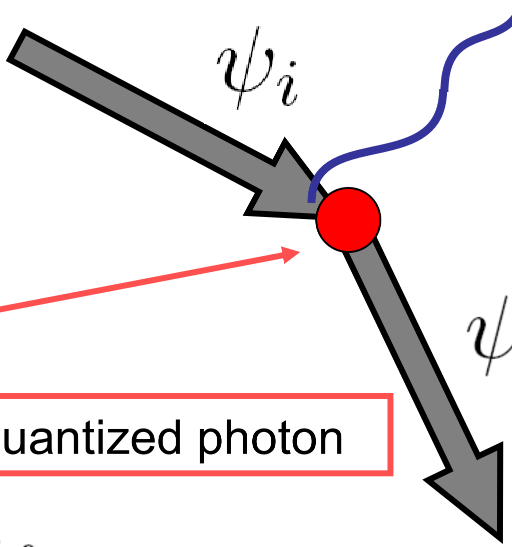
hep-ph/0502231

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

Quantum theory of spin light of neutrino $SL\nu$

Within the **quantum approach**, the corresponding Feynman diagram is the one-photon emission diagram with the **initial** and **final** neutrino states described by the **“broad lines”** that account for the neutrino interaction with matter.



Neutrino magnetic moment interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$\boxed{S_{fi} = -\mu\sqrt{4\pi}\int d^4x\bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)} \,,$$

$$\boxed{\hat{\mathbf{\Gamma}} = i\omega\{[\mathbf{\Sigma}\times\boldsymbol{\varkappa}] + i\gamma^5\mathbf{\Sigma}\}} \,,$$

$k^\mu = (\omega, \mathbf{k}), \boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

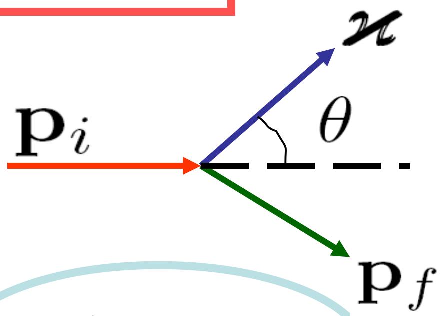
Spin light of neutrino photon's energy

$SL\nu$ transition amplitude after integration :

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}}\, 2\pi\delta(E_f - E_i + \omega) \int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\boldsymbol{\Gamma}}\mathbf{e}^*)e^{i\mathbf{kr}}\psi_i(\mathbf{r})$$

Energy-momentum conservation

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$



For **electron neutrino** moving in matter composed of **electrons**

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$$

$$\omega = \frac{2\alpha m p_i\left[(E_i - \alpha m) - (p_i + \alpha m)\cos\theta\right]}{(E_i - \alpha m - p_i\cos\theta)^2 - (\alpha m)^2}$$

photon energy

- In the radiation process: $s_i = -1 \Longrightarrow s_f = +1$ **neutrino self-polarization**
- For not very high densities of matter , $\tilde{G}_F n/m \ll 1$, in the linear approximation over α

$$\omega = \frac{\beta}{1 - \beta\cos\theta}\omega_0 \quad , \quad \omega_0 = \frac{\tilde{G}_F}{\sqrt{2}}n\beta$$

neutrino speed in vacuum

Spin light transition rate (III)

$SL\nu$

transition rate for different neutrino momentum p
and **matter density parameter** $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$

“relativistic” case

$p \gg m$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^2m, & \text{for } \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^2m^2p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$p \ll m$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^3, & \text{for } \alpha \ll 1, \\ \frac{512}{5}\mu^2\alpha^6p^3, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

neutrino momentum

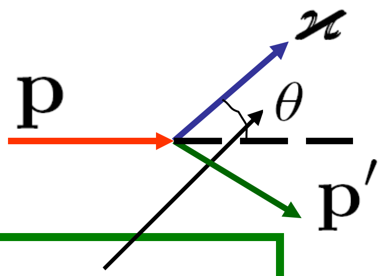
mass

neutrino magnetic moment

Spin light radiation power

$SL\nu$

radiation power angular distribution :



$$I = \mu^2 \int_0^{\pi} \omega^4 \left[(\tilde{\beta}\tilde{\beta}' + 1)(1 - y\cos\theta) - (\tilde{\beta} + \tilde{\beta}')(\cos\theta - y) \right] \frac{\sin\theta}{1 + \tilde{\beta}' y} d\theta$$

$$\tilde{\beta} = \frac{p + \alpha m}{E - \alpha m}, \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}, \quad y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m}, \quad \omega = \frac{2\alpha m p \left[(E - \alpha m) - (p + \alpha m)\cos\theta \right]}{(E - \alpha m - p\cos\theta)^2 - (\alpha m)^2}$$

“relativistic” case

$\boxed{p \gg m}$

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{4}{3}\mu^2\alpha^2 m^2 p^2, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$\boxed{p \ll m}$

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll 1, \\ \frac{1024}{3}\mu^2\alpha^8 p^4, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

Spin light photon average energy

$$\langle\omega\rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

See also:
A.Lobanov,
Phys.Lett.B 619
(2005) 136

"relativistic" case

$p \gg m$

$$\langle\omega\rangle \simeq \begin{cases} 2\alpha\frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

"non-relativistic" case

$p \ll m$

$$\langle\omega\rangle \simeq \begin{cases} 2p\alpha, & \text{for } \alpha \ll 1, \\ \frac{10}{3}p\alpha^2, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

$\alpha \ll \frac{m}{p}$

$$\omega = 2.37 \times 10^{-7} \left(\frac{n}{10^{30} cm^{-3}}\right) \left(\frac{E}{m_\nu}\right)^2 eV$$

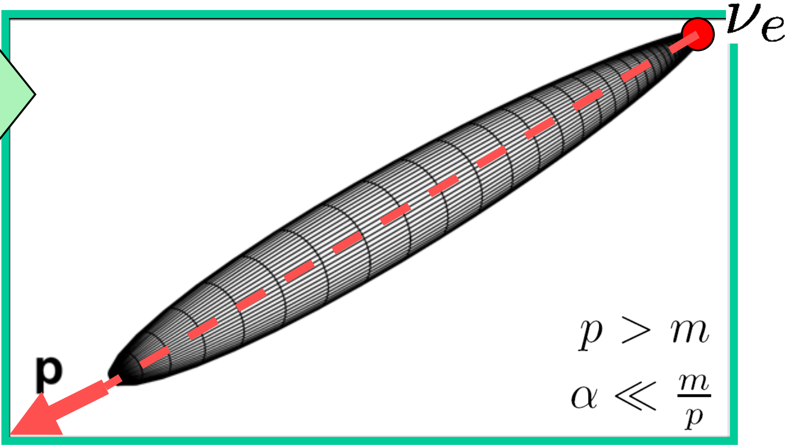
energy range of $SL\nu$

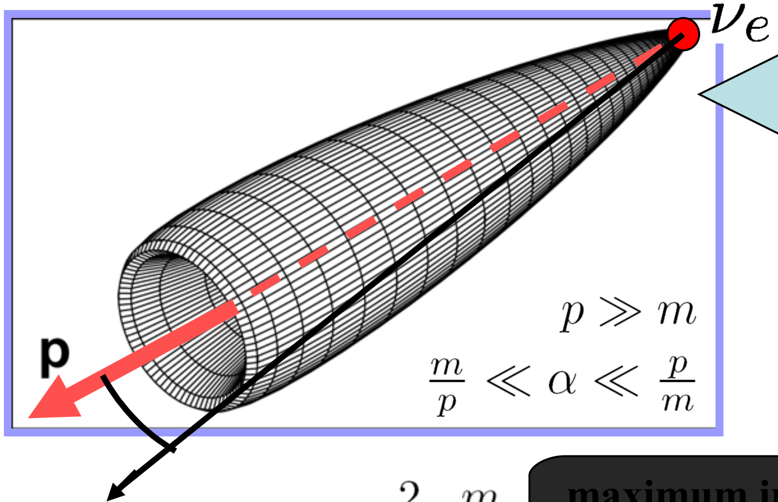
span up to **gamma-rays**

Spatial distribution of radiation power

From the **angular distribution** of $\boxed{SL\nu}$

$$I=\mu^2\int_0^{\pi}\omega^4\left[(\tilde\beta\tilde\beta'+1)(1-y\cos\theta)-(\tilde\beta+\tilde\beta')(\cos\theta-y)\right]\frac{\sin\theta}{1+\tilde\beta' y}d\theta$$





for $p/m=5$ **and** $\alpha=0.01$

$\boxed{n\approx 10^{35}cm^{-3}}$

neutrino momentum

mass

matter density

$$\cos\theta_{max}\simeq 1-\frac{2}{3}\alpha\frac{m}{p}$$

maximum in radiation power distribution

for $p/m=10^3$ **and** $\alpha=100$

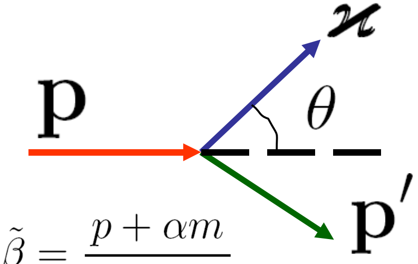
$\boxed{n\approx 10^{39}cm^{-3}}$

increase of matter density

projector-like distribution → **cap-like**

Polarization properties of $SL\nu$ photons (II)

★ Radiation power of **circularly polarized** photons:



$$\boxed{I^{(l)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} S_l \sin\theta d\theta}\;, \quad \tilde{\beta}' = \frac{p'-\alpha m}{E'-\alpha m}\,, \quad \tilde{\beta} = \frac{p+\alpha m}{E-\alpha m}\,,$$

$$y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E-\alpha m - p\cos\theta}{\alpha m}\,,$$

$$\omega = \frac{2(E-\alpha m)(K\beta - 1)}{K^2-1}$$

where

$$\boxed{S_l = \frac{1}{2}\left(1+l\beta'\right)\left(1+l\beta\right)\left(1-l\cos\theta\right)\left(1+ly\right)}$$

$l = \pm 1$ correspond to the photon **right** and **left circular polarizations** .

★ In the limit of **low matter density** $\alpha \ll 1$:

$$E_0 = \sqrt{p^2+m^2}$$

$$\boxed{I^{(l)} \simeq \frac{64}{3}\mu^2\alpha^4 p^4 \left(1 - l\frac{p}{2E_0}\right)}\;, \qquad I^{(+1)} > I^{(-1)}, \text{ however } \quad I^{(+1)} \sim I^{(-1)} \,.$$

★ In **dense matter** $\left(\alpha \gg \frac{m}{p} \text{ for } p \gg m, \text{ and } \alpha \gg 1 \text{ for } p \ll m\right)$:

$$\boxed{\begin{array}{ccc} I^{(+1)} & \simeq & I \\ I^{(-1)} & \simeq & 0 \end{array}}$$

In a dense matter $SL\nu$ is right-circular polarized.

Experimental identification of $SL\nu$ from astrophysical and cosmological sources

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199, **hep-ph/0507200**

- **Fireball model of GRBs** — **B.Zhang, P.Meszaros,** Int.J.Mod.Phys. **A19** (2004) 2385; **T.Piran,** Rev.Mod.Phys. **76** (2004) 1143.
- **Gamma-rays** can be expected to be produced during **collapses** or **coalescence** processes of neutron stars, owing to $\boxed{SL\nu}$ in dense matter.
- Another favorable situation for effective $\boxed{SL\nu}$ production can be realized during

a **neutron star** being **”eaten up”** by the **black hole** at the center of our Galaxy .

For estimation, consider a neutron star with mass $M_{NS} \sim 3M_{\odot}$, $M_{\odot} = 2 \cdot 10^{33} g$

⇒ $n \sim 8 \cdot 10^{38}\ cm^{-3}$, matter density parameter $\alpha \sim 23$,

if $m_{\nu} \sim 0.1\ eV$.

Then for **relativistic neutrinos** $(p \gg m)$

the $\boxed{SL\nu}$ **photon energy** $\langle\omega\rangle \sim \dfrac{1}{3}p$ ← **totally polarized** **gamma-rays.**

It is possible to have $\tau = \frac{1}{\Gamma_{\boxed{SL\nu}}} << $ **age of the Universe ?**

For ultra-relativistic ν
with momentum $p \sim 10^{20} eV$
and magnetic moment $\mu \sim 10^{-10} \mu_B$
in very dense matter $n \sim 10^{40} cm^{-3}$
from

$$\Gamma_{\boxed{SL\nu}} = 4\mu^2\alpha^2 m_\nu^2 p$$

$$p \gg m_{plasmon}$$

recently also discussed by A.Kuznetsov, N.Mikheev, 2006

A.Lobanov, A.S., PLB 2003; PLB 2004
A.Grigoriev, A.S., PLB 2005
A.Grigoriev, A.S., A.Ternov, PLB 2005

$$\alpha m_\nu = \frac{1}{2\sqrt{2}} G_F n (1 + sin^2\theta_W)$$

it follows that

$$\tau = \frac{1}{\Gamma_{\boxed{SL\nu}}} = 1.5 \times 10^{-8} s$$

Spin Light

of Electron in matter

SLe

… a method of studying charged particles interaction in matter…

A.S.,
J.Phys.A: Math. Gen. 39 (2006) 6769

Grigoriev, Shinkevich, Studenikin, Ternov,
Trofimov, hep-ph/0611128,
Russ.Phys.J 50 (2007) 596,
Grav.&Cosm. 14 (2008)

Standard model electroweak interaction of an electron in matter (*e*, *p*, *n*)

Interaction Lagrangian (for **matter composed of electrons, protons and neutrons**)

$$L_{int} = -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1+\gamma_5)\nu_e \underline{-} \bar{e}\gamma^\mu(1 \underline{- 4\sin^2\theta_W} + \gamma_5)e\right]Z_\mu$$

Neutral current interactions contribution to electron potential in electrically neutral matter ($n_e = n_p$)

$$\Delta L^{(e)}_{eff} = -f^\mu\left(\bar{e}\gamma_\mu\frac{1-4\sin^2\theta_W+\gamma^5}{2}e\right),$$

matter current

matter polarization

where

$$f^\mu = \frac{G_F}{\sqrt{2}}(-j^\mu_n + \lambda^\mu_n).$$

A.Studenikin, J.Phys.A: Math. Gen. 39 (2006) 6769

Modified Dirac equation for electron in matter

$$\left\{ i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1-4\sin^2\theta_W+\gamma_5)\tilde{f}^\mu - m_e \right\}\Psi_e(x) = 0,$$

where

$$\tilde{f}^\mu = -f^\mu = \frac{G_F}{\sqrt{2}}(j_n^\mu - \lambda_n^\mu)$$

matter polarization

matter current

It is suppose that there is a macroscopic amount of neutrons in the scale of an electron de Broglie wave length. Therefore, **the interaction of electron with the matter (neutrons) is coherent.**

This is the most general equation of motion of an neutrino in which the effective potential accounts for **neutral-current** interactions with the background electrically neutral matter and also for the possible effects of matter **motion** and **polarization**.

Electron wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon^{(e)}t-\mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\ \sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta}\\ s\varepsilon\eta\sqrt{1-\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\ \sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\eta\sqrt{1-\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta}\end{pmatrix}}$$

A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, I.Trofimov, hep-ph/0611128, Russ.Phys.J 50 (2007) 596

$\eta = \text{sign}(1-s\alpha\frac{m_e}{p})$, $\delta = \arctan(p_2/p_1)$

$$\boxed{E_\varepsilon^{(e)}=\varepsilon\sqrt{\mathbf{p}^2\Big(1-s\alpha_n\frac{m_e}{p}\Big)^2+m_e^2}+c\alpha_n m_e}\ ,\quad \text{where } c=1-4\sin^2\theta_W$$

A.S., J.Phys.A: Math. Gen. 39 (2006) 6769

$$\alpha_n=\frac{1}{2\sqrt{2}}G_F\frac{n_n}{m_e},$$

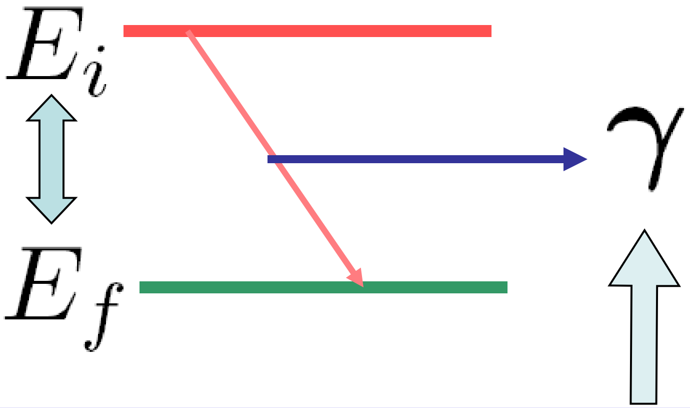
The quantity $\boxed{\varepsilon=\pm1}$ splits the solutions into the two branches that in the limit of **vanishing matter density,** $\boxed{\alpha\rightarrow 0,}$ reproduce the **positive** and **negative-frequency** solutions, respectively.

Quantum theory of spin light of electron (I)

SLe

Spin light of electron in matter
originates from the **two subdivided phenomena:**

the **shift** of the electron **energy levels** in the presence of the background matter, which is different for the two opposite electron helicity states,



$$E_\varepsilon^{(e)} = \varepsilon\sqrt{\mathbf{p}^2\left(1 - s\alpha_n\frac{m_e}{p}\right)^2 + m_e^2} + c\alpha_n m_e$$

$$s = \pm 1$$

the radiation of the photon in the process of the electron transition from the **"excited" helicity state** to the **low-lying helicity state** in matter

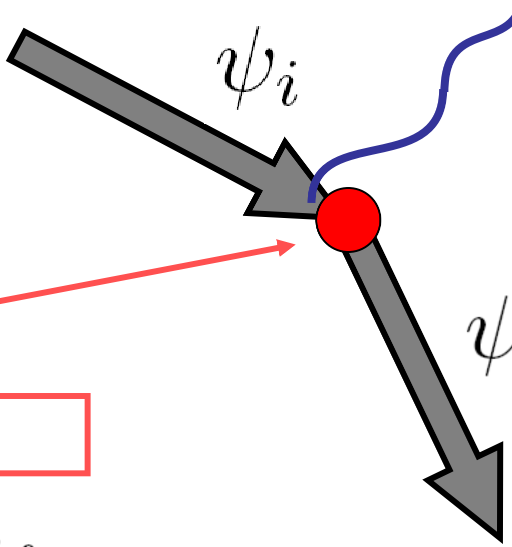
electron-spin self-polarization effect in the matter

A.S., J.Phys.A: Math. Gen. 39 (2006) 6769

Theory of spin light of electron

SLe

The corresponding Feynman diagram is the one-photon emission diagram with the **initial** and **final** electron states described by the **“broad lines”** that account for the electron interaction with matter.



Electron interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$\boxed{S_{fi} = -ie\sqrt{4\pi}\int d^4x\bar{\psi}_f(x)\gamma^\mu e_\mu{}^* \frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)},$$

$k^\mu = (\omega, \mathbf{k})$, $\boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

Order-of-magnitude estimation :

$$\boxed{R = \frac{\Gamma_{SLe}}{\Gamma_{SL\nu}} \sim \frac{e^2}{\omega^2 \mu^2}},$$

A.S.,
J.Phys.A: Math. Gen.
39 (2006) 6769

then for $\mu \sim 10^{-10} \mu_0$ **and** $\omega \sim 5\ MeV$

$$\boxed{R \sim 10^{18}},$$

under these conditions $\boxed{SLe}$

is more effective than $\boxed{SL\nu}$

Grigoriev, Shinkevich, Studenikin, Ternov, Trofimov, hep-ph/0611128, Russ.Phys.J 50 (2007) 596; Grav.&Cosmology 14 (2008)

Exact calculations of $\boxed{SLe}$

Transition rate

$$\boxed{\Gamma = \frac{e^2}{2}\int_0^{\pi} \frac{\omega}{1+\tilde{\beta}'_e\, y} S \sin\theta \, d\theta}$$

and power

$$\boxed{\mathrm{I} = \frac{e^2}{2}\int_0^{\pi} \frac{\omega^2}{1+\tilde{\beta}'_e\, y} S \sin\theta \, d\theta}$$

where $$S = (1 - y\cos\theta)\left(1 - \tilde{\beta}_e\tilde{\beta}'_e - \frac{{m_e}^2}{\tilde{E}\tilde{E}'}\right),$$

$$\tilde{\beta}_e = \frac{p + \alpha_n m_e}{\tilde{E}}, \quad \tilde{\beta}'_e = \frac{p' - \alpha_n m_e}{\tilde{E}'}, \quad \tilde{E} = E - c\alpha_n m_e \;,$$

energy and momentum of final neutrino

$$E' = E - \omega, \quad p' = K_e\,\omega - p,$$

$$K_e = \frac{\tilde{E} - p\cos\theta}{\alpha_n m_e}, \quad y = \frac{\omega - p\cos\theta}{p'}.$$

Spin light of electron in matter (n)

SLe

Transition rate

$$\boxed{\Gamma = \frac{e^2 m^3}{4p^2} \frac{(1+2a)\left[(1+2b)^2 \ln(1+2b) - 2b(1+3b)\right]}{(1+2b)^2 \sqrt{1+a+b}}},$$

and power

$$\boxed{\mathrm{I} = \frac{e^2 m^4}{6p^2} \frac{(1+a)\left[3(1+2b)^3 \ln(1+2b) - 2b(3+15b+22b^2)\right] - 8b^4}{(1+2b)^3}},$$

where $a = \alpha_n^2 + p^2/m_e^2$, $b = 2\alpha_n p/m_e$.

Spin light photon average energy

SLe

$$\langle\omega\rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

In case $\boxed{\alpha_n \gg m_e/p}$

$$\boxed{\langle\omega\rangle \simeq \begin{cases} p, & \text{for } \frac{m_e}{p} \ll \alpha_n \ll \frac{p}{m_e}, \\ \alpha_n m_e, & \text{for } \alpha_n^{-1} \ll \frac{p}{m_e} \ll \alpha_n, \end{cases}}$$

(it is supposed $\ln\frac{4\alpha_n p}{m_e} \gg 1$**)**

- **For relativistic electrons the emitted photons energy is in the range of gamma-rays**

- SLe **photon caries away nearly the whole of the initial electron energy**

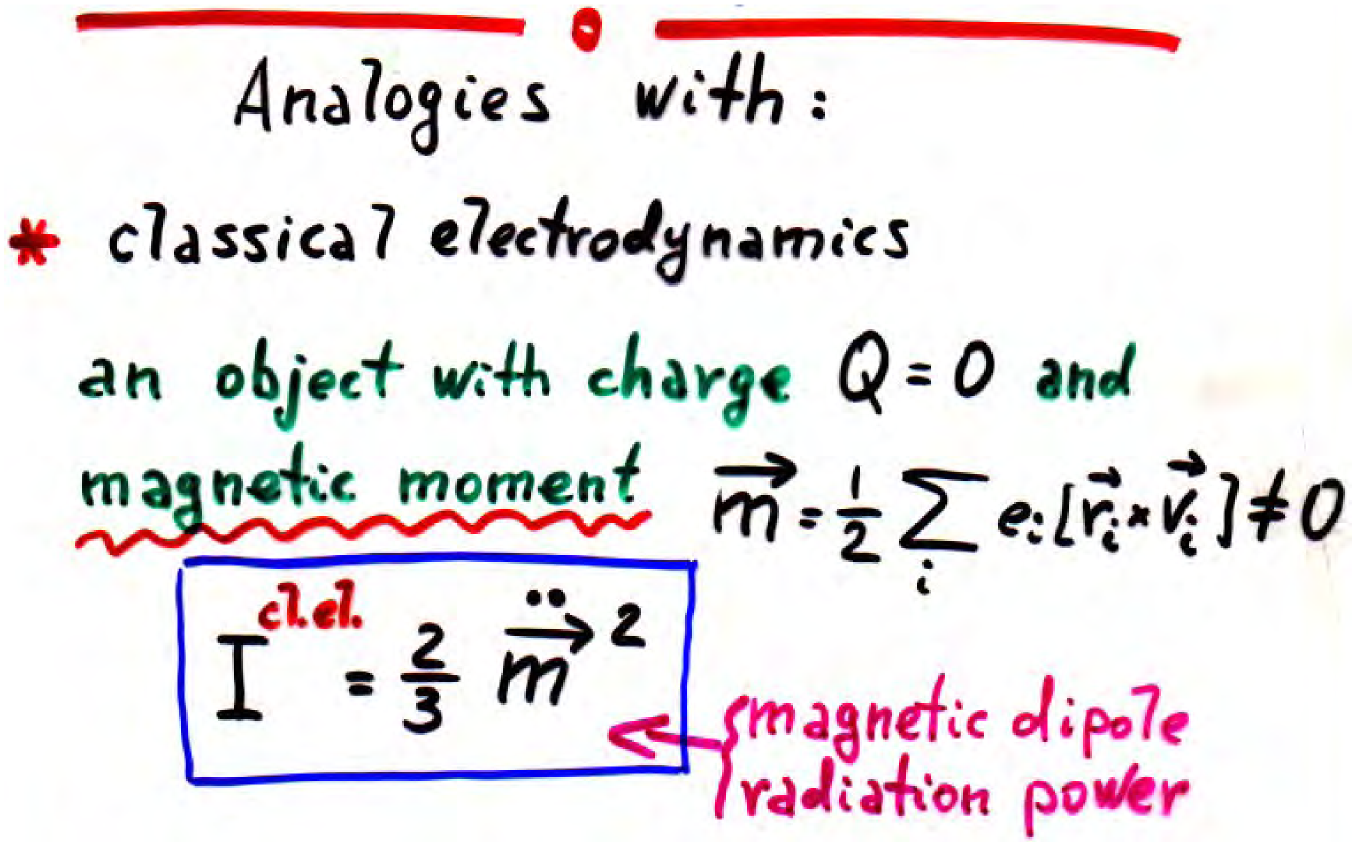
From exact calculations of $\boxed{SLe}$

$$\begin{aligned} n &\sim 10^{37} \div 10^{40} \ \text{cm}^{-3} \\ p &\sim 1 \div 10^{3} \ \text{MeV} \\ m_\nu &= 1 \ \text{eV} \\ \mu &= 10^{-10}\mu_0 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} R_\Gamma &= \frac{\Gamma_{SLe}}{\Gamma_{SL\nu}} \sim 10^{16} \div 10^{19} \\ R_\mathrm{I} &= \frac{\mathrm{I}_{SLe}}{\mathrm{I}_{SL\nu}} \sim 10^{15} \div 10^{19} \end{aligned}$$

Grigoriev, Shinkevich, Studenikin, Ternov, Trofimov, hep-ph/0611128, Russ.Phys.J 50 (2007) 596

New mechanism of electromagnetic radiation

? **Why** **Spin Light** of neutrino $\boxed{SL\nu}$ / of electron $\boxed{SLe}$ **in matter.**



Analogies with :

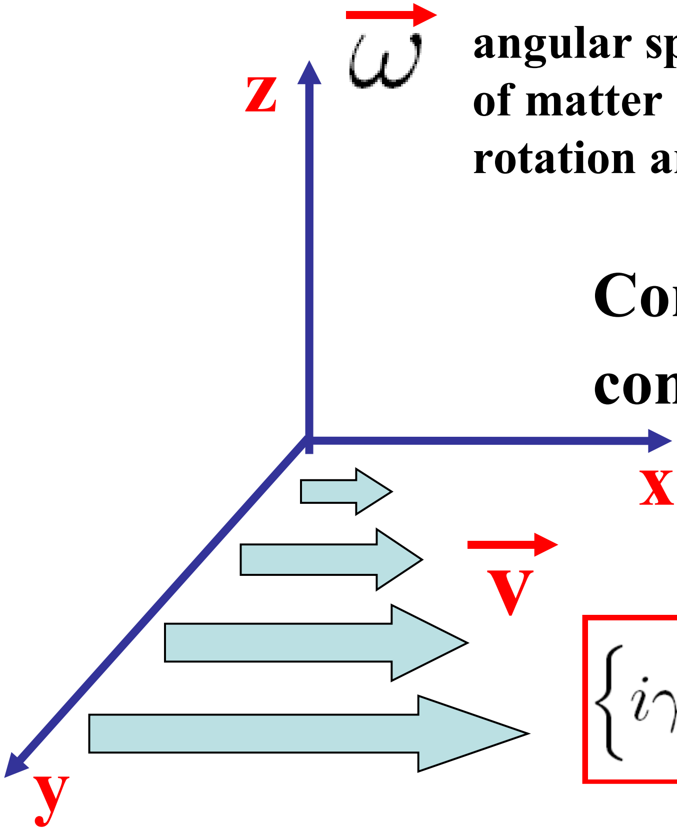
* classical electrodynamics

an object with charge $Q = 0$ and magnetic moment $\vec{m} = \frac{1}{2}\sum_i e_i[\vec{r}_i \times \vec{v}_i] \neq 0$

$$\boxed{I^{cl.el.} = \frac{2}{3}\ddot{\vec{m}}^2}$$

magnetic dipole radiation power

Neutrino energy quantization in moving matter



angular speed of matter rotation around OZ

A.Grigoriev, A.Savochkin,A.Studenikin,
Russ.Phys.J. 8 (2007) 66;

A.Studenikin,
J.Phys.A:Math.Theor. (2008)

Consider ν moving in rotating medium composed of neutrons (generalization s.f.):

ν wave function

$$\boxed{\left\{i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m\right\}\Psi(x) = 0}$$

A.Ternov, A.Studenikin,
Phys.Lett.B 608 (2005) 107

where $f^\mu = -G(n, n\mathbf{v}), \quad \mathbf{v} = (\omega y, 0, 0),$

$$G = \frac{G_F}{\sqrt{2}}$$

neutron number density **speed of matter** **angular speed of rotation**

For ν wave function components $\Psi(x)$:

$$\boxed{\begin{aligned}\left[i\left(\partial_0-\partial_3\right)+Gn\right]\Psi_1+\left[-\left(i\partial_1+\partial_2\right)+Gn\omega y\right]\Psi_2&=m\Psi_3\\ \left[\left(-i\partial_1+\partial_2\right)+Gn\omega y\right]\Psi_1+\left[i\left(\partial_0+\partial_3\right)+Gn\right]\Psi_2&=m\Psi_4\\ i\left(\partial_0+\partial_3\right)\Psi_3+\left(i\partial_1+\partial_2\right)\Psi_4&=m\Psi_1\\ \left(i\partial_1-\partial_2\right)\Psi_3+i\left(\partial_0-\partial_3\right)\Psi_4&=m\Psi_2\end{aligned}}$$

Method of exact solutions → exact solution ?

The problem is reasonable simplified in case of relativistic ν :

$$\boxed{m/p_0 \ll 1}$$

Two pair of wave function components decouple one from each other

and 4 equations → 2 x 2 equations

that couple wave function components in pairs: $\boxed{\left(\Psi_1,\ \Psi_2\right)}$ **and** $\boxed{\left(\Psi_3,\ \Psi_4\right)}$

$\boxed{(\Psi_3, \ \Psi_4)}$ **The second pair of equation (in relativistic case) does not contain matter term** → **sterile right-handed** ν **state** Ψ_R **,**

solution can be written in plain-wave form

$$\boxed{\Psi_R \sim L^{-\frac{3}{2}} \exp\{i(-p_0 t + p_1 x + p_2 y + p_3 z)\}\psi} \ .$$

Exact solution of

$$\boxed{\begin{array}{r} (p_0 - p_3)\, \Psi_3 - (p_1 - ip_2)\, \Psi_4 = 0 \\ -\,(p_1 + ip_2)\, \Psi_3 + (p_0 + p_3)\, \Psi_4 = 0 \end{array}}$$

is

$$\boxed{\Psi_R = \frac{\mathrm{e}^{-ipx}}{L^{3/2}\sqrt{2p_0(p_0 - p_3)}} \begin{pmatrix} 0 \\ 0 \\ -p_1 + ip_2 \\ p_3 - p_0 \end{pmatrix}}$$

where: $px = p_\mu x^\mu, \ \ p_\mu = (p_0, p_1, p_2, p_3)$ **,** $x_\mu = (t, x, y, z)$

with vacuum dispersion relation → **sterile** Ψ_R **.**

$$\boxed{(\Psi_1,\ \Psi_2)}$$

The first pair of equations do contain matter term and corresponds → active left-handed ν , Ψ_L .

Form of equations is similar to case of

electron motion in magnetic field $\boldsymbol{B}$ with vector potential $\boldsymbol{A} = (By, 0, 0)$.

In our case, matter current components

$$\boxed{nv \Rightarrow \boldsymbol{A}}$$

Solution can be written as

$$\boxed{\Psi_L \sim \frac{1}{L} \exp\{i(-p_0 t + p_1 x + p_3 z)\}\psi(y)}$$

Exact solution of

$$\boxed{\begin{aligned}&\left(p_0+p_3+Gn\right)\Psi_1-\sqrt{\rho}\left(\frac{\partial}{\partial\eta}-\eta\right)\Psi_2=0\\&\sqrt{\rho}\left(\frac{\partial}{\partial\eta}+\eta\right)\Psi_1+\left(p_0-p_3+Gn\right)\Psi_2=0\end{aligned}}$$

where

$$\eta=\sqrt{\rho}\left(x_2+\frac{p_1}{\rho}\right),\quad \rho=Gn\omega$$

is

$$\boxed{\Psi_L=\frac{\rho^{\frac{1}{4}}e^{-ip_0t+ip_1x+ip_3z}}{L\sqrt{(p_0-p_3+Gn)^2+2\rho N}}\begin{pmatrix}(p_0-p_3+Gn)\,u_N(\eta)\\-\sqrt{2\rho N}u_{N-1}(\eta)\\0\\0\end{pmatrix}}\quad ,$$

$u_N(\eta)$ are Hermite functions of order N .

Energy spectrum of active left-handed neutrino Ψ_L

$$\boxed{p_0=\sqrt{p_3^2+2\rho N}-Gn,\quad N=0,1,2,\ldots}$$

Antineutrino → **“negative sign” energy eigenvalues**

$$\boxed{\tilde{p}_0=\sqrt{p_3^2+2\rho N}+Gn,\quad N=0,1,2,\ldots}\qquad \rho=Gn\omega$$

ν energy quantization

Lev Landau
(1908-2008)
jubilee

Transversal motion of active relativistic ν is quantized in moving matter like electron energy is quantized in magnetic field (relativistic form of Landau energy levels).

One example: consider antineutrino in rotating neutron matter,

$$\boxed{\tilde{p}_{\perp} = \sqrt{2\rho N}} \qquad \rho = Gn\omega$$

Quantum number N also determines **radius** of antineutrino quasiclassical orbit in moving matter:

$$\boxed{R = \sqrt{\frac{2N}{Gn\omega}}}$$

⟹ **bind orbits inside a Neutron Star !?**

NS:

$$R_{NS} = 10\ km$$
$$n = 10^{37} cm^{-3}$$
$$\omega = 2\pi \times 10^{3}\ s^{-1}$$

radius of trajectory

for this set $R = \sqrt{\frac{2N}{Gn\omega}}$ < $R_{NS} = 10\ km$

if $\boxed{N \leq N_{max} = 10^{10}}$, $\tilde{\nu}$ **with** $N \leq 10^{10}$ can be bound inside the star

thus, $\tilde{\nu}$ **with energy** $\tilde{p}_0 \sim 1\ eV$ **can be bound inside NS**

$N \gg 1$ and $p_3 = 0$

A.Studenikin,
J.Phys.A:Math.Theor.
(2008)

ν quantum states in rotating matter

quasiclassical circular orbits due to central force

$$\boxed{\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m}$$

$$\mathbf{B}_m = \boldsymbol{\nabla} \times \mathbf{A}_m, \quad \mathbf{A}_m = n\mathbf{v}$$

“magnetic field” **vector potential**

“charge”

$$q_m^{(\nu)} = -G$$

matter-induced “Lorentz force”, $\boxed{\mathbf{F}_m^{(\nu)} \perp \boldsymbol{\beta}}$

Generalization to non-constant matter density:

$$\boxed{\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \mathbf{E}_m + q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m,}$$

L.Silva, R.Bingham,
J.Dawson, J.Mendoca,
P.Shukla, Phys.Plasma 7
(2000)

“magnetic field” $\boxed{\mathbf{B}_m = n\boldsymbol{\nabla} \times \mathbf{v} - \mathbf{v} \times \boldsymbol{\nabla} n}$

“electric field” $\boxed{\mathbf{E}_m = -\boldsymbol{\nabla} n - \mathbf{v}\frac{\partial n}{\partial t} - n\frac{\partial \mathbf{v}}{\partial t}}$

e quantum states in rotating matter quasiclassical circular orbits due to central force

Matter-induced "Lorentz force" on electron

A.Studenikin,
J.Phys.A:Math.Theor.
(2008)

$$\boxed{\mathbf{F}_m^{(e)} = q_m^{(e)}\mathbf{E}_m + q_m^{(e)}\boldsymbol{\beta}\times\mathbf{B}_m}$$

We predict that there could be an electromagnetic radiation emitted by an electron moving in radial direction inside a neutrino flow $(m = \nu)$ **emitted from a central part of a star (dipole radiation):**

$$\boxed{I = \frac{2}{3}q_\nu^{(e)}\left[\frac{\mathbf{a}^2}{(1-\beta^2)^2} + \frac{(\mathbf{a}\boldsymbol{\beta})^2}{(1-\beta^2)^3}\right]}$$

acceleration of electron due to mater-induced "Lorentz force"

Developed approach to ν and e :

- **Modified Dirac equations** for neutrino and electron in **matter (background environment)**
- Exact solutions of **modified Dirac equations** in matter
    - **wave functions** and **energy spectra** in matter
- ***Spin light of neutrino*** **in matter** $SL\nu$
    - transition **rate,** radiation **power,** photon **energy**
    - spatial angular **distribution** and **polarization**
- ***Spin light of electron*** **in matter** SLe
- ν ***energy quantization*** **in rotating matter**

. . . Applications to astrophysics and cosmology ?