Neutrino Phenomenology, Facts, and Questions

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Part 1

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

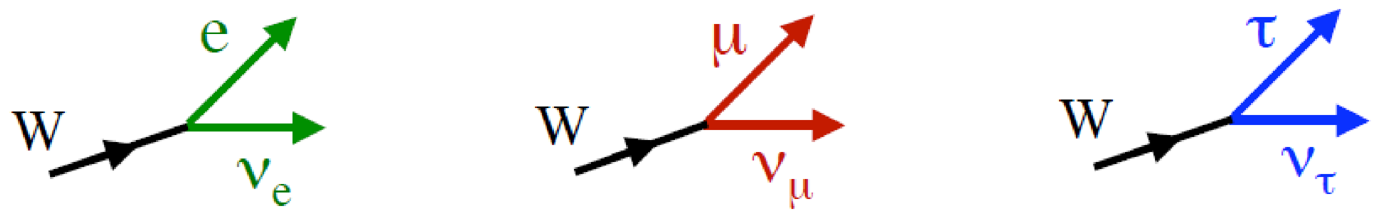
These discoveries come from the observation of

neutrino flavor change (neutrino oscillation).

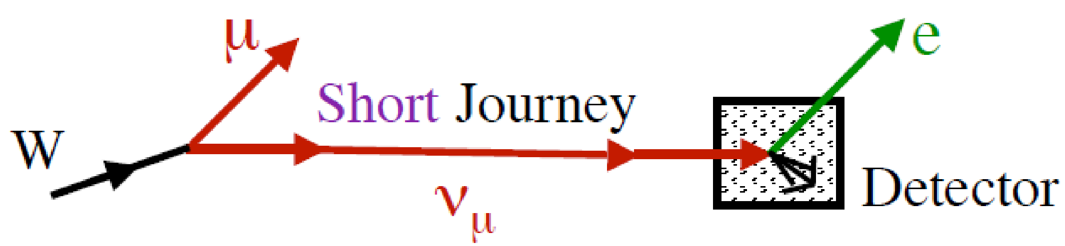
The Physics of Neutrino Oscillation

The Neutrino Flavors

We *define* the three known flavors of neutrinos, ν_e, ν_μ, ν_τ, by W boson decays:



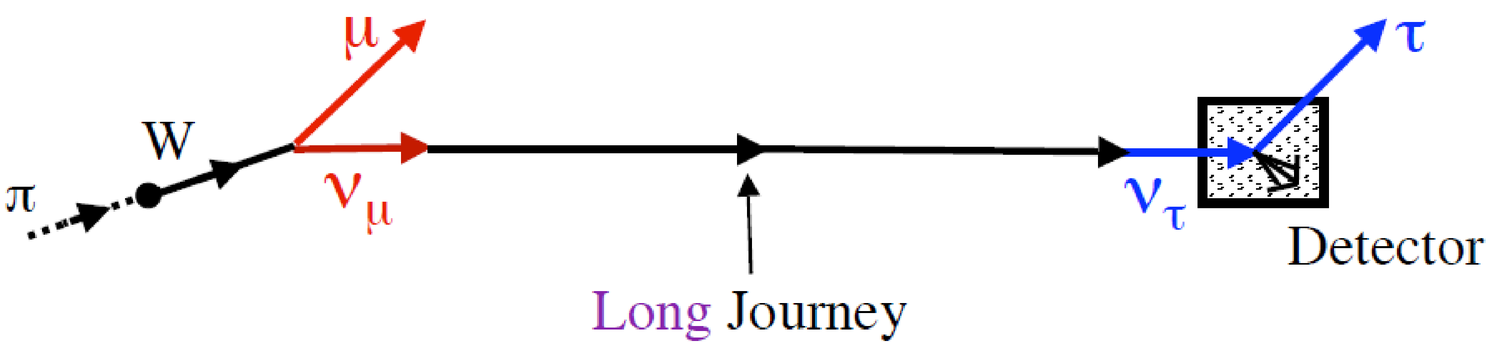
As far as we know, neither



nor any other change of flavor in the $\nu \rightarrow \ell$ ***interaction*** ever occurs. With $\alpha = e, \mu, \tau$, ν_α makes only ℓ_α ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).

Neutrino Flavor Change

If neutrinos have masses, and leptons mix, we can have —



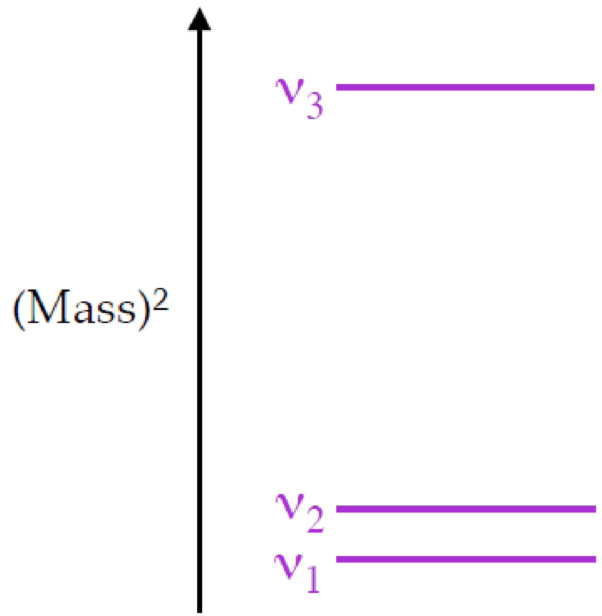
Give ν time to change character

$$\nu_\mu \longrightarrow \nu_\tau$$

The last decade has brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires *Neutrino Masses*

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass (ν_i) $\equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\, |\nu_i> \quad .$$

Neutrino of flavor

$\alpha = e, \mu$, or τ

Neutrino of definite mass m_i

PMNS Leptonic Mixing Matrix

There must be ***at least 3*** mass eigenstates ν_i,
because there are 3 orthogonal neutrinos
of definite flavor ν_α.

This *mixing* is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Left-handed

$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \overline{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

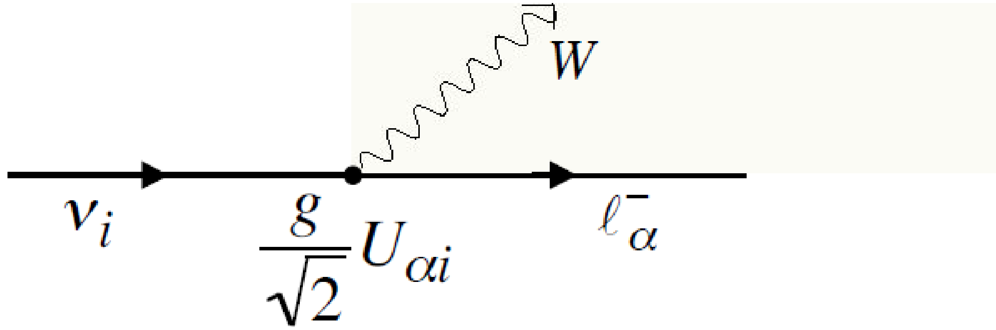
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

If neutrino *masses* are described by an extension of the SM, and there are no new leptons, U is unitary. Then —

$$\mathrm{Amp}\left(W \to \ell_{\alpha} + \nu_{\alpha}; \nu_{\alpha} \to \ell_{\beta} + W\right) \propto \sum_{i=1}^{3} U_{\alpha i}^{*} U_{\beta i} = \delta_{\beta\alpha},$$ as observed.

The Meaning of *U*



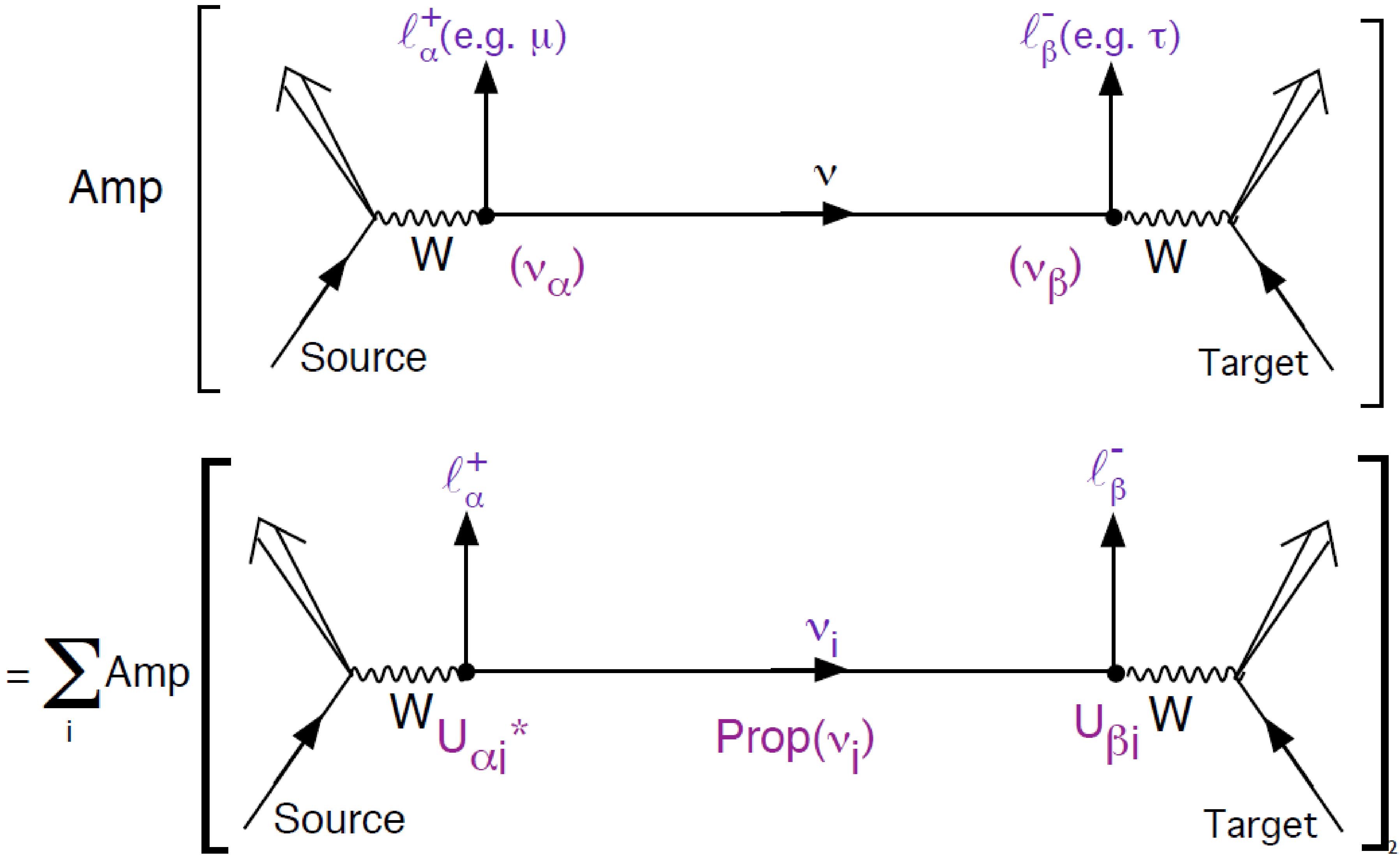
$$
U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array}
\begin{array}{c}
\begin{array}{ccc} \;\;\nu_1 & \;\;\;\;\nu_2 & \;\;\;\;\nu_3 \end{array} \\
\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}
\end{array}
$$

The e row of U: The linear combination of neutrino mass eigenstates that couples to e.

The ν_1 column of U: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

Neutrino Flavor Change (Oscillation) in Vacuum

(Approach of B.K. & Stodolsky)



$$\text{Amp}\,[\nu_\alpha \rightarrow \nu_\beta] = \Sigma U_{\alpha i}{}^* \,\text{Prop}(\nu_i) U_{\beta i}$$

What is Propagator (ν_i) ≡ Prop(ν_i)?

In the ν_i rest frame, where the proper time is τ_i,

$$i\frac{\partial}{\partial \tau_i}|\nu_i(\tau_i)>= m_i|\nu_i(\tau_i)> \quad .$$

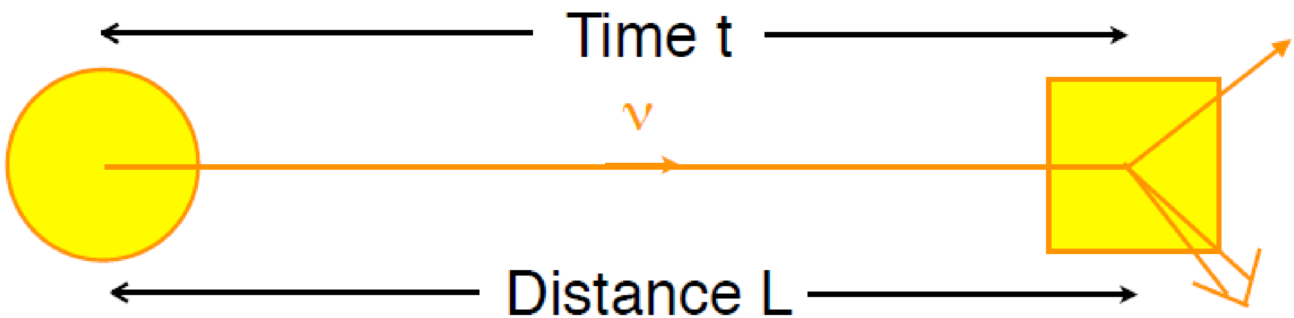
Thus,

$$|\nu_i(\tau_i)>= e^{-im_i\tau_i}|\nu_i(0)> \quad .$$

Then, the amplitude for propagation for time τ_i is —

$$\text{Prop}(\nu_i) \equiv< \nu_i(0)|\nu_i(\tau_i)>= e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t.

They are common to all components of the beam.

For each ν_i, by Lorentz invariance,

$$(E_i, p_i) \times (t, L) = m_i\tau_i = E_i t - p_i L\,.$$

Neutrino sources are ~ constant in time.

Averaged over time, the

$e^{-iE_1 t} - e^{-iE_2 t}$ interference

is —

$$< e^{-i(E_1 - E_2)t} >_t \;=\; 0$$

unless $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent. (Stodolsky)

For each mass eigenstate ,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad .$$

Then the phase in the ν_i propagator *exp[-im*$_i$$\tau_i$*]* is —

$$m_i \tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= \underbrace{E(t - L)}_{\text{Irrelevant overall phase}} + m_i^2 L/2E \quad .$$

What if the neutrino source is ***not*** constant in time?

The relative phase between two mass eigenstates,

$$\delta\phi(21) \equiv (E_2 t - p_2 L) - (E_1 t - p_1 L) \quad ,$$

is unchanged.

(Lipkin)

An approximation to the average speed of the ν_1 and ν_2 waves is

$$\bar{v} \equiv \frac{p_1 + p_2}{E_1 + E_2} \quad .$$

Then the travel time $t \cong L/\bar{v} \quad .$

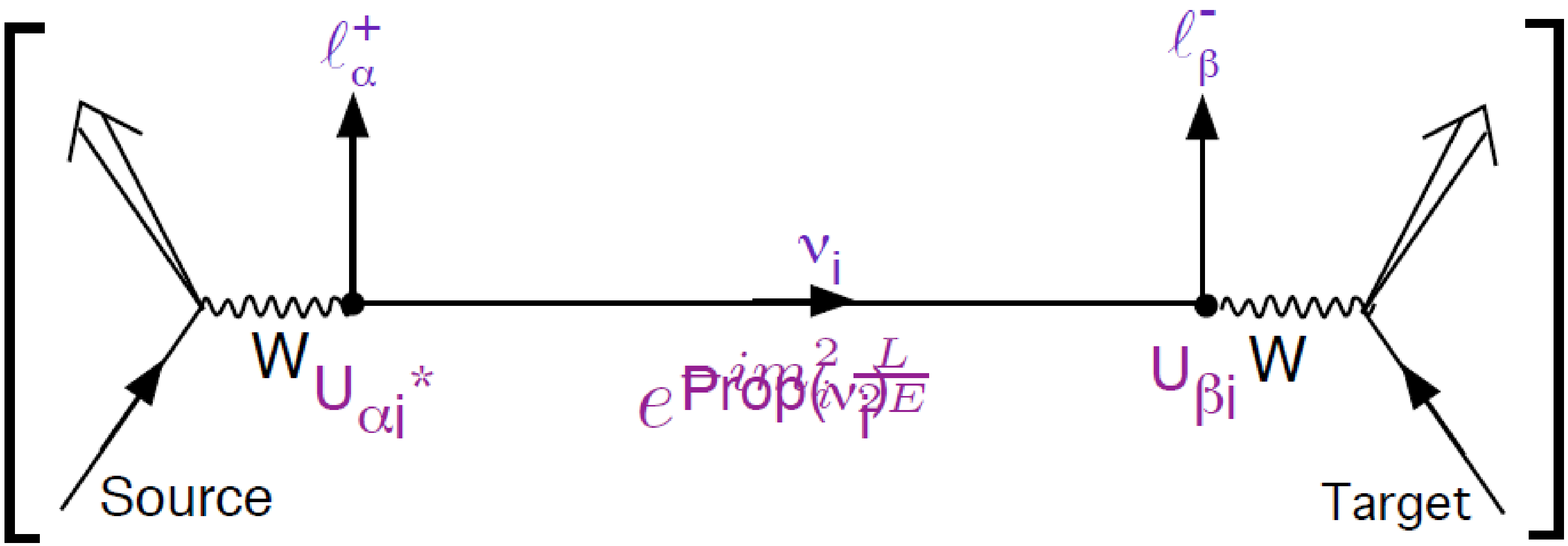
Thus,

$$\delta\phi(21) = (p_1 - p_2)L - (E_1 - E_2)t$$

$$\cong \frac{p_1^2 - p_2^2}{p_1 + p_2}L - \frac{E_1^2 - E_2^2}{p_1 + p_2}L \cong (m_2^2 - m_1^2)L/2E$$

Amp $[\nu_\alpha \rightarrow \nu_\beta]$

$= \sum_i$ Amp



$$= \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

$$\text{where } \Delta m^2_{ij} \equiv m^2_i - m^2_j$$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

$$P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) \stackrel{CPT}{=} P(\nu_\beta \rightarrow \nu_\alpha) = P(\nu_\alpha \rightarrow \nu_\beta; U \rightarrow U^*)$$

Thus,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\underset{(-)}{+} 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

A complex U would lead to the CP violation
$P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) \neq P(\nu_\alpha \rightarrow \nu_\beta)$.

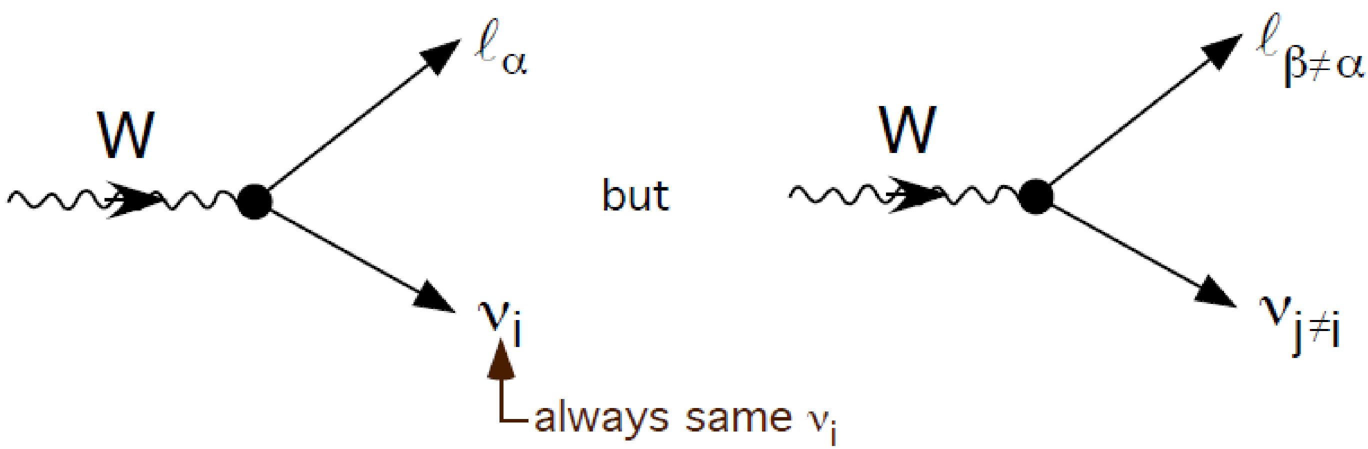
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* $\Rightarrow$ ν Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha, i} = 0, \text{ so that } P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}.$$

Flavor *change* $\Rightarrow$ Mixing

3. One can detect ($\nu_\alpha \to \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2(\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

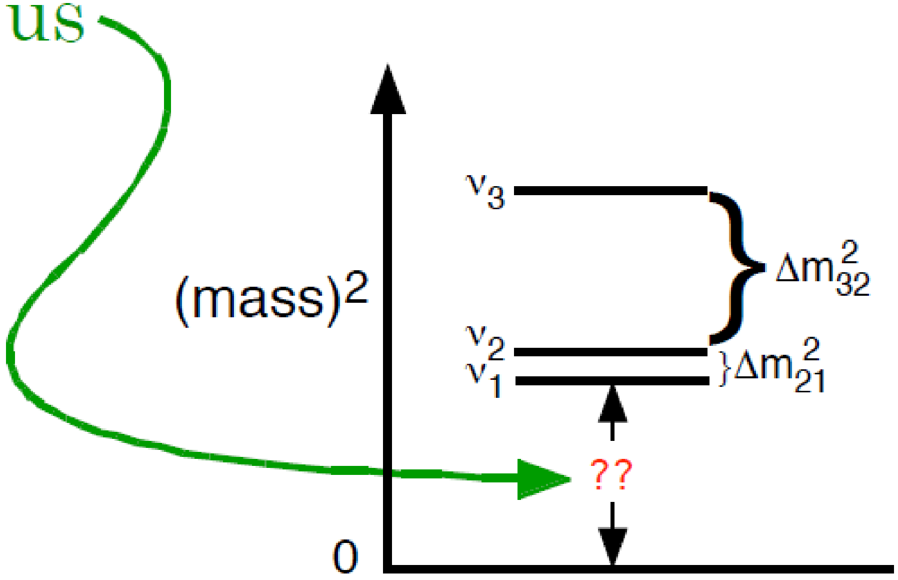
$\sin^2[1.27 \Delta m^2 (\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2(\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. P $(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

8. Assuming all coherent ν_i in a beam have a common momentum p, rather than a common energy E, is a harmless error.

This assumption leads to the same $P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta)$.

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i} U_{\beta i}^* e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}}$$

$$= U_{\alpha 3} U_{\beta 3}^* e^{2i\Delta_{31}} + U_{\alpha 2} U_{\beta 2}^* e^{2i\Delta_{21}} \underbrace{-(U_{\alpha 3} U_{\beta 3}^* + U_{\alpha 2} U_{\beta 2}^*)}_{\text{Unitarity}}$$

$$= 2i[U_{\alpha 3} U_{\beta 3}^* e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U_{\beta 2}^* e^{i\Delta_{21}} \sin \Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m_{ij}^2 \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \right|^2$$

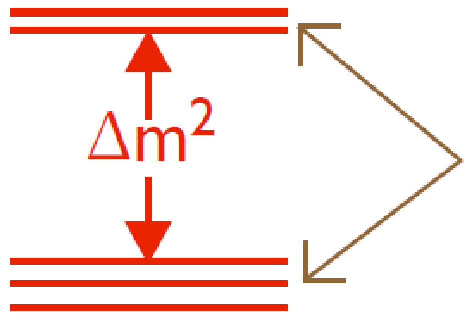
$$= 4[|U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2} U_{\beta 2}|^2 \sin^2 \Delta_{21}$$

$$+2|U_{\alpha 3} U_{\beta 3} U_{\alpha 2} U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \underset{(-)}{+} \delta_{32})] \ .$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3} U^*_{\beta 3} U^*_{\alpha 2} U_{\beta 2})$, a CP – violating phase.

Two waves of different frequencies,
and their $\cancel{\mathrm{CP}}$ interference.

When One Big Δm^2 Dominates



These splittings are invisible if $\Delta m^2 \dfrac{L}{E} = \mathcal{O}(1)$

For $\beta \neq \alpha$,

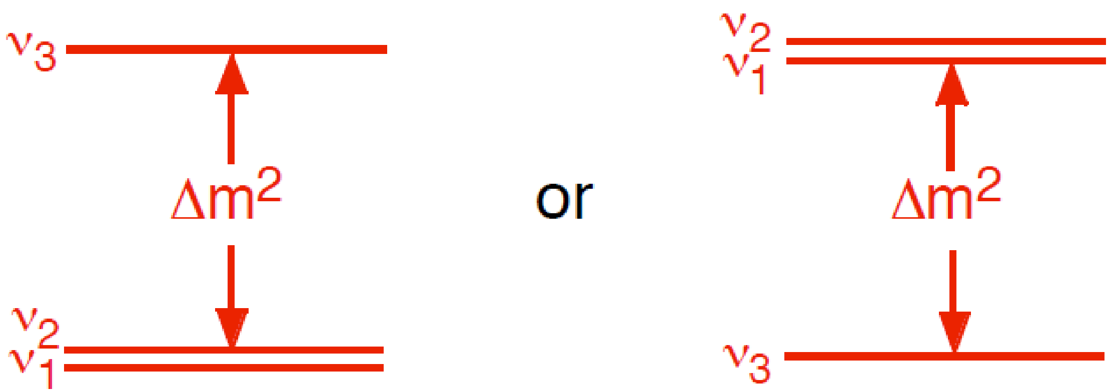
$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \cong S_{\alpha\beta} \sin^2(\Delta m^2 \frac{L}{4E}) \; ; \; S_{\alpha\beta} \equiv 4 \left| \sum_{i \text{ Clump}} U_{\alpha i}^* U_{\beta i} \right|^2 .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \cong 1 - 4T_\alpha(1 - T_\alpha) \sin^2(\Delta m^2 \frac{L}{4E}) \; ; \; T_\alpha \equiv \sum_{i \text{ Clump}} |U_{\alpha i}^*|^2 .$$

"i Clump" is a sum over only the mass eigenstates on one end of the big gap Δm^2.

When the Spectrum Is—



Invisible if $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

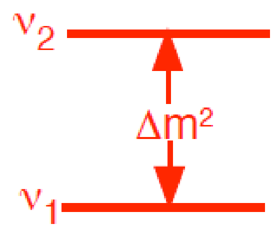
$$\mathrm{P}(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \cong 4|U_{\alpha 3} U_{\beta 3}|^2 \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

For no flavor change,

$$\mathrm{P}(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \cong 1 - 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

Experiments with $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$ can determine the flavor content of ν_3.

When There are Only Two Flavors and Two Mass Eigenstates



Majorana ~~CP~~ phase

$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Mixing angle

$$S_{\alpha\beta} = 4T_\alpha(1 - T_\alpha) = \sin^2 2\theta$$

For $\beta \neq \alpha$, $$P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}) \quad .$$

For no flavor change, $$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}).$$