Are neutrino oscillations a non stationary phenomenon?

S. Bilenky
JINR(Dubna)TUM (Munchen)

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Discovery of neutrino oscillations is one the most important recent discoveries in particle physics

It is a common opinion that small neutrino masses and peculiar neutrino mixing is a signature of a new beyond the Standard Model Physics

In the next generation of neutrino oscillation experiments (Super beam, β-beam, Neutrino Factory) a very high accuracy in the measurement of neutrino oscillation parameters is planned

Are we sure that the theory of neutrino oscillations is fully justified? Are there open problems in the theory?

Investigation of neutrino oscillations is based on the following assumptions

1. Neutrino interaction is the Standard Model CC and NC interaction. The standard leptonic CC is given by

$$j^{CC}_{\alpha}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\,\gamma_{\alpha}\, l_L(x)$$

2. **Neutrino mixing**

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\,\nu_{iL}(x)$$

U is an unitary PMNS mixing matrix and $\nu_i(x)$ is the field of neutrino (Majorana or Dirac) with mass m_k

This last relation is the relation between fields. What are observable consequences for neutrino oscillations?

All existing neutrino oscillation data can be described if we assume that

- The number of massive neutrinos is equal to the number of the flavor neutrinos (three)
- The neutrino transition probability is given by the expression

$$P(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3} U_{l'i} e^{-i\Delta m^2_{ki} \frac{L}{2E}} U^*_{li}|^2$$

$$\Delta m^2_{ki} = m^2_i - m^2_k, k \text{ is fixed}$$

$P(\nu_l \to \nu_{l'})$ depends on Δm^2_{12}, Δm^2_{23}, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Two neutrino oscillation parameters are small

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \simeq 3 \cdot 10^{-2}, \quad \sin^2\theta_{13} \lesssim 5 \cdot 10^{-2}$$

In the leading approximation

$\Delta m^2_{23} \frac{L}{2E} \gtrsim 1$ (atmospheric and LBL experiments)

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{2}\sin^2 2\theta_{23}(1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

$$(\nu_\mu \leftrightarrows \nu_\tau)$$

$\Delta m^2_{12} \frac{L}{2E} \gtrsim 1$ (KamLAND experiment)

$$P(\bar\nu_e \to \bar\nu_e) = 1 - \frac{1}{2}\sin^2 2\theta_{12}(1 - \cos\Delta m^2_{12}\frac{L}{2E})$$

$$(\bar\nu_e \leftrightarrows \bar\nu_{\mu,\tau})$$

Solar neutrino data are described by two-neutrino ν_e survival probability in matter which depends on Δm^2_{12} and $\sin^2\theta_{12}$

In the leading approximation decoupled two-neutrino oscillations in two regions of $\frac{L}{E}$

From the analysis of the data of the atmospheric S-K experiment the following 90% CL ranges were obtained

$$1.9 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.1 \cdot 10^{-3} \mathrm{eV}^2, \quad \sin^2 2\theta_{23} > 0.9.$$

The results of the S-K experiment have been confirmed by the accelerator K2K and MINOS LBL neutrino oscillations experiments. From the analysis of the MINOS data

$$\Delta m^2_{23} = (2.38^{+0.20}_{-0.16}) \cdot 10^{-3} \mathrm{eV}^2, \quad \sin^2 2\theta_{23} > 0.84 \ (90\% \ \mathrm{CL})$$

From the global analysis of the data of reactor KamLAND experiment and the data of the solar neutrino experiments it was found

$$\Delta m^2_{12} = (7.59^{+0.21}_{-0.21}) \cdot 10^{-5} \ \mathrm{eV}^2, \quad \tan^2 \theta_{12} = 0.47^{+0.06}_{-0.05}$$

Neutrino production, propagation and detection in the case of the neutrino mixing

Let us consider a CC decay

$$a \to b + l^{(+)} + \nu$$

The state of the final particles

$$|f\rangle = \sum_i |\nu_i\rangle\ |l^+\rangle|b\rangle\ \langle \nu_i l^+ b|S|a\rangle$$

$|\nu_i\rangle$ is the state of the left-handed neutrino with mass m_i and momentum p_i

In matrix element $\langle \nu_i l^+ b|S|a\rangle$ at neutrino energies $E \gtrsim \mathrm{MeV}$ mass-squared differences can be neglected

$$\frac{\Delta m^2_{12}}{E^2} \sim 10^{-17} \quad \frac{\Delta m^2_{23}}{E^2} \sim 10^{-21}$$

$$\langle \nu_i l^{(+)} b|S|a\rangle \simeq U_{li}^* \ \langle \nu_l l^{(+)} b|S|a\rangle_{\rm SM}$$

$\langle \nu_l l^{(+)} b|S|a\rangle_{\rm SM}$ is the SM matrix element of the decay

$$a \to b + l^{(+)} + \nu_l$$

In the matrix element dependence on ν_i enters through U_{li}^*

$$|f\rangle \simeq |\nu_l\rangle \ |l^+\rangle|b\rangle \ \langle \nu_l l^+ b|S|a\rangle_{\rm SM}$$

$$|\nu_{\mathbf{l}}\rangle = \sum_{\mathbf{i}} \mathbf{U}_{\mathbf{li}}^* \ |\nu_{\mathbf{i}}\rangle$$

"Mixed" flavor neutrino ν_l which is produced together with l^+ is described by the coherent state $|\nu_l\rangle$

Evolution equation in Quantum field theory is the Schrodinger equation for states

$$\frac{d}{dt}|\Psi(t)\rangle = H\ |\Psi(t)\rangle$$

If $|\Psi(0)\rangle = |\nu_l\rangle$ at the time t for neutrino state in vacuum we have

$$|\Psi(t)\rangle = \sum_i |\nu_i\rangle\ e^{-iE_i t} U_{li}^*, \quad E_i = \sqrt{p_i^2 + m_i^2}$$

Developing over the flavor neutrino states we find

$$|\Psi(t)\rangle = \sum_{l'} |\nu_{l'}\rangle\ (\sum_i U_{l'i} e^{-iE_i t} U_{li}^*)$$

Neutrinos detection. Let us consider the CC process

$$\nu_{l'} + N \to l' + X$$

Neglecting extremely small $\frac{\Delta m^2}{E^2}$ terms we have

$$\langle l'X|S|\nu_i N\rangle \simeq \langle l'X|S|\nu_{l'} N\rangle_{\mathrm{SM}}\ U_{l'i}$$

From unitarity of U we find

$$\langle l'X|S|\nu_{l'} N\rangle = \sum_i \langle l'X|S|\nu_i N\rangle U^*_{l'i} \simeq \langle l'X|S|\nu_{l'} N\rangle_{\mathrm{SM}}$$

The normalized $\nu_l \to \nu_{l'}$ transition probability is given by

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i} e^{-i(E_i - E_k)t} U^*_{li}|^2$$

Neglecting in matrix elements $\frac{\Delta m^2_{ik}}{E^2}$ we come to the following conclusion

- States of flavor neutrinos ν_e, ν_μ and ν_τ and neutrino transition probability do not depend on neutrino production and detection processes
- Matrix elements of neutrino production and detection processes are given by the Standard Model

Neutrino oscillations are taking place if neutrino state $|\Psi(t)\rangle$ is a superposition of states of neutrinos with different masses and different energies (non stationary neutrino state)

The non stationary nature of neutrino oscillations was advocated by B.Pontecorvo and his collaborators in early neutrino oscillation papers

In the accelerator K2K and MINOS experiments time of neutrino production and neutrino detection was measured. In the K2K experiment ν_μ's were produced in 1.1 μs spills. After the time $t \simeq L/c \simeq 0.8 \cdot 10^3 \mu s$ ν_μ were detected by the S-K . It was found an agreement with the S-K atmospheric neutrino oscillation results. **Thus, in this experiment non stationary picture of neutrino oscillations was confirmed**

Is this a general feature of neutrino oscillations? Are neutrino oscillations possible also in a stationary case?

Oscillation phase

$$\phi_{ki} = (E_i - E_k)\ t \simeq (p_i - p_k)\ t + \frac{\Delta m^2_{ki}}{2E} t$$

The first term can be comparable with the second one.

$$(p_i - p_k) \simeq a \frac{\Delta m^2_{ki}}{2E} \qquad |a| \lesssim 1$$

Experimental data are described if oscillation phase is given by

$$\phi_{ki} \simeq \frac{\Delta m^2_{ki}}{2E} L$$

Thus, in the approach based on the Schrodinger equation, we need to assume that

$$p_i = p_k = p$$

This was original assumption of B. Pontecorvo and his collaborators

Wave function approach to neutrino propagation

We assume that in a weak process ν_i with momenta p_i^α are produced coherently. Wave function of neutrino is the superposition of plane waves. According to Dirac equation

$$E_i = \sqrt{p_i^2 + m_i^2}$$

Neutrino wave function

$$\Psi_{\nu_l}(x) = \sum_i e^{-ip_i^\alpha x_\alpha} U_{li}^* \, |i\rangle$$

describes propagation in space and time of a superposition of plane waves ($|i\rangle$ describes neutrino with mass m_i and helicity equal to -1)

The transition probability is given by

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i} e^{-i(p_i^\alpha - p_k^\alpha)x_\alpha} U_{li}^*|^2$$

Transitions are due to the fact that different waves after time t and distance $\vec{x}$ gain **different phases**

Oscillation phase

$$\phi_{ik} = (E_i - E_k)t - (p_i - p_k)x$$

General case $E_i \neq E_k$, $p_i \neq p_k$

$$\phi_{ik} \simeq \frac{\Delta m^2_{ik}}{2E}t - (p_i - p_k)(x - t)$$

Independently on the values of the momenta. the second term is equal to zero ($x = t$). For the oscillation phase we obtain the standard expression

$$\phi_{ik} \simeq \frac{\Delta m^2_{ik}}{2E}t$$

Stationary case $E_i = E_k$ Oscillation phase

$$\phi_{ik} = -(p_i - p_k)x = \frac{\Delta m^2_{ik}}{2E}x$$

If propagating neutrinos are described by a coherent superposition of plane waves oscillations are possible also in the stationary case. Oscillation phase is given by the standard expression

Time-energy uncertainty relation for neutrino oscillations

Uncertainty relations are based on the Cauchy-Schwarz inequality

$$\Delta A \, \Delta B \geq \frac{1}{2} |\langle \Psi | \; [A, B] \; | \Psi \rangle|$$

A and B are two Hermitian operators and $|\Psi\rangle$ is any state

The Heisenberg uncertainty relations are a direct consequence of the commutation relations for A and B

For example

$$[p, q] = \frac{1}{i} \rightarrow \Delta p \, \Delta q \geq \frac{1}{2}$$

Has universal form (does not depend on the state $|\Psi\rangle$)

The time-energy uncertainty relation has a different character.

Time in quantum theory is a parameter; there is no operator which corresponds to time

Mandelstam and Tamm method is based on the fact that the **evolution of a quantum system is determined by the Hamiltonian**

For any operator $O_H(t)$ in the Heisenberg representation

$$i\ \frac{d\ O_H(t)}{d\ t} = [O_H(t), H]$$

From this relation and the Cauchy-Schwarz inequality we have

$$\Delta E\ \Delta O_H(t) \geq \frac{1}{2}\ |\frac{d}{dt}\overline{O_H}(t)|$$

Nontrivial constraints only for non stationary processes

We choose

$$O = |\nu_l\rangle\ \langle\nu_l|$$

Assuming that $|\Psi(0)\rangle = |\nu_l\rangle$ we find

$$\overline{O}(t) = P_{\nu_l \to \nu_l}(t)$$

where $P_{\nu_l \to \nu_l}(t)$ is the survival probability

The M-T inequality takes the form

$$\Delta E \geq \frac{1}{2}\ \frac{|\frac{d}{dt} P_{\nu_l \to \nu_l}(t)|}{\sqrt{P_{\nu_l \to \nu_l}(t) - P^2_{\nu_l \to \nu_l}(t)}}$$

If we integrate over the time interval $0 \leq t \leq t_{1\mathrm{min}}$ ($t_{1\mathrm{min}}$ is the time at which the survival probability reaches the first minimum) the time-energy uncertainty relation takes the form

$$\Delta E\ \Delta t \geq \frac{1}{2}\ \left(\frac{\pi}{2} - \arcsin(2\ P_{\nu_l \to \nu_l}(t_{1\mathrm{min}}) - 1)\right), \quad \Delta t = t_{1\mathrm{min}}$$

For $\nu_\mu \to \nu_\mu$ transitions driven by Δm^2_{23} we have

$$\Delta t = t_{1\mathrm{min}} = 2\pi\ \frac{E}{\Delta m^2_{23}}$$

Taking into account that $\sin^2 2\theta_{23} \simeq 1$ we obtain the time-energy uncertainty relation

$$\Delta E\ \Delta t \gtrsim \frac{\pi}{2}$$

It is satisfied in atmospheric S-K and accelerator K2K and MINOS experiments

For $\bar{\nu}_e \to \bar{\nu}_e$ transition driven by Δm^2_{12} we have

$$\Delta E \; \Delta t \gtrsim 2 \;\sin 2\theta_{13}$$

Because $\sin^2 2\theta_{13} \lesssim 2 \cdot 10^{-1}$ T-E uncertainty relation gives much weaker constraint on ΔE in this case

Is time-energy uncertainty relation an universal feature of neutrino oscillations ?

Mössbauer neutrino experiment

It was proposed by Raghavan an experiment on the detection of the tritium $\bar{\nu}_e$ with energy $\simeq 18.6$ keV in the recoilless transitions

$$^{3}\mathrm{H} \to ^{3}\mathrm{He} + \bar{\nu}_{\mathrm{e}}, \quad \bar{\nu}_{\mathrm{e}} + ^{3}\mathrm{He} \to ^{3}\mathrm{H}$$

Oscillation length in such experiment $L^{(23)}_{\mathrm{osc}} \simeq 18.6\ m$

It was estimated by Raghavan that $\Delta E \simeq 8.4 \cdot 10^{-12}$ eV and $\sigma_R \simeq 3 \cdot 10^{-33} \mathrm{cm}^2$

(For much more pessimistic estimates see W. Potzel ArXiv: 0810.2170)

ΔE which drives neutrino transition during the time $t^{(23)}_{1\mathrm{min}}$ must satisfy the inequality

$$\Delta E \geq \frac{1}{2\pi} \sin 2\theta_{13} \frac{\Delta m^2_{23}}{E}$$

If $\sin 2\theta_{13} \neq 0$ a constraint on ΔE

$$\sin^2 2\theta_{13} = 2 \cdot 10^{-1} \text{ (CHOOZ bound)}$$

$$\Delta E \geq 9 \cdot 10^{-9} \text{ eV}$$

$$\sin^2 2\theta_{13} = 10^{-2} \text{ (T2K, Daya Bay,...)}$$

$$\Delta E \geq 2 \cdot 10^{-9} \text{ eV}$$

Estimated energy uncertainty for Mössbauer neutrinos $\Delta E \simeq 8.4 \cdot 10^{-12}$ eV does not saturate time-energy uncertainty relation

If neutrino oscillations will be observed in Mössbauer neutrino experiment in this case the time-energy uncertainty relation is not a characteristic feature of the phenomenon of neutrino oscillations

This is equivalent to the statement that in the case of the observation of neutrino oscillations in the Mössbauer neutrino experiment non stationarity is not an universal feature of neutrino oscillations