Majorana Neutrinos and their Properties

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Neutrino Telescopes
Venice
March 12, 2009

Compelling Evidences for $\nu-$Oscillations: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\,\nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}\ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6

CP-violating phases:

• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980;
J. Schechter, J.W.F. Valle, 1980.

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\text{el}} = 0,\ L_l = 0,\ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0\ ,\quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0\ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta}\ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi\ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP}\ \chi(x)\ U^{-1}_{CP} = \eta_{CP}\ \gamma_0\ \chi(x'),\quad \eta_{CP} = \pm i\ .$$

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}_m^\nu(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}_m^\nu(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

- Example: Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$

$$\mathcal{L}_M^\nu(x) = -\,\frac{1}{2}\,\overline{\nu_{l'R}^c}(x)\; M_{l'l}\;\nu_{lL}(x) + h.c.\;,\;\; \nu_{l'R}^c \equiv C\;(\overline{\nu_{l'L}}(x))^\top$$

- If $M_{l'l} \neq 0$, $L_l \neq const.$, $L \neq const.$
- $\nu_{lL}(x)-$fermions: $M = M^\top$, complex.

$$M^{diag} = U^\top M U\,,\;\; U - \text{unitary (congruent transformation)};\;\; U \equiv U_{\text{PMNS}}$$

$$\chi_j(x) = U_{jl}^\dagger \nu_{lL}(x) + U_{jl}^* \nu_{lR}^c = C\;(\overline{\chi_j}(x))^\top\,,\quad m_j \neq 0$$

CP-invariance: $M^* = M$, M - real, symmetric.

$M^{diag} = (m'_1, m'_2, m'_3)$: $m'_j = \rho_j m_j$, $m_j \geq 0$, $\rho_j = \pm 1$

χ_j: $m_j \geq 0$: $\eta_{CP}(\chi_j) = i\rho_j$

- Dirac Neutrinos: Dirac Mass Term, requires $\nu_R(x)$ - $SU(2)_L$ singlet RH ν fields

$$\mathcal{L}_D^\nu(x) = -\;\overline{\nu_{l'R}}(x)\; M_{Dl'l}\;\nu_{lL}(x) + h.c.\;,\;\; M_D - complex$$

- $\mathcal{L}_D^\nu(x)$ conserves L: $L = const.$

$$M_D = V M_D^{diag} W^\dagger\,,\;\; V, U - \text{unitary}\;(bi - unitary\ transformation)\,,\;\; W \equiv U_{\text{PMNS}}$$

Special Properties of the Currents of $\chi(x)-$Majorana:

$$\bar{\chi}(x)\gamma_\alpha\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0)\,;$$

Has imortant implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_\chi = 0 \quad (\mu_\Psi \neq 0\,)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \; d_\chi = 0 \; (d_\Psi \neq 0, \; if \; CP \; is \; not \; conserved)$$

$\chi(x)$ cannot couple to a real photon (field).

$\chi(x)$ couples to a virtual photon through an anapole moment:

$$(g_{\alpha\beta}\, q^2 - q_\alpha q_\beta)\gamma_\beta\gamma_5 \; F_a(q^2)\,.$$

Properties of Currents Formed by $\chi_1(x)$, $\chi_2(x)$: $\chi_2 \to \chi_1 + \gamma$, $\chi_2 \to \chi_1\chi_1\chi_1$, etc.

$$\bar{\chi_1}(x)\gamma_\alpha(v - a\gamma_5)\chi_2(x) \quad (\bar{\chi_1}(x)\gamma^\alpha(1-\gamma_5)\chi_1(x), ...):$$

- CP is conserved: $v = 0$ ($a = 0$) if $\eta_{1CP} = \eta_{2CP}$ ($\eta_{1CP} = -\eta_{2CP}$)
- CP is not conserved: $v \neq 0$, $a \neq 0$

(Has imortant implications also, e.g. for SUSY neutralino phenomenology:
$e^+ + e^- \to \chi_1 + \chi_2$, $\chi_2 \to \chi_1 + l^+ + l^-$, etc.)

$$\bar{\chi_1}(x)\sigma_{\alpha\beta}(\mu_{12} - d_{12}\gamma_5)\chi_2(x) \quad (F^{\alpha\beta}(x)):$$

- CP is conserved: $\mu_{12} = 0$ ($d_{12} = 0$) if $\eta_{1CP} = \eta_{2CP}$ ($\eta_{1CP} = -\eta_{2CP}$)
- CP is not conserved: $\mu_{12} \neq 0$, $d_{12} \neq 0$

Dirac - Majorana Relation (if any...)

Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$, can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}^{\nu}_{M}(x) = -\,\frac{1}{2}\,\overline{\nu^{c}_{l'R}}(x)\;M_{l'l}\;\nu_{lL}(x) + h.c.\;,\;\;\nu^{c}_{l'R} \equiv C\;(\overline{\nu_{l'L}}(x))^{\mathsf{T}}$$

$\mathcal{L}^{\nu}_{M}(x)$ conserves, e.g. $L' = L_e - L_\mu - L_\tau$ if only $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$

S.T.P., 1982

- Dirac ν, Ψ, is equivalent to two Majorana ν's, $\chi_{1,2}$, having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,,\quad m_1 = m_2 = m_D > 0\,, \eta_{jCP} = i\rho_j\,, \rho_1 = -\rho_2\;\;(C\;(\overline{\chi_j})^{\mathsf{T}} = \rho_j\chi_j)$$

$$\text{Example ZKM } \nu:\;\; \nu_{eL}(x) = \Psi_L = \frac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}\,,\;\; \nu_{\mu L}(x) = \Psi^{C}_{L} = \frac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$$

- Pseudo-Dirac Neutrino: the symmetry of $\mathcal{L}^{\nu}_{M}(x)$ is not a symmetry of $\mathcal{L}_{tot}(x)$

Suppose: $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$, and to "leading order" $m_1 = m_2$, but due to "higher order" corrections $m_1 \neq m_2$, $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects $\sim \Delta m$

- Suppose: $m_1 = m_2$, $\rho_1 = -\rho_2$, but $\chi_{1,2}$ are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L}\cos\phi + \chi_{2L}\sin\phi = \Psi_L\cos\phi' + \Psi^{C}_{L}\sin\phi'$$

All Majorana effects are $\sim m_D\cos\phi'\sin\phi'$

Pontecorvo, 1957, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,, \quad m_1 \neq m_2 > 0\,, \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$ - Majorana, maximal mixing.

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos\theta_C + \Psi_{2L} \sin\theta_C\,,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin\theta_C + \Psi_{2L} \cos\theta_C\,,$$

$\Psi_{1,2}$ - Dirac (composite), θ_C- the Cabbibo angle.

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.
- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.65 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.26$ (2σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.4$ $(2.5) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.033$ $(0.056$ $(0.063))$ 2σ (3σ).

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, arXiv:0804.4857;

T. Schwetz *et al.*, arXiv:0808.2016

Neutrino Oscillation Parameters

parameter	bf	1σ acc.	2σ range	3σ range
Δm^2_{21} [10^{-5} eV^2]	7.65	3%	7.25 – 8.11	7.05 – 8.34
$\lvert\Delta m^2_{31}\rvert$ [10^{-3} eV^2]	2.4	5%	2.18 – 2.64	2.07 – 2.75
$\sin^2\theta_{12}$	0.304	7%	0.27 – 0.35	0.25 – 0.37
$\sin^2\theta_{23}$	0.50	14%	0.39 – 0.63	0.36 – 0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	–	≤ 0.040	≤ 0.056

Best fit values (bf), relative accuracies at 1σ, and 2σ and 3σ allowed ranges of three-flavor neutrino oscillation parameters from a combined analysis of global data.

T. Schwetz *et al.*, arXiv:0808.2016[hep-ph]

• $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $\quad m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \; m^2_{1,2,3} >> \Delta m^2_{\text{atm}}, \; \text{QD}; \; m_j \gtrsim 0.10 \text{ eV}.$$

• Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\text{CP}} \propto J_{\text{CP}} \propto \sin\theta_{13} \sin\delta$

• Majorana phases α_{21}, α_{31}:

– $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

– $\left| < m > \right|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};

– $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

– BAU, leptogenesis scenario: $\alpha_{21,31}$!

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.4) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

$\nu_\odot$, $\Delta m^2_{\mathrm{atm}}$, CHOOZ Data:

- $\theta_{12} = \theta_\odot \sim \frac{\pi}{6}$, $\theta_{23} = \theta_{\mathrm{atm}} \cong \frac{\pi}{4}$, $\theta_{13} < \frac{\pi}{12}$

$$U_{\mathrm{PMNS}} \cong \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1+\lambda)$, $\sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1-\lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\theta_\odot + \theta_c = \pi/4$?

Natural Possibility:

$$U = U^{\dagger}_{\text{lep}}(\lambda)\; U_{\text{bim(tri)}}$$

with

$$U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\text{lep}}(\lambda)$ - from diagonalization of the l^{-} mass matrix,

- $U_{\text{bim(tri)}}$ - from diagonalization of the $\nu-$mass matrix

Further, $\Delta m^2_{\odot} \ll |\Delta m^2_{\text{atm}}|$.

- U_{bim} can be associated with a symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

- $U_{\text{bim(tri)}}$ can be associated with a $\mu - \tau$ symmetry of M_ν

T. Fukuyama, H. Nishiura, 1997; R.N. Mohapatra, S. Nussinov, 1999;...

These symmetries cannot be exact.

In the case of conserved $L' = L_e - L_\mu - L_\tau$:

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$, $\theta_{13} = 0$, $\tan\theta_{23} = M_{e\tau}/M_{e\mu}$,

$m_3 = 0$ - spectrum with IH, $m_1 = m_2$, $\chi_{1,2}$ - equivalent to one Dirac ν, Ψ .

Adding L'-breaking term, e.g. M_{ee}, $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$, leads to $m_1 \neq m_2$ compatible with $\Delta m^2_\odot$.

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^{*}_{e2}\, U^{*}_{\mu 1}\right\} \ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\left\{U_{e1} U^{*}_{e3}\right\}, \quad S_2 = \text{Im}\left\{U_{e2} U^{*}_{e3}\right\} \quad \text{(not unique);} \quad \text{or}$$
$$S'_1 = \text{Im}\left\{U_{\tau 1} U^{*}_{\tau 2}\right\}, \quad S'_2 = \text{Im}\left\{U_{\tau 2} U^{*}_{\tau 3}\right\}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im}\left\{U_{e1} U^{*}_{e3}\right\} \neq 0$ and $\text{Re}\left\{U_{e1} U^{*}_{e3}\right\} \neq 0$.

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP\ :\ \ P_j e^{-i(E_j t - p_j x)} P^{*}_j = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter.

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn(Δm^2_{atm}), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in U_{PMNS} ;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12}+\theta_c=\pi/4$?

– Is $\theta_{23}=\pi/4$, or $\theta_{23}>\pi/4$ or else $\theta_{23}<\pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

If ν_j- Majorana particles, U_{PMNS} **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \to \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \text{(A, Z)} \to \mu^+ + \text{(A, Z} - 2\text{)}$$

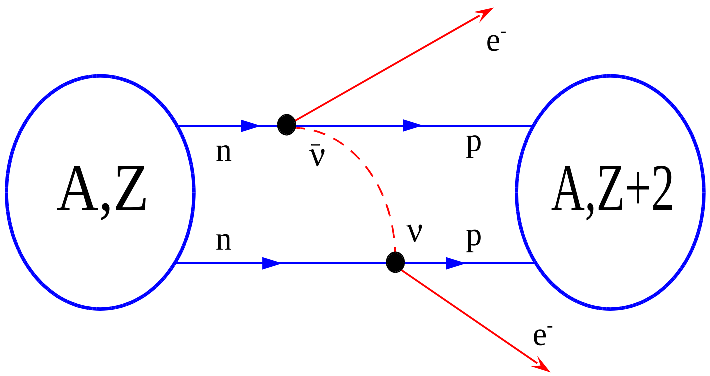
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$\text{(A, Z)} \to \text{(A, Z} + 2\text{)} + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

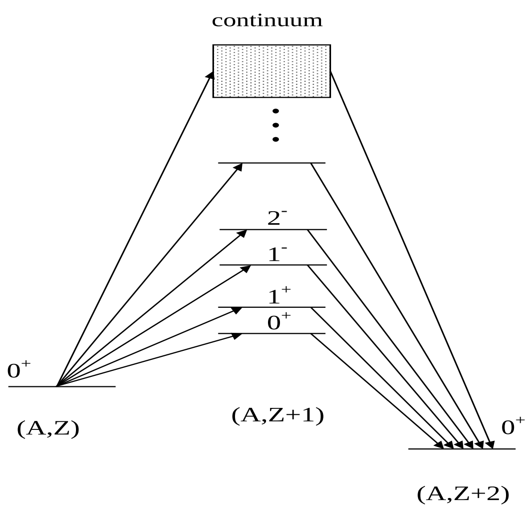
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \to uue^{-}e^{-}(\overline{\nu}_e\overline{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$**,** $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha'_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha'_{31}}\right|,\quad \theta_{12} \equiv \theta_{\odot},\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix; $\alpha'_{31} \equiv \alpha_{31} - 2\delta$

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$|<m>|$: m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of min(m_j), $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering, or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) if $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|<m>| = |<m>| \; (m_{\rm min}, \alpha_{21}, \alpha_{31}; S), \; S = {\rm NO(NH), IO(IH)}.$$

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left| \sqrt{\Delta m^2_{\odot}} \sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}} \sin^2\theta_{13} e^{i\beta} \right| , \ m_1 \ll m_2 \ll m_3 \ \text{(NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}} \left| \cos^2\theta_{12} + e^{i\alpha} \sin^2\theta_{12} \right| , \ m_3 < (\ll) m_1 < m_2 \ \text{(IH)},$$

$$|<m>| \cong m \left| \cos^2\theta_{12} + e^{i\alpha} \sin^2\theta_{12} \right| , \ m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \ \text{(QD)},$$

$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta + 2\delta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$|<m>| \lesssim 5 \times 10^{-3}$ eV, NH;

$\sqrt{\Delta m^2_{13}} \cos 2\theta_{12} \cong 0.013$ eV $\lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055$ eV, IH;

$m \cos 2\theta_{12} \lesssim |<m>| \lesssim m$, $m \gtrsim 0.10$ eV, QD .

Solar neutrino and KamLAND data:

$\cos 2\theta_{\odot} = 0.0$ excluded at > 6 s.d.

Best fit value: $\cos 2\theta_{\odot} \simeq 0.40$

$\cos 2\theta_{\odot} \gtrsim 0.28$, 95% C.L.

Normal hierarchical spectrum:

$$(|<m>|\ )_{\text{max}} \lesssim 0.005 \text{ eV}$$

Inverted hierarchical spectrum:

$$(|<m>|\ )_{\text{min}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos 2\theta_{\odot} \cos^2\theta_{13} \gtrsim 0.01 \text{ eV}$$

$$(|<m>|\ )_{\text{max}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos^2\theta_{13} \lesssim 0.055 \text{ eV}$$

Quasi-degenerate spectrum:

$$(|<m>|\ )_{\text{min}} \simeq m\ (\cos 2\theta_{\odot} \cos^2\theta_{13} - \sin^2\theta_{13}) \gtrsim 0.03 \text{ eV}$$

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|{<}m{>}| = (0.1 - 0.9)$ eV (99.73% C.L.).

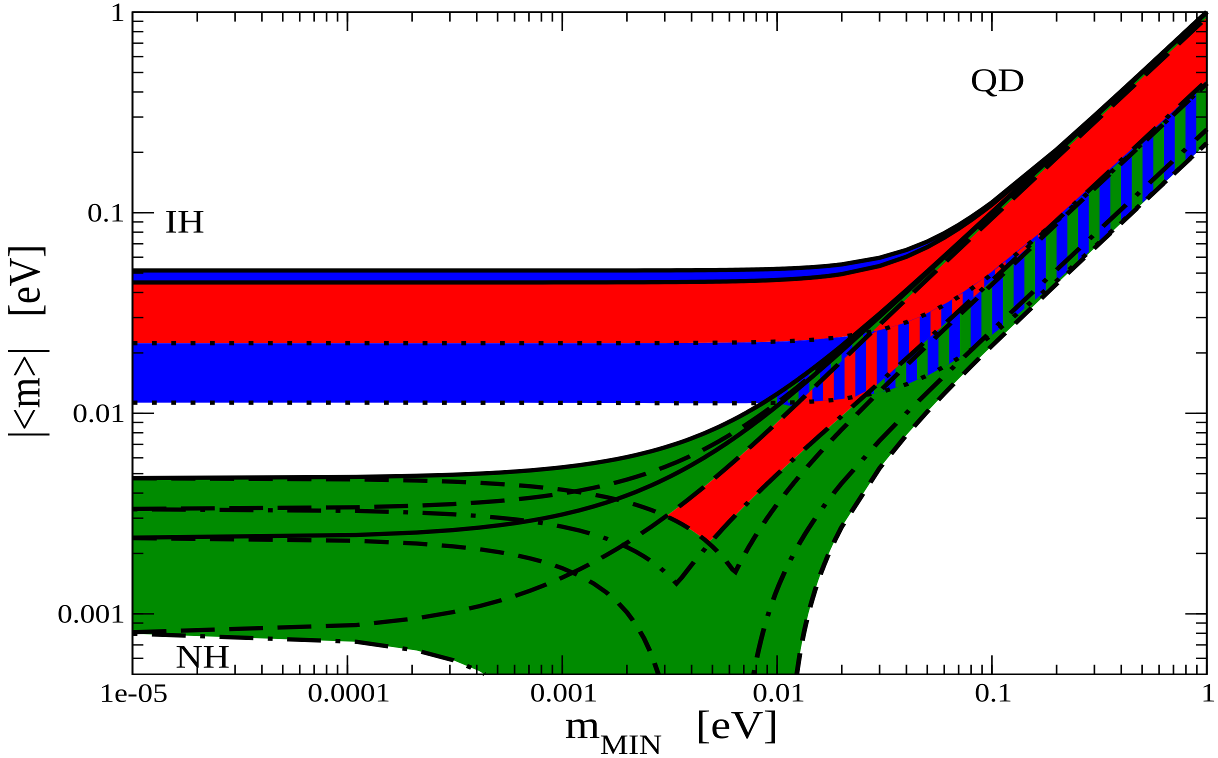
IGEX ^{76}Ge: $|{<}m{>}| < (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|{<}m{>}| <$(0.7–1.2) eV, $|{<}m{>}| <$(0.18–0.90) eV (90% C.L.).

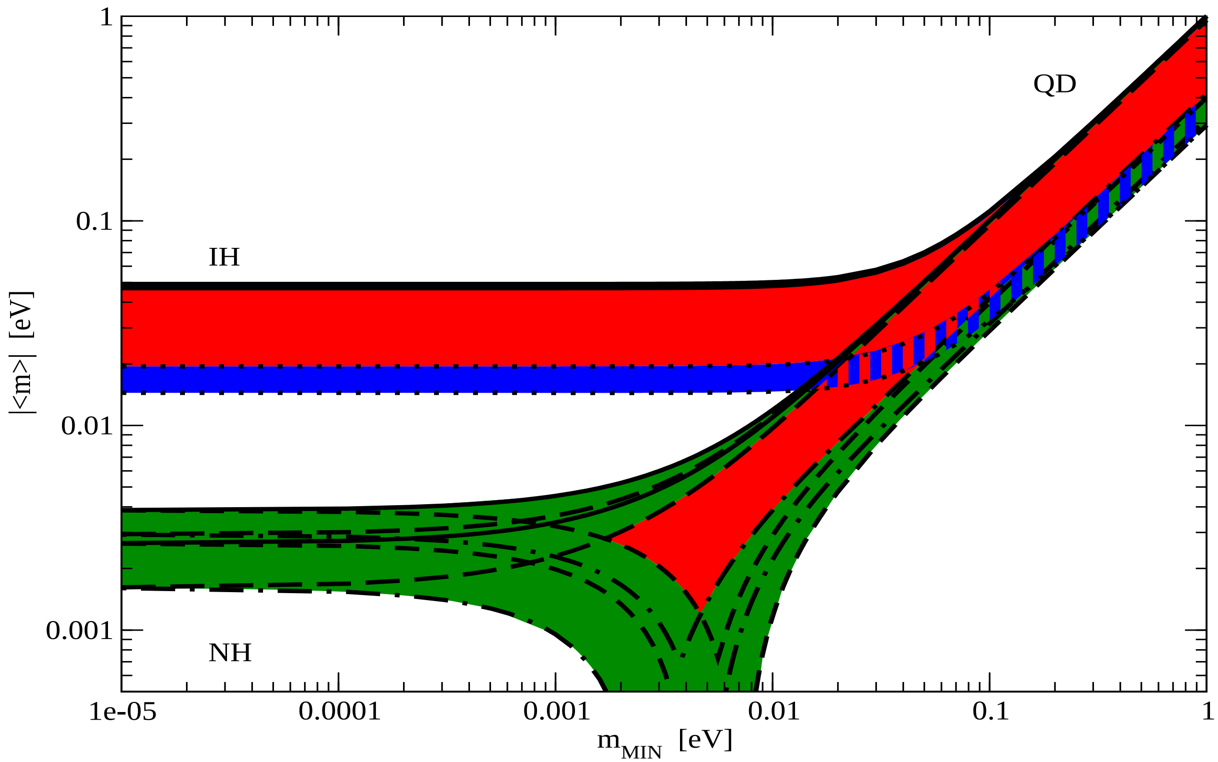
Large number of projects: $|{<}m{>}| \sim (0.01 - 0.05)$ eV

CUORE - ^{130}Te;
GERDA - ^{76}Ge;
SuperNEMO - ^{82}Se,...;
COBRA - ^{116}Cd;
EXO - ^{136}Xe;
MAJORANA - ^{76}Ge;
MOON - ^{100}Mo;
CANDLES - ^{48}Ca;
XMASS - ^{136}Xe.



S. Pascoli, S.T.P., 2007

The current 2σ ranges of values of the parameters used.



$\sin^2\theta_{13} = 0.01 \pm 0.006;\ 1\sigma(\Delta m^2_{\odot}) = 2\%,\ 1\sigma(\sin^2\theta_{\odot}) = 4\%,\ 1\sigma(|\Delta m^2_{\rm atm}|) = 2\%;$

$2\sigma(|{<}m{>}|\ )$ used.

Majorana CPV Phases and $|<m>|$

CPV can be established provided

– $|<m>|$ measured with $\Delta \lesssim 15\%$;

– Δm^2_{atm} (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

• Explains the smallness of $\nu-$masses.

• Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986.

• In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \ , \ \text{etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

• The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x)\,,$$

$$\begin{aligned}
\mathcal{L}_{\text{CC}} &= -\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,\\
\mathcal{L}_{\text{Y}}(x) &= \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,\\
\mathcal{L}_{\text{M}}^{\text{N}}(x) &= -\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.
\end{aligned}$$

ψ_{lL} - LH doublet, $\psi_{lL}^{\text{T}} = (\nu_{lL}\ \ l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\ \ v = 174\ \text{GeV},\ \ v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\ \lambda^T\,M_R^{-1}\,\lambda = U_{\text{PMNS}}^{*}\ m_\nu^{\text{diag}}\ U_{\text{PMNS}}^{\dagger}\ .$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\text{PMNS}})^\dagger/v_u$, all at M_R ; R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j, \;\; C(\overline{N}_k)^T = N_k, \;\; j,k = 1,2,3.$

The CP-symmetry transformation:

$$\begin{aligned} U_{\rm CP}\, N_j(x)\, U_{\rm CP}^{\dagger} &= \eta_j^{NCP}\, \gamma_0\, N_j(x')\,, \quad \eta_j^{NCP} = i\rho_j^N = \pm i\,, \\ U_{\rm CP}\, \nu_k(x)\, U_{\rm CP}^{\dagger} &= \eta_k^{\nu CP}\, \gamma_0\, \nu_k(x')\,, \quad \eta_k^{\nu CP} = i\rho_k^{\nu} = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda_{jl}^{*} = \lambda_{jl}\, (\eta_j^{NCP})^{*}\, \eta^{l}\, \eta^{H*}\,, \quad j = 1,2,3, \;\; l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i, \;\; \eta^H = 1 \quad (\eta^W = 1)$:

$$\begin{aligned} \lambda_{jl}^{*} &= \lambda_{jl}\, \rho_j^N\,, \;\; \rho_j^N = \pm 1\,, \\ U_{lj}^{*} &= U_{lj}\, \rho_j^{\nu}\,, \;\; \rho_j^{\nu} = \pm 1\,, \\ R_{jk}^{*} &= R_{jk}\, \rho_j^N\, \rho_k^{\nu}, \quad j,k = 1,2,3, \;\; l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U_{lk}^{*}\, U_{lm}\,, \;\; k \neq m\,,$$

$$CP: \quad P_{jkml}^{*} = P_{jkml}\, (\rho_j^N)^2\, (\rho_k^{\nu})^2\, (\rho_m^{\nu})^2 = P_{jkml}\,, \quad {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\,R_{jm}\,U^*_{lk}\,U_{lm}\,,\;\; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\,(\rho^N_j)^2\,(\rho^\nu_k)^2\,(\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $P_{123\tau} = R_{12}\,R_{13}\,U^*_{\tau 2}\,U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\,U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\,R_{13}$ corresponds to CP-violation at "high" E.

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma:\ \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2}\ \varepsilon\,\kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k}\, m_k^{1/2}\, U_{lk}^*\right|^2, \quad l = e, \mu, \tau\,.$$

The "One-Flavor" Regime: $M_1 \sim T > 10^{12}$ GeV; $\mathbf{Y}_{e,\mu,\tau}$ - "small"

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l\ H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \left|R_{1k}\right|^2},$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k \left|R_{1k}\right|^2 .$$

Two-Flavour Regime: 10^9 GeV $\lesssim M_1 \sim T \lesssim 10^{12}$ GeV

At $M_1 \sim T \lesssim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \to (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \to N_1$;

$\tau_L^- + \Phi^0 \to \tau_R^-$, $\tau_L^- + \tau_L^+ \to N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable.

Three-Flavour Regime: $M_1 \sim T < 10^9$ GeV

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 \; (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 \, v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2 , \quad l = e, \mu, \tau \, .$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*} \left(\epsilon_2 \, \eta \left(\frac{417}{589} \widetilde{m_2} \right) + \epsilon_\tau \, \eta \left(\frac{390}{589} \widetilde{m_\tau} \right) \right) ,$$

$$\eta \left(\widetilde{m_l} \right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25 \times 10^{-3} \, \text{eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \, \text{eV}}{\widetilde{m_l}} \right)^{-1.16} \right)^{-1} .$$

$Y_{\mathcal{B}} = -(12/37) \, (Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0,$

$$\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{\tau j}^* U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \text{Im}\left(U_{\tau j}^* U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \; R_{1j} R_{1k} = \pm\, |R_{1j} R_{1k}|\,, \\
&= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j)\, |R_{1j} R_{1k}|\, \text{Re}\left(U_{\tau j}^* U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \; R_{1j} R_{1k} = \pm i |R_{1j} R_{1k}|
\end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\text{Im}\left(U_{\tau j}^* U_{\tau k}\right) \neq 0$, $\text{Re}\left(U_{\tau j}^* U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589} \widetilde{m_\tau}\right) - \eta\left(\frac{417}{589} \widetilde{m_2}\right) \right)$$

$m_1 \ll m_2 \ll m_3,\ M_1 \ll M_{2,3};\ \ R_{12}R_{13}$ – real; $\ m_1 \cong 0,\ R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_\odot}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_\odot}{\Delta m^2_{31}}\right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \times \left(1 - \frac{\sqrt{\Delta m^2_\odot}}{\sqrt{\Delta m^2_{31}}}\right) \mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3})$$

$$\mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi,\ \delta = 0$: $\ \mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) = 0$, CPV due to R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2.8 \times 10^{-13}\,|\sin\delta|\left(\frac{s_{13}}{0.2}\right)\left(\frac{M_1}{10^9\ \text{GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \gtrsim 0.11\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \qquad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}}\right) \left(\frac{M_1}{10^9 \text{ GeV}}\right) .$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m^2_\odot/|\Delta m^2_{31}| \cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

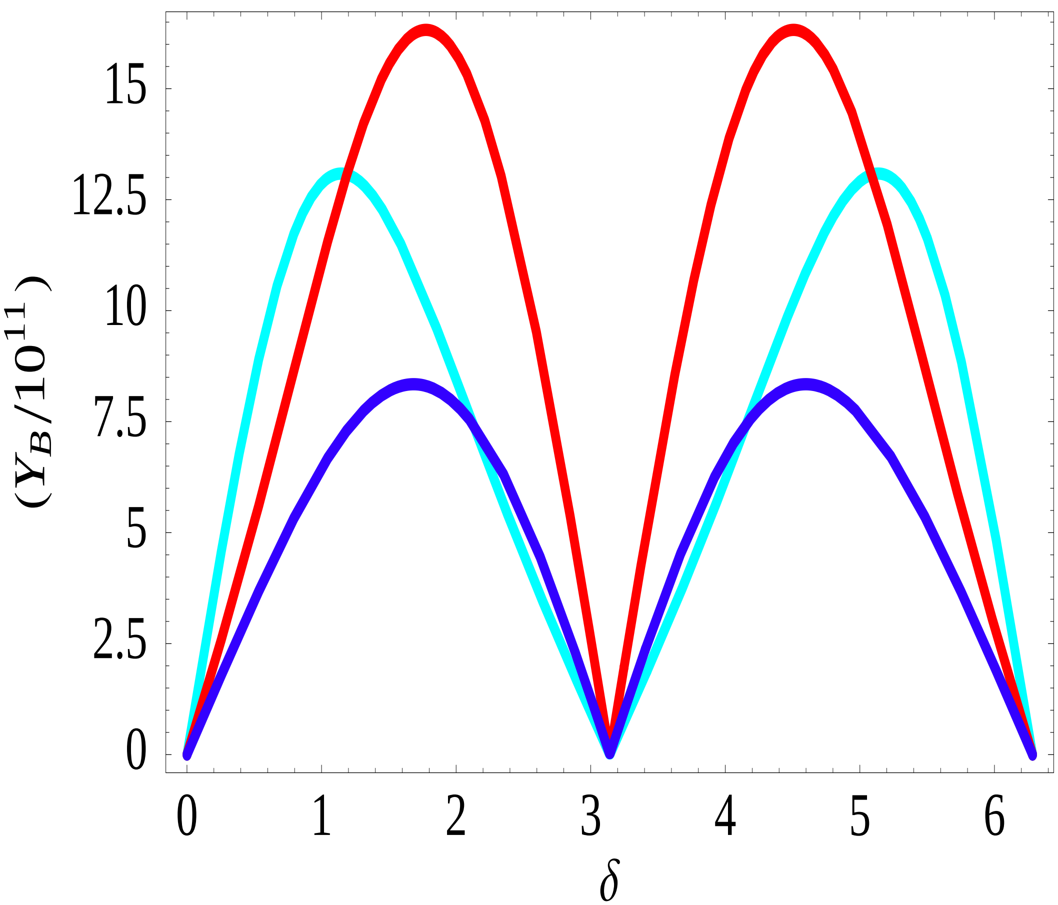
$$|Y_B| \cong 8.1 \times 10^{-12} \, |s_{13} \sin\delta| \left(\frac{M_1}{10^9 \text{ GeV}} \right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02 \,, \qquad \sin\theta_{13} \gtrsim 0.02 \,.$$

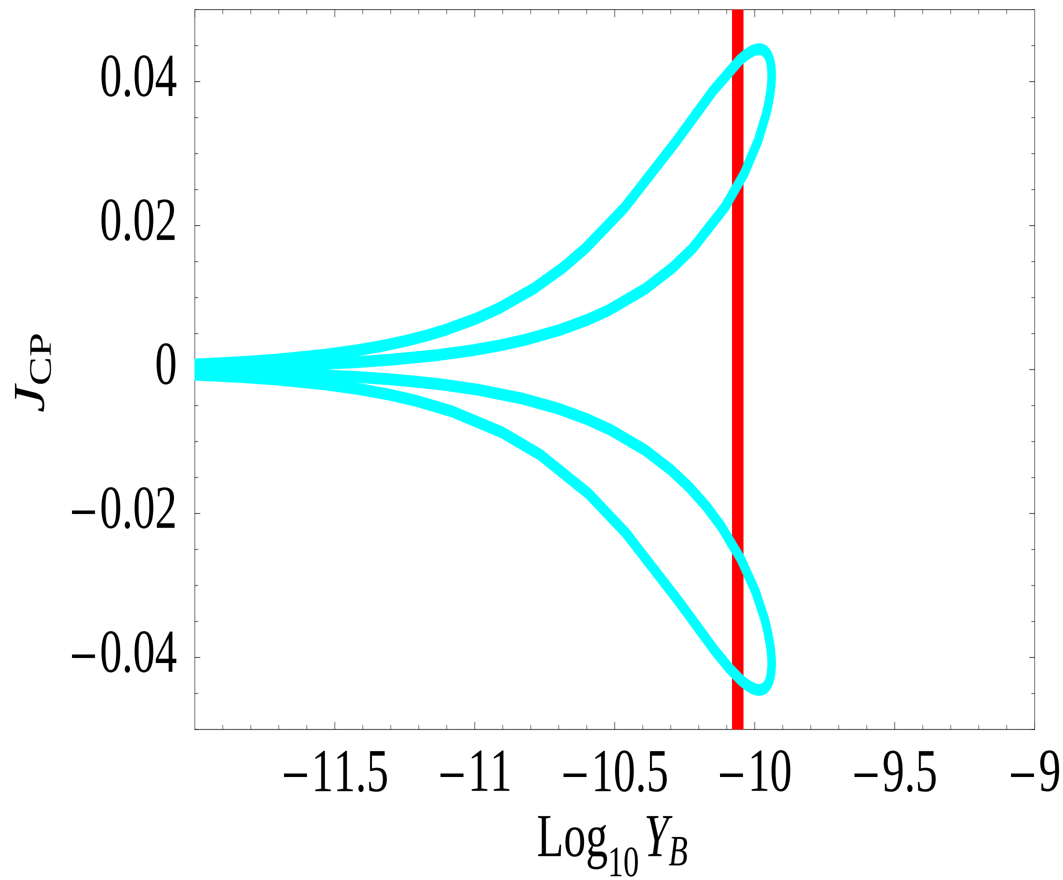
The lower limit corresponds to

$$|J_{CP}| \gtrsim 4.6 \times 10^{-3}$$



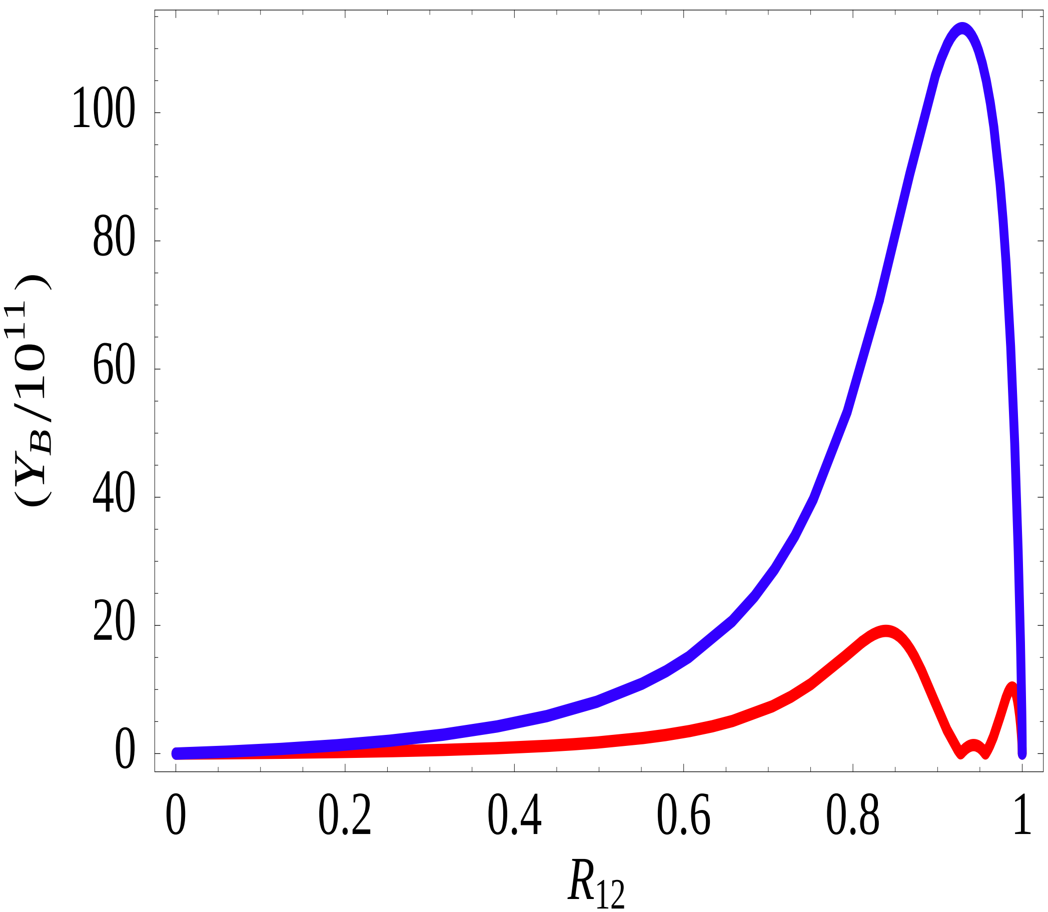
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

S. Pascoli, S.T.P., A. Riotto, 2006.

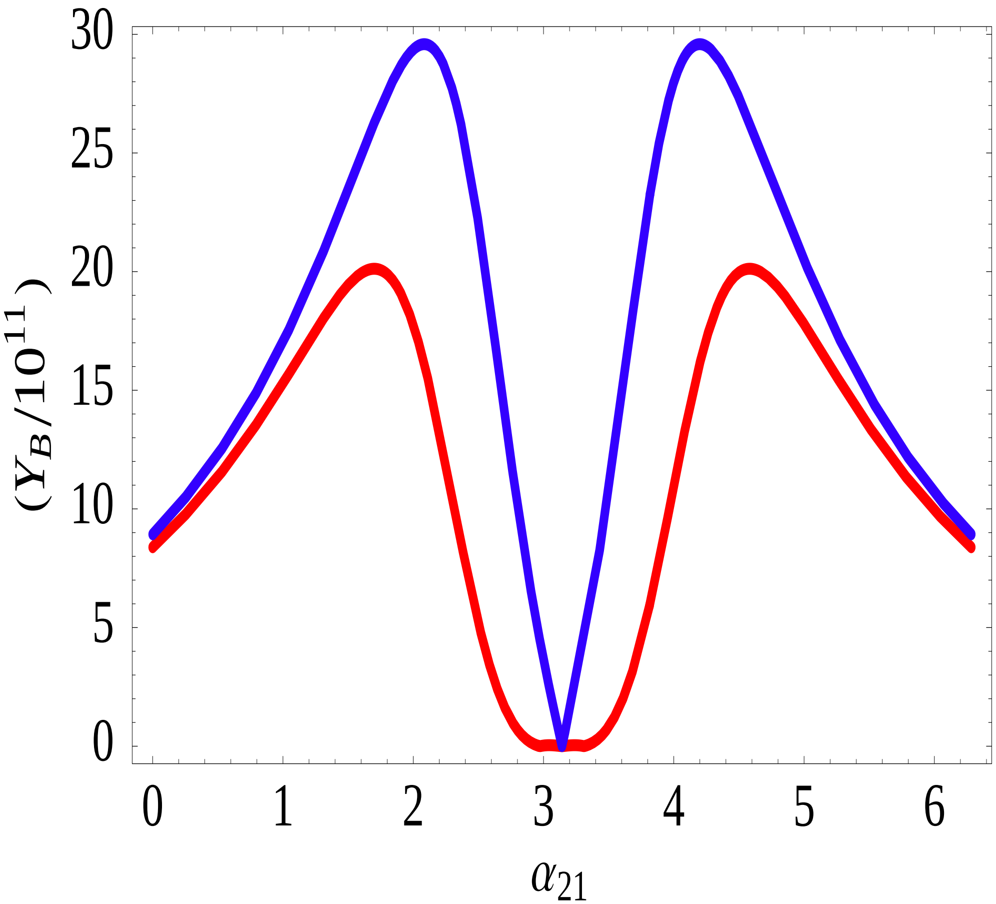


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0 \ (2\pi)$;
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1 \ (-1)$ ($\beta_{23} = 0 \ (\pi)$, $\kappa' = +1$);
The red region denotes the 2σ allowed range of Y_{B}.

S. Pascoli, S.T.P., A. Riotto, 2006.

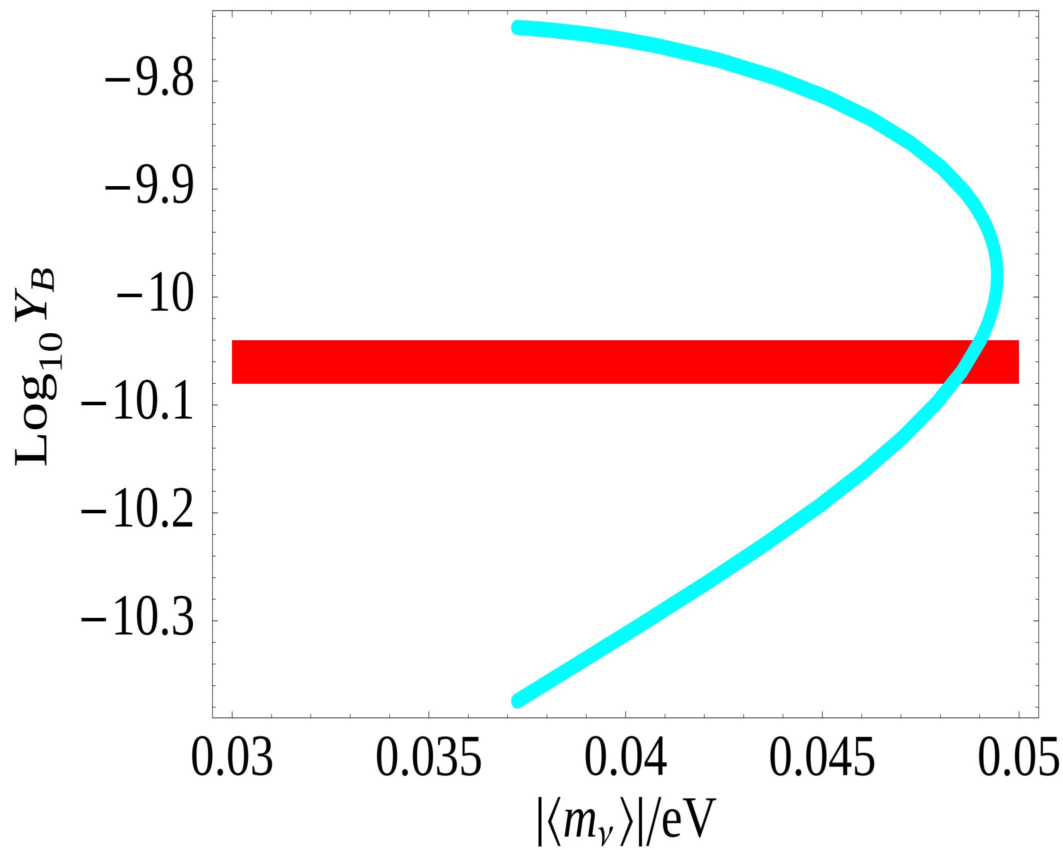


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign$\,(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m^2_\odot$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



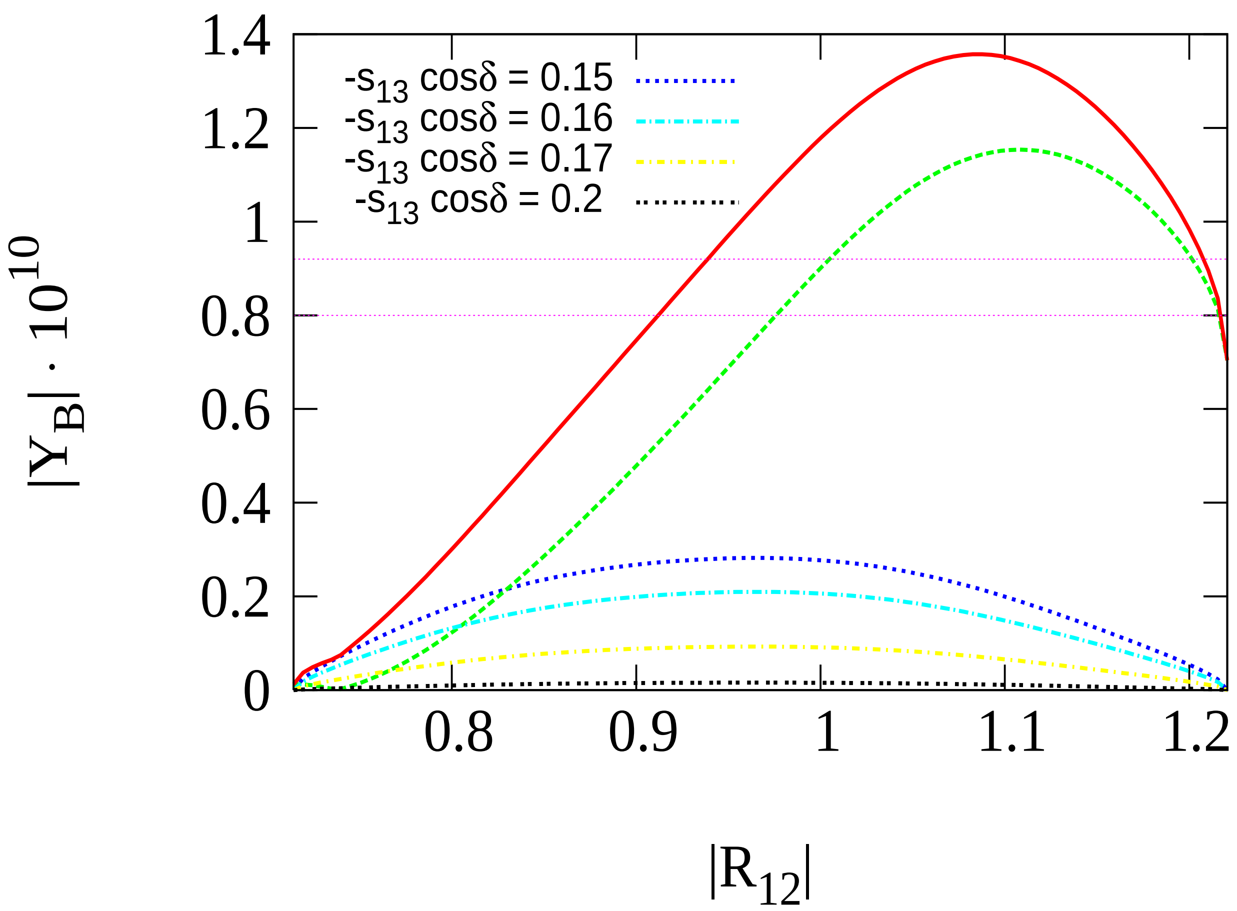
$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



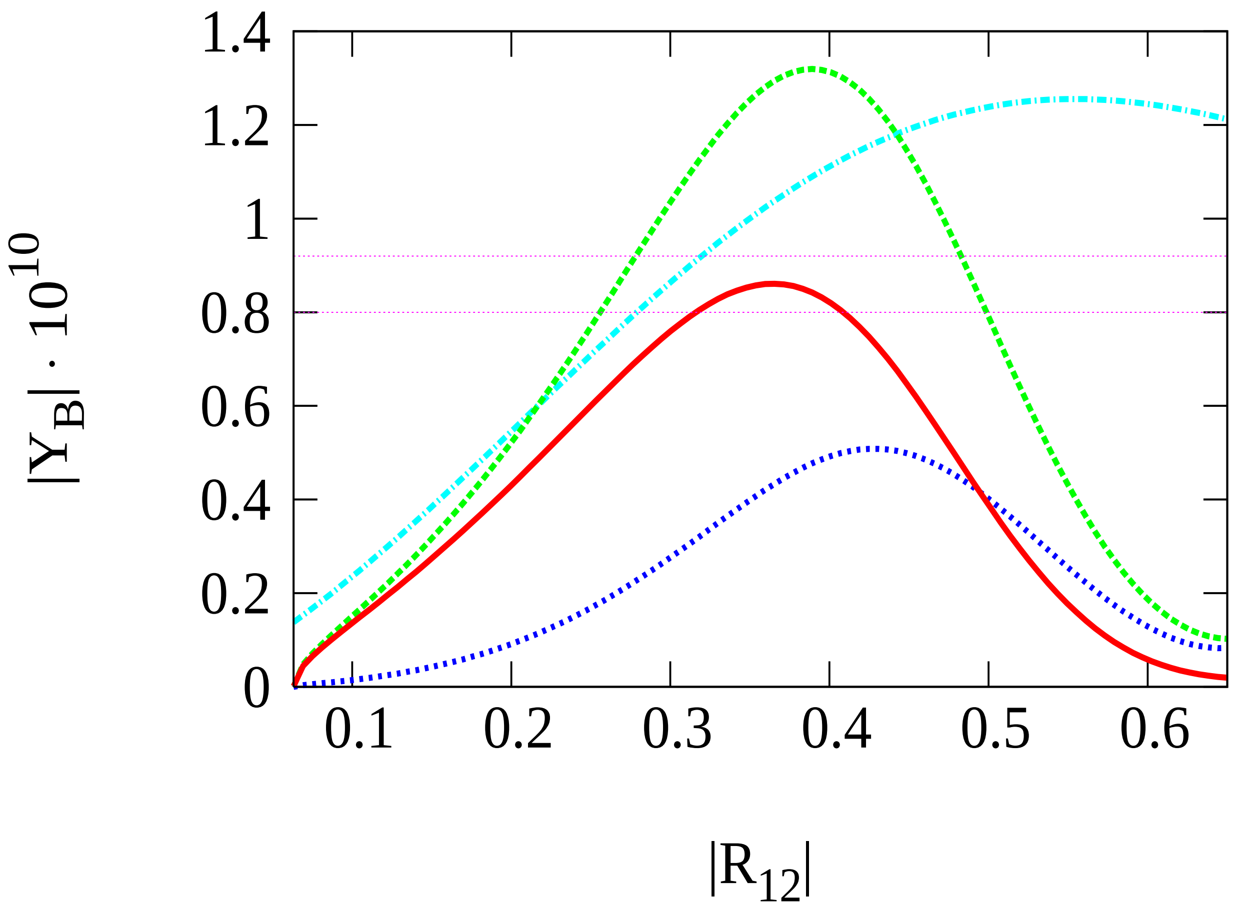
$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV ,
$\alpha_{21} = \pi/2$, $(-s_{13} \cos\delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .

E. Molinaro, S.T.P., 2008



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV ,
$\alpha_{21} = \pi/2$, $s_{13} = 0$, $|R_{11}| \cong 1$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .
Light-blue line: CP-conserving R, $R_{11}R_{12} \equiv ik|R_{11}R_{12}|$, $k = -1$ $|R_{11}|^2 - |R_{12}|^2 = 1$.

E. Molinaro, S.T.P., 2008

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\rm CP}| \gtrsim 2.0\times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\rm CP}| \gtrsim 4.6\times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 3.5\times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$.

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

– Can establish the Majorana nature of ν_j

– Can provide unique information on the ν mass spectrum

– Can provide unique information on the absolute scale of ν masses

– Can provide information on the Majorana CPV phases

The see-saw mechanism provides a link between ν-mass generation and BAU.
Majorana CPV phases in U_{PMNS}: $(\beta\beta)_{0\nu}$-decay, Y_{B}.
Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.