Active-Sterile Solar Neutrino Oscillation

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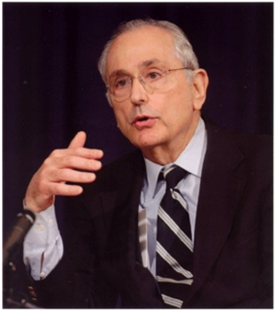
Oct. 27, 2014 @Padova

Overview

1. A brief Introduction on $\nu_{\odot}$ Physics
2. Phenomenology of Light Sterile Neutrinos
3. Active-Sterile $\nu_{\odot}$ Oscillation
4. Conclusion

Section I: A brief Introduction on $\nu_\odot$ Physics

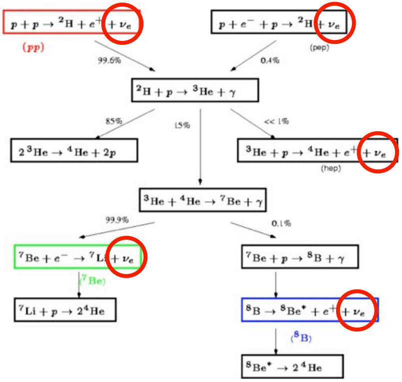
Standard Solar Model



"We mean the solar model that is constructed with the best available physics and input data. All of the solar models we consider, are required to fit the observed luminosity and radius of the Sun at the present epoch, as well as the observed heavy-element-to-hydrogen ratio at the surface of the Sun ." – John Bahcall

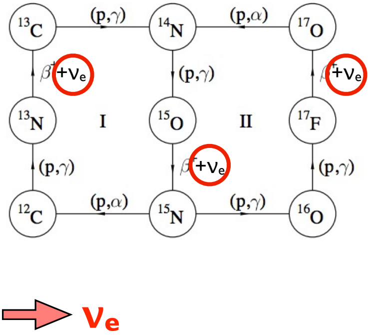
Modern Understanding

pp Chain



CNO Cycle

(contributes ~1% of solar energy)



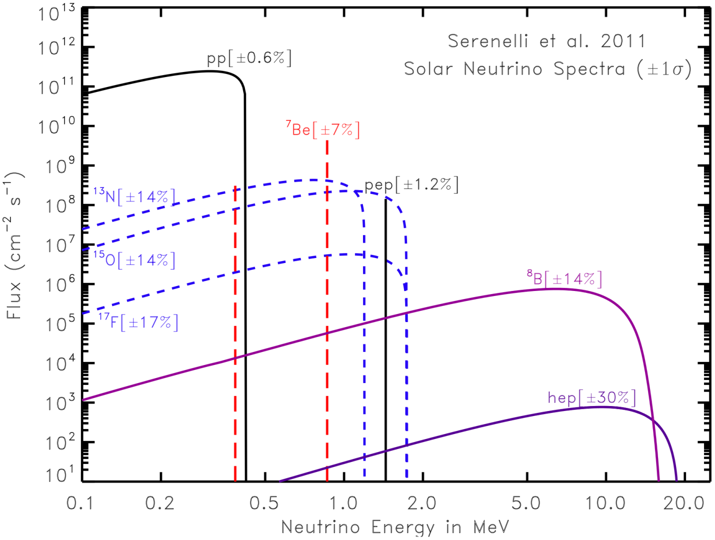
The Astrophysical Journal **687** (2008) 678

$\nu_\odot$ Flux

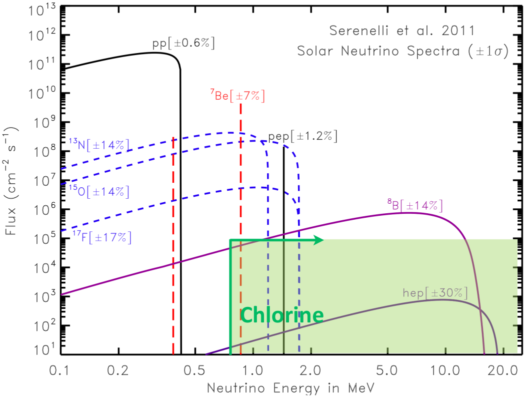
ν flux	GS98	AGSS09	Solar
pp	$5.98(1 \pm 0.006)$	$6.03(1 \pm 0.006)$	$6.05(1^{+0.003}_{-0.011})$
pep	$1.44(1 \pm 0.012)$	$1.47(1 \pm 0.012)$	$1.46(1^{+0.010}_{-0.014})$
hep	$8.04(1 \pm 0.30)$	$8.31(1 \pm 0.30)$	$18(1^{+0.4}_{-0.5})$
^{7}Be	$5.00(1 \pm 0.07)$	$4.56(1 \pm 0.07)$	$4.82(1^{+0.05}_{-0.04})$
^{8}B	$5.58(1 \pm 0.14)$	$4.59(1 \pm 0.14)$	$5.00(1 \pm 0.03)$
^{13}N	$2.96(1 \pm 0.14)$	$2.17(1 \pm 0.14)$	≤ 6.7
^{15}O	$2.23(1 \pm 0.15)$	$1.56(1 \pm 0.15)$	≤ 3.2
^{17}F	$5.52(1 \pm 0.17)$	$3.40(1 \pm 0.16)$	$\leq 59.$
$\chi^2/P^{\rm agr}$	3.5/90%	3.4/90%	—

Neutrino fluxes are given in units of 10^{10}(pp), 10^9(^{7}Be), 10^8(pep, ^{13}N, ^{15}O), 10^6(^{8}B, ^{17}F) and 10^3(hep) cm^{-2} s^{-1}. GS98: higher metallicity model; AGSS09: Lower metallicity model; Solar: Solar neutrino fluxes inferred from all available neutrino data. [Serenelli, etc., 2011]

$\nu_\odot$ Spectrum



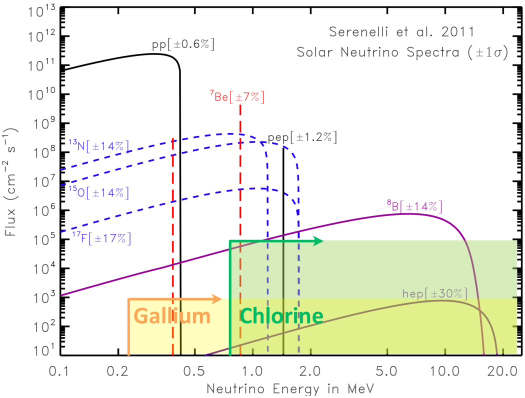
Chlorine Experiments: $\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$



Homestake: USA, 1965-1967, 1970-1994, 4200 mwe, 133 ton Cl,

$$\frac{R_{\rm Cl}^{\rm exp}}{R_{\rm Cl}^{\rm SSM}} = 0.34 \pm 0.03$$

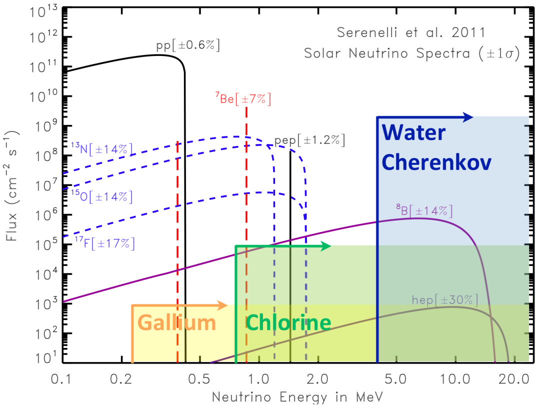
Gallium Experiments: $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$



GALLEX/GNO: Italy, 1991-1997, 1998-2003, 3300 mwe, 30.3 ton Ga
SAGE: USA&Soviet, 1989-2007, 4700mwe, 57ton Ga

$$\frac{R_{\rm Ga}^{\rm exp}}{R_{\rm Ga}^{\rm SSM}} = 0.56 \pm 0.03$$

Water Cherenkov Experiments: $\nu_\alpha + e^- \to \nu_\alpha + e^-$

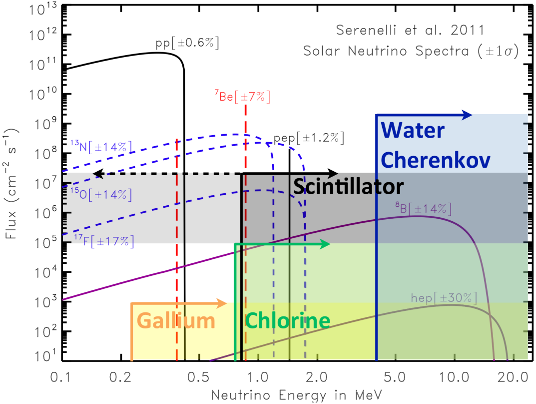


Kamiokande: Japan, I-III: 1983-1995, 2600 mwe, 3000 ton water

Super-Kamiokande: Japan, I-IV: Since 1996, 2600 mwe, 50000 ton water

SNO: Canada, I-III: 1999-2006, 6000 mwe, 1000 ton heavy water

$$\frac{R^{\rm Kam}_{\nu_e}}{R^{\rm SSM}_{\nu_e}} = 0.55 \pm 0.08 \quad , \frac{R^{\rm SKI}_{\nu_e}}{R^{\rm SSM}_{\nu_e}} = 0.465 \pm 0.015$$



Borexino: Italy, since 2007, 3500 mwe, 278 ton scintillator

Designed for LE ν: pep, Be7, CNO, pp

Solar Neutrino Problem

- 1968, Homestake, 1/3 or SSM prediction.
- Helioseismological data shown beautiful agreement with SSM prediction. SSM seemed correct;
- Since 1970s, SNP confirmed by Gallium, Water Cherenkov, Expt.
- 1978-1985, MSW mechanism, flavor conversion, even for SMA;
- 2001, solved by SNO + SK ν_e elastic scattering result
- 2002, VLBL **KamLAND** $\bar{\nu_e}$ disappearance, LMA

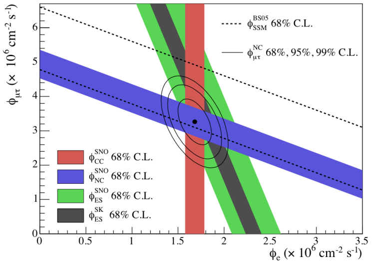
Solution of SNP –SNO

Inclusive appearance at the Sudbury Neutrino Observatory

ES: $\nu_x + e^- \rightarrow e^- + \nu_x$

CC: $\nu_e + {}^2H \rightarrow e^- + 2p$

NC: $\nu_x + {}^2H \rightarrow n + p + \nu_x$



PRL 87 (2001) 071301, PRL 89 (2002) 011301

D_2O phase: 1999 – 2001

$$\frac{R_{CC}^{SNO}}{R_{CC}^{SSM}} = 0.35 \pm 0.02$$

$$\frac{R_{NC}^{SNO}}{R_{NC}^{SSM}} = 1.01 \pm 0.13$$

$$\frac{R_{ES}^{SNO}}{R_{ES}^{SSM}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

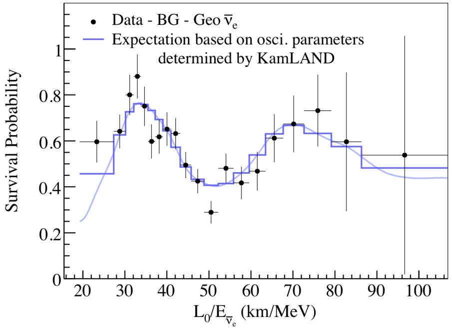
NaCl phase: 2001 – 2002

$$\frac{R_{CC}^{SNO}}{R_{CC}^{SSM}} = 0.31 \pm 0.02$$

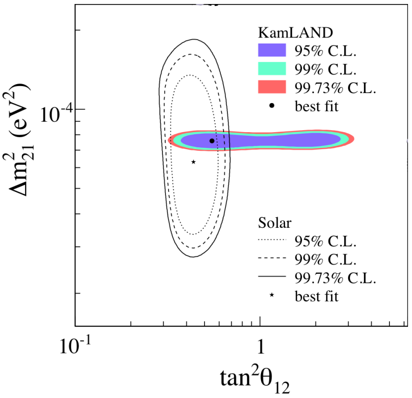
$$\frac{R_{NC}^{SNO}}{R_{NC}^{SSM}} = 1.03 \pm 0.09$$

$$\frac{R_{ES}^{SNO}}{R_{ES}^{SSM}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]



Jap



[KamLAND, PRL 100 (2008) 221803]

Mar.-Oct.,2002, VLBL:180km, 2700mwe

Reactor Expt: $\bar{\nu}_e + p \to e^+ + n$

$$\frac{R^{\rm KamLAND}_{\rm observed}}{R^{\rm KamLAND}_{\rm expected}} = 0.611 \pm 0.085 \pm 0.041$$

N-flavor Neutrino Mixing

- $N = 3 + N_s$, 3 active neutrinos and N_s sterile neutrinos
- Flavor Neutrinos: $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau, \nu_{s_1}, \ldots, \nu_{s_{N_s}}$
- Massive Neutrinos: $\nu_k = \nu_1, \nu_2, \nu_3, \nu_4, \ldots, \nu_N$
- Neutrino Mixing Matrix: U
- For neutrino:

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha\, k} \, |\nu_k\rangle \,,$$

- For anti-neutrino

$$|\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha\, k} \, |\bar{\nu}_k\rangle \,,$$

Parameterization of the Mixing Matrix

- For N-neutrino mixing, U can be parameteried by $(N-1)^2$ parameters. (Ignoring the $N-1$ majorana phases)

$$(N-1)^2 = \frac{N(N-1)}{2}(\text{ Mixing Angle }) + 1 + 2N_s(\text{ Physical Phase })$$

- N=2:

$$U_2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

N=3 and N=4

- N=3:

$$U_3^D = R^{23}(\theta_{23}) \cdot W^{13}(\theta_{13}, \delta) \cdot R^{12}(\theta_{12})$$

$$\equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\equiv \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{i\delta} & c_{13}\, s_{23} \\ s_{12}\, s_{23} - c_{12}\, s_{13}\, c_{23}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, s_{13}\, c_{23}\, e^{i\delta} & c_{13}\, c_{23} \end{pmatrix}$$

- N=4

$$U = W^{34} W^{24} R^{14} R^{23} W^{13} R^{12}$$

The 1st row is important for active-sterile $\nu_\odot$ oscillation:

$$U_{e1} = c_{12} c_{13} c_{14}\, , U_{e2} = s_{12} c_{13} c_{14}\, , U_{e3} = s_{13} e^{-i\eta_{13}} c_{14}\, , U_{e4} = s_{14}\, ,$$

Vacuum Oscillation

- General Expression:

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathrm{Re}[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}] \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$
$$\mp 2\sum_{k>j} \mathrm{Im}[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}] \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right).$$

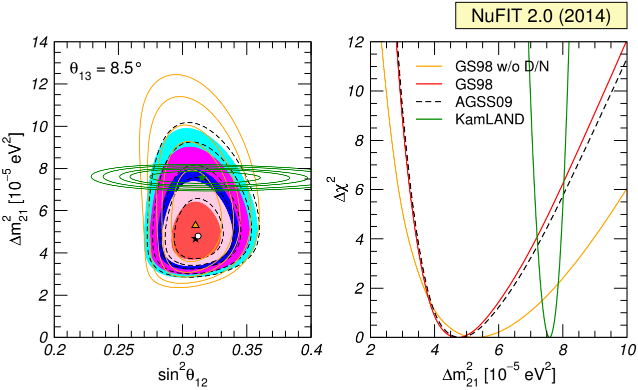
  Surival P. & disppearant expt. ; Transition P. & Appearant expt.

- For N=2, since there is no CP phase, the formulas for neutrino and anti-neutrino are the same.

$$P_{\nu_\alpha \to \nu_\alpha}(L,E) = 1 - P_{\nu_\alpha \to \nu_\beta}(L,E) = 1 - \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right).$$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sin^2 2\vartheta \, \sin^2\left(1.27\frac{\Delta m^2[\mathrm{eV}^2] L[\mathrm{m}]}{E[\mathrm{MeV}]}\right), \qquad (\alpha \neq \beta).$$

Combined Analysis From Latest Solar Data



$$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5}\mathrm{eV}^2$$

$$\theta_{12} = 33.48^{\circ}{}^{+0.78}_{-0.75}$$

Gonzalez-Garcia, etc., arXiv:1409.5439

Beyond SNP

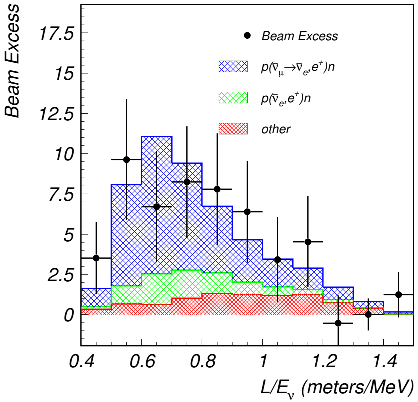
- Precision Measurements of $\nu_\odot$ flux and oscillation parameters
  Hyper-Kamiokande, SNO+, JUNO
- Direct measurement of low energy neutrino
  Be7, pep, CNO: BX, PRD 89:112007 (2014)
  pp: BX, Nature 512, 383386, 2014
- Day-Night Asymmetry: non-zero 2.8σ, SK, 1403.4575
- Solar abundance problem: M. Serenelli et al. 2011 ApJ 743 24
- New physics
  Very light sterile neutrino: de Holanda, et al, PRD 83:113011; Pulido, Das, 1310.0426
  Non-standard MSW Dynamics: Palazzo, PRD 83:101701, 2011
  Non-Standard Models, Solar Neutrinos and Large θ_{13}:
  R. Bonventre, et al. PRD 88:053010, 2013
  Non-standard forward scattering
  Mass-varying neutrinos
  Long-range leptonic forces
  Non-standard solar model

Section II: Phenomenology of Light Sterile Neutrino

Indications of SBL Oscillations Beyond 3ν Mixing

- SNP and atmospheric anomaly lead to standard 3-ν mixing and 2002 Nobel Prize
- Now the same thing happened in SBL sector.
- Oscillation phase: $\frac{\Delta m^2[\mathrm{eV}^2]L[\mathrm{m}]}{E[\mathrm{MeV}]}$
  $L \simeq 1\mathrm{m}, E \simeq 1\mathrm{MeV}$ (or 1km, 1GeV) lead to $\Delta m^2 \simeq 1\mathrm{eV}^2$
- 3 (or 4) SBL anomalies: LSND, Gallium, Reactor (,MiniBooNE)

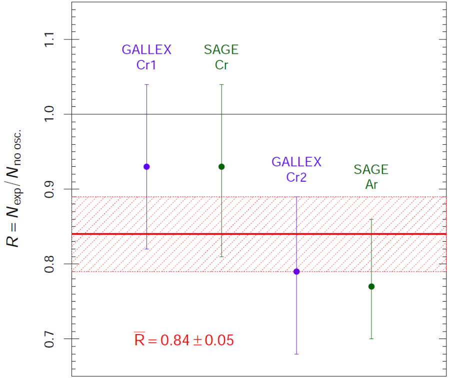
LSND



Accelerator $\bar{\nu_\mu} \to \bar{\nu_e}$, $L \simeq 30m$, $E \simeq 50\mathrm{MeV}$, $\sim 3.8\sigma$ excess, $\Delta m^2 \geq 0.2\mathrm{eV}^2$

A. Aguilar-Arevalo et al. [LSND collab], PRD 64 (2001) 112007

Gallium Anomaly



$\nu_e \to \nu_e$, source: $^{51}\mathrm{Cr}$ and $^{37}\mathrm{Ar}$

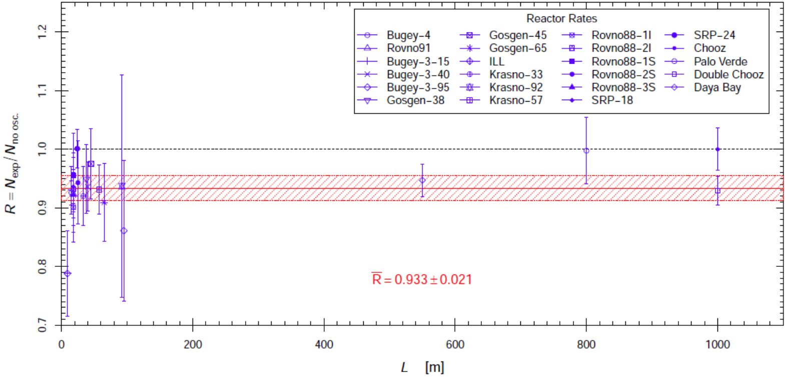
$L \simeq 1m$, $E \simeq 1\mathrm{MeV}$, $\sim 2.9\sigma$ deficit, $\Delta m^2 \geq 1\mathrm{eV}^2$,

SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807; F. Kaether et al., PLB 685 (2010) 4754;

Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344; MPLA 22 (2007) 2499; PRD 78 (2008) 073009; PRC 83 (2011) 065504;

PRD 86 (2012) 113014

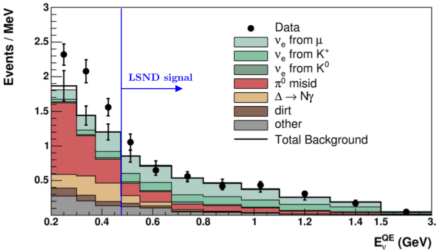
Reactor Anomaly reanalysis (Preliminary 2014)



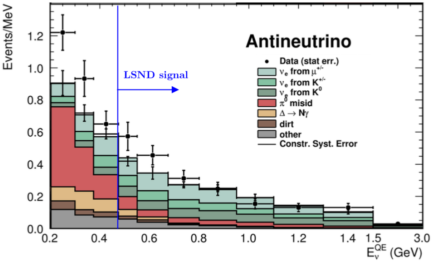
$\bar{\nu_e} \to \bar{\nu_e}$, $L \simeq 10 - 100m$, $E \simeq 4\text{MeV}$, $\sim 3.1\sigma$ deficit, $\Delta m^2 \geq 0.5\text{eV}^2$ Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617;

Mention et al, PRD 83 (2011) 073006; update in White Paper, arXiv:1204.5379

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



- $\bar{\nu}_\mu \to \bar{\nu}_e$:
  $L \simeq 541m$, $E \simeq 200 - 1250\mathrm{MeV}$, $\sim 2.8\sigma$ excess,
  $0.01 < \Delta m^2 < 1\mathrm{eV}^2$, agree with LSND
- $\nu_\mu \to \nu_e$,
  $E \simeq 475 - 1250\mathrm{MeV}$ no significant excess;
  $E \simeq 200 - 475\mathrm{MeV}$ low energy anomaly!

A. Aguilar-Arevalo et al. [MiniBooNE collab], PRL 110 (2013) 161801

Effective SBL Oscillation Probabilities in 3+1 Schemes

General Expression

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) ,$$

$\overset{(-)}{\nu}_e$ disappearance

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2(1 - |U_{e4}|^2)$$

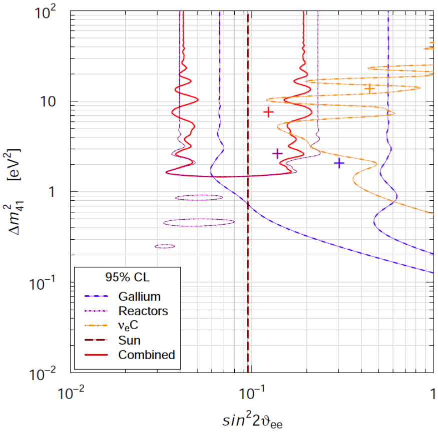
ν_μ disappearance

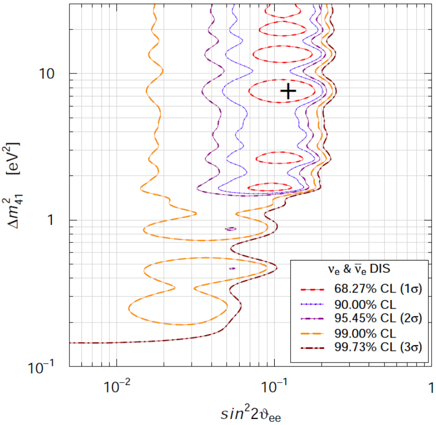
$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$$

$\nu_\mu \to \nu_e$ experiments

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e4}|^2|U_{\mu 4}|^2$$

Global ν_e and $\bar{\nu}_e$ Disappearance (Preliminary 2014)





$\nu_e + ^{12}\mathrm{C} \to ^{12}\mathrm{N}_{g.s.} + e^-$

KARMEN + LSND

Giunti, Laveder, PLB 706 (2011) 200

Conrad, Shaevitz, PRD 85 (2012) 013017

solar ν_e + KamLAND $\bar{\nu}_e$ + θ_{13}

GoF=53%, PGoF=4%

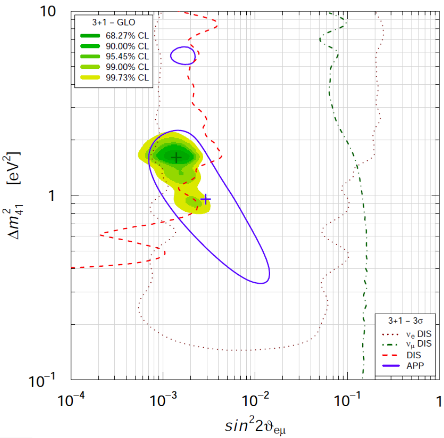
No Osc. excluded at 3.0 σ

$\Delta\chi^2/\mathrm{NDF} = 11.9/2$

Giunti, Li, PRD 80 (2009) 113007

Long, Li, Giunti, PRD 87 (2013) 113004

3+1 Global Fit (Preliminary 2014)



MiniBooNE E > 475MeV

GoF = 26% PGoF = 7%

- APP $\nu_\mu \to \nu_e$ & $\bar{\nu_\mu} \to \bar{\nu_e}$: LSND, MiniBooNE, OPERA, ICARUS, KARMEN, NOMAD, BNL-E776
- DIS ν_e & $\bar{\nu_e}$: Reactors, Gallium, ν_eC , Solar
- DIS ν_μ & $\bar{\nu_\mu}$: CDHSW, MINOS, Atmospheric, MiniBooNE/SciBooNE

No Osc. excluded at 6.3 σ

$\Delta\chi^2/\mathrm{NDF} = 47.7/3$

Summary for sterile fit

- There is only one allowed region in the parameter space around $\Delta m^2_{41} \simeq 1 - 2\text{eV}^2$;
- The crucial indication in favor of short-baseline $\bar{\nu_\mu} \to \bar{\nu_e}$ appearance is still given by the old LSND data;
- The low energy MiniBooNE anomaly cannot be explained by neutrino oscillations;
- MiniBooNE experiment has been inconclusive;
- New better experiments are needed in order to check this signal;
- Reactor $\bar{\nu_e}$ anomaly is alive and exciting;
- Gallium ν_e anomaly strengthened by new cross-section measurements.

Read [Giunti, Laveder, Li, Long, Phys.Rev., 2013, D88, 073008] for detail.

Section III: Active-Sterile $\nu_{\odot}$ Oscillation

Active-Sterile $\nu_\odot$ Osci.

- SBL neutrino: Use vacuum formula, matter effect ignored.
- Solar neutrino :
  - Matter effect is very important. (MSW equation)
  - In the Earth, day-night asymmetry
  - In the sun, MSW resonant effect;
- Previous study worked in 3+1 scheme, assuming U be real.
  C. Giunti and Y. F. Li, Phys. Rev. D 80, 113007 (2009)

  A. Palazzo, Phys. Rev. D 83, 113013 (2011)
- We consider a general scheme of $3+N_s$ neutrino mixing, without any constraint on the neutrino mixing elements, assuming only a realistic hierarchy on the mass-squared differences.

Neutrino probagates in ordinary matter: MSW equation

- Evolution equation in ordinary matter:

$$i\frac{\mathrm{d}}{\mathrm{d}x}\Psi = \left(U\mathcal{M}U^\dagger + \mathcal{V}\right)\Psi$$

where $\Psi = \left(\psi_e, \psi_\mu, \psi_\tau, \psi_{s_1}, \ldots, \psi_{s_{N_s}}\right)^T$ is the flavor transition amplitudes, U is the $N \times N$ neutrino mixing matrix, and

$$\mathcal{M} = \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E}, \frac{\Delta m^2_{31}}{2E}, \frac{\Delta m^2_{41}}{2E}, \ldots, \frac{\Delta m^2_{N1}}{2E}\right) \quad ,$$

$$\mathcal{V} = \mathrm{diag}(V_{\mathrm{CC}} + V_{\mathrm{NC}}, V_{\mathrm{NC}}, V_{\mathrm{NC}}, 0, \ldots, 0) \quad ,$$

- The charge-current (CC) and neutral-current (NC) neutrino matter potentials are defined as:

$$V_{\mathrm{CC}} = \sqrt{2}G_{\mathrm{F}}N_e \simeq 7.63 \times 10^{-14}\,\frac{N_e}{N_{\mathrm{A}}\,\mathrm{cm}^{-3}}\;\mathrm{eV} \quad , V_{\mathrm{NC}} = -\frac{1}{2}\sqrt{2}G_{\mathrm{F}}N_n$$

Difficulties and solutions

- Problem 1: $N \times N$ Schrodinger Equation is too large, impossible to solve analytically
- Solution: Assuming normal hierarchy, for solar and terrestrial matter profile, we have:

$$V_{\rm CC} \sim |V_{\rm NC}| \lesssim \frac{\Delta m^2_{21}}{2E} \ll \frac{|\Delta m^2_{k1}|}{2E} \quad \text{for} \quad k \geq 3 \quad ,$$

Thus, in the vacuum basis $\Psi^{\rm V} = \left(\psi^{\rm V}_1, \ldots, \psi^{\rm V}_N\right)^T = U^\dagger \Psi$, the full evolution equation can be truncated to a 2×2.

Difficulties and solutions

- Problem 2: For $N \leq 3$, $V_{\rm NC}$ can be absorbed as a common phase by the amplitude. But For $N \geq 4$, both $V_{\rm CC}$ and $V_{\rm NC}$ take place.
- Solution:
  - For terrestrial matter profile:
    It's reasonable to consider $V_{\rm NC}$ is proportional to $V_{\rm CC}$, which is the so-called "Constant electron fraction approximation.";
  - For solar matter profile:
    Numerical calculations tell us that the approximation is still valid.

Solar Neutrino Oscillation Probabilities in the daytime

- Final result:

$$\overline{P}^{\mathrm{S}}_{\nu_e\to\nu_\beta} = \cos^2\chi_e \cos^2\chi_\beta \overline{P}^{(2\nu)}_{\nu_e\to\nu_\beta} + \sum_{k=3}^{N} |U_{ek}|^2 |U_{\beta k}|^2 \quad ,$$

where

$$\overline{P}^{(2\nu)}_{\nu_e\to\nu_\beta} = \frac{1}{2} + (\frac{1}{2} - P_{12}) \cos 2\Theta_e^0 \cos 2\theta_\beta \quad ,$$

$$\cos 2\Theta_e^0 = \cos 2\theta_e \cos 2\omega^0 - \cos\Phi_e^0 \sin 2\theta_e \sin 2\omega^0 \quad ,$$

$$\Phi_e^0 = \phi_{e1} - \phi_{e2} + \varphi^0 \quad ,$$

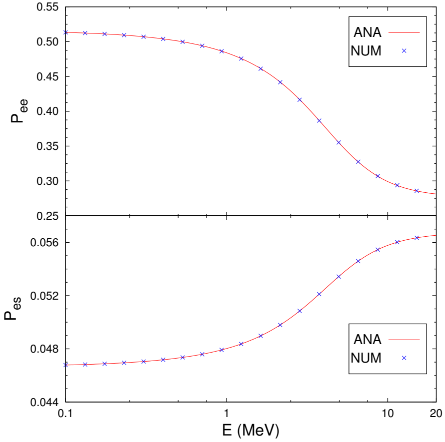
φ^0 is a symbol of CP-phase; P_{12} the crossing probability in resonant region, extremely small for LMA.

- Here,

$$\begin{cases} U_{\beta 1} &= \cos\theta_\beta \ \cos\chi_\beta \ e^{i\,\phi_{\beta 1}} \\ U_{\beta 2} &= \sin\theta_\beta \ \cos\chi_\beta \ e^{i\,\phi_{\beta 2}} \end{cases} \quad \text{with} \quad \cos^2\chi_\beta = |U_{\beta 1}|^2 + |U_{\beta 2}|^2 \quad ,$$

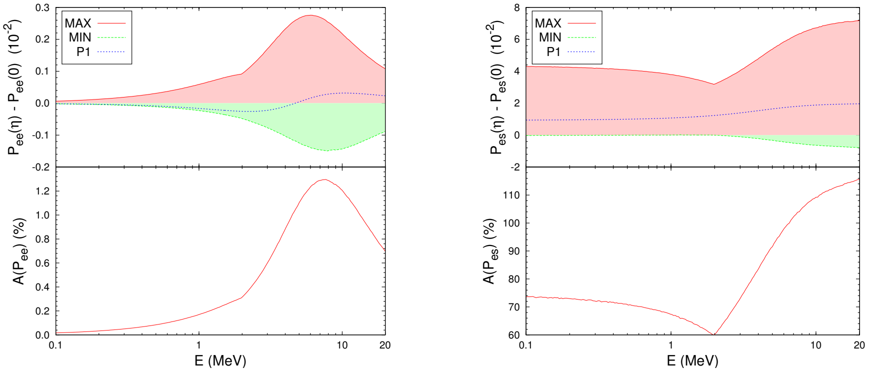
Read [Long, Li, Giunti, Phys. Rev. D 87, 2013, 113004] for detail

Energy spectra of the analytical and numerical probabilities



- Pee is suppressed slightly by active-sterile oscillation.
- Pes is small, but can be measured by future precise $\nu_\odot$ experiment.
- "Constant electron fraction approximation." is validated.

Exploring the maximum effect of CP-violating phase



$$\text{P1}: \quad \begin{cases} \Delta m^2_{12} \simeq 7.54 \times 10^{-5}\,\text{eV} \\ \theta_{12} \simeq 33.6^\circ, \quad , \theta_{23} \simeq 39.1^\circ, \quad , \theta_{13} \simeq 9.0^\circ \\ \theta_{14} = \theta_{24} = \theta_{34} = 10^\circ, \\ \eta_{13} = 35^\circ, \quad \eta_{24} = 75^\circ, \quad \eta_{34} = 115^\circ \quad , \end{cases}$$

- CP-violating phase will effect the oscillation behavior for $N \geq 4$, a completely new phenomenology
- In the case of 3+1 neutrino mixing, can reach the level of 1% for the Pee and may be as large as 100% for the Pes.

Solar Neutrino Oscillation Probabilities in the nighttime

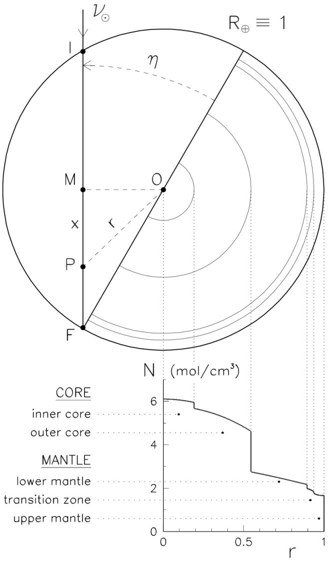
- The nighttime oscillation probabilities:

$$\overline{P}^{\rm SE}_{\nu_e\to\nu_\beta} = \overline{P}^{\rm S}_{\nu_e\to\nu_\beta} - \cos^2\chi_e(1-2P_{12})\cos 2\Theta^0_e R_{2\beta}$$

where R is the regeneration factor of active-sterile solar neutrino oscillations:

$$R_{2\beta} = P^{\rm E}_{\nu_2\to\nu_\beta} - P^{\rm V}_{\nu_2\to\nu_\beta} \quad ,$$

- We develop three different ways to calculate $P^{\rm E}_{\nu_2\to\nu_\beta}$:
  - Perturbative approximation
  - Slab approximation
  - Slab+Pert



Three different ways to calculate $P^{\mathrm{E}}_{\nu_2\to\nu_\beta}$

- The three different way:
  - Pert: Contains double integral, the most accurate method, most time-consuming.
  - Slab: Accurate and fast enough for practical analysis, but not analytically explicit.
  - Slab+Pert Of the worst accuracy, but analytically explicit, helpful for analytical discussion.
- Perturbative approximation:

$$P^{\mathrm{E}}_{\nu_2\to\nu_\beta}(x_f) \simeq P^{\mathrm{V}}_{\nu_2\to\nu_\beta} + \cos^2\chi_\beta \sin 2\theta_\beta \sin 2\xi \times \int_{x_i}^{x_f} V(x) \sin\left[2\Delta(x_f, x) + \Phi_\beta\right] \mathrm{d}x \,.$$

$$\Delta(x_f, x_i) = \int_{x_i}^{x_f} \delta_{\mathrm{M}} \mathrm{d}x \quad ,$$

Long, Li, Giunti, JHEP, 2013, 1308, 056

Three different ways to calculate $P^{\mathrm{E}}_{\nu_2\to\nu_\beta}$

- Slab approximation:

$$\Psi_2^{\mathrm{V}}(x_n) \simeq S^{\mathrm{V}}_{\mathrm{slab}} \Psi_2^{\mathrm{V}}(x_{-n}) \quad ,$$

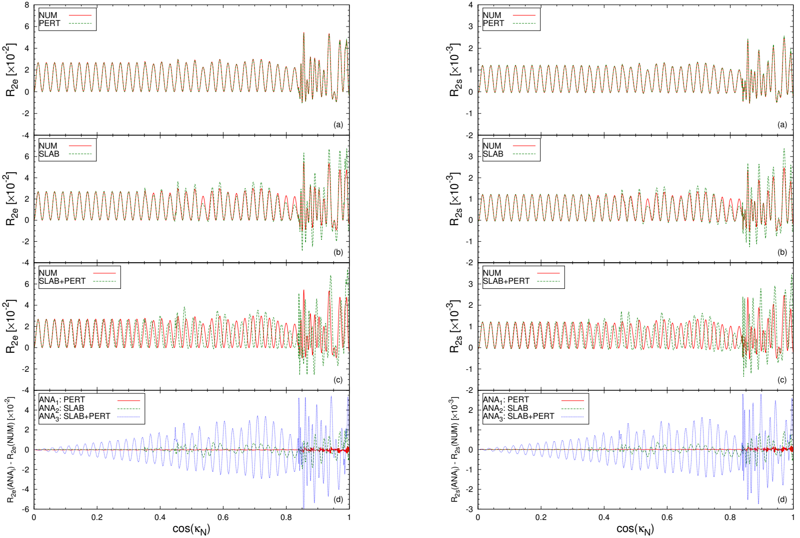
where S^{V}_k is the S-matrix of the k-th layer and can be calculated easily.

$$\begin{aligned} S^{\mathrm{V}}_{\mathrm{slab}} &= S^{\mathrm{V}}_n S^{\mathrm{V}}_{n-1} \dots S^{\mathrm{V}}_1 S^{\mathrm{V}}_{-1} \dots S^{\mathrm{V}}_{-(n-1)} S^{\mathrm{V}}_{-n} \\ &= S^{\mathrm{V}}_n S^{\mathrm{V}}_{n-1} \dots S^{\mathrm{V}}_1 S^{\mathrm{V}}_1 \dots S^{\mathrm{V}}_{n-1} S^{\mathrm{V}}_n \quad . \end{aligned}$$

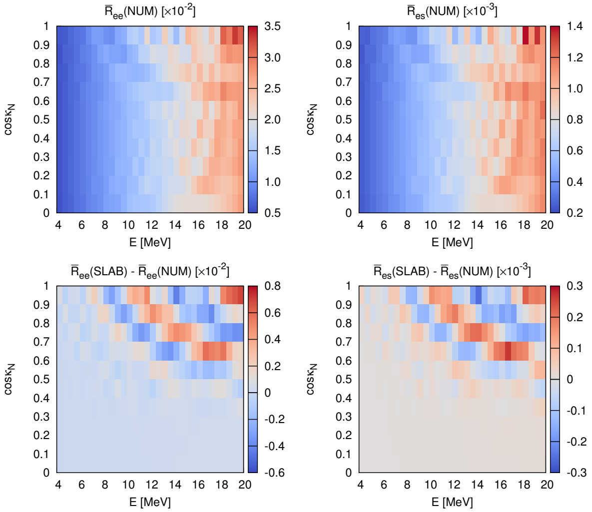
- Perturbative plus slab:

$$\begin{aligned} P^{\mathrm{E}}_{\nu_2\to\nu_\beta}(x_n) \simeq P^{\mathrm{V}}_{\nu_2\to\nu_\beta} &+ \cos^2\chi_\beta \sin 2\theta_\beta \times \\ &\sum_{k=1}^{n} \frac{V_k}{\delta} \sin 2\xi_k \sin(\delta L_n + \Phi_{\beta(k)}) (\sin\delta L_k - \sin\delta L_{k-1}) \quad , \end{aligned}$$

Comparison of Three Methods

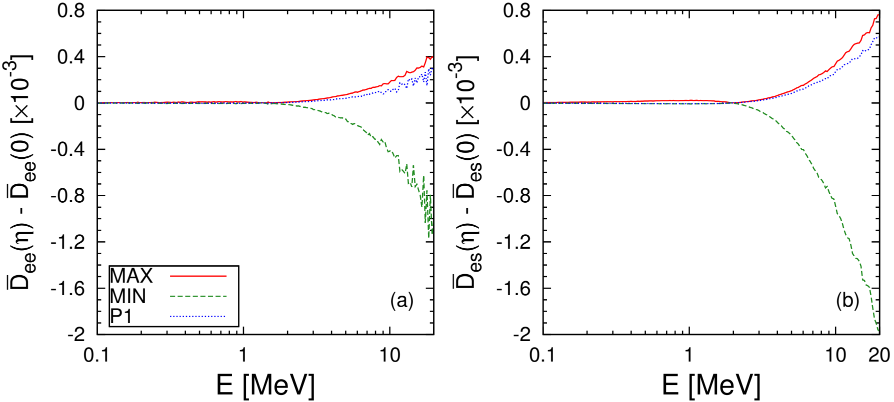


Practical simulation for the SLAB method



- Our analytical result is acurrate enough for practical use.
- DNA for Pee, 3.5×10^{-2}
- DNA for Pee, 1.4×10^{-3}
- The former one might be detected by present SK experiment.

Possible effects of CP-violating phases



The annual averages DNA induced by CP-violating phases: 10^{-3} in the HE region, might be observed in future high-precision.

Conclusion

- $\nu_\odot$ physics has enter the era of precision measurement
- There are many interesting sub-leading effects for $\nu_\odot$;
- No conclusive evidence for sterile neutrino yet, better experiments are needed to check LSND signal;
- Both daytime and night neutrino oscillation probabilities were calculated in the general $3 + N_s$ scheme
- Active-Sterile Solar Oscillation can be detected in future solar experiments;
- CP-violating phase contribute to $\nu_\odot$ osci.
- Day-night effect will be confirmed in near future.

Thank you!