Thermal history of neutrinos

Basic equations:

(A) Cosmological, $ds^2 = dt^2 - a^2(t) dr^2$

$$H^2 = \frac{8\pi}{3} \frac{\rho}{m_{pl}^2} + \frac{k}{a^2}; \qquad H = \frac{\dot{a}}{a}$$

can be neglected

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{m_{pl}^2}(\rho + 3p);$$

$$\dot{\rho} = -3H(\rho + p) \Longleftrightarrow \mathcal{D}_\mu T^\mu_\nu = 0$$

$$\dot{k} = -Hk \qquad \text{(red-shift)}$$

(B) Thermodynamical (in thermal equilibrium)

$$f_\gamma = \frac{2}{e^{\omega/T} - 1}\ ; \quad f_\nu = \frac{1}{e^{E/T} + 1}\ ;$$

$$n_\gamma = \int \frac{d^3p}{(2\pi)^3} f_\gamma = 0.24\,T^3\ ; \quad n_\nu = \frac{3}{4}\cdot\frac{n_\gamma}{2}\ ;$$

$$\rho_\gamma = \frac{\pi^2}{30}\cdot 2T^4\ ; \qquad \rho_\nu = \frac{7}{8}\,\frac{\rho_\gamma}{2}\ ;$$

If $n_\nu \neq n_{\bar\nu}$ (lepton asymmetry)

then $\mu = -\bar\mu \neq 0$ $\qquad \xi = \frac{\mu}{T} = \text{const}$ for adiabatic expansio.

$$f_\nu = \frac{1}{e^{\frac{E}{T} - \xi} + 1}\,.$$

Conservation of entropy (if $\mu = 0$):

$$\frac{d}{dt}\left[a^3\,\frac{\rho + p}{T}\right] = 0\,.$$

Conditions for thermal equilibrium

$$\begin{pmatrix}\text{Expansion}\\ \text{rate}\end{pmatrix} < \begin{pmatrix}\text{Reaction}\\ \text{rate}\end{pmatrix}$$

$$H < \Gamma$$

$$H = \sqrt{\frac{8\pi}{3}\frac{\rho}{m_{pl}^2}}\,, \quad \rho = \frac{\pi^2}{30} g_* T^4 + \text{massive particles} \sim e^{-m/T}$$

$$g_* = 2 + 2\cdot 2\cdot\frac{7}{8} + 3\cdot 2\cdot\frac{7}{8} + \underbrace{\frac{7}{4}(N_\nu - 3)}_{\text{all other new relativistic species}}$$

γ $\quad e^+e^-$ $\quad \nu_e+\nu_\mu+\nu_\tau+\bar\nu_e+\bar\nu_\mu+\bar\nu_\tau$

$$g_*^{\text{standard}} = 10.75$$

Simple estimate $\Gamma \approx \frac{\dot n}{n} = \sigma n$

$\sigma \simeq G_F^2 T^2, \quad n \simeq T^3$

ν are in thermal equilibrium between themselves and e^+e^- at $T > 1$ MeV (roughly)

More accurately from kinetic equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial p}\dot{p} = \frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p} = I_{coll}$$

$$I_{coll} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3\cdot 2E_2}\,\frac{d^3p_3}{(2\pi)^3\cdot 2E_3}\cdot\frac{d^3p_4}{(2\pi)^3\cdot 2E_4}\cdot$$

$(1+2 \leftrightarrow 3+4)$

$$(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,|A_{1+2\to 3+4}|^2$$

(if T-inv is true)

$$[-f_1 f_2 (1-f_3)(1-f_4) + f_3 f_4 (1-f_1)(1-f_2)].$$

New variables: $x = ma$, $y = pa$

In many cases $a \approx \frac{1}{T}$, $\dot{T} = -HT$

In Bolzmann approximation and keeping only direct reactions:

$$Hx\frac{\partial f}{\partial x} = -\gamma f \qquad \gamma = c\,\frac{G_F^2\, y}{3\pi^3 x^5}$$

(?) What if T-invariance is broken?

Decoupling with respect to annihilation $\nu\bar{\nu} \leftrightarrow e^+e^-$:

$$\begin{cases} g_L = s_W^2 \pm \frac{1}{2} \\ g_R = s_W^2 \end{cases}$$

$T_{\nu e} \approx 3\,\text{MeV}$, $T_{\nu_{\mu,\tau}} \approx 5\,\text{MeV}$ $\quad c = 16(g_L^2 + g_R^2)$

(below these T n_ν^{tot} is conserved)

⊕ Elastic νe-scattering: $\quad c = 80(g_L^2 + g_R^2)$

$T_{\nu e} \approx 1.9\,\text{MeV}$, $T_{\nu_{\mu,\tau}} \approx 3.1\,\text{MeV}$

(below these possible $T_\nu \neq T_e = T_\gamma$)

Complete decoupling from all possible interactions: $\quad c = 80(1 + g_L^2 + g_R^2)$

$T_{\nu e} \approx 1.35\,\text{MeV}$, $T_{\nu_{\mu,\tau}} \approx 1.5\,\text{MeV}$

(essential for coherence breaking of oscillating ν)

? What happens if inverse reaction are taken into account?

In equilibrium (at $T > \sim$ MeV) and if $n_\nu = n_{\bar\nu}$ ($\mu = 0$, vanishing asymmetry)

$$n_\nu + n_{\bar\nu} = \frac{3}{4} n_\gamma$$

(if $m_\nu \neq 0$ and it is Dirac mass what about right-handed states?)

$\Downarrow$

today ?

When $T \lesssim m_e$: $e^+e^- \to 2\gamma$ and heats up photons.

ν are decoupled, $n_\nu a^3 = \text{const}$,

$T_\nu \sim 1/a$

Conservation of $e\gamma$-entropy:

$$\left(a^3 T_\gamma^3\right)_{in} \left(2 + \frac{7}{2}\right) = 2 \left(a^3 T_\gamma^3\right)_{fin}$$

Eqn. of state: $P_{\gamma,e} = \frac{1}{3} \rho_{\gamma,e}$

$\left(T_\nu / T_\gamma\right)_{in} = 1$, $\quad a = c/T_\nu$

$T_\gamma = 2.7$
$T_\nu = 1.9$
$m \to 0$

$\left(T_\gamma / T_\nu\right)_{fin} = \left(\frac{11}{4}\right)^{1/3}$, $\quad \frac{n_\nu + n_{\bar\nu}}{n_\gamma} = \frac{3}{4} \cdot \frac{4}{11} = \frac{3}{11}$

Gerstein - Zeldovich bound:

Today: $(n_{\nu_a}+n_{\bar\nu_a}) = \frac{3}{11} n_\gamma = 2\cdot 56\ /cm^3 = 112/cm^3$

for each species a

Energy (mass) density of neutrinos if $m_\nu \neq 0$:

$$\rho_\nu = 112 \sum_a m_{\nu_a}\ /cm^3 \Leftrightarrow \rho_c = \frac{3H^2}{8\pi} m_{pl}^2 \approx 10\,\mathrm{keV}\,\frac{h^2}{cm^3}$$

$$H = 100 \frac{km}{sec\,Mpc}\cdot h$$

$h \approx 0.7$

$\Omega = \rho/\rho_c$

$$\underbrace{\Omega_\nu < \Omega_m \approx 0.3}_{<0.1} \longrightarrow \sum_a m_{\nu_a} < 12\,eV$$

$\Omega_\nu < 0.3\,\Omega_m$ since LSS is not formed at RD stage and small scales are suppressed

$\longrightarrow \sum_a m_{\nu_a} < 4\,eV$ and if masses are degenerate $m_{\nu_a} < 1.3\,eV$

Recent WMAP+LSS $\Rightarrow \sum m_\nu < 0.7\,eV$

? T_ν today? Is $f_\nu = f_{eq}$?

Non-equilibrium heating of neutrinos

Massless particles maintain their equilibrium spectrum even if interactions are off:

$$f_{eq} = f\left(\frac{E}{T}\right) \underset{\text{for } m=0}{=} f\left(\frac{p}{T}\right) = f(y)$$

Kin eqn : $\quad H x \partial_x f(x,y) = I_{coll}$

$I_{coll}[f_{eq}] = 0$ by definition

If $I_{coll} = 0$, $\quad H x \partial_x f = 0$, true for f_{eq} if $m = 0$

That's why CMBR has equilibrium spectrum.

This is not true for ν, even if $m_\nu = 0$.

In equilibrium, when $T_\nu = T_e$
$\nu e \leftrightarrow \nu e$ and $\nu\bar{\nu} \leftrightarrow e^+e^-$
do not distort ν-spectrum.

However, this is not so at $T < m_e$
when $T_e > T_\nu \Rightarrow$ residual annihilation and scattering $\Rightarrow$ $\underline{\delta f \neq 0}$

$e^+e^- \to \nu\bar{\nu}$ more efficient at higher energies $\Rightarrow$ spectrum distortion

$$\frac{\delta f_{\nu e}}{f_{\nu e}} \approx 3 \cdot 10^{-4} \frac{E}{T} \left(\frac{11}{4} \frac{E}{T} - 3 \right)$$

$\delta \rho_{\nu e} \approx 0.95\%$, $\delta \rho_{\nu_{\mu,\tau}} \approx 0.40\%$

All together $\oplus$ plasma corrections to $e^\pm, \gamma \Rightarrow$

$\Delta N_\nu = 0.04$

or $\underline{N^\nu_{tot} = 3.04}$

Excitation of righanded states

1. If Majorana mass, $m_M \nu_c^T \nu$, then only ν_L are in plasma
2. Dirac mass, $m_D \bar{\nu} \nu$, $\Rightarrow$ Both ν_L and ν_R are possible

$$(1+\gamma_5)\nu = (1+\gamma_5)\begin{pmatrix} 1 \\ \frac{\vec{\sigma}\vec{p}}{E+m} \end{pmatrix} = \left(1-\frac{m}{2E}\right)\left(1+\frac{\vec{\sigma}\vec{p}}{p}\right) + \frac{m}{2E}\left(1-\frac{\vec{\sigma}\vec{p}}{p}\right)$$

$$\Gamma_R = \frac{m^2}{4E^2}\Gamma_L \qquad \Gamma_L = \sigma_W n \sim G_F^2 T^5$$

for $T < m_{W,Z}$

$T > m_{W,Z}$: $\quad \begin{matrix} W \\ Z \end{matrix} \leftrightarrow \begin{matrix} \nu \ell \\ \nu\bar{\nu} \end{matrix}$

$$\Gamma_R \approx \left(\frac{m_\nu}{T}\right)^2 \frac{\Gamma_W^\nu n_W + \Gamma_Z^\nu n_Z}{0.1 T^3}$$

ν_R in equilibrium if $m > 2\,\text{keV}$

If new interactions with ν_R, remember about entropy suppression (g_*^{fin}/g_*^{in})

I. BBN.

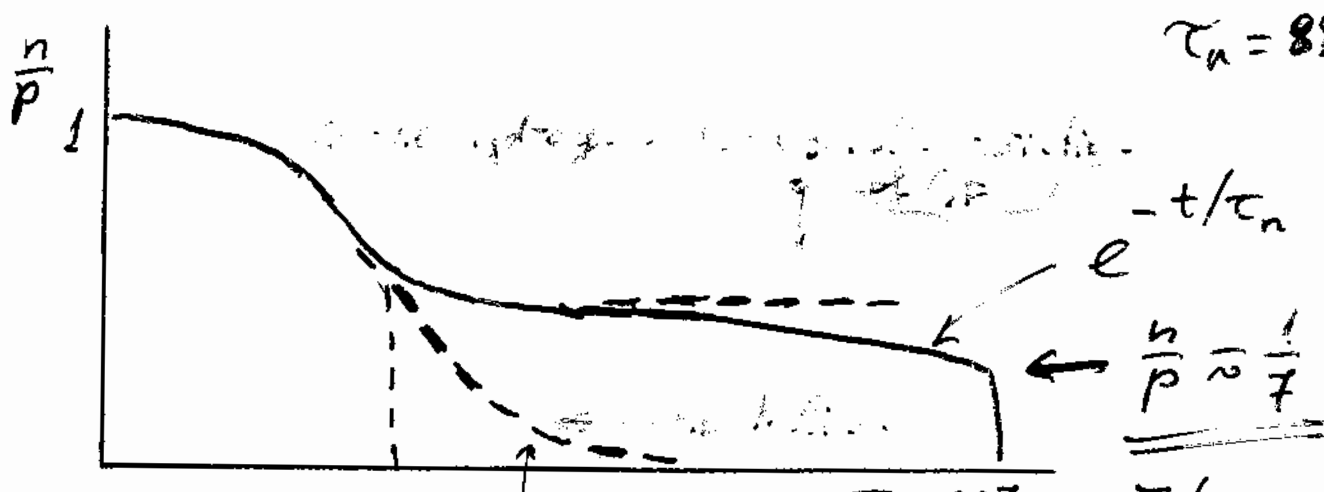
$t = 1 - 10^3$ sec, $T = 1 - 0.07$ MeV

Produced ^2H $\sim 3 \cdot 10^{-5}$ by number

^4He $\sim 24\%$ by mass

^7Li $\sim 3 \cdot 10^{-10}$ by number

$$\left[\begin{array}{l} e^- + p \leftrightarrow n + \nu \\ \bar{\nu} + p \leftrightarrow n + e^+ \end{array}\right.$$



$e^{-\Delta m/T}$

$\Delta m = 1.3$ MeV

or $e^{-\frac{\Delta m}{T} - \frac{\mu_{\nu e}}{T}}$

$\xi = \mu/T$

$\left.\begin{array}{l} p + n \to {}^2H + \gamma \\ {}^2H + N \to {}^3He, {}^3H \end{array}\right\}$ etc

formation of light elements, depends upon only previously unknown parameter

$\eta_{10} = 10^{10} N_B / N_\gamma$

CMBR: $\eta_{10} \approx 6$

Neutron-proton transformation

$$\begin{cases} \nu + n \leftrightarrow p + e^-, \\ e^+ + n \leftrightarrow p + \bar{\nu}, \end{cases} \qquad n \to p + e^- + \bar{\nu}.$$

Assumed: $f_{n,p} = e^{-E/T + \xi}$, all other particles in thermal equilibrium,

$$f = (e^{E/T} + 1)^{-1}. \quad \text{Why? Check it.}$$

$(n_p + n_n) a^3 = \text{const} \Leftarrow$ conservation of baryons

Kinetic equation: $\quad r = \dfrac{n_n}{n_n + n_p} \equiv \dfrac{n}{n+p}$

$$\dot{r} = \frac{(1 + 3g_A^2) G_F^2}{2\pi^3} \left[A - (A+B) r \right] \qquad (*)$$

In the limit $m_e = 0$:

$$\begin{cases} B = 48 T^5 + 24 \Delta m T^4 + 4 (\Delta m)^2 T^3 \\ A = B \exp(-\Delta m / T) \leftarrow \text{(in equilibrium for any } m_e) \end{cases}$$

$$\Delta m = m_n - m_p = 1.3 \text{ MeV}$$

In accurate calculations $T_\nu \neq T_\gamma$ is taken into account and many other effects.

$$A = \int_0^\infty dE_\nu E_\nu^2 E_e p_e f_e(E_e)\,[1 - f_\nu(E_\nu)]\,|_{E_e = E_\nu + \Delta m} +$$

$$\int_{m_e}^\infty dE_e E_\nu^2 E_e p_e f_{\bar\nu}(E_\nu)\,[1 - f_{\bar e}(E_e]\,|_{E_\nu = E_e + \Delta m} +$$

$$\int_{m_e}^{\Delta m} dE_e E_\nu^2 E_e p_e f_{\bar\nu}(E_\nu) f_e(E_e)\,|_{E_\nu + E_e = \Delta m}\,,$$

$$B = \int_0^\infty dE_\nu E_\nu^2 E_e p_e f_\nu(E_\nu)\,[1 - f_e(E_e)]\,|_{E_e = E_\nu + \Delta m} +$$

$$\int_{m_e}^\infty dE_e E_\nu^2 E_e p_e f_{\bar e}(E_e)\,[1 - f_{\bar\nu}(E_\nu)]\,|_{E_\nu = E_e + \Delta m} +$$

$$\int_{m_e}^{\Delta m} dE_e E_\nu^2 E_e p_e\,[1 - f_{\bar\nu}(E_\nu)]\,[1 - f_e(E_e)]\,|_{E_\nu + E_e = \Delta m}$$

two reactions and decay are included.

Solution: can be easily done numerically

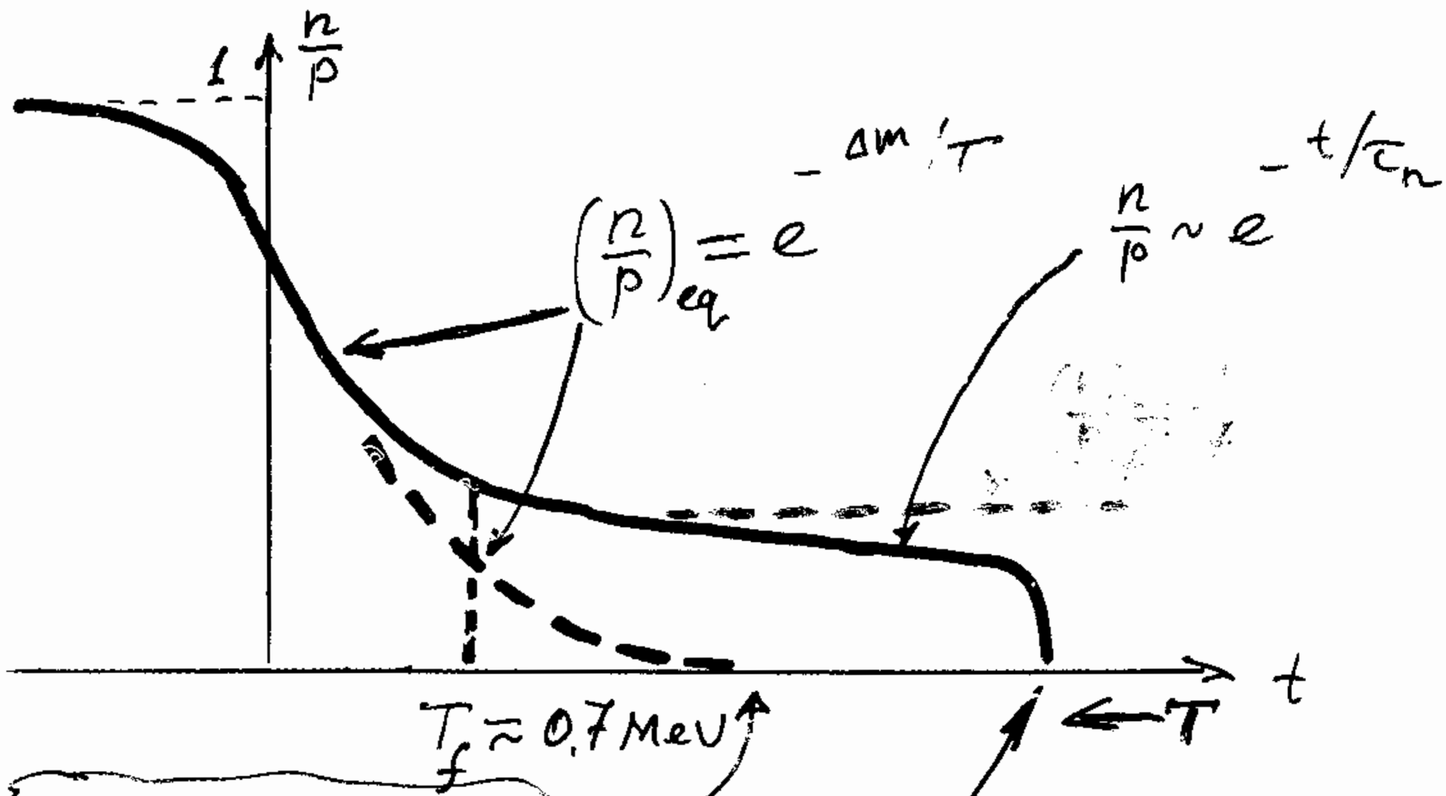
Approximate: $\Gamma(n \leftrightarrow p) \sim G_F^2 T^5$, $H \sim T^2/m_{pe}$

$$T_{np}^{f} = 0.7 \left(\frac{g_*}{10.75}\right)^{1/6} \text{MeV} \leftarrow \text{freezing temperature}$$

$T > T_f$: $\quad \frac{n}{p} = \exp\left(-\frac{\Delta m}{T}\right)$.

$T < T_f$: $\quad \frac{n}{p} = \text{const} \cdot \exp\left(-t/\tau_n\right)$

$\tau_n = 889$ sec



We are lucky that weak interactions are weak, otherwise only hydrogen would exist

nucleosynthesis

$T_{BBN} \sim (60-70)$ keV

BBN temperature

$$n + p \leftrightarrow D + \gamma$$

In equilibrium $\underline{f_{D,n,p} = e^{-\frac{E}{T} + \xi}}$

$$\underline{\xi_D = \xi_n + \xi_p}$$

$$n_D = 3\, e^{\xi_D - \frac{m_D}{T}} \left(\frac{m_D T}{2\pi}\right)^{3/2}$$

spin states

$$e^{\xi_p} = \frac{n_p}{2} e^{\frac{m_p}{T}} \left(\frac{2\pi}{m_p T}\right)^{3/2}, \quad e^{\xi_n} = \frac{n_n}{2} \dots$$

$$n_D = \frac{3}{4} n_n n_p\, e^{E_b/T} \left(\frac{2\pi m_D}{m_p m_n T}\right)^{3/2}$$

$$E_b = (m_D - m_p - m_n)(-1)$$

$$\underline{n_p = 10^{-10} \eta_{10} n_\gamma}, \quad \underline{n_\gamma = 0.24 T^3}$$

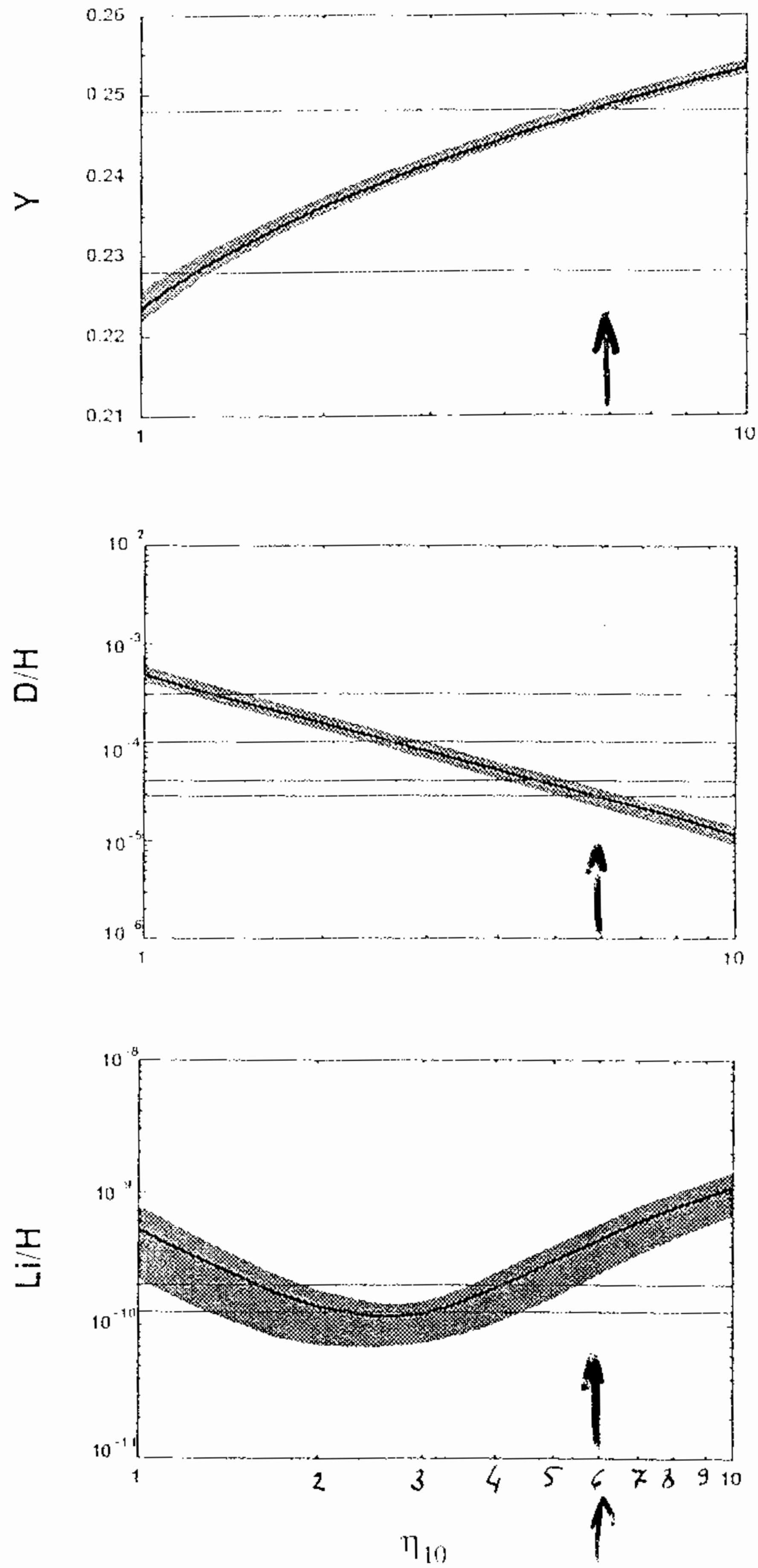
T_{BBN} from condition $(n_D/n_n) \approx 1$.

$$(E_b/T) = \ln\left[\frac{10^{10} \cdot 10^{3/2}}{0.18\, \eta_{10} \cdot (4\pi)^{3/2} \cdot (0.06)^{3/2}}\right]$$

sic

$$\boxed{T_{BBN} \simeq \frac{66\ \text{keV}}{1 - 0.03 \ln \eta}}$$

$\eta_{10} = 5\text{-}6$.



weakly depends on $\eta = \frac{n_B}{n_\gamma}$ 10^{10}

Baryometer (before CMBR data)

CMBR

From Olive, Steigman, Walker, 1999

Uncertainties in theoretical predictions

Reaction rate uncertainties translate into uncertainties in theoretical predictions:

Krauss & Romanelli 1990, Smith et al 1993, Kernan & Krauss 1994 -
Monte-Carlo evaluation of uncertainties;
Fiorentini et al 1998 - Semi-analytical evaluation of the error matrix;
Nollet & Burles 2000, Cyburt et al 2001, Descouvemount et al 2003 (Private Comunication), Cuoco et al. 2003 - Re-analysis of nuclear data with less conservative approach. New data (NACRE + LUNA $d(p,\gamma)^3He$).
→ Factor 2 reduction of theoretical errors.

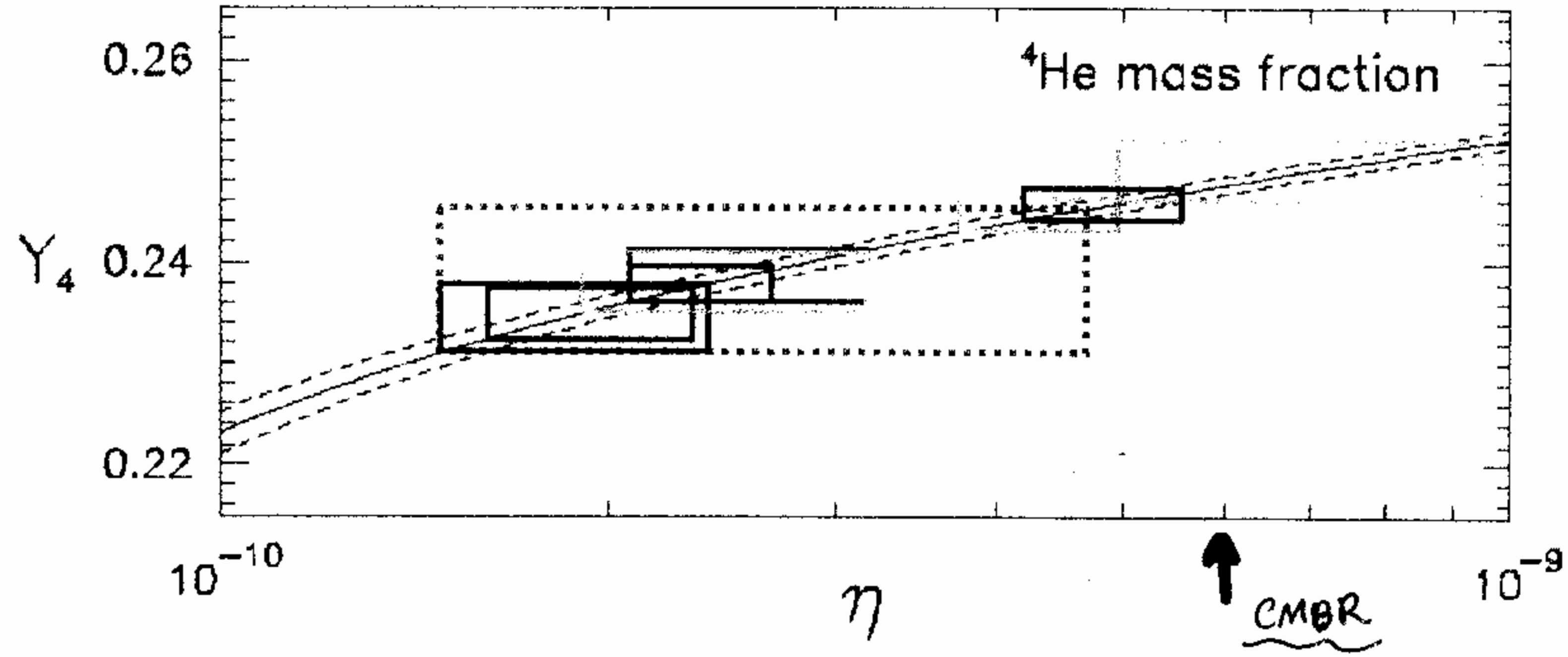
Summarizing ...

- Theoretical uncertainties at the level of:

0.2%
5%
5%
15%

- (see later) Theoretical uncertainties are sub-dominant with respect to systematic observational errors.

Observations: Helium-4

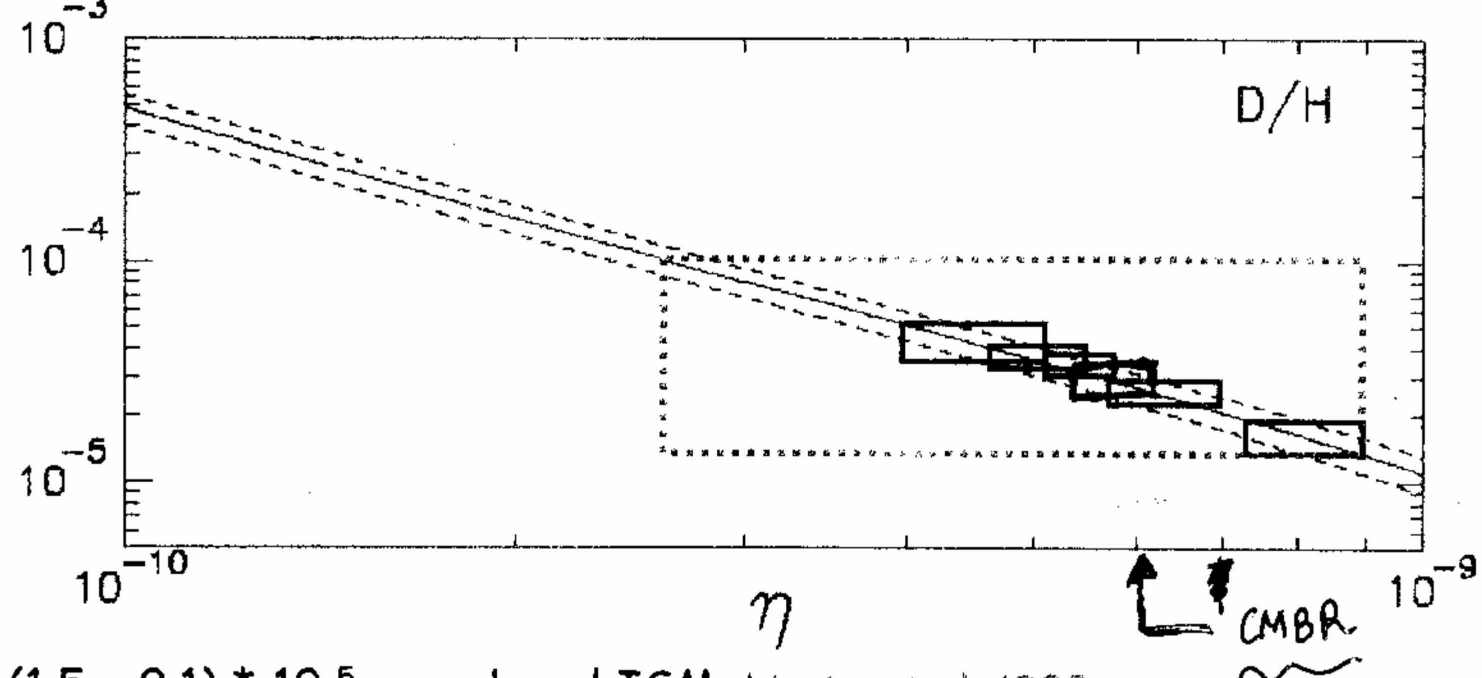


Yp =

0.2452 ± 0.0015	Izotov et al. 1998	Average of the 2 most metal poor BCGs
	Izotov et al. 1999	Regression using 45 BCGs
	Gruenwald et al. 2002	Ionization corr. to Izotov et al. 1999
	Ballantyne et al 2000	Ionization corr. to Izotov et al. 1999
0.235 ± 0.003	Olive et al. 1997	Regression using 62 BCGs
0.238 ± 0.002	Fields and Olive 1998	Re-analysis (update) of Olive et al. 1997
0.2345 ± 0.0026	Peimbert et al 2000	HII reg. of the Small Magellanic Cloud
0.2384 ± 0.0025	Peimbert et al 2001	Average of the 5 most metal poor BCGs

$Y_p = 0.238 \pm 0.002\ (\text{stat}) \pm 0.005\ (\text{syst})$ Fields & Sarkar - PDG 2002

Observations: deuterium



$(D/H) = (1.5 \pm 0.1) * 10^{-5}$	Local ISM	Linsky et al 1998		
$(20 \pm 5) * 10^{-5}$	z=0.701	Webb et al	1997	LLS
$(4.0 \pm 0.65) * 10^{-5}$	z=2.504	O'Meara et al	2001	LLS
$(3.25 \pm 0.3) * 10^{-5}$	z=3.572	O'Meara et al	2001	LLS
$(D/H) = (2.54 \pm 0.23) * 10^{-5}$	z=2.356	O'Meara et al	2001	DLA
$(1.65 \pm 0.35) * 10^{-5}$	z=2.701	Pettini, Bowen	2001	DLA
	z=3.025	D'Odorico et al	2001	DLA
	z=3.025	Levshakov et al	2001	DLA
$(2.78$ [illegible]$) \cdot 10^{-5}$	[illegible]	Kirkman et al	200[illegible]	

Role of neutrinos in BBN

1. Additional neutrino family, $\Delta N_\nu = N_\nu - 3 \neq 0$

a) $T_f(n \leftrightarrow p) \sim g_*^{1/6} \Longrightarrow \left(\frac{n}{p}\right) \Uparrow \qquad \frac{n}{p} = e^{-\frac{\Delta m}{T}}$

b) T_{BBN} would be reached earlier,

$t \sim (g_*^{1/2} T^2)^{-1} \Rightarrow$ more neutrons survive decay

$$\Delta N_\nu = 1 \Longrightarrow \frac{\Delta He}{He} \approx 5\% \Uparrow$$

Observational limits:

optimists $\quad \Delta N_\nu < 0.2$

pessimists $\quad \Delta N_\nu < 1$

valid for any form of relativistic matter.

$$\rho = \frac{m_{Pl}^2}{t^2} \to g_* T^4$$

$$g_* = 10.75 + \frac{7}{4}(N_\nu - 3)$$

KAII

2. Neutrino degeneracy (i.e. $\xi \neq 0$)

a) $\xi_{\mu,\tau} \neq 0$ equivalent to $\underline{\Delta N_\nu > 0}$

$\nu_{\mu,\tau}$

$$\frac{\rho(\xi)}{\rho(0)} = 1 + \underbrace{\frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4}$$

b) $\xi_{\nu_e} \neq 0 \Rightarrow \underline{\left(\frac{n}{p}\right)_f \sim e^{-\xi_{\nu e}} \cdot e^{-\Delta m/T}}$

Bounds: $|\xi_{\mu,\tau}| < 1.5$ if $\Delta N < 1$

$|\xi_e| < 0.1$ $\left(-0.02 < \xi_e < 0.1\right)$?

If combined variation is allowed $\Rightarrow$ rise of $\xi_{\mu,\tau}$ may be compensated by ξ_e (unnatural?):

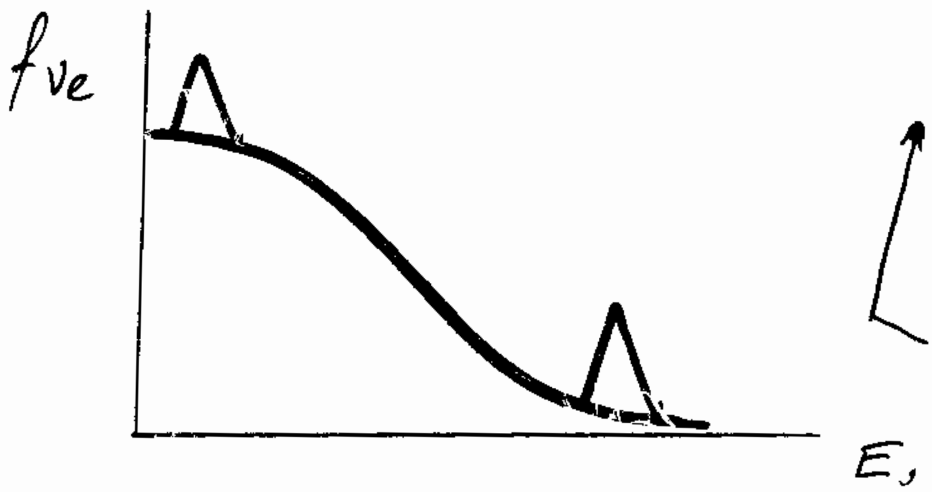
$0 < \xi_e < 1.4$, $|\xi_{\mu,\tau}| < 40$.

If CMBR is used:

$$\boxed{\begin{array}{c} -0.01 < \xi_e < 0.2 \\ |\xi_{\tau,\mu}| < 2.6 \end{array}}$$

(for $\Delta N_\nu < 0.3$)

3. Spectrum distortion of ν_e.



may be observed by Planck?

1. Excess at high $E \Rightarrow (n/p)\uparrow$

More n is destroyed in $\nu + n \to p + e^-$
and more n is produced in $\bar{\nu} + p \to n + e^+$
but $(n/p) \approx 1/7$.

2. Excess at low $E \Rightarrow (n/p)\downarrow$

because production of n in
$\bar{\nu} + p \to n + e^+$ is suppressed by
threshold 1.8 MeV.

3. Total excess of number of $\nu_e \Rightarrow$

lower freezing temperature of (n/p)
so $(n/p)\downarrow$ (the same effect if G_F
was larger at BBN)

time variation of fundamental constants? $\alpha = \alpha(t)$ can be constrained

$\nu(t) \to$ changes what?

BBN limit on N_ν

Lisi, Sarkar, FLV 1999

$Y_2 = (3.4 \pm 0.3) \times 10^{-5}$
$Y_4 = 0.245 \pm 0.004$

BBN can be used to constraint any new light degree of freedom in thermal equilibrium in the early Universe:

$$N_\nu = 3 + f_{B,F} \sum_i \left(\frac{T_i}{T_\nu} \right)^4$$

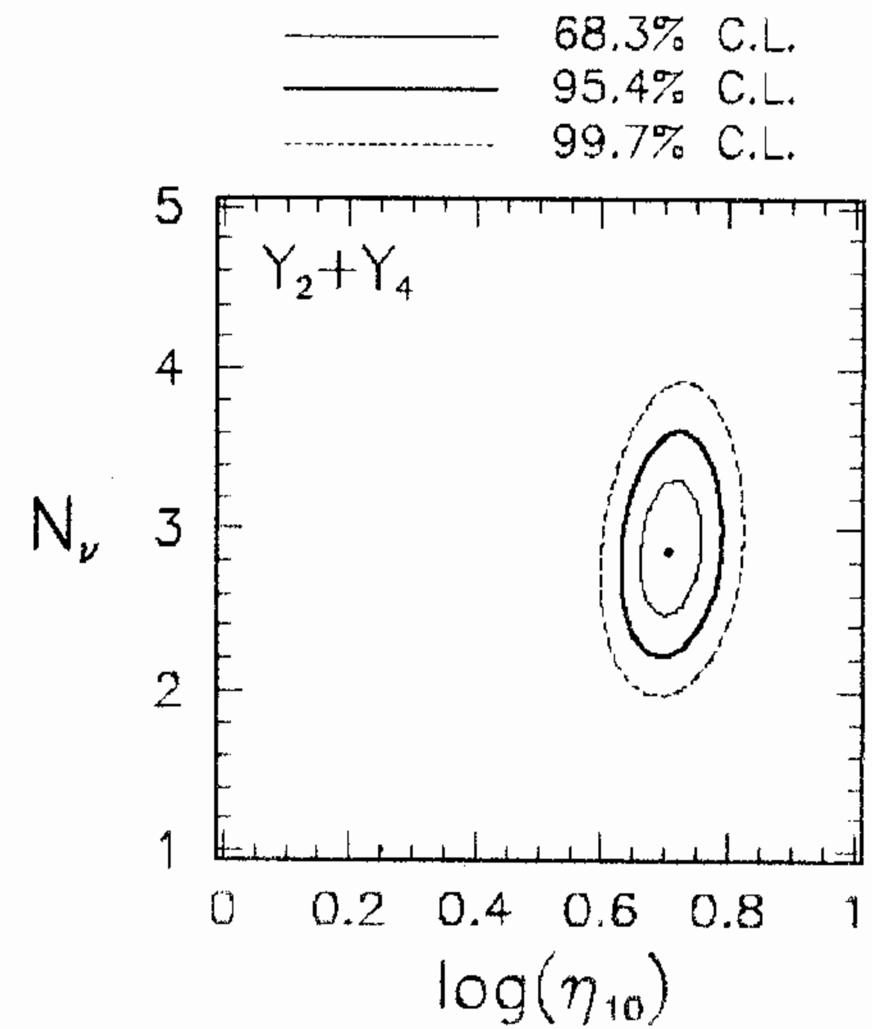
$f_B = 8/7$ (bosons);
$f_F = 1$ (fermions)

Observational limits:

Optimistic: $\Delta N < 0.2$

Pessimistic: $\Delta N < 1.0$

The limit on ΔN is mainly determined by ^{4}He.



$N_\nu < 3$?

Note that [illegible] → [illegible] requires [illegible]

Neutrino oscillations in the early universe.

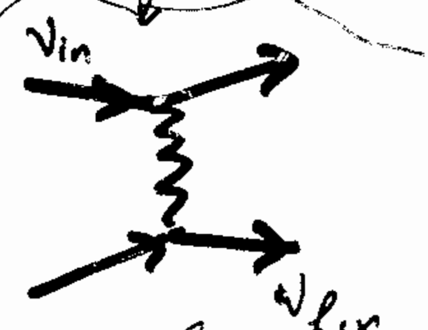
$$i\partial_t \Psi = \left(\mathcal{H}_{vac} + V_{eff}\right)\Psi$$

coherence break is essential and density matrix is to be used

$$V^{(a)}_{eff} = \pm C\eta^{(a)} G_F T^3 + C_2^{(a)} \frac{G_F^2 T^4 E}{\alpha}$$

charge asymmetric.
$A \sim \langle J^{weak}_\mu \rangle$

Dominates in stars
but
subdominant in the early universe for "normal" value of $\eta \sim 10^{-9}$



non-local
charge symmetric

$T < m_\mu$: $C_1 = 0.95$, $C_2^e = 0.61$

$C_2^{\mu,\tau} = 0.17$

$n^{(e)} = 2n + n + n + n$ ...

$$\underset{\sim}{A} = \begin{bmatrix} |A_{ee}|^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} |A_{ann}|^{\cdot} \\ 0 \end{bmatrix}$$

for active - sterile mixing; other both diagonal terms are non-

to the coherence breaking terms by elastic scattering of oscillating neutrinos on lep tons in plasma (i.e. on electrons, positrons and other active neutrinos which are co mixed with the neutrinos in question) is given by [54]:

$$\dot{\rho} = \left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)\rho \;=\; i\left[\mathcal{H}_m + V_{eff}, \rho\right] + \int d\tau(\bar{\nu}, l, \bar{l})\left(f_l f_{\bar{l}} A A^+ - \frac{1}{2}\left\{\rho, AA^+\right\}\right) +$$

$$\underline{\int d\tau(l, \nu', l')\left(f_{l'} B \rho' B^+ - \frac{1}{2} f_l \left\{\rho, BB^+\right\}\right)} \qquad (28$$

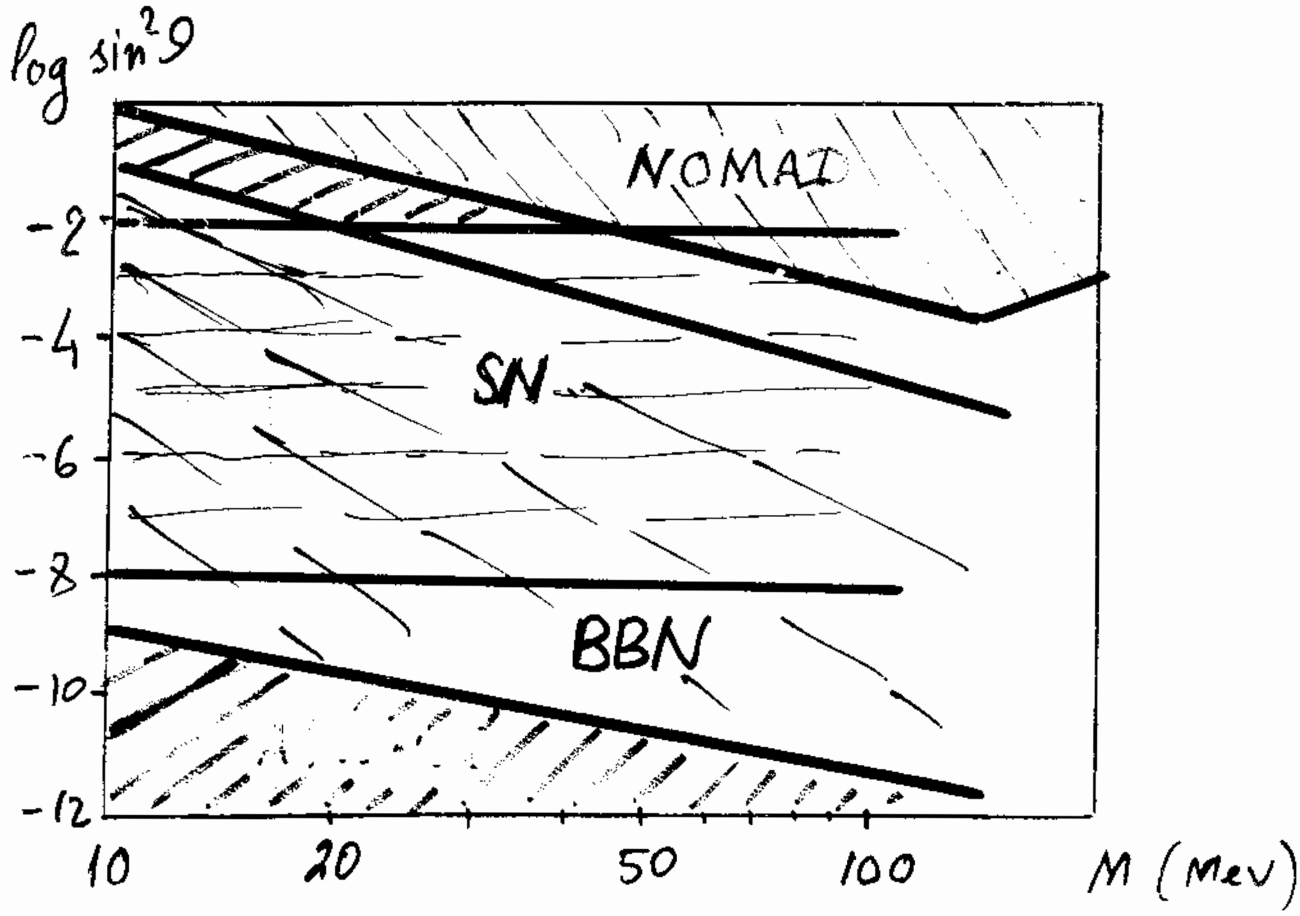
where ρ is the density matrix of the oscillating neutrinos, f_l is the distribution func tion of other leptons in the plasma, $d\tau$ is the phase space element of all particle participating in the reactions except for the neutrinos in question - it is given by eqs. (43,44). The first commutator term in the r.h.s. is first order in the interaction It is the usual contribution from refraction index that does not break coherence. The last two terms are second order in the interaction and are related respectively to annihilation, $\nu\bar{\nu} \leftrightarrow l\bar{l}$, and elastic scattering, $\nu l \leftrightarrow \nu' l'$. The quantum statistics fac tors, $(1-f)$, and $(I-\rho)$ are neglected here. They can be easily reconstructed, see ref. [599]. In the interaction basis and in the case of active-sterile mixing the matrice A and B have only one non-zero entry in the upper left corner equal to the ampli tude of annihilation or elastic scattering respectively; the upper '+' means Hermitian conjugate.

The contribution of the similar coherence breaking terms but related to self interaction of the oscillating neutrinos is presented in ref. [599]. It has more compli cated and lengthy matrix structure.

The equation for the evolution of the diagonal components of the density matrix with the coherence breaking taken into account, has the same form as the equation for the distribution function of non-oscillating particles (42), while the role of coherence breaking terms in the evolution of non-diagonal terms is essentially given by $\dot{\rho}_{as} = -\Gamma\rho_{as}$ [54, 597, 114, 599] as follows from eq. (282).

The complete form of the coherence breaking terms in kinetic equations is quite

Heavy ν, mixed with active ones

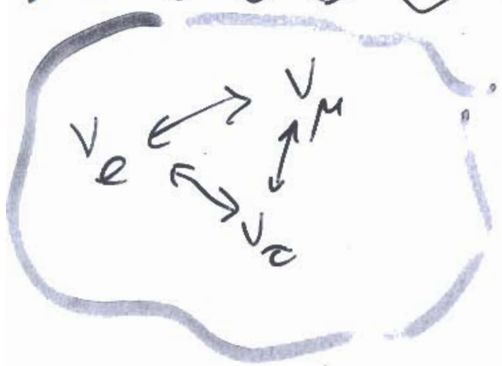


All above mentioned effects are important: change of N_ν,
drop of ν_e/γ
distortion of ν_e spectrum
change $T(t)$ because of annihilation

$\nu_h \to e^+e^-\nu$
$\pi\nu$

S. Hansen, S. Pastor
D. Semikoz, G. Raffelt
A. D.

Mixing of active neutrinos



In the standard model —

— thermal equilibrium is early established,

$$\rho_{ab} = \delta_{ab} f_{eq} \quad \text{or} \quad \rho_{\{\mu,e,\tau\}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} f_{eq}$$

in mass basis

but masses are different

as if oscillations are absent

→ equilibrium solution of kinetic equations with or without oscillations

B. Initial non-equilibrium state due to large, <u>non-conserved</u> by oscillations, L_e, L_μ and/or L_τ.

$B \to 0$ in thermal equil.

$|\xi_{\mu,\tau}| < 2.6$, $|\xi_e| < 0.1$

(previous bounds)

allows noticeable impact on cosmology, LSS, CMBR

$$\Gamma_{\Delta L} \sim \Gamma_{weak} \cdot \sin^2 2\vartheta_m ,$$

$$\tan 2\vartheta_m = \frac{\sin 2\vartheta_{vac}}{\cos 2\vartheta_{vac} + V_{eff}/2\delta E}$$

$$\delta E = \frac{\delta m^2}{2E}$$

If $\overset{\xi}{\eta} \sim 1$, $V_{eff}/\delta E \ll 1$

and oscillations are strongly suppressed in primeval plasma.

<u>wrong</u> !!!

Due to $(\nu-\nu)$-scattering refraction index contains large non-diagonal terms. (Pantaleone) [refraction index $- 1 =$

$= V_{eff} / E$]

$\vec{h} = h_+ + h_-$, strong non-linearity

$$V_{eff} = \frac{1}{2}\left(h_0 + \vec{\sigma}\vec{h}\right), \quad \vec{h}_- = V_{asym} \int dp\, p^2 \left(\vec{P} - \vec{\bar{P}}\right)$$

large off-diag terms in V_{eff}

$$\boxed{\rho = \frac{1}{2}\left(P_0 + \vec{\sigma}\vec{P}\right)}$$

initially $h_{x,y} = 0$ but quickly rises

Approximate analytic solution is possible if $h_- / \delta E \gg 1$:

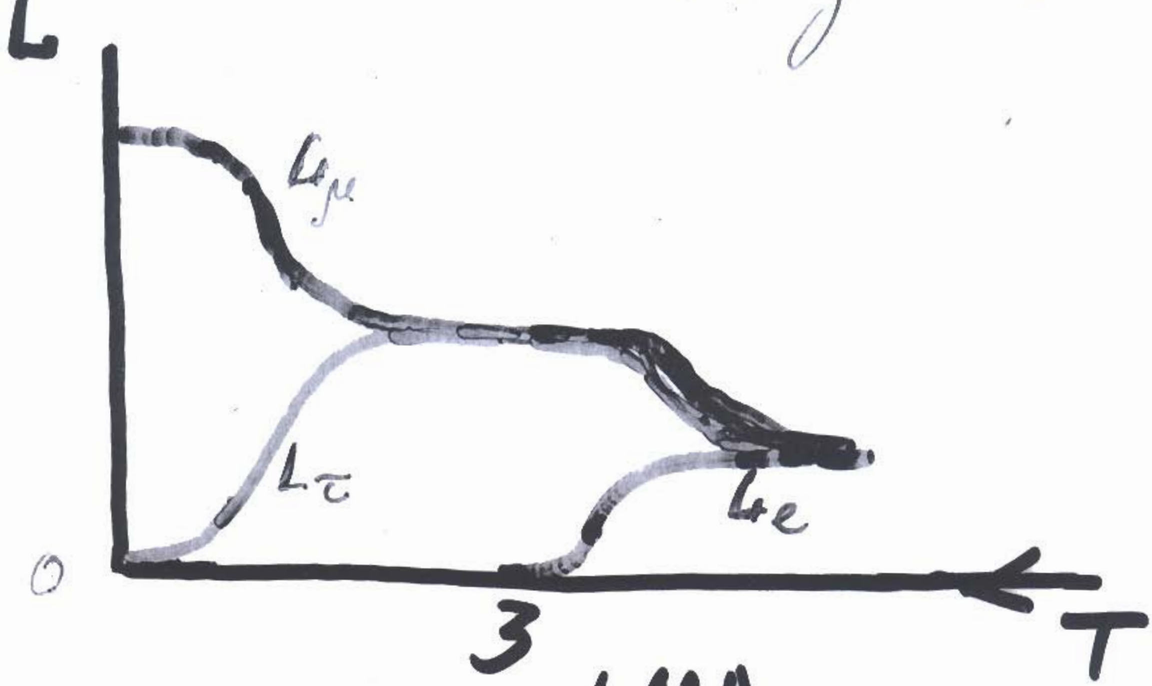
$$Ha \frac{d}{da}\left(n_\mu - n_\tau\right) = -\Gamma\left(n_\mu - n_\tau\right) \cdot \underbrace{\frac{\sin^2 2\theta\, I_E^2}{c_2^2 I_E^2 + I_\gamma^2}}$$

not suppressed by large V_{eff}

$$I_E = \int d^3p \left(P_z + \bar{P}_z\right) \delta E,$$

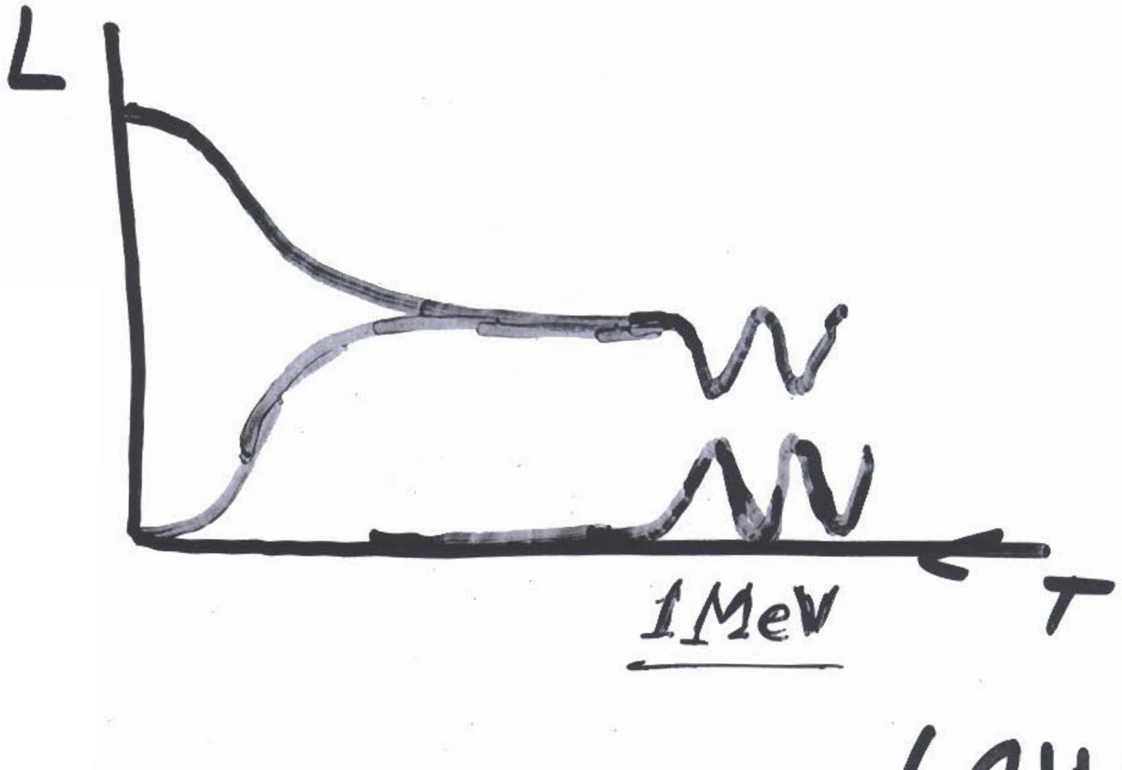
$$I_\gamma = \int d^3p \left(P_z + \bar{P}_z\right) \Gamma$$

Analytic estimates



LMA

$|\xi_a| < 0.1$



LOW

$|\xi_{e,\mu,\tau}| < 0.07$

Numerical solution of the complete system of integrodifferential kinetic equations for $2 \cdot (3\times 3)$ density matrices

$$i\left(\partial_t - Hp\,\partial_p\right)\rho = [\mathcal{H}_1, \rho] + \{\mathrm{Im}\,\mathcal{H}_2, \rho\}$$

effective potential containing

$\int (\rho-\bar{\rho})\, d^3p$

coherence breaking

collision integrals

fast oscillating off-diagonal matrix elements

$\mathcal{H}_1$ and $\mathcal{H}_2$ depend upon ρ

(G_F and G_F^2 not written explicitely)

Two-step process

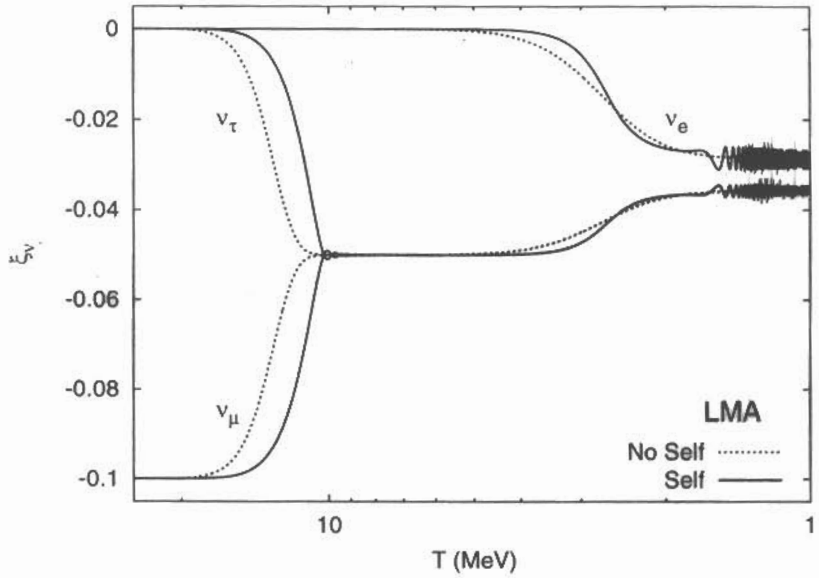
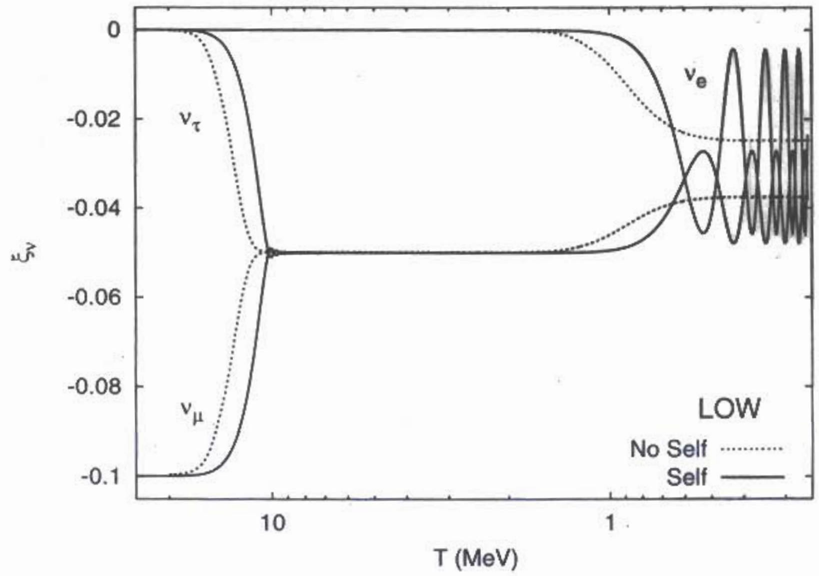


Fig. 5. Evolution of the neutrino degeneracy parameters for the LMA case, $\theta_{13} = 0$, and the initial values $\xi_e = \xi_\tau = 0$ and $\xi_\mu = -0.1$.



← interesting effect on BBN

All modes (with different E) have the same frequency

Usually $\omega \sim \frac{\delta m^2}{E}$

Fig. 6. Evolution of the neutrino degeneracy parameters for the LOW case, $\theta_{13} = 0$, and the initial values $\xi_e = \xi_\tau = 0$ and $\xi_\mu = -0.1$.

Onset of oscillations at max ϑ_{matter}

$$T_{onset} \approx 20 \, \mathrm{MeV} \left(\frac{\delta m^2}{\mathrm{eV}}\right)^{1/6}$$

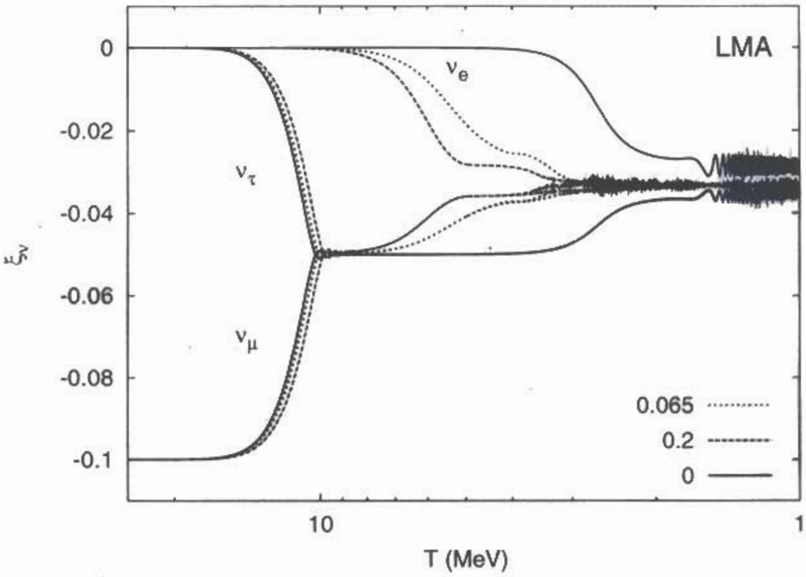


Fig. 7. Evolution of the neutrino degeneracies for different values of $\tan^2\theta_{13}$ in the LMA case for the initial conditions $\xi_e = \xi_\tau = 0$ and $\xi_\mu = -0.1$.

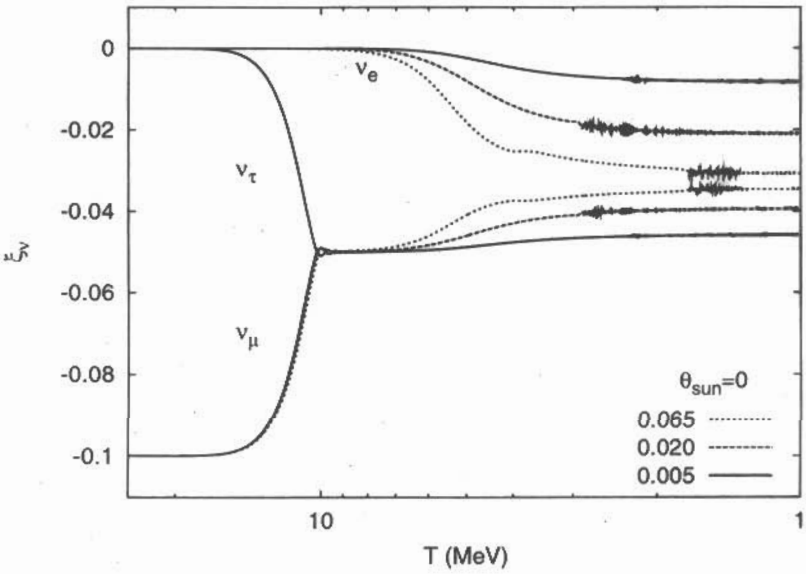


Fig. 8. Evolution of the neutrino degeneracies for different values of $\tan^2\theta_{13}$ in the limit $\Delta m^2_{\rm sun} \simeq 0$ and $\theta_{\rm sun} \simeq 0$ for the initial conditions $\xi_e = \xi_\tau = 0$ and $\xi_\mu = -0.1$.

<u>Results</u>: all primordial cosmological charge asymmetries would be equally redistributed among 3 flavors e, μ, τ and thus $\underline{|\xi_{\nu_a}| < 0.07}$ for any $a = e, \mu, \tau$, if LMA solution is realized.

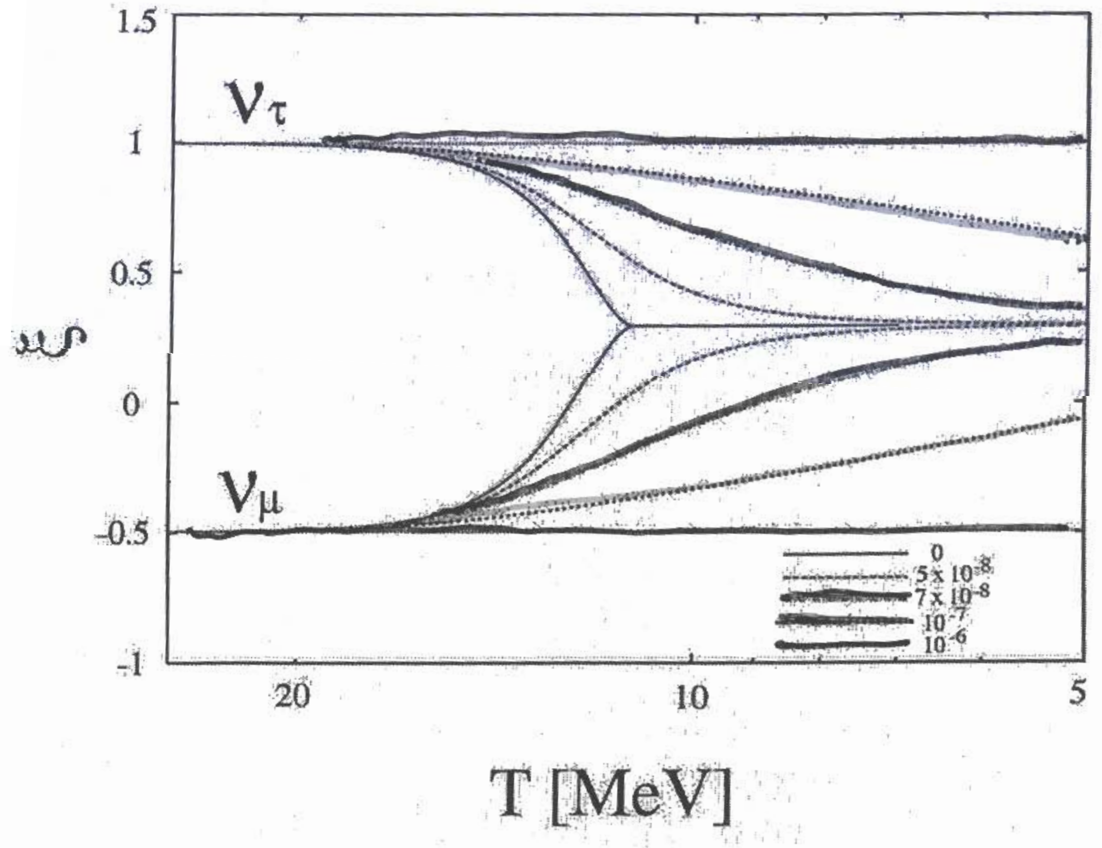
Cosmological lepton asymmetry is too small to play noticeable role in the present day universe (e.g. CMBR, LSS, ...)

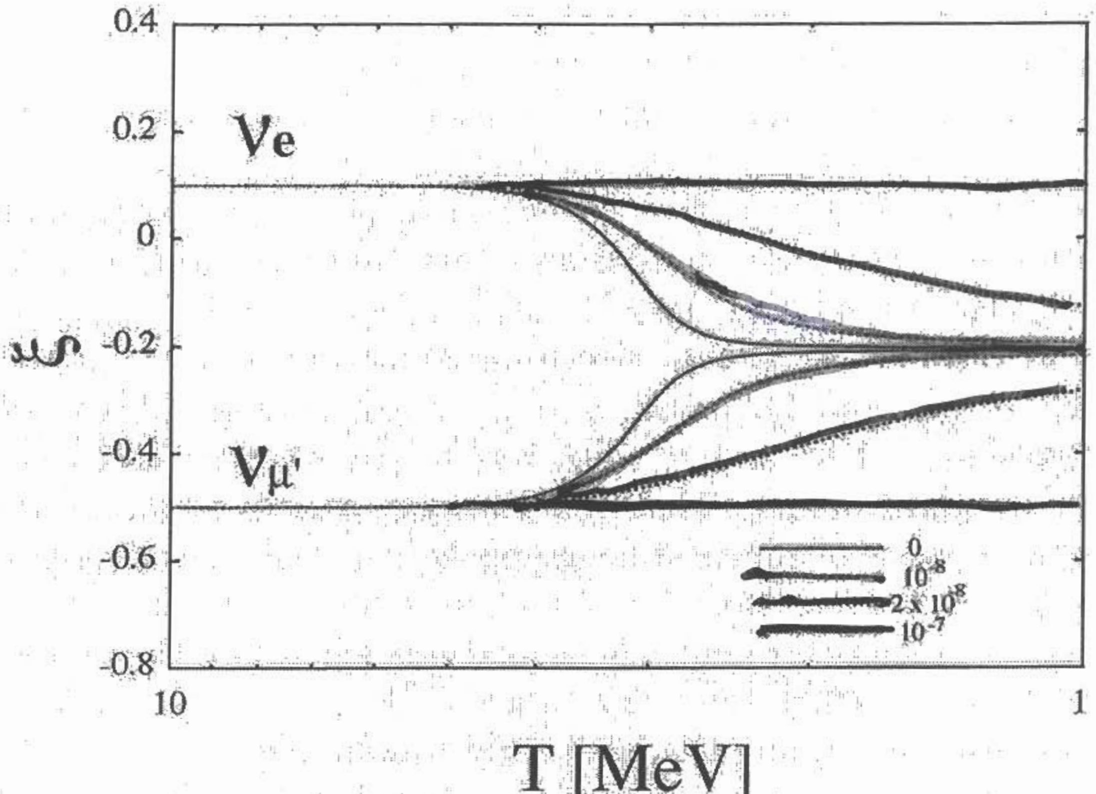
<u>Is</u> it possible to avoid this bound and to have cosmologically significant lepton asymmetry?

<u>Yes</u> if new interaction of ν exists, e.g. ν - majoron coupling: $\underline{g \chi \nu C \nu}$ with $g \sim 10^{-5} \Rightarrow$ <u>with strong breaking</u> <u>of flavor symmetry</u> (compatible with astrophysics and 2β)

A.D.
F. Takahashi

Necessary:
strong breaking of flavor symmetry in $\nu\nu f$ - coupling
(if the flavor symmetry is even weakly broken the transitions are strongly suppressed)





NM 10

$\underline{\text{IV}}$. **BBN bounds on $\nu_a \leftrightarrow \nu_s$** (AD + F. Villante) YAK

(previously derived bounds – for fixed a and no mixing between ν_a) → i.e. 1+1 mixing

RB + AD

Physical effects:

1. Creation of additional neutrino species
   $\Delta N_\nu = N_{\nu_s}$ (if in equilibrium)
2. Depletion of $\nu_e \to \nu_s$ (out of equilibrium, refil ν_e by expence of ν_e thru $\nu_e \bar{\nu}_e \to \nu_\mu \bar{\nu}_\mu$ } when refilling by $\ell\bar{\ell} \to \nu_e \bar{\nu}_e$ is weak)
3. Resonance generation of lepton asymmetry of active neutrinos (at the expence of ν_s)
   for small $\vartheta_{\text{vac}} < 10^{-2}$
4. Spectrum distortion of ν_e (for small values of δm^2)

Technique: numerical solution of exact integrodifferential kinetic equations for oscillating neutrinos (for a small number of test points ⇔ oscillating functions, large computational time) and quick solution in stationary point and resonance approximation (can be 3 resonances)

Master equation: (for density matrix of ν)

$$i\dot{\rho} = \underbrace{[\mathcal{H}^{(1)}, \rho]_-}_{\text{contains vacuum term and } V_{eff}} - i \underbrace{\{\mathrm{Im}\,\mathcal{H}^{(2)}, \rho\}}_{\text{coherence breaking terms}}$$

Mixing matrix: $\nu_\alpha = U_{\alpha j} \nu_j$

$\nu_{\alpha = e,\mu,\tau,s}$ = flavor eigenstates,

$\nu_{j=1,2,3,4}$ = vacuum eigenstates.

$$\frac{d\rho}{dt} = \partial_t \rho - H p \partial_p \rho$$ in FRW background, $H = \dot{a}/a$

in terms of $x = m_0 a = \frac{m_0}{T}$ and $y = \frac{p}{T}$:

for adiabatic expansion

$$\frac{d\rho}{dt} = \underline{\underline{H x \partial_x \rho}}$$

integro - diff eqs

$$\begin{cases} i\dot{\rho}_{ss} = \sum_a \mathcal{H}^{(1)}_{sa} (\rho_{as} - \rho_{sa}) ; \\ i\dot{\rho}_{aa} = \sum_b \mathcal{H}^{(1)}_{ab} (\rho_{ba} - \rho_{ab}) - \mathcal{H}^{(1)}_{sa} (\rho_{as} - \rho_{sa}) - i \underline{I_{coll}[\rho]} \\ i\dot{\rho}_{ab} = \mathcal{H}^{(1)}_{as} \rho_{sb} - \mathcal{H}^{(1)}_{sb} \rho_{as} + \sum_c \left(\mathcal{H}^{(1)}_{ac} \rho_{cb} - \mathcal{H}^{(1)}_{cb} \rho_{ac} \right) - \\ \qquad - \underline{i \gamma_{ab} \rho_{ab}} ; \\ i\dot{\rho}_{sa} = \mathcal{H}^{(1)}_{ss} \rho_{sa} - \mathcal{H}^{(1)}_{sa} \rho_{ss} + \sum_b \left(\mathcal{H}^{(1)}_{sb} \rho_{ba} - \mathcal{H}^{(1)}_{ba} \rho_{sb} \right) - \\ \qquad - i \gamma_{as} \rho_{sa} . \end{cases}$$

$$\mathcal{H}^{(1)}_{ab} = V^{eff}_{ab} + \sum_{j=1,4} \frac{m_j^2}{2E} u_{ja} u_{jb}$$

also contains $\underline{\int d^3 p}$

$$I^{el}_{coll} = \frac{1}{2E_\ell} \sum \int d\tau (\ell_2, \ell_3, \ell_4) |A_{el}|^2 \left[\rho_{aa}(p_1) f_e(p_2) - \rho_{aa}(p_3) f_e(p_4) \right]$$

$$I^{ann}_{coll} = \ldots$$

(all in Bolzmann approximation)

$$\gamma = \frac{1}{2E} \int d\tau_{fin} \left[A^2_{el} + A^2_{ann} \right] f(p_2)$$

may depend upon ρ_{aa}

Methods of solution:

1. "Brute force", no assumptions except for Bolzmann statistics; 18 collision integrals $f_{\nu_e} = f_{\nu_e}(x, y)$ with $f_{in} = e^{-y}$ too much time

2. Stationary point.

$$\underbrace{H x \partial_x f_{\alpha\beta} = L(f_{\alpha\beta}, f_{\alpha\alpha}) - \Gamma f_{\alpha\beta}}_{\alpha \neq \beta} \leftarrow \text{ensures stability}$$

$$\hookrightarrow L(f_{\alpha\beta}, f_{\alpha\alpha}) = \Gamma f_{\alpha\beta} \leftarrow \text{algebraic relation,}$$

to be substituted into $H x \partial_x f_{aa} = \ldots$

⊕ kinetic equilibrium, i.e. $\underline{f_{aa} = e^{-y} \cdot c(x)}$

Good approximation if $\delta m^2 > 10^{-6}$ and no resonances. → differential eqs (not integro-differential)

3. Resonance approximation if $m_{\nu_s} > m_{\nu_a}$ (1, 2, or 3 resonances) – saddle point in addition to upper limit.

Approximate methods well agree with the exact results in several test points

Nn15

Non-resonance case

- Simple analytic estimates are possible.

- Sterile neutrinos production rate roughly given by:

$$\Gamma_s = (\Gamma_{act} / 4) \tan^2(2\theta_{matter})$$

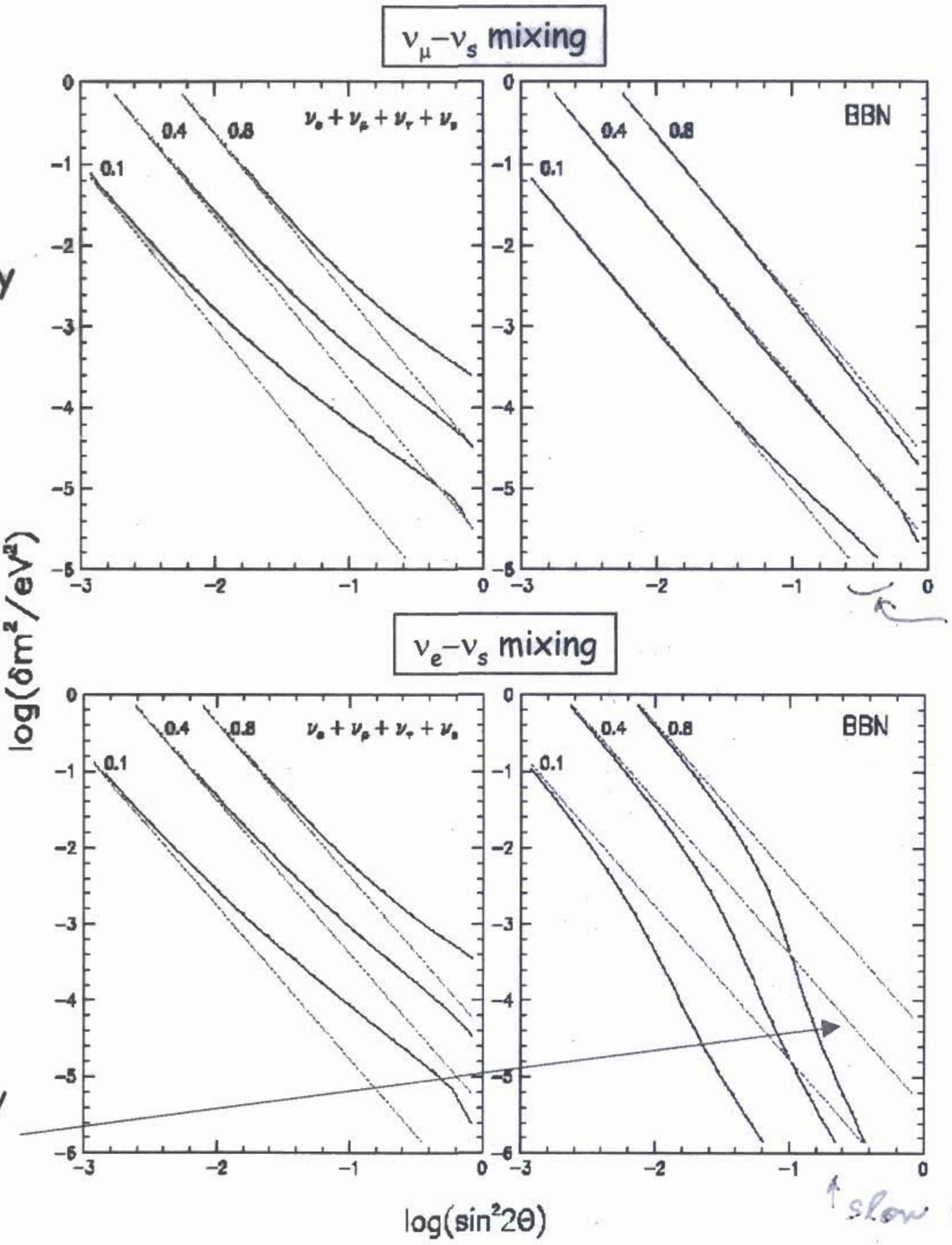
ν_μ - ν_s mixing

$$\left(\frac{\delta m^2}{eV^2}\right)\sin^4(2\theta) = 1.7\cdot 10^{-5}\ln^2(1-\Delta N_\nu)$$

ν_e - ν_s mixing

$$\left(\frac{\delta m^2}{eV^2}\right)\sin^4(2\theta) = 3.2\cdot 10^{-5}\ln^2(1-\Delta N_\nu)$$

- In previous estimates, ν_e number density reduction and ν_e spectrum distortion are neglected. Relevant effect for small mass differences.



Dolgov, FLV, hep-ph/0308083

conservation of $\nu_\mu + \nu_s$ (slow repopulation)

slow repopulation of ν_e

Resonance Case

- Resonance case →

  $m_s < m_a$ in the small mixing angle limit

- More complicated picture. Calculations usually performed (e.g. by Enqvist et al. 1992) in single momentum approximation.

- Complete calculation shows larger effects.

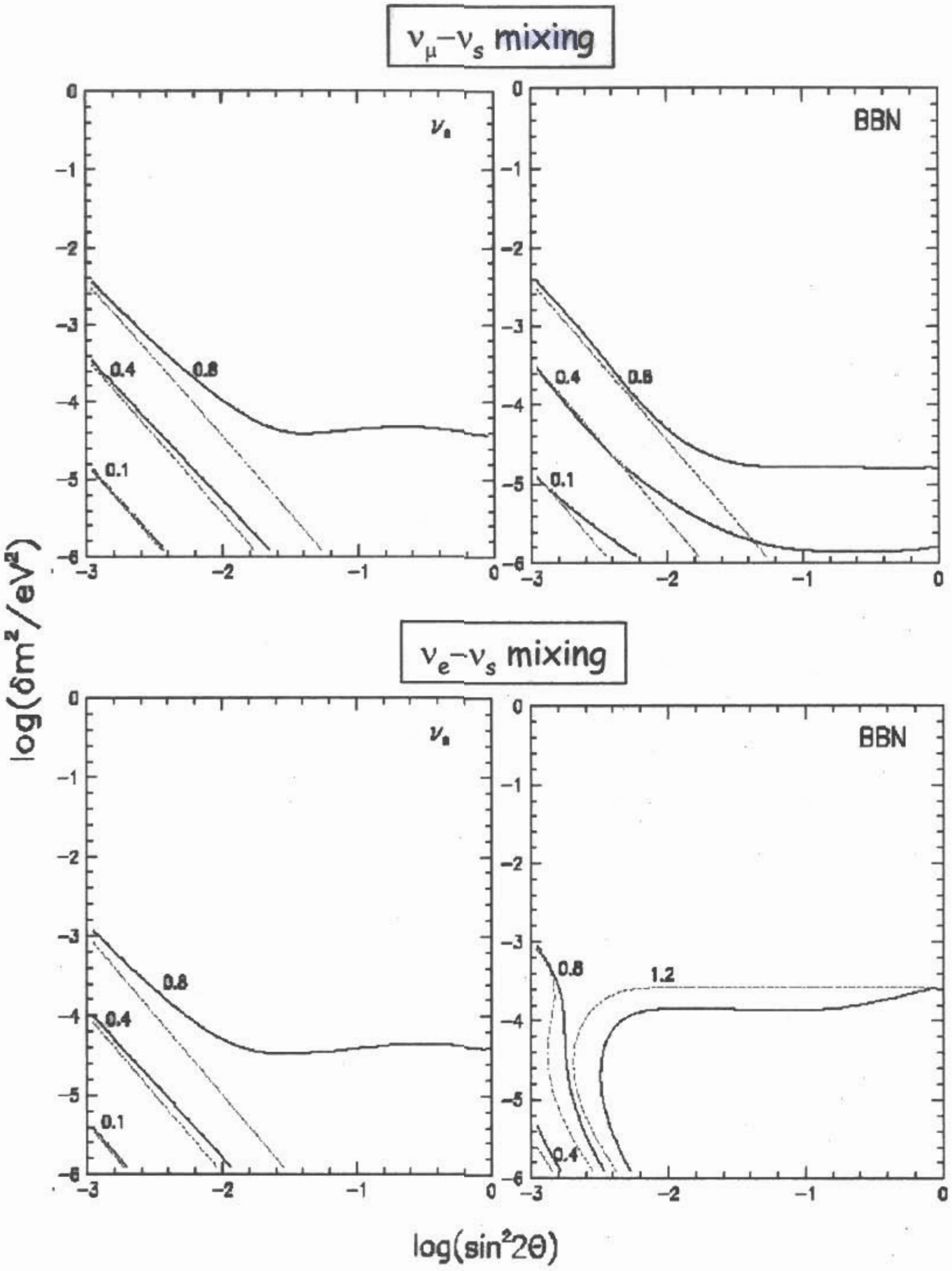
- Small mixing angle behaviour can be understood analytically (Landau-Zener approach).

ν_μ - ν_s mixing

$$\left(\frac{\delta m^2}{eV^2}\right)\sin^4(2\theta) = 1.9\cdot 10^{-9}\ln^2(1-\Delta N_\nu)$$

ν_e - ν_s mixing

$$\left(\frac{\delta m^2}{eV^2}\right)\sin^4(2\theta) = 5.2\cdot 10^{-10}\ln^2(1-\Delta N_\nu)$$



Dolgov, FLV, hep-ph/0308083

NΠ16

Active + Sterile oscillations

Active neutrinos are now known to be strongly mixed. Their mutual transformation should be taken into account together with $\nu_{act} - \nu_s$ oscillations. One expects:

- Stronger limits for "$\nu_{\mu\tau}$-ν_s mixing" (ν_e depletion as a result of ν_e-ν_μ mixing)
- Depending on masses and mixings, "new" resonances among active and steriles neutrinos.

Complicated problem in the general case. If we assume small active-sterile mixing:

$$\begin{pmatrix} \nu_{e\mu\tau} \\ \nu_s \end{pmatrix} = \begin{pmatrix} U^{act}_{3\times3} & \eta \\ \varepsilon & 1 \end{pmatrix} \cdot \begin{pmatrix} \nu_{123} \\ \nu_4 \end{pmatrix} \qquad \eta = -U^{act}_{3\times3} \cdot \varepsilon$$

$$U^{act}_{3\times3} = U_{23} \cdot U_{13} \cdot U_{12}$$

$$\theta_{12} = \theta_{solar}; \qquad \theta_{23} = \theta_{atmo}$$

$$\delta m^2{}_{21} = \delta m^2{}_{solar}; \qquad \delta m^2{}_{32} = \delta m^2{}_{atmo}$$

In the assumption θ_{13}=0, one can take advantage of the symmetry between ν_μ and ν_τ, reducing to:

$$\begin{pmatrix} \nu_e \\ \nu'_\mu \\ \nu'_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 & \eta'_1 \\ -s_{12} & c_{12} & 0 & \eta'_2 \\ 0 & 0 & 1 & \eta'_3 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & 1 \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

2 active + 1 sterile mixing

1 active + 1 sterile mixing

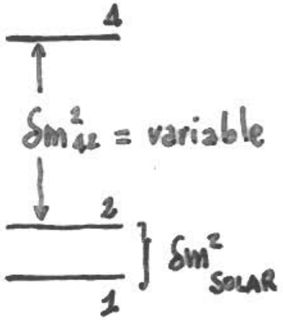
N717

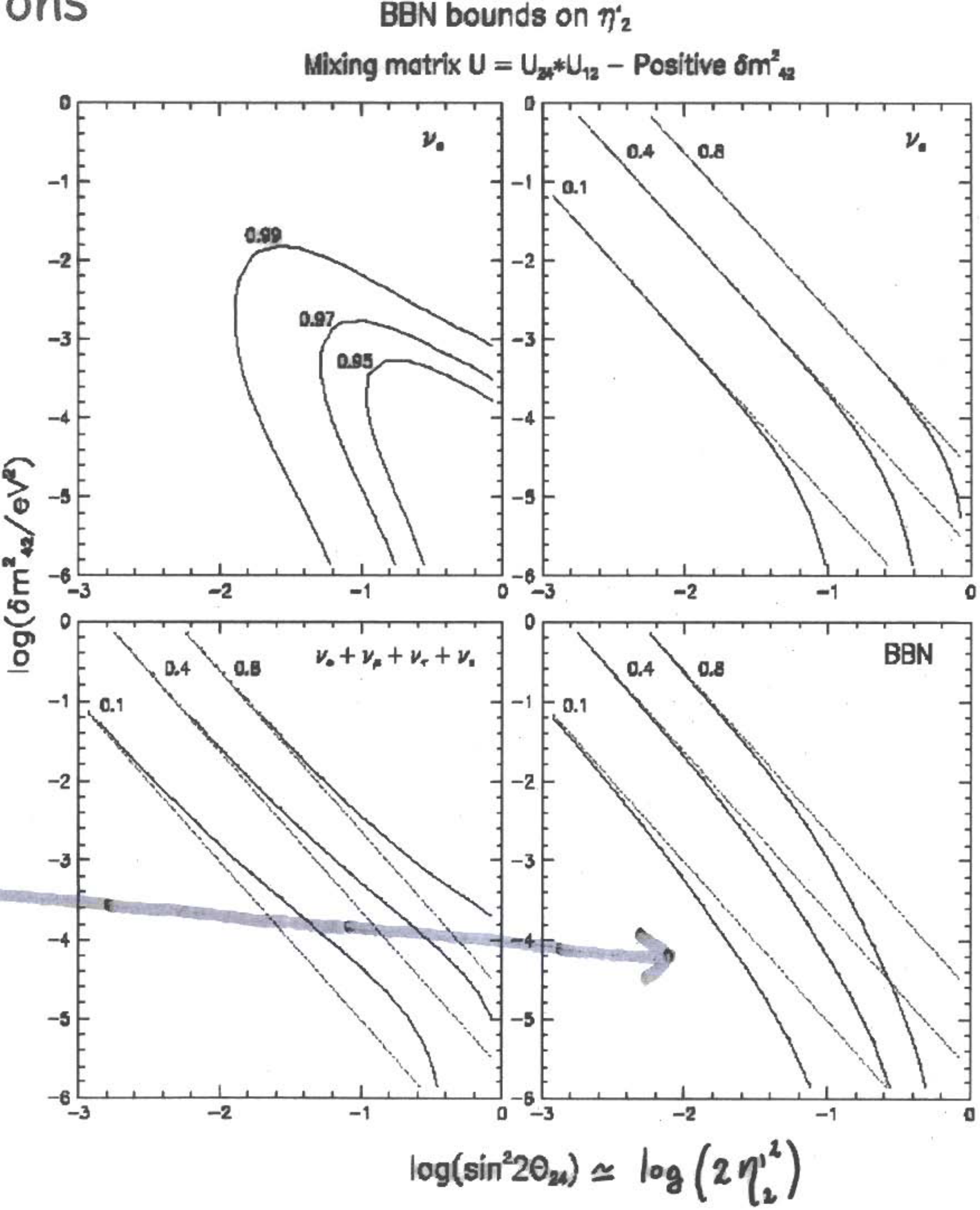
Active + Sterile oscillations

Dolgov, FLV, hep-ph/0308083

Mixing matrix $U = U_{24} \cdot U_{12}$

– positive δm^2_{42}





stronger than 1+1

weaker

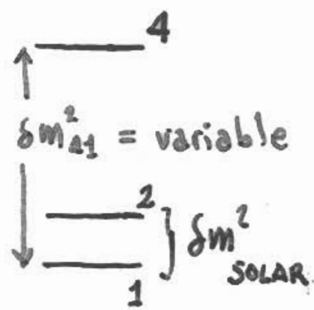
stronger limits for "$\nu_{\mu\tau}$ - ν_s mixing"
ν_e depletion as a result of ν_e - ν_4 mixing

Active + Sterile oscillations

Dolgov, FLV, hep-ph/0308083

Mixing matrix $\quad U = U_{14} \cdot U_{12}$

- positive δm^2_{41}



$m_4 > m_2$

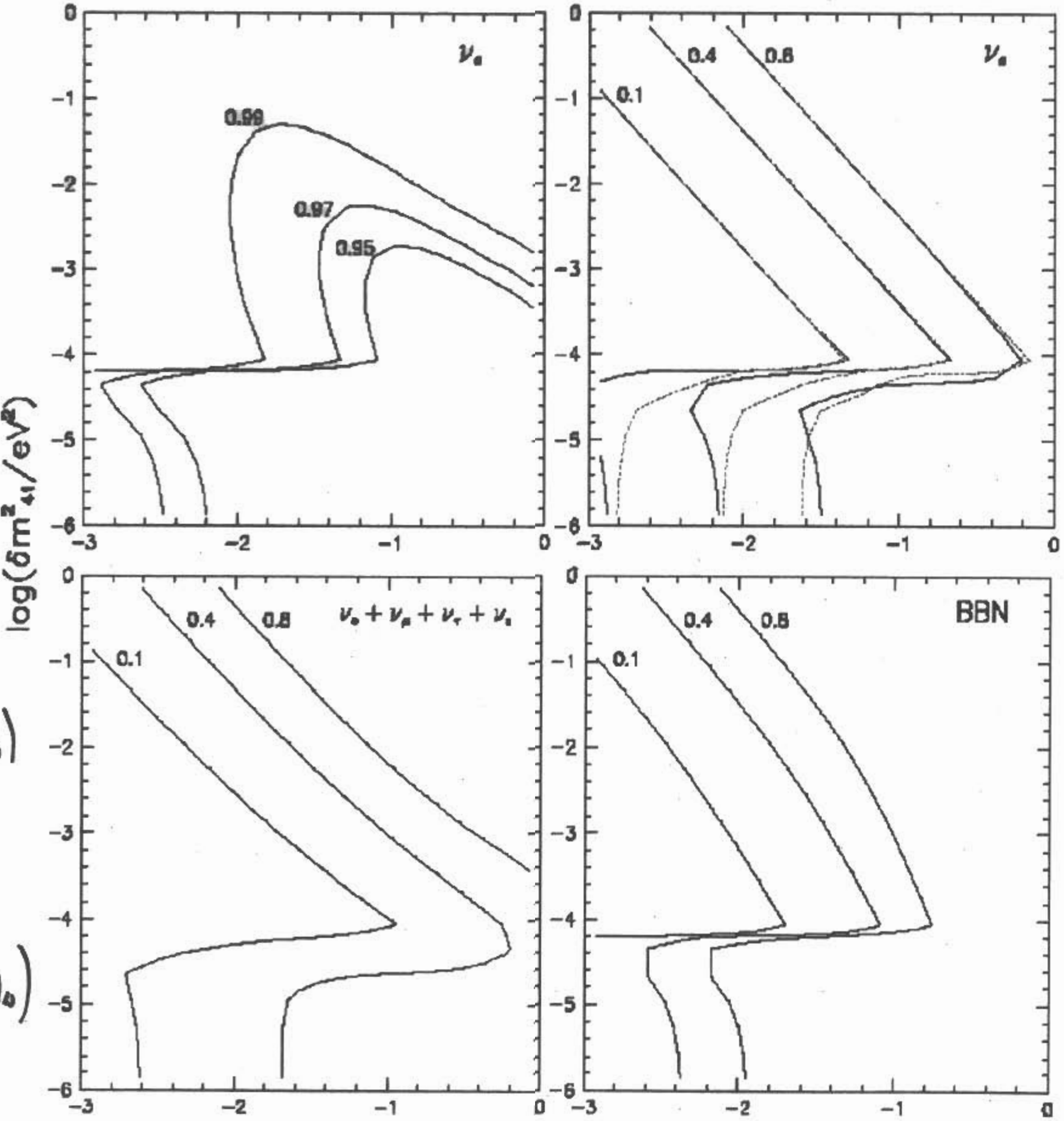
$$\left(\delta m^2_{41} / eV^2\right) \cdot \left(2\eta_1'\right)^4 = 3.16 \cdot 10^{-5} \ln^2\left(1 - \Delta N_\nu\right)$$

$m_4 < m_2$ Resonant ν_s production

$$\left(\frac{|\delta m^2_{42}|}{eV^2}\right) \cdot \left[2\eta_1' \sin(\theta_{12})\right]^4 = 5.2 \cdot 10^{-10} \ln^2\left(1 - \Delta N_\nu\right)$$

BBN bounds on η'_1

Mixing matrix $U = U_{14} * U_{12}$ – Positive δm^2_{41}



$\log(\sin^2 2\theta_{14}) \simeq \log(2\eta_1'^2)$

NΠ19

NΠ20

Summary

- Theory: -robust, well defined, very precise
  -theoretical uncertainties, at present, sub-dominant with respect to observational errors

- Observations: **two problems:**
  - systematics
  - evolutionary effects

- Theory.vs.data: $(\Omega_B h^2)_{BBN} = 0.0214 \pm 0.0020$

  $\Delta N_\nu \leq 1$

  $|\xi| \leq 0.05$ for any neutrino flavour

$(\delta m^2 / eV^2) \sin^4(2\theta) < 3.2 * 10^{-5} \ln^2(1-\Delta N\nu)$ $\nu_e - \nu_s$ No Res.

$(\delta m^2 / eV^2) \sin^4(2\theta) < 5.2 * 10^{-10} \ln^2(1-\Delta N\nu)$ Res.

$(\delta m^2 / eV^2) \sin^4(2\theta) < 1.7 * 10^{-5} \ln^2(1-\Delta N\nu)$ $\nu_{\mu\tau} - \nu_s$ No Res.

$(\delta m^2 / eV^2) \sin^4(2\theta) < 1.9 * 10^{-9} \ln^2(1-\Delta N\nu)$ Res.

Important effects related to mixing between active neutrinos

Generation of large lepton asymmetry by ν-oscillations.

Assumptions: only one $\Rightarrow$

existence of sterile neutrino, ν_s, mixed with any active one with $\theta < 10^{-2}$, $|\delta m^2| > 10^{-2}\, eV^2$ *

Resonance oscillations create lepton asymmetry in active ν sector

$L = (n_\nu - n_{\bar\nu}) / (n_\nu + n_{\bar\nu}) \sim 1$ $(0.3 - 0.4)$ $.37$

$\{\xi_\nu \neq 0$ but $\xi_{\bar\nu} = 0 \Leftarrow$ possible$\}$ $\hookrightarrow 0.1$ each ν_a max

If there are small inhomogeneities in e.g. baryon asymmetry $\delta B/B \sim 10^{-5}$ (with $B \sim 10^{-9} - 10^{-10}$) then $(\delta L/L) \sim 1$ would be generated.

* Observations allow 30–50% admixture of ν_s to ν_a

If mixing angle is ~~m~~ large equilibrium between ν_a and ν_s and $\bar\nu_a$ and $\bar\nu_s$ is quickly established and <u>no asymmetry</u> between ν_a and $\bar\nu_a$ is generated.

For small mixing only resonance transitions are essential.

$$W = \frac{\delta m^2}{2E_R} + \bar{C}_2 \frac{G_F^2 T^4 E_R}{\alpha} \pm \bar{C}_1 \eta G_F T^3 = 0$$

$$E_R = \sqrt{\frac{\alpha}{2C_2}} \frac{(-\delta m^2)^{1/2}}{G_F T^2} \left(1 \pm \eta C_1 \sqrt{\frac{\alpha}{2C_2}} \frac{T}{\sqrt{\delta m^2 (-1)}}\right)$$

$\delta m^2 < 0$

$$\frac{E_R}{T} = y_R \sim 10^4 \text{MeV} \left(\frac{\delta m^2}{eV^2}\right)^{1/2} \left(\frac{\text{MeV}}{T}\right)^3$$

$y_R \sim T^{-3}$ (if asymmetry is small)

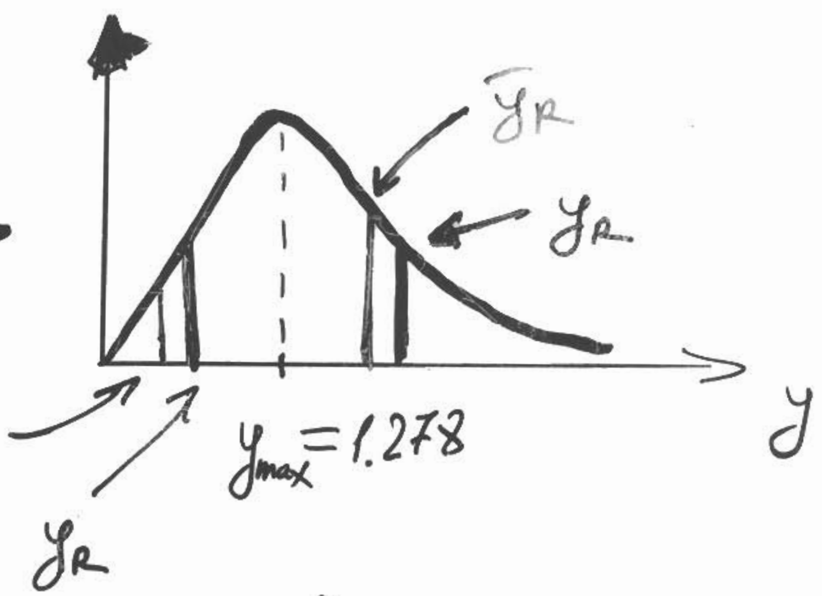
At fixed T transition $\nu_a \to \nu_s$ and $\bar\nu_a \to \bar\nu_s$ takes place only for $y = \frac{p}{T} = y_{res}(T)$

Evolution of number density:

$$\dot{n} = \Gamma_{Res} \cdot y_R^2 f_{eq}(y_R) \quad , \quad y = \frac{E}{T}, \quad f_{eq} = \frac{1}{e^y + 1}$$

$$\Gamma_{Res} = \frac{\Gamma \sin^2 2\vartheta}{W^2 + \Gamma^2/4} \Big|_{W=0} \sim \frac{1}{\Gamma} \sim \frac{1}{y}$$

$\dot{n} \sim y_R f_{eq}(y_R)$



When $y_R < y_{max}$ $\quad \dot{n} > \dot{\bar{n}}$ and ν_a disappear faster than $\bar{\nu}_a \Longrightarrow L \Downarrow$

When $y_R > y_{max}$ $\quad \dot{n} < \dot{\bar{n}}$ and $L \Uparrow$

$\dot{L} \sim L \Longrightarrow$ exponential decrease and later exponential rise

because $y_R - \bar{y}_R \sim L$ (for small L)

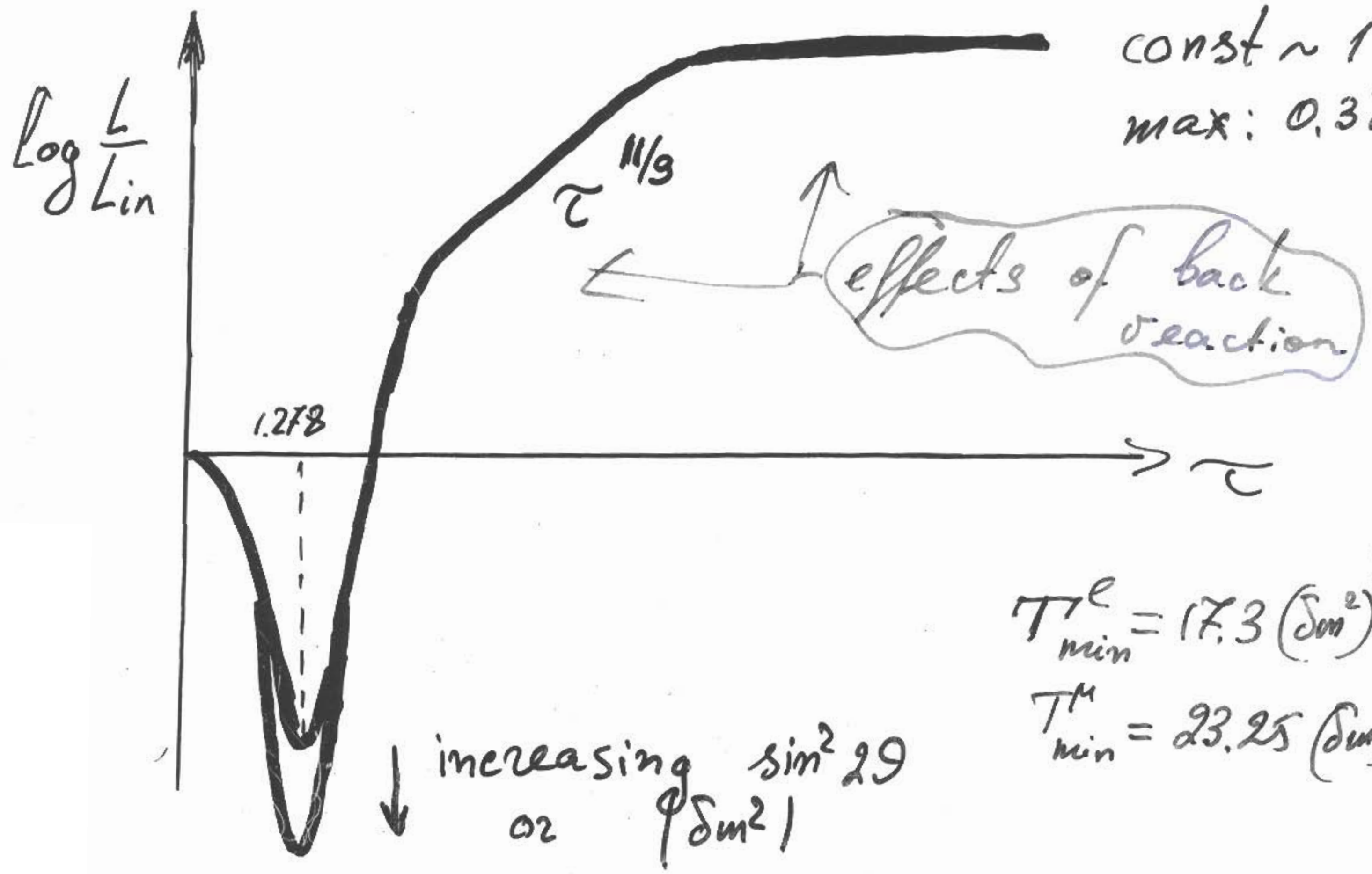
Evolution equation:

$$\frac{dL}{d\tau} = A(\tau; L)\, L \qquad \text{for } L \ll 1,\ A = A(\tau)$$

$$A(\tau) = C\tau^{5/3} f_{eq}(\tau)\left[-1+\tau\left(1-f_{eq}(\tau)\right)\right]$$

$$C \approx 10^{10}\left(\delta m^2/eV\right)^{3/2} \sin^2 2\vartheta_{vac}$$

$$\tau = (\sim 10^4)\left(\delta m^2/eV\right)^2 \left(1\,MeV/T\right)^3$$



$$T^{e}_{min} = 17.3\left(\delta m^2\right)^{1/6}\ MeV$$

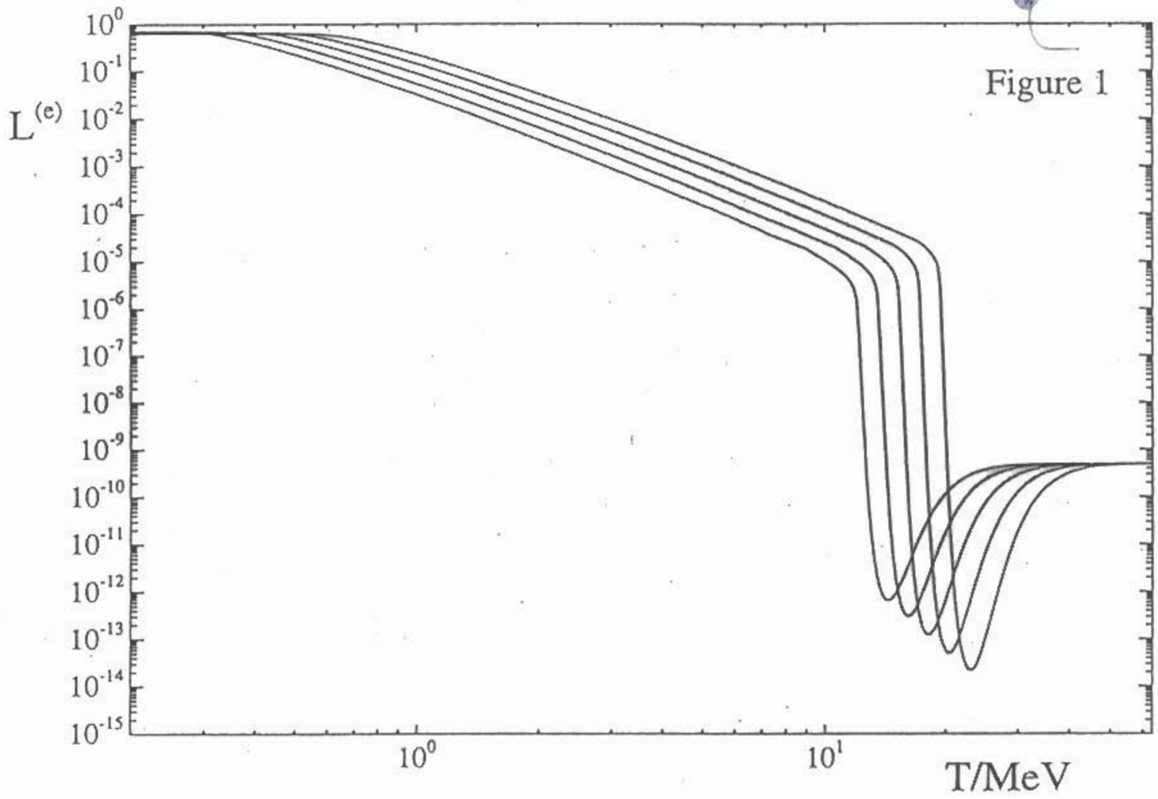
$$T^{\mu}_{min} = 23.25\left(\delta m^2\right)^{1/6}$$

$$\left(\frac{L_{min}}{L_{in}}\right) \sim \exp\left\{-(7-8)\left(\frac{\sin 2\vartheta}{10^{-4}}\right)^2 \cdot \left(\delta m^2\right)^{1/6}\right\}$$

NM24

K48

Di Bari, Foot



$\sin 2\vartheta_{vac} = 10^{-4}$; $\eta_{in} = 5 \cdot 10^{-10}$

$\delta m^2 = -\frac{1}{4}, -\frac{1}{2}, -1, -2, -4 \text{ eV}^2$

$T_{min}(\delta m^2) \longrightarrow$ corresponds to the same $\tau \sim T^{-3}(\delta m^2)^2$

In a range of k, $k_{min} < k < k_{max}$

such that

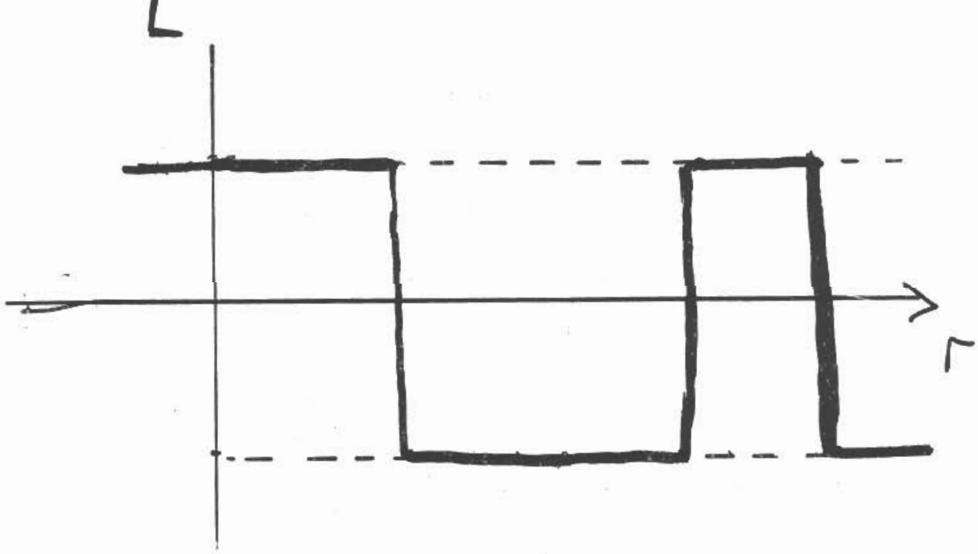
$$\delta B\, k^2 \int_{t_{in}}^{\tau} dt' \mathcal{D}(t')\, e^{-k^2 \int_{t'}^{\tau} \mathcal{D}(t'')\, dt''} > L_{min}$$

(roughly)

the sign of L_∞ is determined by the sign of δB

($B \sim 5 \cdot 10^{-10}$, $\delta B / B < 10^{-3}$ from CMBR

"normal" $\delta B / B \sim 10^{-5}$?)



$\hookrightarrow$ 3D

Historical comments:

1. Possible exponential rise of L thru res. transitions: R. Barbieri + AD
but concluded back reaction terminate it at $\sim 10^{-5}$
Enquist et al $L \to 0$

2. Foot, Thomson, Volkas $L \nearrow (0.3-0.4)$
numerical integration

3. Shi $\longrightarrow$ chaotic rise

4. AD + S. Hansen + S. Pastor + D. Semikoz
$\Rightarrow$ chaoticity = numerical instability
did not calculate

5. Analytical treatment: 4 $\Rightarrow$ weak rise
AD - error corrected $\Rightarrow$ agreed with 2
No chaoticity

6. Kainulainen, Sorri $\Rightarrow$ chaotic in averaged eqs
Foot, Volkas, di Bari $\Rightarrow$ no chaoticity in coinciding range of parameters in exact eqs.
but chaoticity in unstable (numerically) regions

Kainulainen, Sorri
chaotic in exact eqs.

NП27

Asymptotic value of lepton asymmetry:

$$L = \frac{n_\nu - n_{\bar\nu}}{n_\gamma}\Bigg|_{\text{in thermal equl. with } \nu,\ \text{i.e. } T_\nu = T_\gamma}$$

$$n_\gamma = \frac{4}{3}\, n_\nu \cdot 2$$

$$L_{max} = \frac{3}{8} = 0.375$$

$\sin^2 2\theta = 10^{-8}$, $\delta m^2 = -1 \Rightarrow L_\infty = 0.35$

$\sin^2 2\theta = 10^{-9}$, $\delta m^2 = -10 \Rightarrow L_\infty = 0.27$

$|L_\infty|$ does not depend upon initial value since the main increase is achieved at power regime (from $\sim 10^{-5}$

Spatial variation

(P. di Bari)

$$\dot{L}_{va} = \mathcal{A}(\tau)\left[2L_{va} + \bar{L} + \delta B(x)\right] + \mathcal{D}(\tau)\nabla^2 L_{va}$$

$$\mathcal{A}(t) = \alpha(\tau - \tau_c)$$

$$L_{va} = \bar{L}\int_{t_{in}}^{\tau} dt'\,\underline{a(t')}\, e^{2\int_{t'}^{\tau} dt''\,\underline{a(t'')}} + (\sim L_{in})$$

$$+ \int_{t_{in}}^{\tau} dt'\,a(t')\, e^{2\int_{t'}^{\tau} dt''\,a(t'')} \int d^3k\, e^{i\vec{k}\vec{x}}\, \delta\tilde{B}(k, t_{in}) \cdot e^{-k^2\int_{t'}^{\tau} dt''\,\mathcal{D}(t'')}$$

k-modes

$$\Longrightarrow e^{2\int_{t_{in}}^{\tau} dt''\,a(t'')}\left[\bar{L}\,\delta^{(3)}(\vec{k}) + \frac{1}{2}B_k\, e^{-k^2\int_{t_{in}}^{\tau} dt''\,\mathcal{D}(t'')}\right]$$

contribute both $a>0$ and $a<0$

$$+ \frac{1}{2}\delta B_k\, k^2 \int_{t_{in}}^{\tau} dt'\, e^{\int_{t'}^{\tau} dt''\,a(t'') - k^2\int_{t'}^{\tau}\mathcal{D}(t'')\,dt''}\, \mathcal{D}(t')$$

is not strongly suppressed for negative $a(t'')$

Integral is dominated by t'' where $a(t'') > 0$. Exponential rise for large τ is the same.

NΠ29

Neutrino and large scale structure

If $m_\nu = 0 \Rightarrow$ general suppression
because $\delta\rho$ started to rise
at MD – stage
and more energy is in ν, later is
MD – stage

$$\left(\frac{\delta\rho}{\rho}\right)_{Now} \approx \left(\frac{\delta\rho}{\rho}\right)_{initial} \cdot (z+1)_{MD}$$

If $m_\nu \neq 0$ can they make all
dark matter with any assumption
of spectrum of initial $(\delta\rho/\rho)$
(usually assumed to be flat)
and spectrum of ν (non-thermal?)

Tremain - Gunn bound

If ν's make all DM in galaxies

$\Downarrow$

degenerate gas, $\quad p_F = m_\nu v_F$

$$n = n_\nu + n_{\bar\nu} = \frac{2p_F^3}{6\pi^2}$$

$$M_{tot}(R) = \frac{4\pi}{3} R^3 m_\nu n = \frac{2}{9\pi} m_\nu (R p_F)^3$$

Virial theorem:

$$v_F^2 = \frac{G_N M_{gal}}{R}$$

$$\hookrightarrow m_\nu = \underline{\underline{80\,eV}} \left(\frac{300\ km/sec}{v_F}\right)^{1/4} \left(\frac{1\,kpc}{R}\right)^{1/2}$$

GZ - bound $\underline{\underline{m_\nu < 5\,eV}}$

If ν are Bosons (!?) they would work well for LSS

Neutrino free streaming

Neutrinos are very relativistic at decoupling and move with speed of light till $T_\nu \approx m_\nu/3$

$$\ell_{\substack{free\\stream}} \approx 2t\,(T_\nu = m_\nu/3)$$

$$M_{\substack{free\\stream}} \approx 0.1\,\frac{m_{pe}^3}{m_\nu^2} \approx \underline{5\cdot 10^{17} M_\odot \left(\frac{1\,eV}{m_\nu}\right)^2}$$

If density perturbations are common for all particles (e.g. adiabatic ones created by inflation) neutrino out-stream would leave behind less power at small scales.

1. The larger Ω_ν, the stronger effect.
2. The larger fraction of relativistic matter (hot matter), the later structure formation begins.

Observations of structures at high z permit to obtain $\sum m_\nu < 2\text{-}3$ eV ? (will) (< 1eV)

HCDM - model ⇒ suppressed small scales

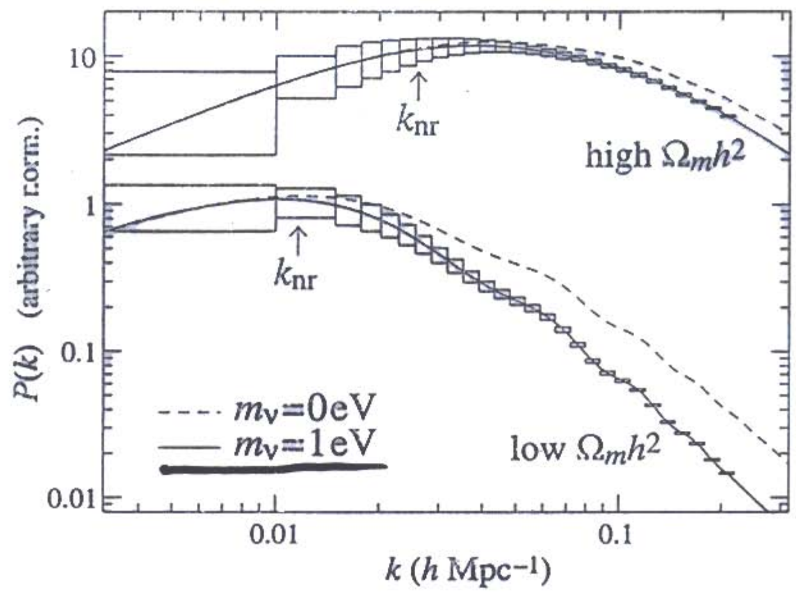


Figure 21: Effect of a 1 eV neutrino on the bright red galaxy (BRG) power spectrum compared with expected precision of the SDSS BRG survey (1σ error boxes). Upper curves: an $\Omega_m = 1.0$, $h = 0.5$, $\Omega_b h^2 = 0.0125$, $n = 1$ model with and without a 1 eV neutrino mass. Lower curves: the same but for an $\Omega_m = 0.2$, $h = 0.65$ model.

(from Hu et al, astro-ph 9806362)

Sloan Digital Sky Survey (SDSS) may be sensitive to $m_\nu \sim 0.1\,eV$ (if all other cosmological parameters are known)

The larger is the mass of ν, the stronger is effect.

But why?

Recent CMBR at high ℓ
(WMAP+baloons) (small scales)

measures $(\delta\rho/\rho)$ at the same scales
as data on LSS

$\Downarrow$

much better accuracy

$\Downarrow$

$\sum m_\nu < 0.7\,eV$

$\Downarrow$

$m_\nu < 0.23\,eV$ surprisingly strong!

with this mass
ν is about non-relativistic
at recombination

Effects of ν on angular spectrum of CMBR

Take CMBFast and see ...

Positions of the peaks depends upon:
1) the geometry of the universe
2) horizon at recombination ← this in turn depends on $\underline{\underline{R\varnothing \leftrightarrow M\varnothing}}$ and $\underline{\underline{N_\nu, m_\nu}}$

Height of the peaks depends upon equation of state and if $M\varnothing + R\varnothing$ they are <u>higher</u> (resonance amplifications of oscillations)

Analysis of WMAP data:

$$N_\nu = 1.5 - 6 \quad (\text{or } 7)$$

WMAP + Numerical simulation of LSS (with $m_\nu = 10^6 M_\odot$)

$$\Longrightarrow \quad \underline{m_\nu < 0.23\,eV}$$

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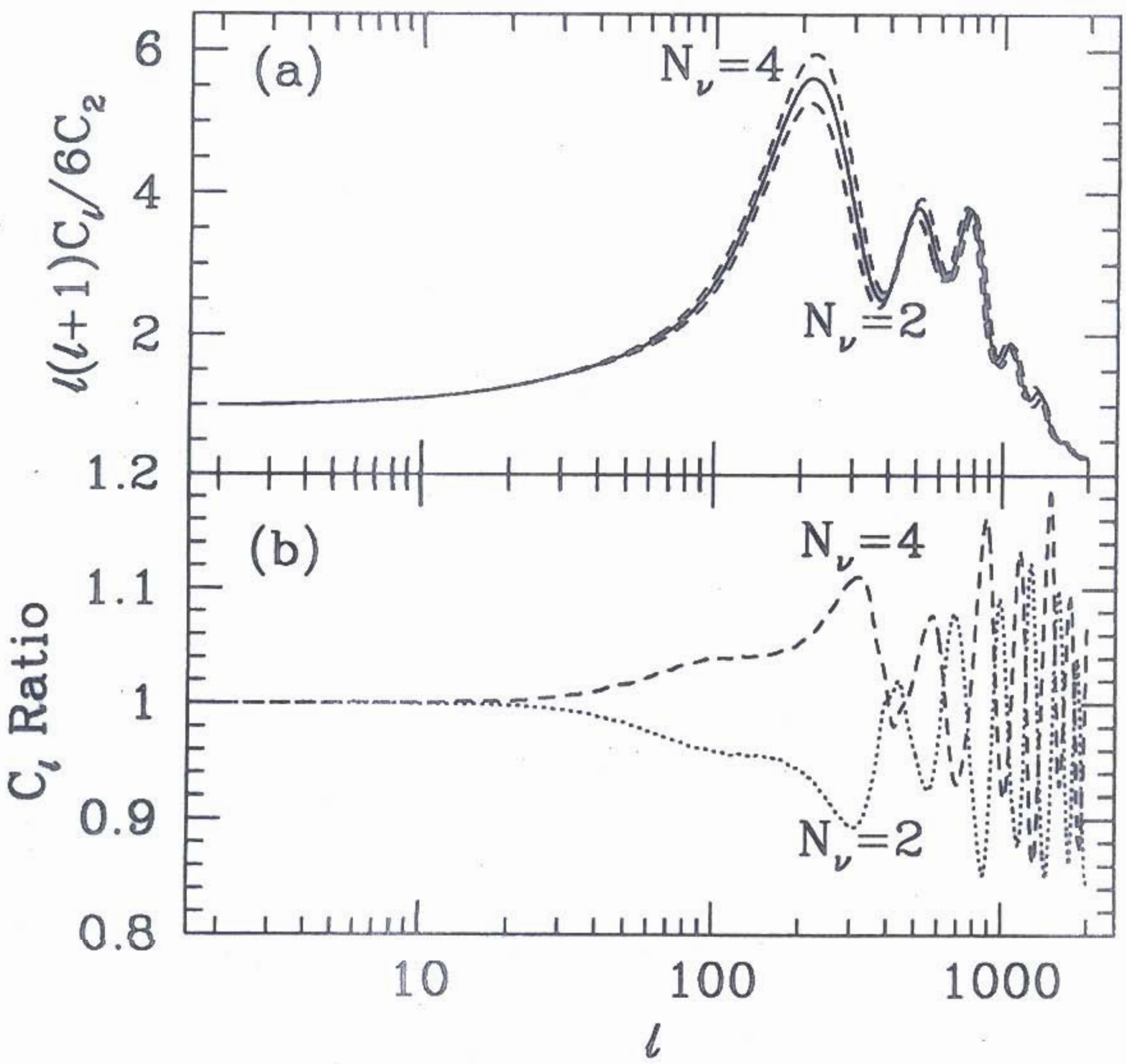

ımple of an angular spectrum of CMB anisotropies with
pecies, $k_\nu = 2, 3, 4$. (b) The ratio of C_l for $k_\nu = 2$, 4 rel
2])

t MD-stage. Indeed from the Newtonian equation