Advanced Quantum Field Theory – Syllabus AY 2018/19

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Oral examen: *topics between* **square** *brackets are excluded from the examen. Only main results, without* **details** *of proofs, are required for subjects between* **round** *brackets. The proposed* **exercises**, *see separate file, are part of the examen and serve as personal check of your comprehension.*

1) Perturbative and non-perturbative aspects of quantum field theories. Operatorial approach and functional integral approach. (Haag's theorem and non-existence of the interaction picture [EF]. Dyson's argument for QED [ZI: 15.5], non-Borel summability of the perturbative series, Lautrup's *renormalons* [L1], asymptotic series. Triviality-problem of the $\lambda\varphi^4$-theory. Wightman's axiomatic approach and the reconstruction theorem [SW], [S4]. From Schwinger's $SU(2)$ model to 't Hooft's proof of the renormalizability of YM theories).

2) Symmetries and quantum-mechanically consistent interactions. The paradigm of *canonical quantization.* Unitary implementation of global symmetries in quantum theories. Invariances of the S-matrix. Poincaré group and *internal* symmetries. Coleman-Mandula theorem [S1: 2.5]. Representations of the Lorentz group and spin. [Duality between scalar fields and antisymmetric potentials]. General properties of theories with particles of spin $0 \leq s \leq 1$, $3/2 \leq s \leq 2$ and $2 < s$.

3) Classical field theories. Action and equations of motion: relativistic invariance, global symmetries and Nöther theorem, locality, renormalizable Lagrangians. Theories with scalars and fermions: Yukawa couplings, chirality and mass. (Lie groups and Lie algebras: simple and compact groups, structure constants, irreducible representations, direct products of representations, Dynkin index, Casimir invariants. Adjoint representation and Jacobi identity). *Invariant tensors* and their relation with direct products of irreducible representations. General construction of invariant polynomials of fields. Most general Lagrangian of scalars and fermions invariant under global transformations of a Lie group [S3].

4) Classical Yang-Mills theories. Theories with local abelian gauge symmetries. QED with scalar charged fields. Theories with local non-abelian gauge symmetries. YM vector potentials: transformations laws and their consistency, infinitesimal transformations, covariant derivatives of matter fields. YM curvature: covariance, Bianchi identity, YM Lagrangian. Uniqueness of the coupling constant. Non-simple gauge groups. General renormalizable Lagrangian invariant under local gauge transformations. Chiral gauge couplings. Equations of motion. Nöther currents and covariant currents. Conserved *color* charges and their algebra. Self-interaction of YM fields and comparison with *General Relativity.*

5) Functional integral technics. The functionals Z, W and Γ. The Euclidean space as the appropriate framework. Conventions for correlation functions and Fourier transforms. Translation invariance. Background field method and classical limit $\Gamma \to I$. *Linear* classical symmetries and their quantum implementation: proof of *Furry's theorem*, mass protection through chiral symmetry, flavor conservation. Ward identity in QED as consequence of local gauge invariance: form of divergent counterterms and renormalizability, transversality of the photon vacuum polarization $\Gamma^{\mu\nu}$ [and relation with current conservation; relation between $\Gamma^{\mu\nu}$ and the correlation function $\langle 0|Tj^\mu j^\nu|0\rangle$ [L2: 103, 104]]; proof of the identities $Z_{1r} = Z_{2r}$. (Functional determinants for real and complex, commuting and anticommuting, scalar fields, and for spinor fields. One-loop effective action as a determinant. Effective scalar potential. Coleman-Weinberg mechanism for $\lambda\varphi^4$-theory and radiative spontaneous symmetry breaking; dimensional transmutation). [Remarks on the mechanism in *scalar* QED [WII: 16.2], [IZ: 9.2.2, 11.2.2]].

6) Perturbative methods and renormalizability. Derivation of the Feynman rules from the Lagrangian of a generic quantum field theory. The concept of *strictly renormalizable* theories. Application to *scalar* QED: ultraviolet divergences of the four-point scalar correlation function. (Proof of locality of the one-loop divergences in a generic local quantum field theory. Criterion for the determination of the superficially divergent correlation functions. Determination of all power-counting renormalizable couplings in a generic D-dimensional space-time). Non-renormalizable theories and super-renormalizable theories [IZ: 8.1.3]. The $\lambda\varphi^3$-theory in $D = 6$ as a *limit* theory. [The *Gross-Neveu* model in $D = 3$ as a non-perturbatively consistent theory, although non-renormalizable by power counting].

7) $\lambda\varphi^3$-model in $D = 6$ and higher-order renormalizability. Superficial divergences and sub-divergences. Explicit one-loop renormalization: computation of Z, Z_1, the β-function, and the anomalous dimension γ; running coupling constant and *asymptotic freedom* [S2: 14, 16]. (Explicit solution of the Callan-Symanzik equation and the phenomenon of *dimensional transmutation*). Determination of one-loop counterterms. (Two-loop diagrams for the propagator and the three-point function: local and non-local divergences, the problem of *nested* and *overlapping* divergences. Proof of the cancelation of all non-local divergences at two loops, and the role of the subtraction of one-loop divergences. Subtraction of *remnant* superficial local divergences at two loops [C: 5.2]). [n-point correlation functions with $n > 3$ and higher loops].

8) Quantization of YM theories. Negative norm states in YM theories and the problem of manifest covariance. The gauge-fixing problem in the functional integral approach. (Canonical quantization in the axial gauge, and the corresponding expression of gauge-invariant correlation functions in the functional integral approach from *first principles* [WI: 9.2], [WII: 15.4], [IZ: 12.2.1]). Gauge invariance of the functional measure over matter fields and YM potentials [WII: 15.4]. Faddeev-Popov quantization method: functional δ-function identities and FP determinant [P: 3.1]. Independence of gauge-invariant correlation functions of the gauge-fixing. The Lautrup *weighting* functionals $H[b]$. The Feynman-Lorenz λ-gauge. (Weinberg's theorem). Connection of the FP method with the axial gauge and *first principles* [WII: 15.5]. FP determinant and ghost fields, anticommutation statistics, ghost number conservation. The Nakanishi-Lautrup auxiliary fields B^a [WII: 15.6].

9) BRST symmetry and physical states. Missing gauge invariance of the FP action. BRST symmetry as the key ingredient ensuring the *strict* renormalizability of a YM theory and the decoupling of non-physical states. BRST transformations as a global one-parameter anticommuting symmetry group. Nilpotency of the transformations and invariance of the action. The FP action as a *trivial cocycle*. *On-shell* nilpotency in the absence of auxiliary fields. The conserved BRST charge Q in canonical quantization and its properties. Requirement of gauge-fixing independence of correlation functions of *physical* operators and the Kugo-Ojima condition for physical states. Q commutes with the S-matrix. The physical positive-definite Hilbert space $\mathcal{H}$ as the *cohomology* of Q. (Canonical quantization of the asymptotic fields and their commutation relations with Q. Proof of the absence of ghosts and non-physical gauge bosons in $\mathcal{H}$ [WII: 15.7]. Conserved BRST current. Explicit expression of Q and relation with the Gupta-Bleuler condition in QED).

10) Slavnov-Taylor identities. The problem of *strict* renormalizability of the FP-gauged-fixed YM action. Non-linearity of the BRST transformations, and the external currents K. ST identity for the functionals S and Γ. (Schwinger-Dyson equations [IZ: 10.1]). ST identities for the *reduced* functionals $\widetilde{S}$ and $\widetilde{\Gamma}$. Non-propagation of the gauge-fixing terms. Dependence of $\widetilde{\Gamma}$ on $K^{a\mu}$ and $\overline{C}^a$. Transversality of the exact gluon vacuum polarization $\Gamma^{ab}_{\mu\nu}(p)$. [Relation between the transversality of $\Gamma^{ab}_{\mu\nu}(p)$ and the quantum conservation of the color currents $j^{a\mu}$]. Correlation functions as *invariant* tensor fields. Preservation of the ST identities under renormalization: (general solution of the equations $\widetilde{S} * \widetilde{S} = 0$ and $\widetilde{S} * F + F * \widetilde{S} = 0$), and the structure of the divergent counterterms. *Strict* renormalizability of non-abelian gauge theories at all orders in perturbation theory. Validity of the ST identities for the renormalized theory [WII: 16.4, 17.1, 17.2], [IZ: 12.4.3].

11) Perturbative one-loop analysis of non-abelian gauge theories. Derivation of the Feynman rules [IZ: 12.2.3]. ST identities for the ratios between renormalization constants [IZ: 12.3.4]. Renormalization and transversality of the gluon vacuum polarization: (explicit evaluation of the diagrams with gluon loops, fermion loops, and ghost loops). The role of the ghost fields w.r.t. preservation of unitarity. One-loop divergences of the fermion two-point function and of the fermion-fermion-gluon correlation function. (Computation of the renormalization constants Z_3, Z_ψ, $Z_{1\psi}$). Determination of the β-function. Running coupling constant, asymptotic freedom, and dimensional transmutation in QCD. The scale $\Lambda_{\rm QCD}$. Comparison between the theories QED, QCD, $\lambda\varphi^4$ in $D=4$, and $\lambda\varphi^3$ in $D=6$ [R1: 9.8], [R2: 8.6, 8.8], [S2: 73], [PS: 16.5]. (Contribution to the β-function of scalar fields. β-functions and running of the coupling constants α_1, α_2 and α_3 in the Standard Model of elementary particles. $SU(5)$-Grand Unification and merging of the coupling constants in the minimal supersymmetric Standard Model. $N=4$ Super-YM theory as a perturbatively finite theory with all-order vanishing β-function).

12) BRST quantization of gauge theories with spontaneous symmetry breaking. [Goldstone theorem and Higgs mechanism revisited, massive gauge bosons. The FP method applied to the 't Hooft gauge: massive ghost fields. BRST symmetry and Kugo-Ojima condition on physical states. Decoupling of ghosts and goldstone bosons].

13) Anomalies. [Origin of quantum violation of a classical symmetry: symmetry breaking ultraviolet regularizations. *Consistent* anomalies and Wess-Zumino consistency condition. Trivial anomalies and finite counterterms. Locality and finiteness of anomalies. Non-abelian chiral gauge theories and ABBJ anomalies. The chiral Dirac determinant. The 't Hooft-Veltman dimensional regularization of chiral theories in $D=2n$ [B1: 4.3.3] and non-invariance of the regularized action. Anomalies versus violation of current conservation. Explicit evaluation of the ABBJ anomaly in $D=2$. Anomalies in $D \geq 4$ and triangle diagrams. Anomaly cancelation in the Standard Model [PS: 20.2]. Bardeen's theorem and its generalizations. Anomalies of *global* symmetries. *Covariant* anomalies. [B1], [B2], [S2: 75, 76]].

14) Instantons. Perturbative approach to the functional integral. Field configurations connected continuously to the absolute minimum of the Euclidean action. YM theories in Euclidean space-time. Pure-gauge YM connections. Relevant configurations and finiteness of the action. Classification of YM connections through the Pontryagin invariant n. The homotopy group $\pi_3(G)$ of a Lie group G. (The Maurer-Cartan invariant). Instantons as absolute minima of the action at fixed n. Instantons for $G=SU(2)$ relative to $n=1$. [*Moduli* as collective coordinates. The case $G=SU(N)$]. Non-perturbative character of the weighting factor $e^{-8\pi^2 n/g^2}$. ϑ-vacua and the strong CP problem. [The *axial* $U(1)$ problem]. [W2: 23.4, 23.5 23.6], [VV].

Main textbooks:

• C. Itzykson, J.-B. Zuber, *Quantum Field Theory*, New York, McGraw-Hill Book Co, 1987.

• S. Weinberg, *The Quantum Theory of Fields II – Modern applications*, Cambridge, Cambridge University Press, 2005.

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[VV] S. Vandoren, P. van Nieuwenhuizen, *Lectures on Instantons*, arXiv:0802.1862 [hep-th].

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[WII] S. Weinberg, *The Quantum Theory of Fields II - Modern applications*, Cambridge, Cambridge University Press, 2005.

[ZI] E. Zeidler, *Quantum Field Theory I - Basics in Mathematics and Physics*, Berlin, Springer-Verlag, 2005.