QUANTUM FIELD THEORY II – Syllabus – A.A. 2017/18

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Oral examen: *topics between* **square** *brackets are excluded from the examen. Only main results, without details of proofs, are required for subjects between* **round** *brackets. The proposed exercises, see separate file, are part of the examen and serve as personal check of the comprehension.*

1) Perturbative and non-perturbative aspects of quantum field theories. Operatorial approach and functional integral approach. (Haag's theorem and non-existence of the interaction picture [EF]. Lattice regularization. Asymptotic series, non-Borel summability of QED [L1] and *renormalons*, Dyson's argument [ZI: 15.5]. *Triviality-problem* of $\lambda\varphi^4$-theory). [Wightman's axiomatic approach and the *reconstruction theorem* [SW], [S4]].

2) Symmetries and quantum-mechanically consistent interactions. The paradigm of *canonical quantization.* Unitary implementation of global symmetries in quantum theories. Invariances of the S-matrix. Poincaré group and *internal* symmetries. Coleman-Mandula theorem [S1: 2.5]. Representations of the Lorentz group and spin. (Duality between scalar fields and antisymmetric potentials). General properties of theories with particles of spin $0 \leq s \leq 1$, $3/2 \leq s \leq 2$ and $2 < s$.

3) Classical field theories. Action and equations of motion: relativistic invariance, global symmetries and Nöther theorem, locality, renormalizable Lagrangians. Theories with scalars and fermions: Yukawa couplings, chirality and mass. (Lie groups and Lie algebras: simple and compact groups, structure constants, irreducible representations, Dynkin index, Casimir invariants). Adjoint representation and Jacobi identity. *Invariant tensors* and their relation with products between representations. Construction of invariant polynomials of fields. Most general Lagrangian of scalars and fermions invariant under global transformations of a Lie group [S3].

4) Classical Yang-Mills theories. Theories with local abelian gauge symmetries. QED with scalar charged fields. Theories with local non-abelian gauge symmetries. YM vector potentials: transformations laws and their consistency, infinitesimal transformations, covariant derivatives of matter fields. YM curvature: covariance, Bianchi identity, YM Lagrangian. Uniqueness of the coupling constant. Non-simple gauge groups. General renormalizable Lagrangian invariant under local gauge transformations. Chiral gauge couplings. Equations of motion. Nöther currents and covariant currents. (Conserved *color* charges and their algebra). Self-interaction of YM fields and analogy with *General Relativity.*

5) Functional integral technics. The functionals Z, W and Γ. The Euclidean space as the correct framework. Conventions for correlation functions and Fourier transforms. Translation invariance. *Background field method* and classical limit $\Gamma \to I$. *Linear* classical symmetries and their quantum implementation: (proof of *Furry's theorem*), mass protection through chiral symmetry, flavor conservation. *Ward identity* in QED as consequence of local gauge invariance: form of divergent counterterms and renormalizability, transversality of the photon vacuum polarization $\Gamma^{\mu\nu}$ [and relation with current conservation; relation between $\Gamma^{\mu\nu}$ and the correlation function $\langle 0|Tj^\mu j^\nu|0\rangle$ [L2: 103, 104]]; proof of the identities $Z_{1r} = Z_{2r}$. (Functional determinants for real and complex – commuting and anticommuting – scalar fields and for spinor fields. One-loop effective action as a determinant and *effective scalar potential. Coleman-Weinberg mechanism* for $\lambda\varphi^4$-theory and radiative spontaneous symmetry breaking; dimensional transmutation). [Remarks on the mechanism in *scalar* QED [WII: 16.2], [IZ: 9.2.2, 11.2.2]].

6) Perturbative methods and renormalizability. Derivation of Feynman rules for a generic local quantum field theory in D dimensions. The concept of *strictly renormalizable* theories. Application to *scalar* QED: ultraviolet divergences of the four-point scalar correlation function. (Proof of locality of one-loop divergences in a generic local theory. Criterion for the determination of divergent correlation functions and determination of *power-counting* renormalizable couplings in a D-dimensional space-time). Non-renormalizable theories and (super)-renormalizable theories [IZ: 8.1.3]. The $\lambda\varphi^3$-theory in $D = 6$ as a *limit* theory. [The *Gross-Neveu* model in $D = 3$ as a non-perturbatively consistent theory, although non-renormalizable by power counting].

7) $\lambda\varphi^3$-model in $D = 6$ and higher order renormalizability. Superficial divergences and subdivergences. Explicit one-loop renormalization: computation of Z, Z_1, β-function and anomalous dimension γ; *running coupling constant* and *asymptotic freedom* [S2: 14, 16]. (Explicit solution of the Callan-Symanzik equation and the phenomenon of *dimensional transmutation*). Determination of one-loop counterterms. (Two-loop diagrams for the propagator and the three-point function: local and non-local divergences, the problem of *nested* and *overlapping* divergences. Proof of cancelation of non-local divergences at two loops and role of subtraction of one-loop divergences. Subtraction of *remnant* superficial local divergences at two loops [C: 5.2]. n-point correlation functions with $n > 3$ and higher loops).

8) Quantization of YM theories. Negative norm states in YM theories and the problem of manifest covariance. The gauge-fixing problem in the functional integral approach. (*Canonical* quantization in axial gauge and corresponding expression of gauge-invariant correlation functions in the functional integral approach from *first principles* [WI: 9.2], [WII: 15.4], [IZ: 12.2.1]). Gauge invariance of the functional measure over matter fields and YM potentials [WII: 15.4]. *Faddeev-Popov* quantization method: functional δ-function identities and FP determinant [P: 3.1]. Independence of gauge-invariant correlation functions of the gauge-fixing. The Lautrup *weighting* functionals $H[b]$. The Feynman-Lorenz λ-gauge. (*Weinberg's theorem*). Connection with axial gauge and *first principles* [WII: 15.5]. FP determinant and *ghost* fields, anticommutation statistics and ghost number conservation. The *Nakanishi-Lautrup* auxiliary fields B^a [WII: 15.6].

9) BRST symmetry and physical states. Missing gauge invariance of the FP action. Classical and quantum BRST symmetry as key ingredient ensuring *strict* renormalizability and non-propagation of non-physical states. BRST transformations as a global one-parameter anticommuting symmetry group. Nilpotency of the transformations and invariance of the action. The FP term as *trivial cocycle*. *On-shell* nilpotency in the absence of auxiliary fields. The BRST charge Q in canonical quantization and its properties. Requirement of gauge-fixing independence of correlation functions of *physical* operators and the Kugo-Ojima condition on physical states. Q commutes with the S-matrix. The physical positive definite Hilbert space $\mathcal{H}$ as *cohomology* of Q. (Canonical quantization of asymptotic fields and their commutation relations with Q. Proof of absence of ghosts and non-physical gauge bosons in $\mathcal{H}$ [WII: 15.7]. Conserved BRST current. Explicit expression of Q and relation with the Gupta-Bleuler condition in QED).

10) Slavnov-Taylor identities. The problem of *strict* renormalizability of the FP-gauged-fixed YM action. Non-linearity of BRST transformations and external currents. Slavnov-Taylor identities for the functionals S and Γ. (Schwinger-Dyson equations [IZ: 10.1]. ST identities for the *reduced* functionals $\widetilde{S}$ and $\widetilde{\Gamma}$. Non-propagation of the gauge-fixing terms. Dependence of $\widetilde{\Gamma}$ on $K^{a\mu}$ and $\overline{C}^a$).

Transversality of the gluon vacuum polarization $\Gamma^{ab}_{\mu\nu}$. [Relation between the transversality of $\Gamma^{ab}_{\mu\nu}$ and the quantum conservation of color currents]. (Symmetries of correlation functions as *invariant* tensor fields). Preservation of ST identities under renormalization: (general solution of the equations $\widetilde{S} * \widetilde{S} = 0$ and $\widetilde{S} * F + F * \widetilde{S} = 0$), structure of divergent counterterms, *strict* renormalizability of non-abelian gauge theories at all orders in perturbation theory, validity of ST identities for the renormalized theory [WII: 16.4, 17.1, 17.2], [IZ: 12.4.3].

11) Perturbative one-loop analysis of non-abelian gauge theories. Derivation of Feynman rules [IZ: 12.2.3]. ST identities for the ratios between renormalization constants [IZ: 12.3.4]. Renormalization and transversality of the gluon vacuum polarization: (explicit evaluation of the diagrams with gluon loops, fermion loops and ghost loops). The role of ghosts w.r.t. preservation of unitarity. One-loop divergences of the fermion propagator and of the fermion-fermion-gluon correlation function. (Computation of the renormalization constants Z_3, Z_ψ, $Z_{1\psi}$). Derivation of the β-function. *Running coupling constant*, asymptotic freedom and dimensional transmutation in QCD. The scale Λ_{QCD}. Comparison between the theories QED, QCD, $\lambda\varphi^4$ in $D = 4$ and $\lambda\varphi^3$ in $D = 6$ [R1: 9.8], [R2: 8.6, 8.8], [S2: 73], [PS: 16.5]. (Contribution to the β-function of scalar fields. β-functions and running of the coupling constants α_1, α_2 and α_3 in the Standard Model. $SU(5)$-Grand Unification and merging of coupling constants in the minimal supersymmetric Standard Model). [Super-YM theory as a perturbatively finite theory with all-order vanishing β-function].

12) BRST symmetry and spontaneous symmetry breaking. [Goldstone theorem and Higgs mechanism revisited, massive gauge bosons. The FP method applied to the 't Hooft gauge: massive ghost fields. BRST symmetry and Kugo-Ojima condition on physical states: decoupling of ghosts and goldstone bosons].

13) Anomalies. [Origin of quantum violation of a classical symmetry: symmetry breaking ultraviolet regularizations. *Consistent* anomalies and Wess-Zumino *consistency condition.* Trivial anomalies and finite counterterms. Locality and finiteness of anomalies. Non-abelian chiral gauge theories and ABBJ anomalies. The chiral Dirac determinant. The 't Hooft-Veltman dimensional regularization of chiral theories in $D = 2n$ [B1: 4.3.3] and non-invariance of the regularized action. Anomalies versus violation of current conservation. Explicit evaluation of the ABBJ anomaly in $D = 2$. Anomalies in $D \geq 4$ and triangle diagrams. Anomaly cancelation in the Standard Model [PS: 20.2]. Bardeen's theorem and its generalizations. Anomalies of *global* symmetries. *Covariant* anomalies. [B1], [B2], [S2: 75, 76]].

14) Instantons. Perturbative approach to the functional integral and configurations connected continuously to the absolute minimum of the action. YM theories in euclidean space-time. *Pure-gauge* YM connections. Relevant configurations and finiteness of the action. Classification of YM connections through the Pontryagin invariant n. The homotopy group $\pi_3(G)$ of a Lie group G. (The Maurer-Cartan invariant). Instantons as absolute minima of the action at fixed n. Instantons for $G = SU(2)$ relative to $n = 1$. [*Moduli* as collective coordinates. The case $G = SU(N)$]. Non-perturbative character of the weighting factor $e^{-8\pi^2 n/g^2}$. ϑ-vacua and *strong CP* problem. [The *axial* $U(1)$ problem]. [W2: 23.4, 23.5 23.6], [VV].

Main textbooks:

- C. Itzykson, J.-B. Zuber, *Quantum Field Theory*, New York, McGraw-Hill Book Co, 1987.
- S. Weinberg, *The Quantum Theory of Fields II - Modern applications*, Cambridge, Cambridge

University Press, 2005.

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[CL] T.-P. Cheng, L.-F. Li, *Gauge theory of elementary particle physics.*

[EF] J. Earman, D. Fraser, *Haag's theorem and its implications for the foundations of quantum field theories*, Erkenntnis **64** (2006) 305, http://philsci-archive.pitt.edu/id/eprint/2673.

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[R1] L.H. Ryder, *Quantum Field Theory*, Cambridge, Cambridge University Press, 1996. Second edition.

[R2] P. Ramond, *Field theory: a modern primer*, Boulder Colorado, Westview Press, 1997. Second edition.

[S1] M.F. Sohnius, *Introducing Supersymmetry*, Phys. Rep. **128** (1985) 39.

[S2] M. Srednicki, *Quantum Field Theory*, Cambridge, Cambridge University Press, 2007.

[S3] M. Slansky, *Group theory for unified model building*, Phys. Rep. **79** (1981) 1.

[S4] F. Strocchi, *Selected topics on the general properties of Quantum Field Theory.*

[S5] F. Strocchi, *Elements of Quantum Mechanics of infinite systems.*

[SW] R.F. Streater, A.S. Wightman, *PCT, spin and statistics, and all that*, Reading, Benjamin-Cummings, 1980. Third editon.

[VV] S. Vandoren, P. van Nieuwenhuizen, *Lectures on Instantons*, arXiv:0802.1862 [hep-th].

[WI] S. Weinberg, *The Quantum Theory of Fields I - Foundations*, Cambridge, Cambridge University Press, 2005.

[WII] S. Weinberg, *The Quantum Theory of Fields II - Modern applications*, Cambridge, Cambridge University Press, 2005.

[ZI] E. Zeidler, *Quantum Field Theory I - Basics in Mathematics and Physics*, Berlin, Springer-Verlag, 2005.