



**Dipartimento di Fisica e Astronomia "Galileo Galilei" - DFA**  
**Anno Accademico 2018/19**  
**Registro lezioni del docente LECHNER KURT**

**Attività didattica**

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**ADVANCED QUANTUM FIELD THEORY [SCP7081759]**

**Corso di studio: PHYSICS [SC2382]**

**Periodo di svolgimento: Primo Semestre**

**Docente titolare del corso: LECHNER KURT matr. 004893**

**Riepilogo registro docente:**

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**LECHNER KURT matr. 004893**

Docente interno - Professori Associati

Insegnamento attribuito per: COMPITO ISTITUZIONALE GRATUITO

**Stato registro docente: Bozza**

**Studenti presenti nella 1 settimana: 15 , in media 10**

**Ore inserite: 52 ore**

**Ore previste dall'offerta didattica: 48 ore**

**Gruppi di studenti con i quali è stata svolta l'attività - ore per gruppo**

- prevista per tutti gli studenti (senza gruppi associati) - 52 ore

**Ore inserite per tipologia di attività**

52 ore lezione :

- prevista per tutti gli studenti (senza gruppi associati) - 52 ore

**Osservazioni:**

Firma del docente del corso:.....

Data:.....



**Dettaglio delle attività svolte:**

**ADVANCED QUANTUM FIELD THEORY [SCP7081759]**

**01/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Introduction

**Descrizione estesa:**

1) Aim of the course. Different methods of formulating perturbative quantum field theories: canonical quantization and functional integral. Haag's theorem. Dyson's argument for the divergence of the perturbative series of QED. Failure of Borel summability. Asymptotic series. 2) Non-perturbative analysis and lattice quantization. Triviality of  $\lambda\phi^4$  theory. Wightman's axiomatic approach and the reconstruction theorem. From Schwinger's SU(2) model for electroweak unification to 't Hooft's proof of the renormalizability of non-abelian gauge theories.

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**02/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Symmetries in relativistic quantum field theories

**Descrizione estesa:**

1) Lie groups and Lie algebras of symmetries in classical theories, and their implementation in quantum field theory. Unitary representations of symmetry groups. 2) Coleman-Mandula theorem and internal symmetries. Irreducible representations of the Lorentz group, particles, spin, and physical degrees of freedom. Consistency of relativistic quantum field theories for particles with different spin.

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**08/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Classical field theories. Lie groups and Lie algebras

**Descrizione estesa:**

1) General properties of Lagrangians of classical field theories. Spinors and scalars. Yukawa couplings. Chirality and Majorana fermions. 2) Compact and simple Lie groups and Lie algebras of matrices. Structure constants and Jacobi identity. Unitary representations. Canonical form of the Killing form. Completely antisymmetric structure constants.

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**09/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Tensor calculus for Lie groups

**Descrizione estesa:**

1) Adjoint representation and its properties. Casimir and Dynkin invariants. Examples. BCH formula. Conjugate representation. 2) Tensor products of representations and their reduction. Contractions of tensors. Invariant tensors and number of singlets in tensor products of representations.

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**15/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Applications of the tensor calculus for Lie groups. Non abelian gauge theories

**Descrizione estesa:**

1) The rules of the Lie-group tensor calculus. Invariant products of spinors and vectors for the Lorentz group. Majorana mass. A fundamental property of the adjoint representation. Invariant tensors from traces of generators of the Lie algebra. Examples of invariant polynomials of matter fields. 2) Generic Lagrangian invariant under global non-abelian gauge transformations of a group  $G$ . Local gauge transformations. YM connection and covariant derivatives. Gauge transformation of the YM connection: consistency and universality.

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**16/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

YM fields and general gauge invariant Lagrangians

**Descrizione estesa:**

1) Properties and universality of the YM vector potentials. YM field strength and its properties. 2) Generic Lagrangians invariant under local gauge transformations. YM action. Compact and non-compact gauge groups. Non-simple gauge groups. Chiral gauge couplings. Equations of motion and color Noether currents.

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**22/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Color conservation. Functional integral and background field method

**Descrizione estesa:**

1) Conserved color currents and covariant color currents. Color charge of gauge fields and their self-interaction. Comparison with General Relativity. The functionals  $Z$ ,  $W$  and  $\Gamma$ . Euclidean space. 2) Green functions in Fourier transform. Conventions. The background field method as a self-consistent reorganization of the perturbative expansion.

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**23/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Symmetries of quantum field theories in the functional integral approach

**Descrizione estesa:**

1) Transfer of linear infinitesimal quasi-symmetries of classical actions to the vertex functional  $\Gamma$ . Generalization to finite and discrete symmetries. Furry's theorem. Chiral protection of the mass terms for fermions. Ward identity in QED for  $\Gamma$ . 2) Transversality of the exact photon propagator: unitarity, vanishing of the photon mass, current conservation, absence of quantum corrections to the gauge fixing. Renormalizability of QED in compatibility with gauge invariance.

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**29/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

lezione non tenuta per allerta meteo

**Descrizione estesa:**

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**30/10/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

lezione non tenuta per allerta meteo

**Descrizione estesa:**

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**05/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Functional determinants

**Descrizione estesa:**

1) Functional determinants for commuting real and complex scalar fields, and for anticommuting spinor fields. Partition function in the presence of an external electromagnetic field for spinor and scalar QED. The Dirac determinant. 2) Determinants for anticommuting real scalar fields. One-loop effective action as a functional determinant. Application to the Coleman-Weinberg mechanism (to be continued).

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**06/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Coleman-Weinberg mechanism. Feynman rules for a generic QFT

**Descrizione estesa:**

1) Computation of the one-loop effective action in  $\lambda\phi^4$  for a constant field via the trace of the logarithm of a quadratic operator. Renormalization via differentiation. Derivation of the Coleman-Weinberg potential and physical consequences: radiative SSB and dimensional transmutation. 2) Derivation of the Feynman rules for a generic renormalizable QFT. Application to scalar QED. Power-counting renormalizable and strictly renormalizable QFT's.

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**12/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Renormalizability of a generic QFT. Locality of counterterms

**Descrizione estesa:**

1) Four-scalar correlation function in SQED: example of a non-strictly renormalizable QFT. Theorem on the locality of the divergent part of correlation functions at higher loops. 2) Proof of the theorem at one-loop. Determination of the degree of superficial divergence of a correlation function in a generic local QFT in  $D$  dimensions. Renormalizable, super-renormalizable, and non-renormalizable theories. Examples.

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**13/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Perturbative renormalizability versus non-perturbative formulations. Lambda-phi-3 in D=6 as a laboratory.

**Descrizione estesa:**

Renormalizability of QFT's in dimensions  $D = 3, 4, 6$ . Non-perturbative aspects and Gross-Neveu model. Lambda-phi-3 theory in  $D = 6$ . General organization of the renormalization process in dimensional regularization. 2) One-loop renormalization: explicit computation of  $Z$ ,  $Z_1$  and  $\delta m^2$ .

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**19/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Renormalization group in lambda-phi-3. Higher order renormalization and locality

**Descrizione estesa:**

1) Lambda-phi-3 theory in  $D=6$ : beta function and anomalous dimension and their physical meaning, running coupling constant and dimensional transmutation, asymptotic freedom, solution of the Callan-Symanzik equation, resummation of leading logs, limits of the perturbative approach. 2) Two-loop renormalization of the theory: proof of locality of two-loop divergences, nested and overlapping sub-divergences of two- and three-point functions. Renormalizability via differentiation.

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**20/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Quantization of YM theories: main problems and quantization from first principles

**Descrizione estesa:**

Quantization of YM theories. Local gauge invariance and non-physical degrees of freedom. Non-perturbative aspects, heuristic factorization of the infinite volume of the gauge group. Perturbative aspects, non-invertible kinetic operator of the YM potential. 2) Classical gauge fixing versus preservation of gauge invariance at the quantum level. Covariant and non-covariant gauge fixings. Preservation of the Gupta-Bleuler condition during time evolution in QED. Quantization of YM theories from first principles in axial gauge.

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**26/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Faddeev-Popov quantization of YM theories

**Descrizione estesa:**

1) Haar measure for compact Lie groups. Gauge invariance of the functional measure over matter fields. Fundamental delta-function identity for a generic gauge-fixing function. FP-determinant. 2) Non-gauge invariance of the FP-determinant. Correlation functions of gauge invariant operators for a generic gauge-fixing and for a generic weighting function. Weinberg's theorem for the universality of these correlation functions. Axial gauge and comparison with the correlation functions derived from first principles. Lorenz-gauge. Triviality of the FP-determinant in abelian gauge theories for linear gauge fixings.

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**27/11/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Ghost fields and BRST symmetry

**Descrizione estesa:**

1) The FP-determinant as an integral over ghost fields. Ghost Lagrangian and gauge-fixing Lagrangian. Ghost number conservation. Lorenz gauge and shift symmetry. Ghost fields and the spin-statistics theorem. NL-formalism. 2) Violation of gauge-invariance of the FP-Lagrangian. BRST symmetry: anticommuting transformation parameter, transformation of the fields and their nilpotency, BRST invariance of the total Lagrangian, the FP-action as a BRST-exact action.

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**03/12/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Gauge-independence of physical correlation functions and BRST charge

**Descrizione estesa:**

1) Comments on BRST symmetry: full set of quantum Lagrangian fields, non-linearity of the transformations, reduced BRST transformations without Nakanishi-Lautrup fields. Gauge-fixing dependence of generic correlation functions. Indefinite-norm Hilbert space and ghost states. Gauge-fixing-independence of correlations functions of physical operators. The BRST charge  $Q$ . 2) Nilpotency of  $Q$ . The physical Hilbert space as the cohomology of  $Q$ . Statement of the Kugo-Ojima theorem.

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**04/12/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Kugo-Ojima construction of the physical Hilbert space

**Descrizione estesa:**

1) Asymptotic LSZ-limit in QFT. Canonical quantization and Fock space of the asymptotic theory. Commutation relations between the BRST charge  $Q$  and the creation and annihilation operators. Transverse, longitudinal, and temporal gluons. 2) General solution of the K-O condition. The cohomology of  $Q$  is constituted by transverse gluons and quarks; ghosts and non-physical gluons are  $Q$ -exact states. Explicit form of  $Q$  from the Noether theorem. Relation with the Gupta-Bleuler condition.

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**10/12/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Slavnov-Taylor identity

**Descrizione estesa:**

1) The problem of strict renormalizability of the FP-gauge-fixed action and of the recovering of BRST-symmetry in the functional-integral quantization. The external currents  $K$  and the ST-identity for the extended action  $S$  and for the 1PI functional  $\Gamma$ . Explicit form of  $S$ . Total derivatives in the functional integral approach. 2) Reduced ST-identities for  $S$  tilde and for  $\Gamma$  tilde. The gauge-fixing term receives no quantum corrections. Factorization of colored correlation functions in terms of  $G$ -invariant tensors.

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**11/12/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Slavnov-Taylor identity and renormalizability of YM theories

**Descrizione estesa:**

1) Proof that the exact gluon two-point function is transverse using the ST-identities. UV-divergences of the extended 1PI effective action  $\Gamma$ . General solution of the ST-identity  $S^*S=0$ .  $S = B + \Delta$  times  $K$ . 2) General solution of the equation  $S^*F=0$ . Multiplicative renormalization of the ST-identity for  $\Gamma$  at one and two loops; iteration at higher loops.  $\Gamma_R * \Gamma_R=0$ .

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**17/12/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 09:30

**Ora fine:** 11:15

**Ore accademiche:** 2

**Argomento:**

Explicit one-loop renormalization of YM theories (1)

**Descrizione estesa:**

1) YM theories are strictly renormalizable, as consequence of the ST identity for Gamma. ST identities for the renormalization constants Z. Feynman rules for YM theories. 2) General procedure for the perturbative renormalization of a correlation function. Gluon two-point function: explicit evaluation of the diagram with a gluon loop. Vanishing of massless tadpoles.

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**18/12/2018 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Explicit one-loop renormalization of YM theories (2)

**Descrizione estesa:**

1) Renormalization of the gluon two-point function: ghost loop, fermion loop, transversality of the correlation function. Determination of  $Z_3$ . 2) Renormalization of the fermion two-point function and of the fermion-fermion-gluon vertex. Determination of  $Z_\psi$  and  $Z_{1\psi}$ . Derivation of the beta function.

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**08/01/2019 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 10:30

**Ora fine:** 12:15

**Ore accademiche:** 2

**Argomento:**

Beta functions, asymptotic freedom, coupling constant unification in GUT theories

**Descrizione estesa:**

1) Beta function in YM theories with fermions and running coupling constant. Asymptotic freedom, Lambda-QCD, deep inelastic scattering. Behavior of running coupling constants in QED, QCD,  $\lambda_\phi=3$ ,  $\lambda_\phi=4$ . 2) Remarks on finiteness of SYM theory. SU(5) GUT and SSB to the SM-gauge group. Normalization of the hypercharge generator. The three beta functions of the SM-gauge group and missing unification of the coupling constants. Beta functions of the MSSM and unification of the coupling constants at the GUT scale.

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**09/01/2019 - lezione -**

**Docente:** LECHNER KURT

**Ora inizio:** 15:30

**Ora fine:** 17:15

**Ore accademiche:** 2

**Argomento:**

Instantons

**Descrizione estesa:**

1) Perturbative approach and field configurations continuously connected with the trivial minimum of the action. The Pontryagin invariant  $n$  classifies disconnected sectors of YM potentials. Chern-Simons form. Asymptotic form of the potentials and the maps from  $S^3$  to  $G$ . Homotopy groups. 2) Third homotopy group of a compact, connected, and simple Lie group. Instantons as the minima of the action for fixed  $n$ : self-duality equation and YM equation.  $SU(2)$  instantons. Non-analyticity and non-perturbative aspects of instanton contributions. Theta vacua and the strong CP problem.

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