1) dal teorema di Gauß

19/11/2001

$$E(r) = \begin{cases} \dfrac{\rho r}{2\varepsilon_0}, & r < R \\[2ex] \dfrac{\rho R^2}{2\varepsilon_0 r}, & r > R \end{cases}$$

$$V(r) = -\frac{\rho R^2}{2\varepsilon_0} \ln r, \quad r > R$$

a) $$V(P_1) - V(P_2) = -\frac{\rho R^2}{2\varepsilon_0}\left[\ln a - \ln 2a\right] = \frac{\rho R^2}{2\varepsilon_0}\ln 2 \Rightarrow \boxed{\rho = \frac{2\varepsilon_0 \Delta}{R^2 \ln 2}}$$

b) $$E_{int}(r) = \frac{\rho r}{2\varepsilon_0} = \frac{\Delta}{R^2 \ln 2} \cdot r$$

c) $$\left.\begin{aligned} F_{est} &= F_{centripeta} \\ Q E_{est}(r) &= m\frac{v^2}{r} \end{aligned}\right\} \quad Q \cdot \frac{\rho R^2}{2\varepsilon_0 \cancel{r}} = m\frac{v^2}{\cancel{r}} \Rightarrow \boxed{Q = \frac{2\varepsilon_0 m v^2}{\rho R^2}} = \frac{\ln 2 \cdot m v^2}{\Delta}$$

(negativa)

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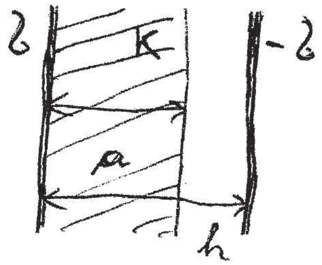
2)



$$D = b; \quad E(x) = \frac{1}{\varepsilon_0 K(x)} D$$

$$\boxed{\begin{aligned} a &= \tfrac{2}{3} h \\ K &= 3 \end{aligned}}$$

a) $$E = \begin{cases} \dfrac{b}{K\varepsilon_0} & x < a \\[2ex] \dfrac{b}{\varepsilon_0} & a < x < h \end{cases}$$

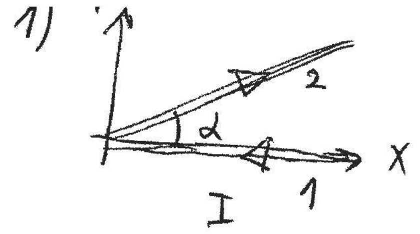
b) $$\Delta V = \int E\,dx = \frac{b}{K\varepsilon_0} a + \frac{b}{\varepsilon_0}(h - a) = \frac{5}{9}\frac{b}{\varepsilon_0} \cdot h$$

$$C' = \frac{Q}{\Delta V} = \frac{b \cdot A}{\Delta V} = \boxed{\frac{9}{5}\frac{\varepsilon_0 A}{h}} = \frac{9}{5} C$$

c) $$f = \frac{\frac{1}{2}\frac{Q^2}{C'} - \frac{1}{2}\frac{Q^2}{C}}{\frac{1}{2}\frac{Q^2}{C}} = \boxed{-\frac{4}{9}}$$

11/01/200

1)



a) $\vec{B}_1 = \frac{\mu_0 I}{4\pi b}\vec{u}_y$, $\vec{B}_2 = \frac{\mu_0 I}{4\pi b}\left(\operatorname{sen}\alpha\,\vec{u}_x - \cos\alpha\,\vec{u}_y\right)$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi b}\left(\operatorname{sen}\alpha\,\vec{u}_x + (1-\cos\alpha)\,\vec{u}_y\right)$$

$$\operatorname{tg}\varphi = \frac{B_y}{B_x} = \frac{1-\cos\alpha}{\operatorname{sen}\alpha} = \operatorname{tg}\frac{\alpha}{2} \Rightarrow \boxed{\alpha = 2\varphi = \frac{\pi}{4}}$$

b) $|\vec{B}| = \frac{\mu_0 I}{4\pi b}\sqrt{\operatorname{sen}^2\alpha + (1-\cos\alpha)^2} = \frac{\mu_0 I}{2\pi b}\operatorname{sen}\frac{\alpha}{2}$

c) $\Delta U = \mu B - (-\mu B) = \boxed{2\mu|\vec{B}|}$

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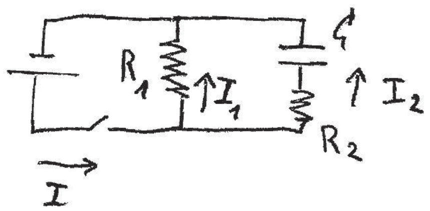
2)



a) $I(0) = V_0\,\frac{R_1 + R_2}{R_1 \cdot R_2}$

b) $V_0 = R_2 I_2 + \frac{Q}{C}$; $I_2 = \frac{dQ}{dt} \Rightarrow \frac{dQ}{dt} + \frac{Q}{R_2 C} = \frac{V_0}{R_2}$ con $\underline{\underline{Q(0) = 0}}$, $(\tau \equiv R_2 C)$

Soluzione: $Q(t) = C V_0\left(1 - e^{-t/\tau}\right)$

$$I_2(t) = \frac{dQ}{dt} = \frac{V_0}{R_2}e^{-t/\tau}$$

$$I_2(t_1) = \frac{V_0}{R_2}e^{-t_1/\tau} = \frac{V_0}{4R_2} \Rightarrow \tau = \frac{t_1}{\ln 4} \text{ e } \boxed{C = \frac{\tau}{R_2} = \frac{t_1}{R_2\ln 4}}$$

c) $I_1(t) = \frac{V_0}{R}$; $I(t) = I_1(t) + I_2(t)$

1) 

a) $\vec{\mu} = I \frac{\sqrt{3}}{4} L^2 \vec{u}_z = \mu \vec{u}_z$

29/01/2002

b) $\underline{\underline{Q=(0,0,b)}}$; $B_x = B_y = 0$

$$B_z = -\frac{\mu_0 \mu}{4\pi}\left(\frac{1}{r^3} - \frac{3z^2}{r^5}\right) = -\frac{\mu_0 \mu}{4\pi}\left(\frac{1}{b^3} - \frac{3b^2}{b^5}\right) = \frac{\mu_0 \mu}{2\pi b^3}$$

Formule per il campo magnetico creato da un dipolo $\vec{\mu} \parallel z$.

c) $\underline{\underline{P=(c_1, c_2, 0)}}$; $B_x = B_y = 0$

$$B_z = -\frac{\mu_0 \mu}{4\pi} \frac{1}{(c_1^2 + c_2^2)^{3/2}}$$

$$m\vec{a} = q\vec{v} \times \vec{B} = q(v_y \vec{u}_y) \times (B_z \vec{u}_z) = q v_y B_z \vec{u}_x$$

$$v_y = \frac{ma}{q|B_z|} = \frac{ma\, 4\pi (c_1^2 + c_2^2)^{3/2}}{q \mu_0 \mu}$$

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2) a) $\vec{\nabla} \times \vec{E} = 0$

$$(\vec{\nabla} \times \vec{E})_z = \partial_x E_y - \partial_y E_x = -\frac{b}{x^2 y^3 z^2} + \frac{2a}{x^2 y^3 z^2} = \frac{(2a - b)}{x^2 y^3 z^2} = 0 \Rightarrow b = 2a \text{ etc.}$$

$$\boxed{\begin{matrix} b = 2a \\ c = 2a \end{matrix}}$$

b) $\vec{E} = -\vec{\nabla} V \Rightarrow V(\vec{x}) = \frac{a}{x y^2 z^2} + d$; $V(1,1,1) = 2 \Rightarrow d = 1\,V$ [$a = 1\,V m^5$

$$\boxed{V(x,y,z) = \frac{1\,V m^5}{x y^2 z^2} + 1\,V}$$

26/02/2002

1) a) $\left.\begin{aligned} Q_1 + Q_2 &= Q_0 \\ \frac{Q_1}{C_1} &= \frac{Q_2}{C_2} \end{aligned}\right\} \quad Q_1 = \frac{C_1}{C_1 + C_2} Q_0 \, ; \quad Q_2 = \frac{C_2}{C_1 + C_2} Q_0$

$U_E = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$

b) $\Delta E = \frac{1}{2} \frac{Q_0^2}{C_1} - \left(\frac{1}{2} \frac{Q_1^2}{C_1} + \frac{1}{2} \frac{Q_2^2}{C_2} \right) = \frac{1}{2} \frac{C_2}{C_1 (C_1 + C_2)} Q_0^2$

$V = \frac{Q}{C}$

c) $Q_1(t) = Q_1 e^{-t/RC_1}$

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2) a) Le traiettorie sono eliche con assi paralleli all'asse del solenoide, ed a raggio e passo costanti.

b) $B = \mu_0 n I$ (parallelo all'asse)

$v_T = v \,\mathrm{sen}\,\varphi = \frac{v}{2}$

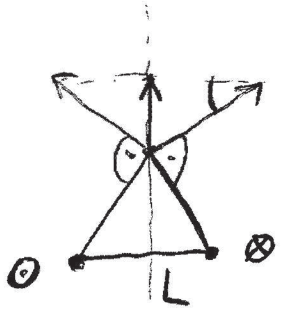
Raggio di ciclotrone: $R = \frac{m v_T}{e B} = \frac{m v}{2 e B} \Rightarrow \boxed{v_{MAX} = \frac{2 e B R}{m}}$

c) $e E = e v_0 B \Rightarrow B = \frac{E}{v_0} = \mu_0 n I \Rightarrow \boxed{I = \frac{E}{v_0 \mu_0 n}}$

25/06/2002

MOD A

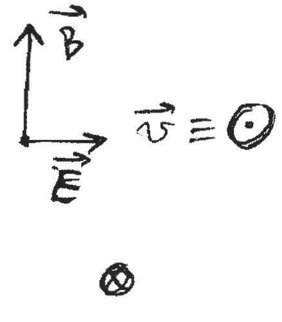
1)



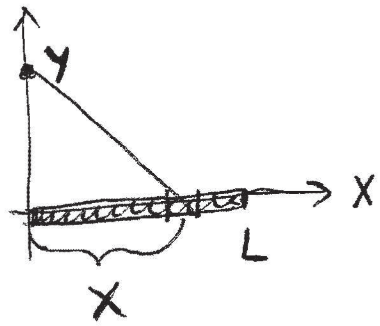
a) $B = \frac{\mu_0 I}{2\pi L} (2 \cdot \operatorname{sen} 30°)$

$= \frac{\mu_0 I}{2\pi L}$

b) $E = v B$



2)



a) $V(Y) = \frac{1}{4\pi\varepsilon_0} \int_0^L \frac{\lambda(X)\, dX}{\sqrt{X^2+Y^2}}$

$= \frac{k}{4\pi\varepsilon_0} \left(\sqrt{Y^2+L^2} - |Y| \right)$

b) $E_y = -V'(Y) = \frac{k}{4\pi\varepsilon_0} \left(\operatorname{sgn}(Y) - \frac{Y}{\sqrt{Y^2+L^2}} \right)$

c) $\mathcal{E} = T - |Q|\, V(Y) = \text{cost.} = T_\infty = \frac{1}{2} m v_\infty^2 \geq 0$

Valutando nell'origine: $T_0 - |Q|\, V(0) \geq 0$

$$T_0 \geq \frac{|Q|\, k L}{4\pi\varepsilon_0}$$

[illegible]

a) $Q = 4\pi \int_0^R r^2 \rho(r)\, dr = 4\pi A \dfrac{R^{\alpha+3}}{\alpha+3}$

$$\underline{\underline{A = \frac{(\alpha+3)\, Q}{4\pi R^{\alpha+3}}}}$$

b) Per $r < R$, Gauß: $\displaystyle\int_r \vec{E}\cdot d\vec{\Sigma} = \frac{Q_r}{\varepsilon_0}$

$$4\pi r^2 E(r) = \frac{1}{\varepsilon_0}\, 4\pi A\, \frac{r^{\alpha+3}}{\alpha+3}$$

$$E(r) = \frac{A}{\varepsilon_0}\, \frac{r^{\alpha+1}}{\alpha+3}$$

$$\Rightarrow \boxed{\alpha = -1}$$

$A = \dfrac{Q}{2\pi R^2}$;

$$E(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2} & r > R \\[2ex] \dfrac{Q}{4\pi\varepsilon_0 R^2} & r < R \end{cases}$$

c) $$V(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r} & r > R \\[2ex] -\dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{r}{R^2} - \dfrac{2}{R}\right) & r < R \end{cases}$$

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2) a) $$B(x) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mu_0 (n I\, ds)}{2}\, \frac{R^2}{\left((x-s)^2 + R^2\right)^{3/2}}$$

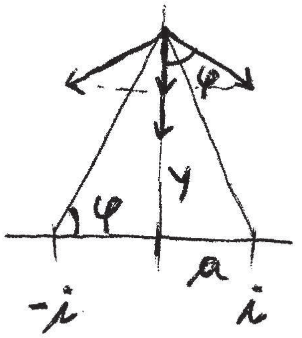
$$= \frac{\mu_0 n I}{2} R^2 \int_{-\frac{L}{2}-x}^{\frac{L}{2}-x} \frac{dt}{(t^2+R^2)^{3/2}} = \frac{\mu_0 n I}{2} \int_{-\frac{\frac{L}{2}+x}{R}}^{\frac{\frac{L}{2}-x}{R}} \frac{dz}{(1+z^2)^{3/2}}$$

$$= \frac{\mu_0 n I}{2} \left[\frac{\frac{L}{2} - x}{\sqrt{R^2 + \left(\frac{L}{2} - x\right)^2}} + \frac{\frac{L}{2} + x}{\sqrt{R^2 + \left(\frac{L}{2} + x\right)^2}} \right]$$

b) $\lim_{L\to\infty} B(x) = \mu_0 n I$ (campo di un solenoide infinito)

Soluzione 27/04

1) a)

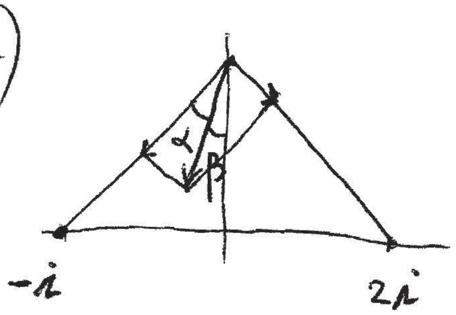


$$B = 2\left(\frac{\mu_0 i}{2\pi\sqrt{a^2+y^2}}\right)\cos\varphi$$

$$\cos\varphi = \frac{a}{\sqrt{a^2+y^2}}$$

$$\underline{\underline{B = \frac{\mu_0 a i}{\pi(a^2+y^2)}}}$$

b)



$$\operatorname{sen}\alpha = \frac{1}{\sqrt{5}}$$

$$\operatorname{sen}\beta = \operatorname{sen}\left(\frac{\pi}{4}-\alpha\right) = \frac{1}{\sqrt{2}}(\cos\alpha - \operatorname{sen}\alpha) = \underline{\underline{\frac{1}{\sqrt{10}}}}$$

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o

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2) a)

$$\left.\begin{aligned} i_3 &= i_1 + i_2 \\ \mathcal{E}_1 - \mathcal{E}_2 &= R(i_2 + i_3) \\ \mathcal{E}_2 &= 2 i_1 R - i_2 R \end{aligned}\right\} \qquad \begin{aligned} i_1 &= \frac{\mathcal{E}_1 + \mathcal{E}_2}{5R} \\ i_2 &= \frac{2\mathcal{E}_1 - 3\mathcal{E}_2}{5R} \\ i_3 &= \frac{3\mathcal{E}_1 - 2\mathcal{E}_2}{5R} \end{aligned}$$

b)

$$\left.\begin{aligned} P_1 &= \mathcal{E}_1 i_3 = \frac{\mathcal{E}_1(3\mathcal{E}_1 - 2\mathcal{E}_2)}{5R} \geq 0 \\ P_2 &= -\mathcal{E}_2 i_2 = \frac{\mathcal{E}_2(3\mathcal{E}_2 - 2\mathcal{E}_1)}{5R} \geq 0 \end{aligned}\right\} \qquad \underline{\underline{\frac{2}{3}\mathcal{E}_2 \leq \mathcal{E}_1 \leq \frac{3}{2}\mathcal{E}_2}}$$

c)

$$V_B - V_A = R(i_3 + i_1) = 0 \implies \underline{\underline{\mathcal{E}_2 = 4\mathcal{E}_1}}$$