compitino 22/04/2002

1) I fluisce in verso orario.

$|\mathcal{E}| = \dot{\phi} = v\ell B$ $\quad \begin{cases} v\ell B = RI + Q/C & (1) \\ m\dot{v} = -I\ell B & (2) \end{cases}$ $\quad Q(0) = 0\,;\ v(0) = v_0$ $\quad I = \dfrac{dQ}{dt}$

$F_{magn} = -I\ell B$

a) $I(0) = \dfrac{v_0 \ell B}{R}\,;\quad a(0) = -\dfrac{\ell B I(0)}{m} = -\dfrac{(\ell B)^2 v_0}{Rm}$

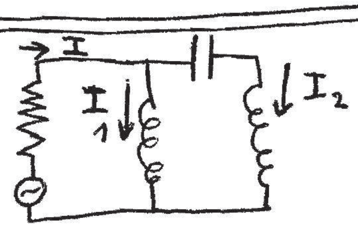
b) Derivando (1) e inserendo (2): $\quad \dot{I} + \dfrac{m + (\ell B)^2 C}{mRC} I = 0$

$$I(t) = I(0)e^{-t/\tau}\,;\quad \tau = \frac{mRC}{m + (\ell B)^2 C}$$

c) $\dot{v} = -\dfrac{\ell B}{m} I = -\dfrac{\ell B}{m} I(0) e^{-t/\tau} \Rightarrow$

$$v(t) = \frac{\ell B \tau}{m} I(0)\left[e^{-t/\tau} - 1\right] + v_0 \xrightarrow[t\to\infty]{} -\frac{\ell B \tau I(0)}{m} + v_0 = \overbrace{\frac{m v_0}{m + (\ell B)^2 C}}^{v_\infty}$$

Da (1): $Q_\infty = C\ell B v_\infty = \dfrac{m v_0 C \ell B}{m + (\ell B)^2 C}\,;$ $\quad \left[\text{Oppure: } Q_\infty = \displaystyle\int_0^\infty I(t)\,dt = I(0)\tau\right]$

2) 

$$Z = R + \frac{i\omega L\left(i\omega L + \frac{1}{i\omega C}\right)}{i\omega L + \left(i\omega L + \frac{1}{i\omega C}\right)} = R + i\,\frac{\omega^2 LC - 1}{2\omega^2 LC - 1}\,\omega$$

a) $\operatorname{Im} Z = 0 \Rightarrow \omega_0 = \dfrac{1}{\sqrt{LC}}$

b) $(i\omega L) I_1 = \left(i\omega L + \dfrac{1}{i\omega C}\right) I_2 \Rightarrow \dfrac{I_1}{I_2} = 1 - \dfrac{1}{\omega^2 LC} = \dfrac{1}{2} \Rightarrow i_1 = \dfrac{1}{2} i_2$ $\quad [i = \mathrm{Re}$

$$f = \frac{\frac{1}{2}L\langle i_1^2\rangle}{\frac{1}{2}L\langle i_2^2\rangle} = \frac{\langle i_1^2\rangle}{\langle i_2^2\rangle} = \frac{1}{4}\,;\quad I_1 + I_2 = I \Rightarrow I_2 = \frac{2}{3} I = \frac{2}{3}\frac{\mathcal{E}}{|Z|} e^{i(\omega t -}$$

c) $i_2 = \mathrm{Re}\, I_2 = \dfrac{2}{3}\dfrac{\mathcal{E}}{|Z|}\cos(\omega t - \varphi) = \dfrac{d}{dt} Q_2(t) \Rightarrow Q_2(t) = \overbrace{\dfrac{2\mathcal{E}}{3\omega|Z|}}^{Q_{MAX}} \sin(\omega t - \varphi) \Rightarrow Q_{MAX} = \sqrt{\dfrac{2L}{2L + 9R^2C}}$

Soluzione

12/06/2002

a) $\operatorname{sen}\vartheta_{MIN} = \frac{\lambda}{a} K = 0{,}75 K \Rightarrow \underline{\underline{\operatorname{sen}\vartheta_{MIN} = \frac{3}{4}}} \quad (K=1)$

b) $\operatorname{sen}\vartheta_{MAX} = \frac{\lambda}{d} k = 0{,}15 k \quad (k = 0,1,2,3,4,\cancel{5},6)$ $\quad$ $\underline{\underline{6 \text{ massimi}}}$

c) $$I(\vartheta_{MAX}) = I_0 N^2 \left(\frac{\operatorname{sen}\left(\frac{\pi a \operatorname{sen}\vartheta_{MAX}}{\lambda}\right)}{\frac{\pi a \operatorname{sen}\vartheta_{MAX}}{\lambda}} \right)^2 = I_0 N^2 \left(\frac{\operatorname{sen} \pi k/5}{\pi k/5} \right)^2$$

$$\underline{\underline{\frac{I(\vartheta_6)}{I(\vartheta_0)} = \left(\frac{\operatorname{sen} 6\pi/5}{6\pi/5} \right)^2}} \approx 5\%$$

[NB: $k=4$ è più luminoso di $k=6$]

d) $\lambda \to \bar{\lambda} = \frac{\lambda}{n}$; $\operatorname{sen}\vartheta_{MIN} = \frac{\bar{\lambda}}{a} K = 0{,}625 K = 0{,}625$

$\operatorname{sen}\vartheta_{MAX} = \frac{\bar{\lambda}}{d} k = 0{,}125 k \quad (k = 0,1,2,3,4,\cancel{5},6,7,(8))$

$\underline{\underline{8 \text{ massimi}}}$ (7 effettivi)

2) a) $$\left.\begin{aligned} 2nd &= \left(N + \tfrac{1}{2}\right)\lambda_1 \\ 2nd &= (N+1)\lambda_2 \end{aligned}\right\}$$ Uguagliando: $N = \dfrac{\lambda_2 - \frac{1}{2}\lambda_1}{\lambda_1 - \lambda_2} = 7$

$\boxed{n = 1{,}33 = \frac{4}{3}}$

$\underline{\underline{d = 3\lambda_2 = 1{,}35\ \mu m = 1.350\ nm}}$

b) $2nd = \left(\tilde{N} + \frac{1}{2}\right)\lambda_{MAX} \Rightarrow \lambda_{MAX} = \dfrac{2nd}{\tilde{N} + \frac{1}{2}} = \dfrac{3.600\ nm}{\tilde{N} + \frac{1}{2}}$

$\lambda_{MAX} \leq 800\ nm$, quindi $\tilde{N} = 4$ e $\underline{\underline{\lambda_{MAX} = 800\ nm}}$

23/06/200[illegible]
MOD B

1) a)
$$\left.\begin{array}{l} M\dot{v} = Mg - IaB \\ I = \frac{1}{R}\dot{\phi} = \frac{aBv}{R} \end{array}\right\} \Rightarrow \dot{v} + \frac{v}{\tau} = g$$

$$\tau = \frac{MR}{(aB)^2}$$

b)
$$v(t) = g\tau\left(1 - e^{-t/\tau}\right) \quad ; \quad I(t) = \frac{aB}{R} v(t)$$

c)
$$Q = \int_0^\tau I(t)\,dt = \frac{aB}{R} g\tau \int_0^\tau \left(1 - e^{-t/\tau}\right) dt = \frac{aBg}{eR}\tau^2$$

d)
$$I_\infty = \frac{aB}{R} v_\infty = \frac{aB}{R} g\tau = \frac{Mg}{aB}$$

$$P_\infty = R I_\infty^2 = \left(\frac{Mg}{aB}\right)^2 R \; \left[= Mg\, v_\infty\right]$$

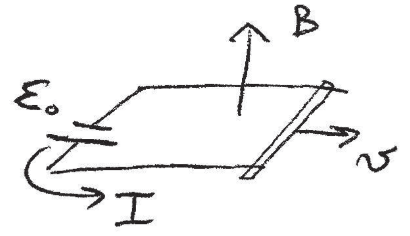
2) a) Risoluzione di due righe (con $n=1$): $\frac{\Delta\lambda}{\lambda} \geq \frac{1}{N}$

$$N \geq \frac{\lambda}{\lambda_2 - \lambda_1} = N_{MIN} = 3645$$

$$d_M = \frac{L}{N_{MIN}} = 13{,}72\,\mu m$$

b)
$$\operatorname{sen}\vartheta_{MAX} = n\frac{\lambda}{d_M} = n \cdot 0{,}048 \leq 1 \Rightarrow n_{MAX} = 20$$

Compito 12/07/02



1) $\mathcal{E}_0 - vBL = RI$

$m\dot{v} = IBL$

a) $v(0) = 0 \Rightarrow I(0) = \dfrac{\mathcal{E}_0}{R}$

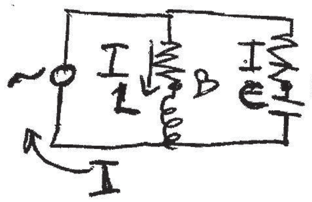
b) $\dot{v} + \dfrac{1}{\tau} v = \dfrac{BL\mathcal{E}_0}{mR}$; $\tau \equiv \dfrac{mR}{(BL)^2}$

$v(t) = \dfrac{\mathcal{E}_0}{BL}\left(1 - e^{-t/\tau}\right)$; $I(t) = \dfrac{\mathcal{E}_0}{R} e^{-t/\tau}$

c) $v_\infty = \dfrac{\mathcal{E}_0}{BL}$

$I_\infty = 0$

d) $Q = \displaystyle\int_0^\infty I(t)\,dt = \frac{\mathcal{E}_0}{R}\tau = \frac{m\mathcal{E}_0}{(BL)^2}$



$\omega^2 = \dfrac{1}{LC}$

$Z_1 = R + i\omega L$

$Z_2 = R + \dfrac{1}{i\omega C}$

2) a) $Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2} = \dfrac{1}{2}\left(R + \dfrac{L}{RC}\right) = 75\,\Omega$

$\dfrac{\mathcal{E}_0}{Z}\cos(\omega t) = i(t)$ (reale!)

b) $I_L = \dfrac{Z_2}{Z_1+Z_2} I = \dfrac{R + \frac{1}{i\omega C}}{2R} \dfrac{\mathcal{E}_0}{Z} e^{i\omega t}$

$i_L = \mathrm{Re}\, I_L = \dfrac{\mathcal{E}_0}{2Z} \dfrac{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}{R}\cos(\omega t - \alpha)$; $\mathrm{tg}\,\alpha = \dfrac{1}{RC\omega}$

$I_C = \dfrac{Z_1}{Z_1+Z_2} I = \dfrac{R + i\omega L}{2R}\dfrac{\mathcal{E}_0}{Z} e^{i\omega t}$

$\underline{(\alpha = \beta)}$

$i_C = \mathrm{Re}\, I_C = \dfrac{\mathcal{E}_0}{2Z}\dfrac{\sqrt{R^2+\omega^2 L^2}}{R}\cos(\omega t + \beta)$; $\mathrm{tg}\,\beta = \dfrac{\omega L}{R}$

c) $V_A - V_B = R(i_C - i_L) = -\omega L \dfrac{\mathcal{E}_0}{Z}\,\mathrm{sen}(\omega t)$

Soluzione 11/7/02

1) a) $\sin\vartheta_i = n \sin\vartheta_t$; ϑ_t è massimo per $\vartheta_i = \frac{\pi}{2}$

$$n = \frac{1}{\sin\vartheta_t} = \underline{\underline{\frac{5}{4}}}$$

b) Massimi : $\cos\vartheta_t = \left(N + \frac{1}{2}\right)\frac{\lambda}{2nd} = \left(N + \frac{1}{2}\right)\cdot\frac{1}{8} < 1 \Rightarrow \underline{\underline{N = 7}}$

$$\cos\vartheta_t = \frac{15}{16} \Rightarrow \sin\vartheta_t = \frac{\sqrt{31}}{16} \text{ e } \underline{\underline{\sin\vartheta_i = n\sin\vartheta_t = \frac{5\sqrt{31}}{64}}} \simeq 0{,}43$$

c) Minimi : $\cos\vartheta_t = \tilde{N}\frac{\lambda}{2nd} = \frac{\tilde{N}}{8}$

Deve essere anche : $\sin\vartheta_t \leq \frac{1}{n} \Leftrightarrow \cos\vartheta_t \geq \sqrt{1 - \frac{1}{n^2}} = \frac{3}{5} = 0{,}6$

Quindi : $\underline{\underline{0{,}6 \leq \cos\vartheta_t \leq 1}}$, e $\underline{\underline{\tilde{N} = 5, 6, 7, 8}} \Rightarrow \underline{\underline{4 \text{ Minimi}}}$

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2) a) $\mathcal{E} = RI + L\frac{dI}{dt} \Rightarrow I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$; $\tau = \frac{L}{R}$

b) $U_B = \frac{1}{2} L I^2(t)$

c) $U_J = \int_0^t R I^2(t')\,dt' = \frac{\mathcal{E}^2}{R}\left[t - \frac{3}{2}\tau - \frac{\tau}{2}e^{-2t/\tau} + 2\tau e^{-t/\tau}\right]$

d) $U_{gen} = U_B + U_J$, oppure

$$U_{gen} = \int_0^t \mathcal{E} I(t')\,dt' = \frac{\mathcal{E}^2}{R}\left[t + \tau e^{-t/\tau} - \tau\right]$$

Soluzione 25/09/0

1) a) $R(t) = \frac{R\,\varsigma}{\Sigma}\left(\vartheta(t)+2\right)$; $\vartheta(t) = \frac{\alpha}{2}t^2$

b) $\phi = BR^2\frac{\vartheta}{2}$; $\mathcal{E}(t) = \dot{\phi} = \frac{BR^2}{2}\alpha t$

c) $I(t) = \frac{\mathcal{E}}{R} = \frac{\alpha B R \Sigma}{\varsigma}\;\frac{t}{\alpha t^2+4}$

$\frac{dI}{dt} = 0 \Rightarrow t_0 = \frac{2}{\sqrt{\alpha}}$; $\underline{\underline{\vartheta(t_0) = 2 \text{ rad}}}$

(massimo)

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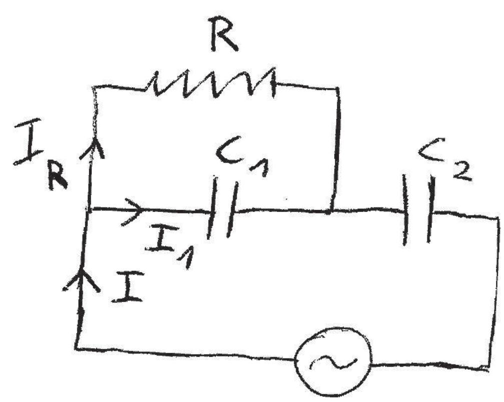
2) a) $Z = \frac{1}{\frac{1}{R}+i\omega C_1} + \frac{1}{i\omega C_2} = \left(\frac{1}{2}-i\right)R$

$I = \frac{\mathcal{E}_0}{Z} = \frac{2}{5}(1+2i)\frac{\mathcal{E}_0}{R}$

Modulo (ampiezza) : $|I| = \frac{2}{\sqrt{5}}\;\frac{\mathcal{E}_0}{R}$

Sfasamento : $\operatorname{tg}\varphi = 2$

b) $\left.\begin{array}{l} I_R + I_1 = I \\ R I_R = \frac{1}{i\omega C_1} I_1 \end{array}\right\}$ $I_R = \frac{1}{1+i} I$



$|I_R| = \sqrt{\frac{2}{5}}\;\frac{\mathcal{E}_0}{R}$

c) $P = \frac{1}{2}\mathcal{E}_0 |I| \cos\varphi$; $\cos\varphi = \frac{1}{\sqrt{1+\operatorname{tg}^2\varphi}} = \frac{1}{\sqrt{5}}$

$P = \frac{1}{5}\;\frac{\mathcal{E}_0^2}{R} = \left[\frac{1}{2}R|I_R|^2\right]$

$E = P \cdot 60\,s = 193{,}6$ Joule