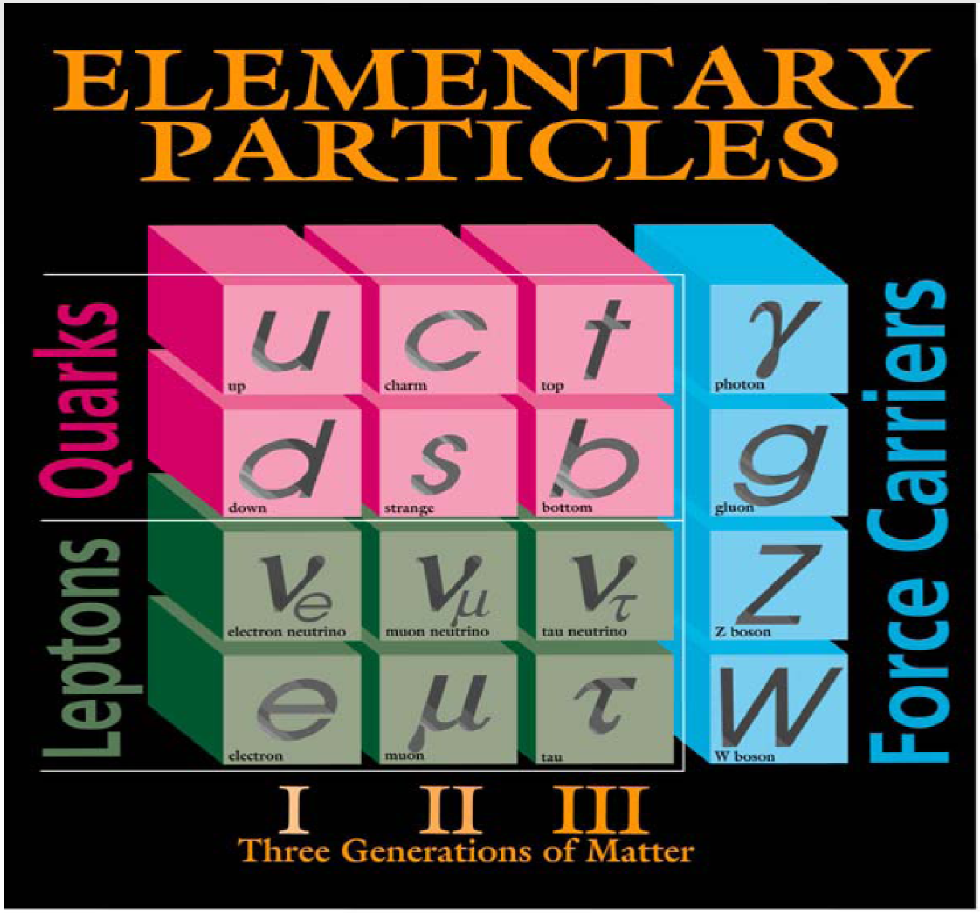
# The Top Quark Mass

Outline :

- Introduction on the Top quark
- Top mass measurement
- The Standard Model and the Top mass



Fermilab 95-759

# Introduction

An afternoon symposium in celebration of the 10th anniversary of the discovery of the Top Quark at Fermilab by the CDF and D0 collaborations. A reception in the Wilson Hall atrium will follow the symposium.

Details at http://www.fnal.gov/pub/news05/TopTurnsTen.html

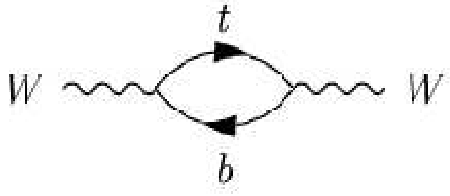
Top quark discovered in 1995 by CDF and D0

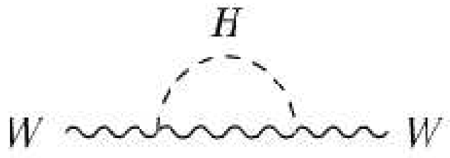
Missing particle as today it is the Higgs

Important to undestand what we missing in the Standard Model

# Top Quark Mass

- Top quark was expected to be very heavy, mt~170 GeV
- It decays with a very short lifetime $\tau \sim \hbar/\Gamma \sim 10^{-25}$s it has no time to hadronize $\tau^{QCD} \sim 10^{-24}$s
- It is possible to study the bare quark: measure the mass of the quark important SM parameter
- Important ingredient for SM precison tests: $B \to X_s\gamma$ and $K_L \to \pi^0\nu\nu$
- It helps in understanding the Higgs sector due to the interreletionship with W and H

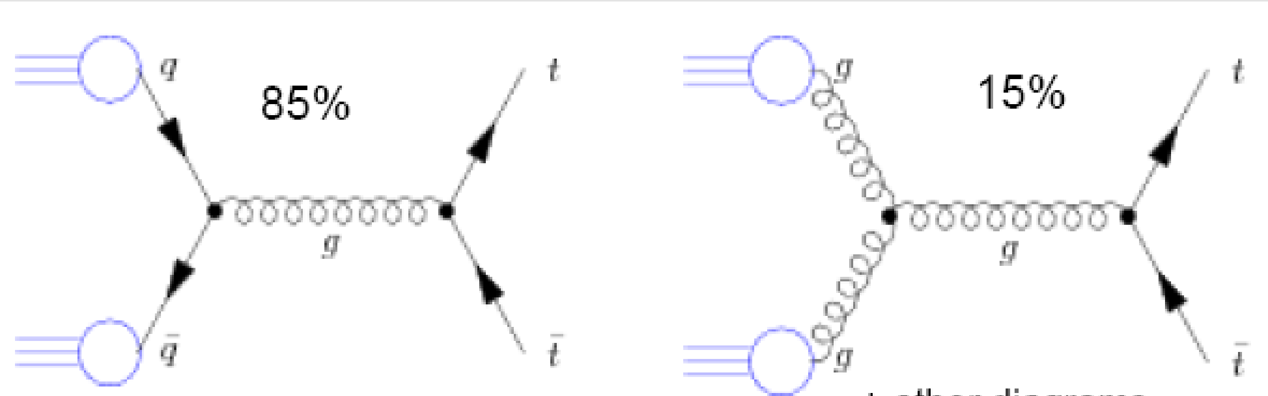




W propagator, and hence its coupling to the H vacuum expectation value (Mw) is affected through internal loops with top quark and H

# Top Quark Production & Decay

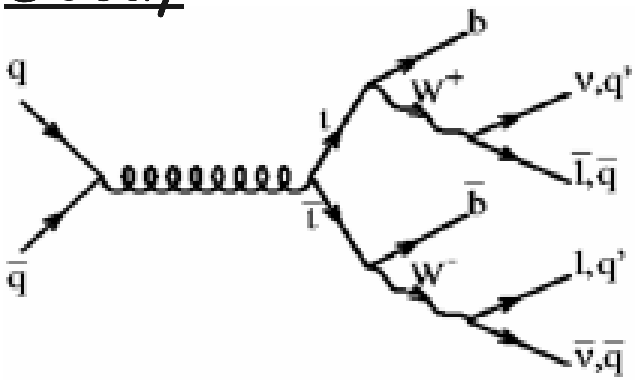
## Production

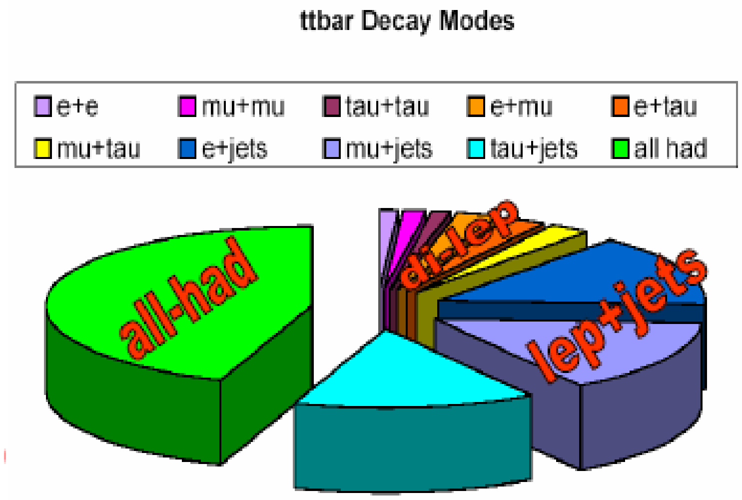


+ other diagrams

At Tevatron top quark is mainly produced in pairs

## Decay





| | |
|---|---|
| Di-leptons | 5% |
| Leptons+jets | 30% |
| All-hadronic | 44% |
| Others | 21% |

# Top Quark Final States

Each final state has specific characteristics and require a specific approach:

- Di-lepton: low yield, low background, well defined leptonic signature
- Lepton+jets: higher yield, moderate background, lepton signature + MET + jets
- All hadronic: highest yield, huge background, only jets

# Data Collection: Lepton Triggers

Dilepton and single lepton channels use “high pt muon” and " high pt electron” triggers.

High_PT_muon trigger:

Level 1 : stub in the muon chambers

Level 2: track reconstructed in the central chamber Pt>15 GeV/c
point to a stub

Level 3: muon recostructed with offline quality

High_PT_electron trigger:

Level 1: em. calorimetric tower Et>8 GeV & proto-track

Level 2: em. jet Et>18 GeV & Ehad/Eem <0.125 & track point to a jet

Level 3: em jet reconstructed with L0 energy calibration, i.e
pedestal subtraction

# Data Collection: Hadronic Trigger

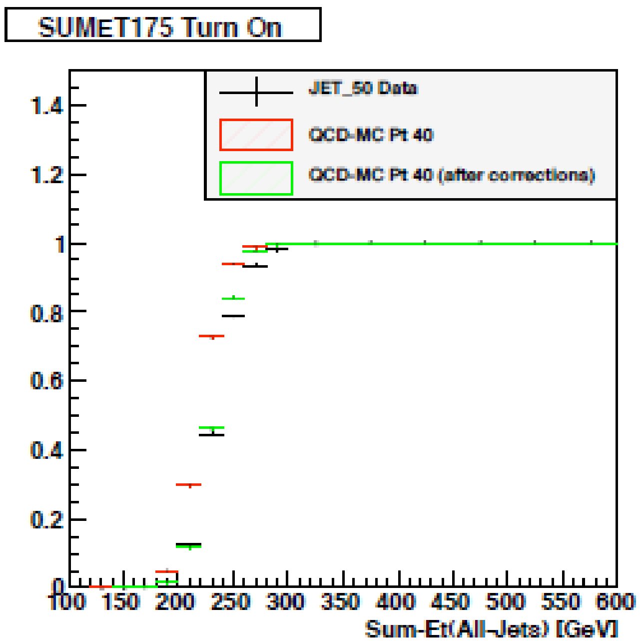
Hadronic channel uses "Multi jets" trigger

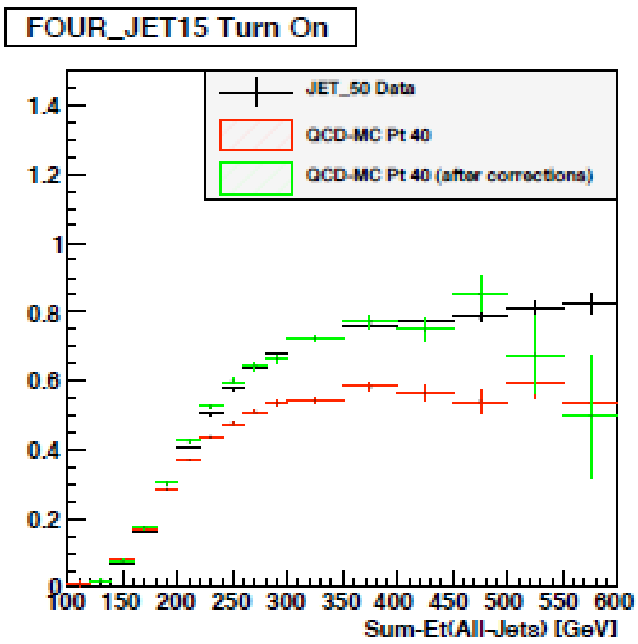
Multi_jets trigger:

Level 1: calorimetric tower Et>20 GeV

Level 2: Four jets Et>15 GeV & Total Et>175 GeV

Level 3: Jets reconstructed with L0 energy calibration

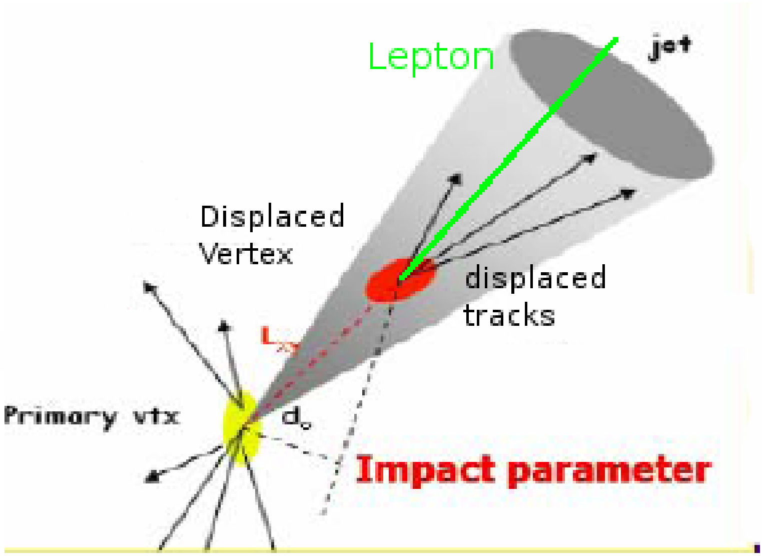


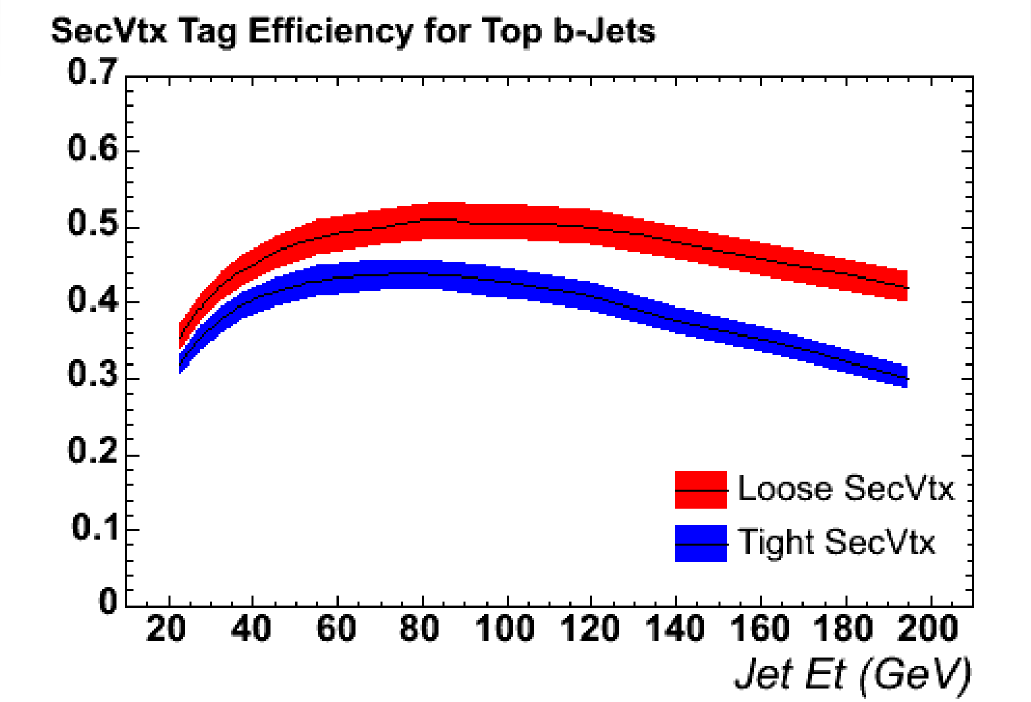


Trigger efficiecies are nedeed then to simulate Monte Carlo.

# Top Quark Mass Measurement Ingredients

- Jets in the final states → Precise jet energy determination
- Final states always with b-quark → b-tag algorithm used to find it
  - Select tracks with high impact parameter wrt primary vertex
  - Fit to identify a secondary vertex
  - Cut on decay lenght $L_{xy}$

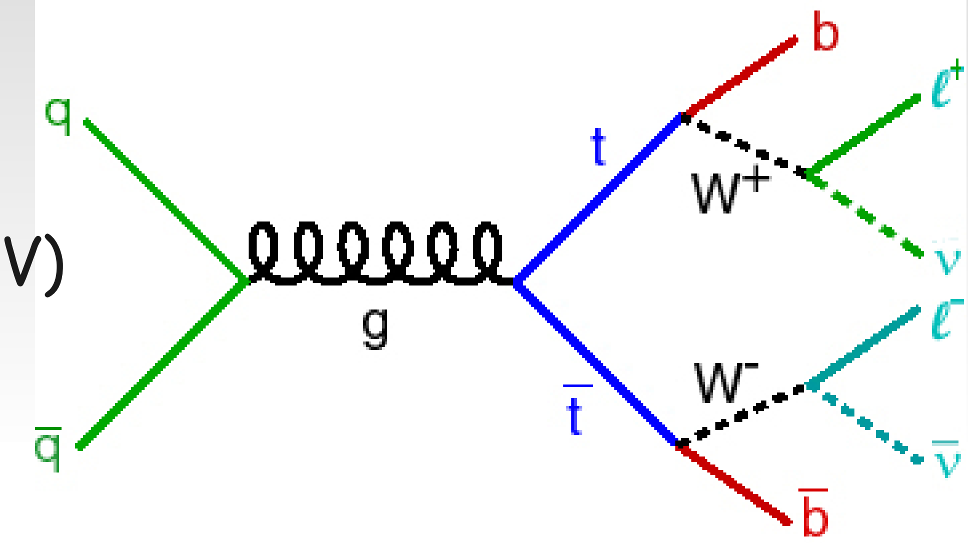




# Top Quark Events Selection: Dileptons

Requirements:

- two high $P_T$ opposite charge isolated leptons (Pt>15 GeV)
- at least 2 high $E_T$ jets ($E_T$>20 GeV)
- at least one vertex b-tag
- MET> 25 GeV



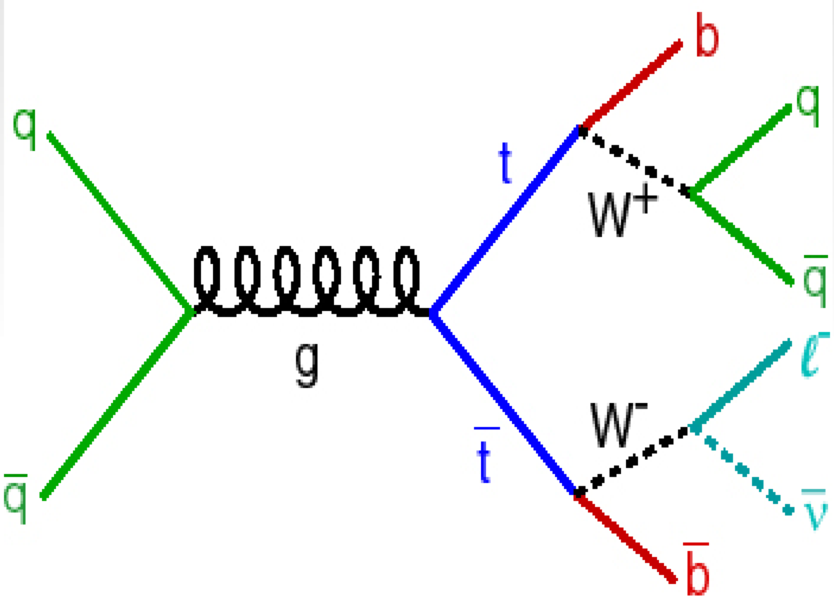
Major Backgrounds

- 🟩 Drell-Yan $Z/\gamma^*$: calculated using control region
- 🟩 Di-boson: WW,WZ,ZZ
- 🟩 QCD: fake leptons

# Top Quark Events Selection: leptons+jets

Requirements:

- one high $P_T$ isolated leptons (Pt>20 GeV)
- at least 4 high $E_T$ jets ($E_T$ >20 GeV)
- at least one b-tag
- MET>20 GeV



Major Background

- W+jets:
  - Heavy Flavour fraction and shape dermined with MC & data
  - light flavour from data
- Other contributions from non-W

# Matrix Element Method Introduction

Observables: measured momenta of jets and leptons

Question: for an observed set of kinematic variables $x$ what is the most probable top mass

Method: start with an observed set of events of given kinematics and find maximum of the likelihood, which provides the best measurement of top quark mass

Our sample is a mixture of signal and background

$$\boxed{P_{evt}(x,m_t) = f_{top} \cdot P_{\mathrm{sgn}}(x,m_t) + (1 - f_{top}) \cdot P_{bkg}(x)}$$

$P_{bkg}(X)$ depends on the decay channel

# Matrix Element Method

probability to observe a set of kinematic variables *x* for a given top mass

***$d^n\sigma$*** is the differential cross section
Contains **matrix element** squared

***W(x,y)*** is the probability that a parton level set of variables ***y*** will be measured as a set of variables ***x***.
Parton Energy ↔ Jet Energy

$$P_{sgn}(x;m_t) = \frac{1}{\sigma(m_t)} \int d^n\sigma(y;m_t)\, dq_1\, dq_2\, f(q_1)\, f(q_2)\, W(x,y)$$

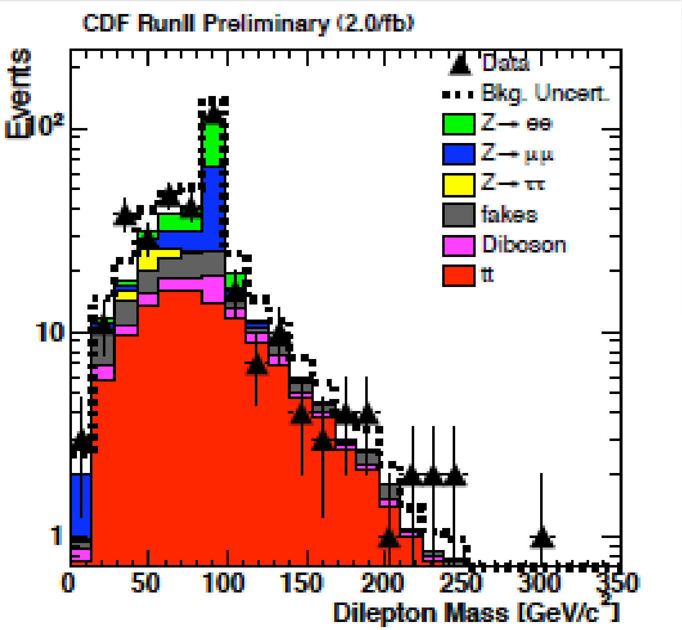
Normalization depends on $m_t$
Includes acceptance effects

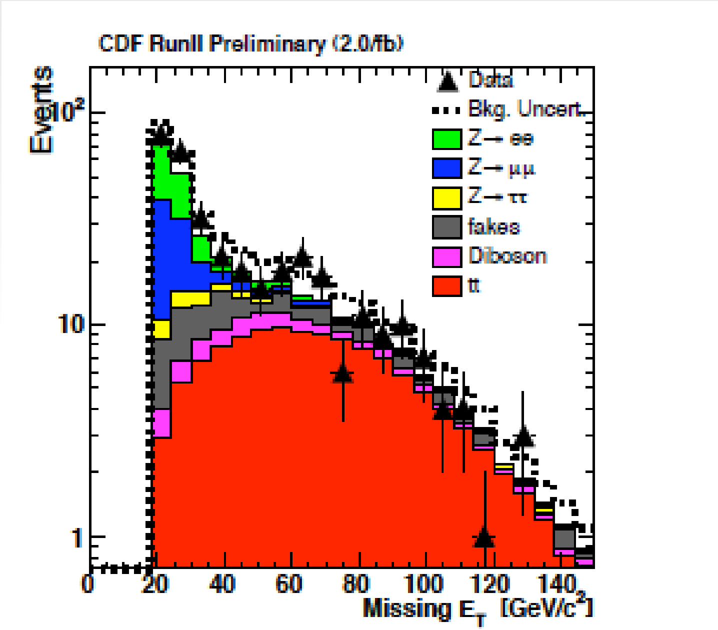
***f(q)*** is the probability distribution than a parton will have a momentum ***q***

Integrate over unknown:
kinematical variable $q_1,q_2$ of initial states parton and final states parton y
Approximations: LO matrix element and *qq->t$\underline{t}$* process only (no gluon fusion – 15%)

# Dileptons Events Analysis

Matrix Element and Neural Network are applied to discriminate signal from background





The information contained in an event regarding top mass is expressed as conditional probability

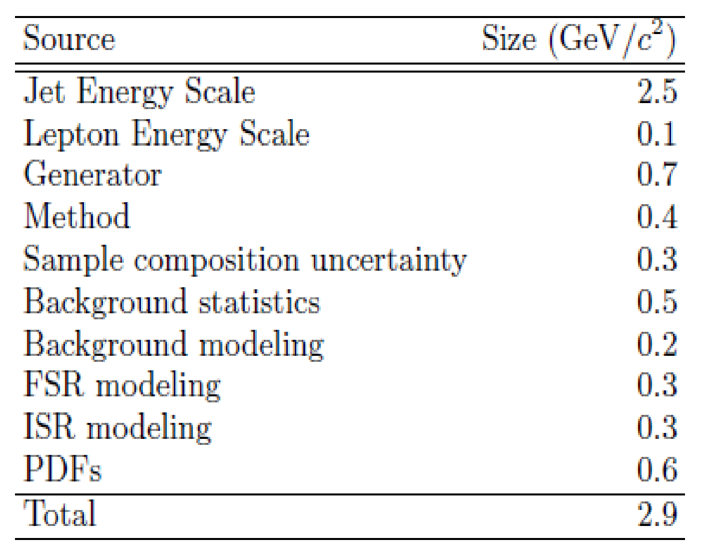
$$P(\mathrm{x}|M_t) = \frac{1}{\sigma(M_t)}\frac{d\sigma(M_t)}{d\mathrm{x}}$$

Mt is the top pole mass, x is the vector of measured quantities

# Dileptons Events Result

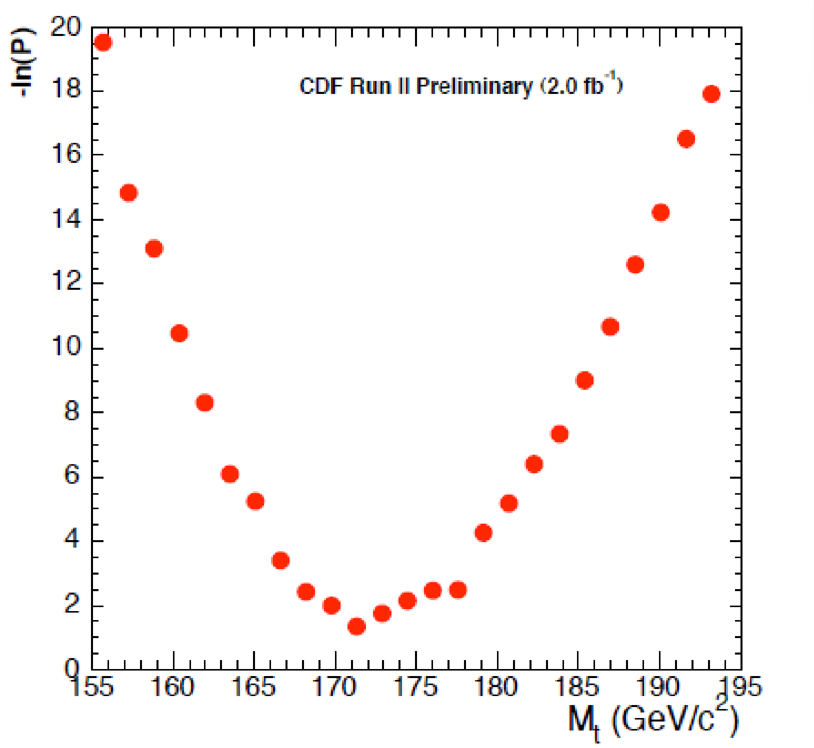
The simple formula is then modified to include background

Systematics error are evaluated and included in the fitter



| Source | Size ($\mathrm{GeV}/c^2$) |
|---|---|
| Jet Energy Scale | 2.5 |
| Lepton Energy Scale | 0.1 |
| Generator | 0.7 |
| Method | 0.4 |
| Sample composition uncertainty | 0.3 |
| Background statistics | 0.5 |
| Background modeling | 0.2 |
| FSR modeling | 0.3 |
| ISR modeling | 0.3 |
| PDFs | 0.6 |
| Total | 2.9 |

Fit posterior probability to extract Mt

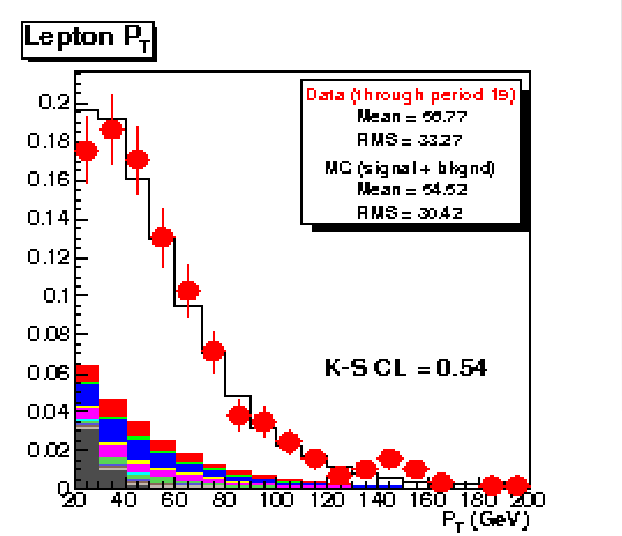


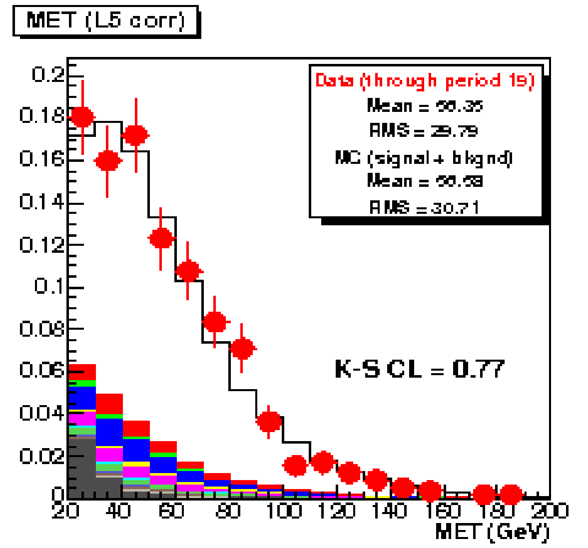
$$M_{top} = 171.2 \pm 2.7(\mathrm{stat.}) \pm 2.9(\mathrm{syst.})\ \mathrm{GeV}/c^2$$

# Lepton+jets analisys

Sample composition after event selection

| Background | 1 tag | $\geq$ 2 tags |
|---|---|---|
| non-$W$ QCD | $23.4 \pm 20.4$ | $1.6 \pm 2.3$ |
| $W$+light mistag | $22.1 \pm 5.7$ | $0.4 \pm 0.2$ |
| diboson ($WW$, $WZ$, $ZZ$) | $5.5 \pm 0.6$ | $0.5 \pm 0.1$ |
| $Z \rightarrow ee, \mu\mu, \tau\tau$ | $3.6 \pm 0.5$ | $0.3 \pm 0.1$ |
| Sum of above 3 | $31.2 \pm 5.8$ | $1.2 \pm 0.2$ |
| $W + b\bar{b}$ | $32.4 \pm 12.5$ | $6.6 \pm 2.2$ |
| $W + c\bar{c}$ | $19.4 \pm 6.7$ | $0.9 \pm 0.3$ |
| $W + c$ | $10.3 \pm 3.6$ | $0.5 \pm 0.2$ |
| Single top s-chan | $2.4 \pm 0.3$ | $0.9 \pm 0.1$ |
| Single top t-chan | $2.7 \pm 0.3$ | $0.7 \pm 0.1$ |
| Sum of above 5 | $67.2 \pm 21.8$ | $9.5 \pm 2.6$ |
| Total background | $121.8 \pm 31.7$ | $12.3 \pm 4.4$ |
| Events observed | 459 | 119 |





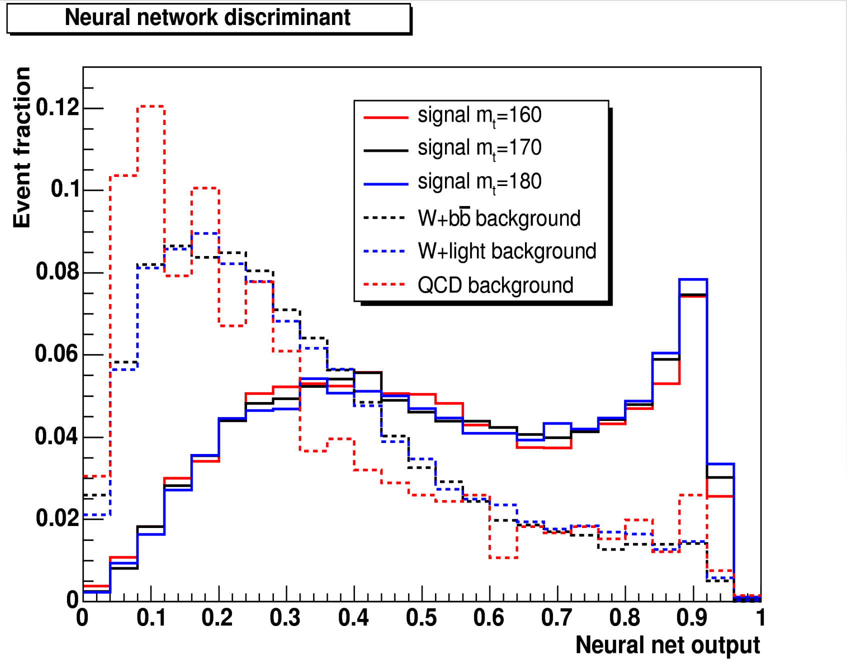
Signal likelihood calculation is performed by integrating over the matrix element using the following formula:

$$L(\vec{y}\,|\,m_t, \Delta_{\mathrm{JES}}) = \frac{1}{N(m_t)}\frac{1}{A(m_t, \Delta_{\mathrm{JES}})}\sum_{i=1}^{24} w_i L_i(\vec{y}\,|\,m_t, \Delta_{\mathrm{JES}})$$

# Lepton+jets Analysis

For each event calculate the background fraction for that event fbg(q) = B(q)/(S(q)+B(q)), where q is neural network output for that event.

The background is subtracted to have the signal probablity for each event



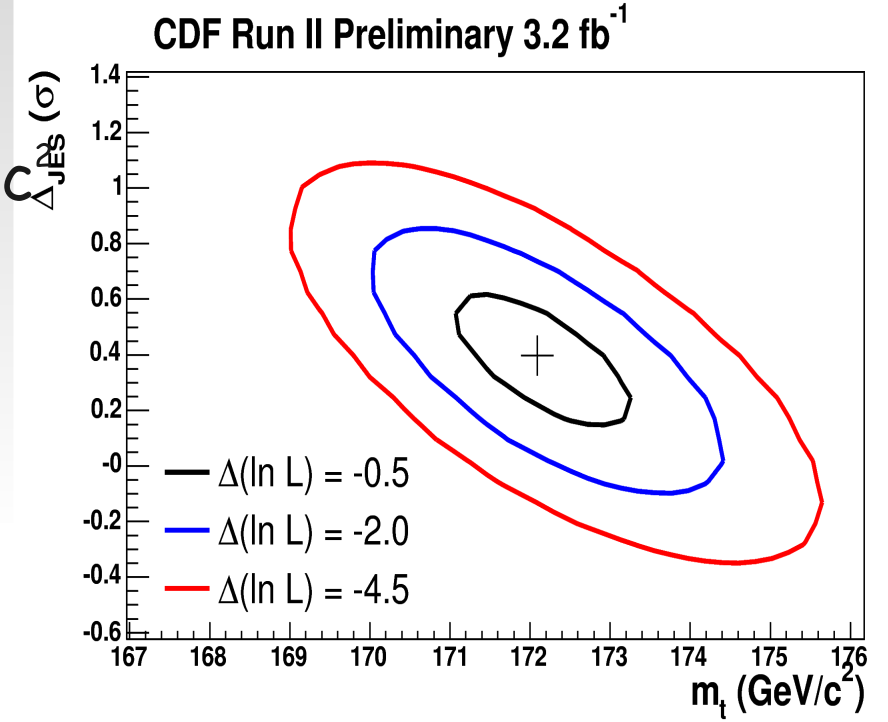
log Lsig(mt,JES) = Σi[log Li(mt,JES)] - nbg log Lavg(mt, JES | bck)

# Lepton+jets Result

The results of the fit are

$m_t$=172.1 ±1.2(stat) ±0.7(JES) GeV/c²

ΔJES = 0.40 ± 0.26 σ



Adding systematics

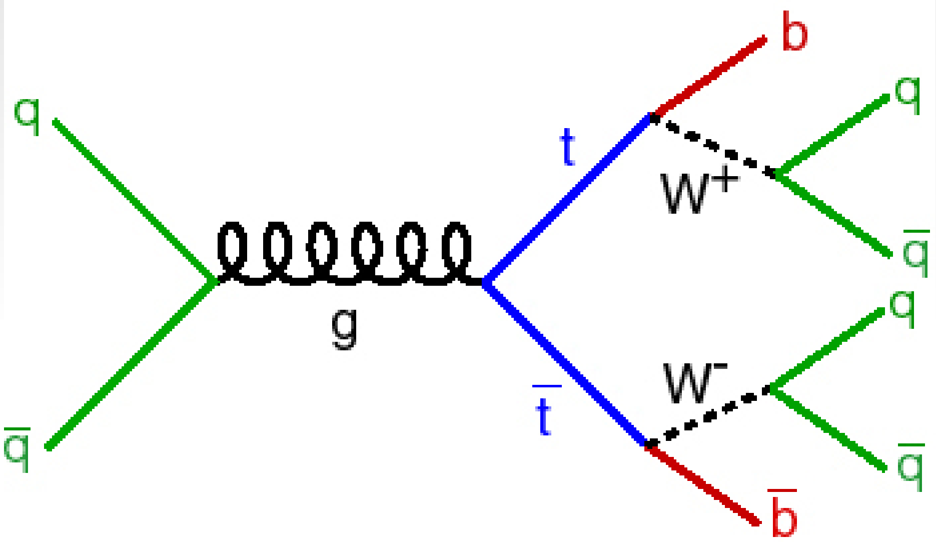
| Systematic source | Systematic uncertainty (GeV/$c^2$) |
|---:|:---:|
| Calibration | 0.2 |
| MC generator | 0.5 |
| ISR and FSR | 0.3 |
| Residual JES | 0.5 |
| $b$-JES | 0.4 |
| Lepton $P_T$ | 0.2 |
| Multiple hadron interactions | 0.1 |
| PDFs | 0.2 |
| Background | 0.5 |
| Color reconnection | 0.4 |
| Total | 1.1 |

mt = 172.1 ± 0.9 (stat.) ± 0.7 (JES) ± 1.1 (syst.) GeV/c2

# Top Quark Events Selection: all jets

**Requirements:**

- at least 6 high $E_T$ jets ($E_T$ >15 GeV)
- at least one b-tag
- Small MET
- No leptons



**Dominant Background**

- QCD multi-jets

# Template Method

**Method:** build top mass and JES template for signal and background
Use the templates as pdf in the Likelihood.
Extract top mass and JES

Reconstruct the event kinematic by minimizing:

$$\chi^2 = \frac{(m_{jj}^{(1)} - M_W)^2}{\Gamma_W^2} + \frac{(m_{jj}^{(2)} - M_W)^2}{\Gamma_W^2} + \frac{(m_{jjb}^{(1)} - m_t^{rec})^2}{\Gamma_t^2} + + \frac{(m_{jjb}^{(2)} - m_t^{rec})^2}{\Gamma_t^2} + \sum_{i=1}^{n} \frac{(p_{T,i}^{fit} - p_{T,i}^{meas})^2}{\sigma_i^2}$$

mjj = invariant mass of two light jets

mjjb = invariant mass of three jets

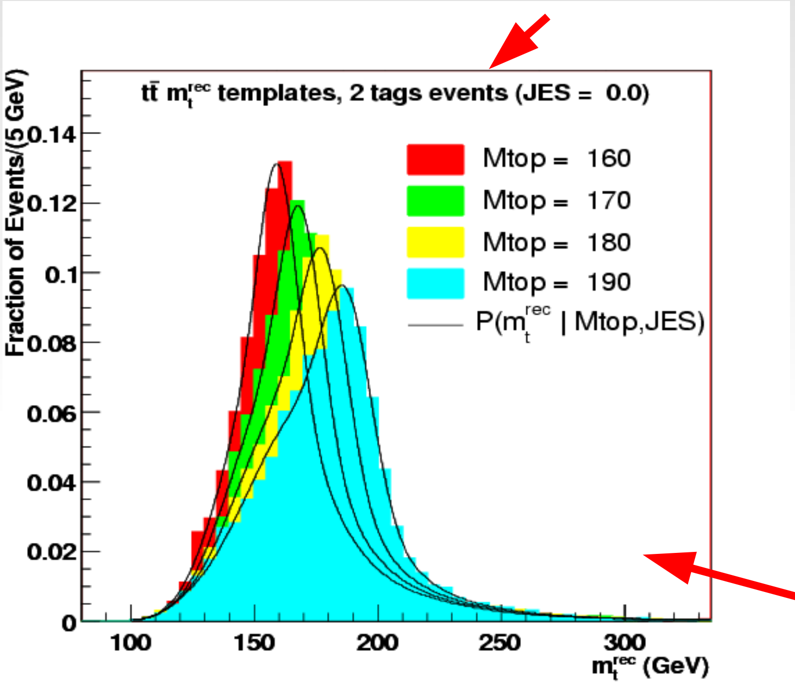
$P_T^{fit}$ =top transv. momentum

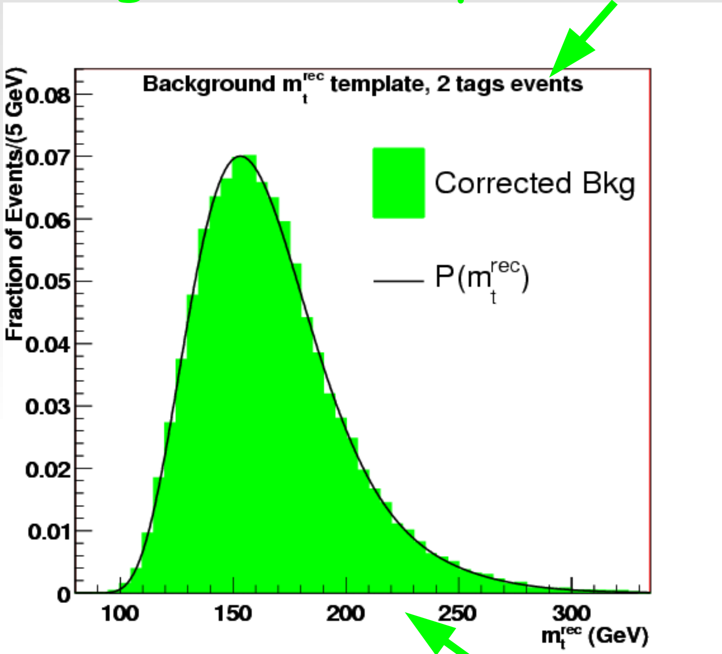
For each permutation we obtain $m_t^{rec}$ this forms the template for signal (MC) and background (data)

# Quark Top Mass: All Hadronic

Signal template: Monte Carlo data

Background template: data





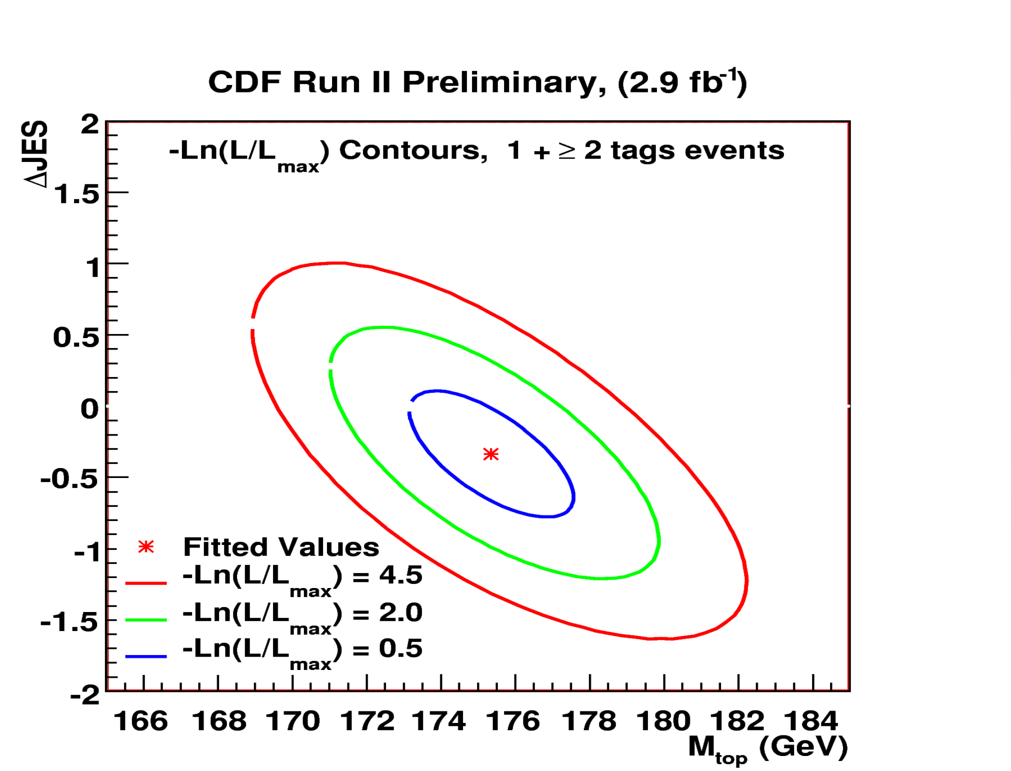
$$L_{tot} = L_{M_{top}} + L_{JES}$$

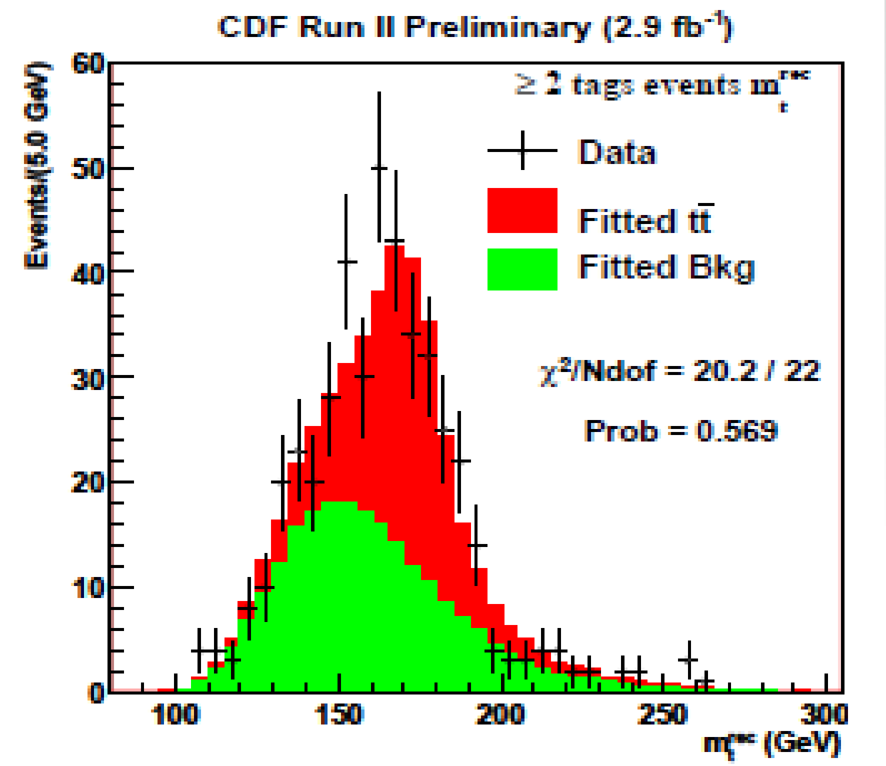
$$\mathcal{L}_{M_{top}} = \prod_{i=1}^{N_{obs}} \frac{n_s \cdot P_{sig}^{m_t^{rec}}(m_{t,i} \,|\, M_{top}, \mathrm{JES}) + n_b \cdot P_{bkd}^{m_t^{rec}}(m_{t,i})}{n_s + n_b}$$

$$\mathcal{L}_{\mathrm{JES}} = \prod_{i=1}^{N_{obs}} \frac{n_s \cdot P_{sig}^{m_W^{rec}}(m_{W,i} \,|\, M_{top}, \mathrm{JES}) + n_b \cdot P_{bkd}^{m_W^{rec}}(m_{W,i})}{n_s + n_b}$$

The Likelihood has two terms: one for the mass and one for JES

# All Hadronic result

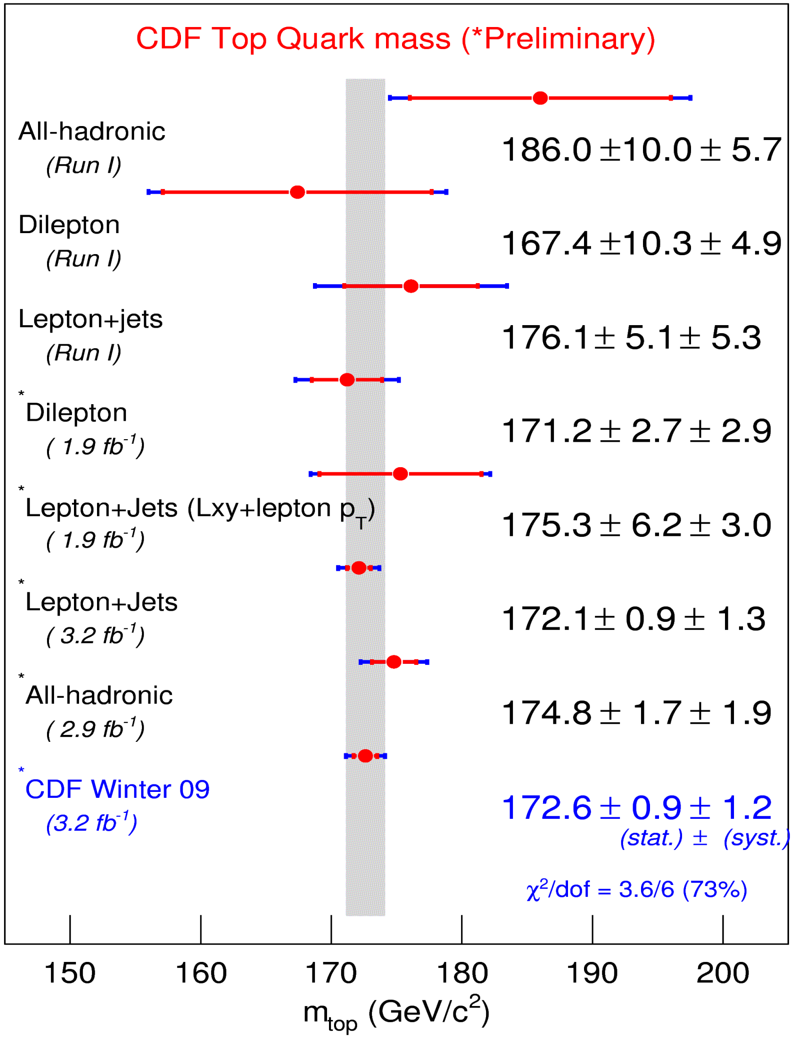




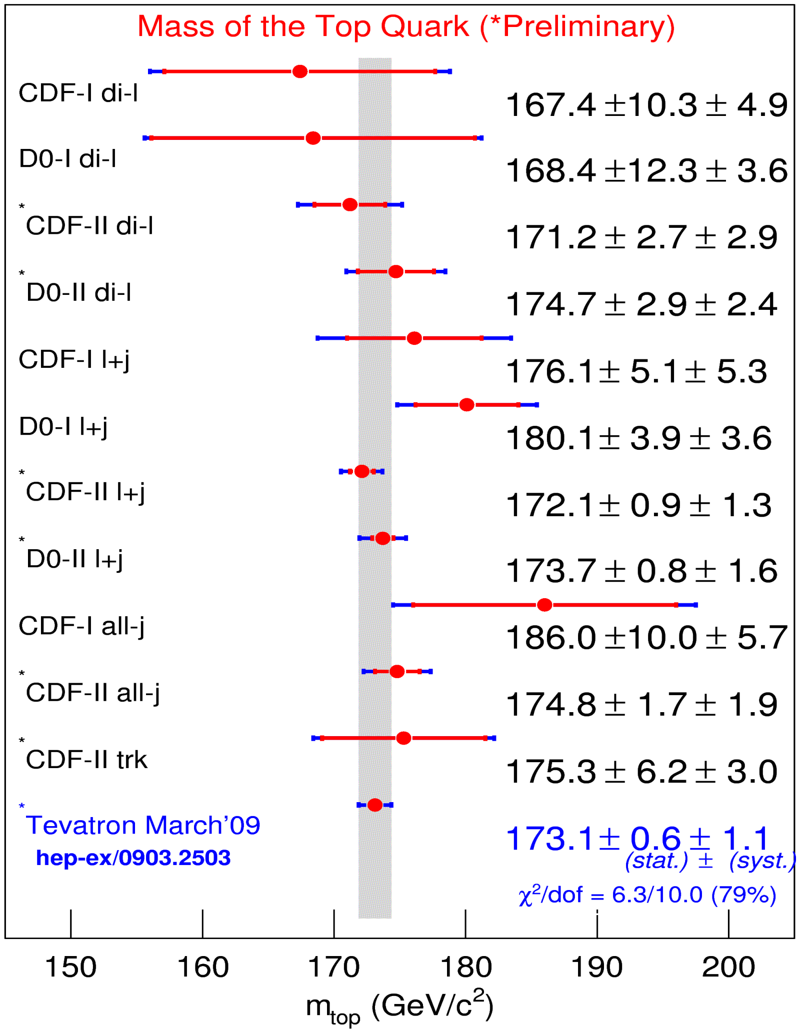
$M_{top}$ = 174.8 ± 1.7 (stat) ± 1.9 (syst) GeV/$c^2$

$$\delta M_{top}/M_{top} \simeq 1.5\%$$

# Top Quark mass combination



172.6±0.9(stat)±1.2(syst) GeV/c$^2$

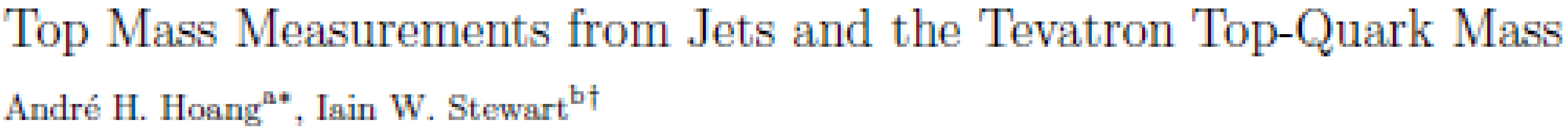


173.1±0.6(stat)±1.1(syst) GeV/c$^2$

# Top Quark Mass: What are we measuring?

- All Mtop measurements make heavy usage of Monte Carlo
- So we measure the MC top mass!
- Usually in the MC the parameter we calibrate to is the top-quark pole mass.
- All say It;s pole mass with $\Lambda_{QCD}$~200 MeV/c$^2$

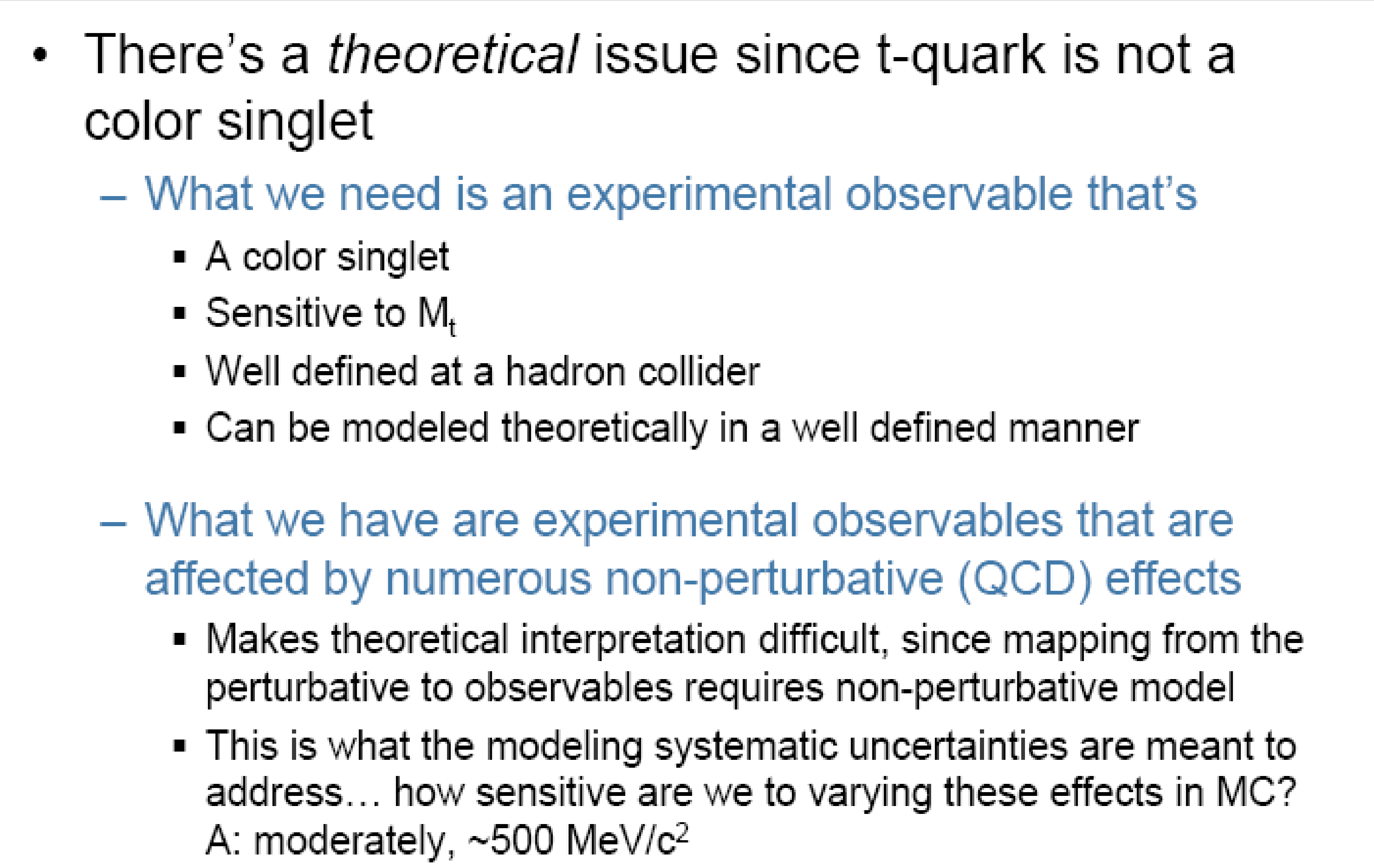
The complete discussion in http://arxiv.org/abs/0808.0222v2



Top Mass Measurements from Jets and the Tevatron Top-Quark Mass

André H. Hoang[a*], Iain W. Stewart[b†]

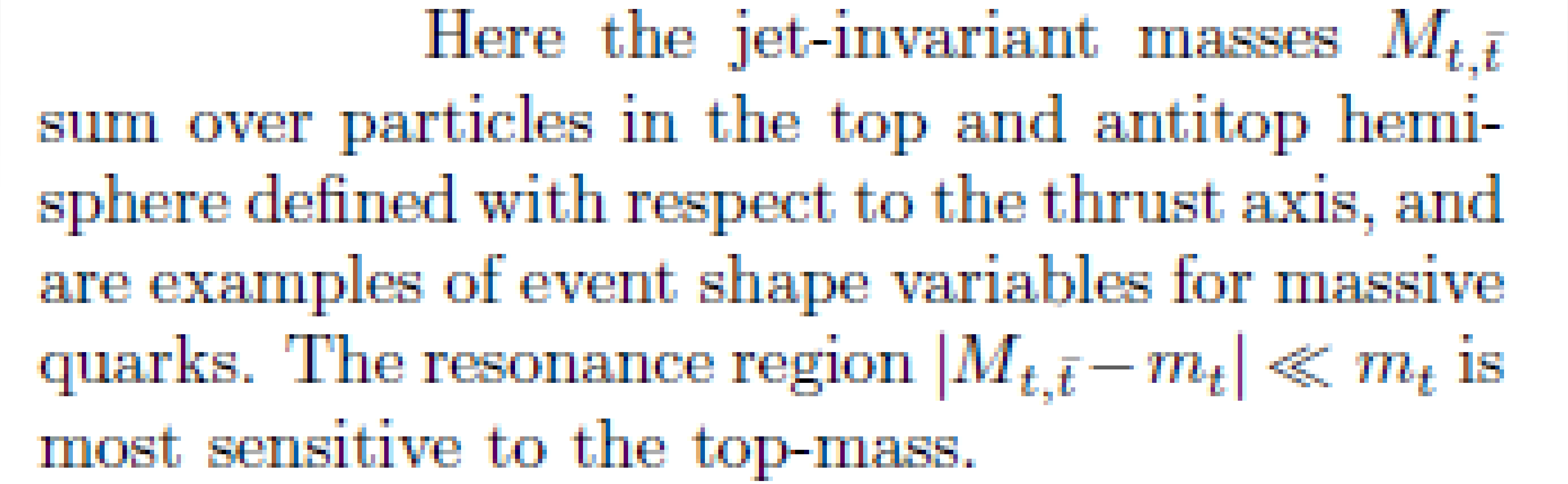
# Quark Top Mass: What are we measuring?



- There's a *theoretical* issue since t-quark is not a color singlet
  - What we need is an experimental observable that's
    - A color singlet
    - Sensitive to $M_t$
    - Well defined at a hadron collider
    - Can be modeled theoretically in a well defined manner
  - What we have are experimental observables that are affected by numerous non-perturbative (QCD) effects
    - Makes theoretical interpretation difficult, since mapping from the perturbative to observables requires non-perturbative model
    - This is what the modeling systematic uncertainties are meant to address… how sensitive are we to varying these effects in MC? A: moderately, ~500 MeV/$c^2$

# Quark Top Mass: What are we measuring?

The authors address the issue in $e^+e^-$ collider: $e^+e^-$->tt √s>>mt choosing a mass schema.



Here the jet-invariant masses $M_{t,\bar{t}}$ sum over particles in the top and antitop hemisphere defined with respect to the thrust axis, and are examples of event shape variables for massive quarks. The resonance region $|M_{t,\bar{t}} - m_t| \ll m_t$ is most sensitive to the top-mass.

# Quark Top Mass: What are we measuring?

The situation at hadron collider is more complicated:

- initial state radiation
- jet reconstructed as MC parton shower → "the implementation of the parton shower for top quark and the size of the shower cutoff determines the top mass definition used in MC"

A relation among the MC mass and the that defined in the Mass Schema is obtained.

$$\text{top mass } \overline{m}_t(\overline{m}_t) = 163.0 \pm 1.3^{+0.6}_{-0.3}\ \text{GeV}$$

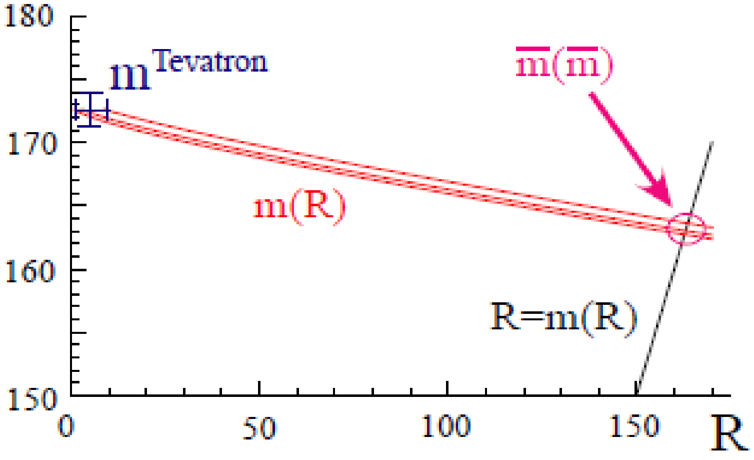
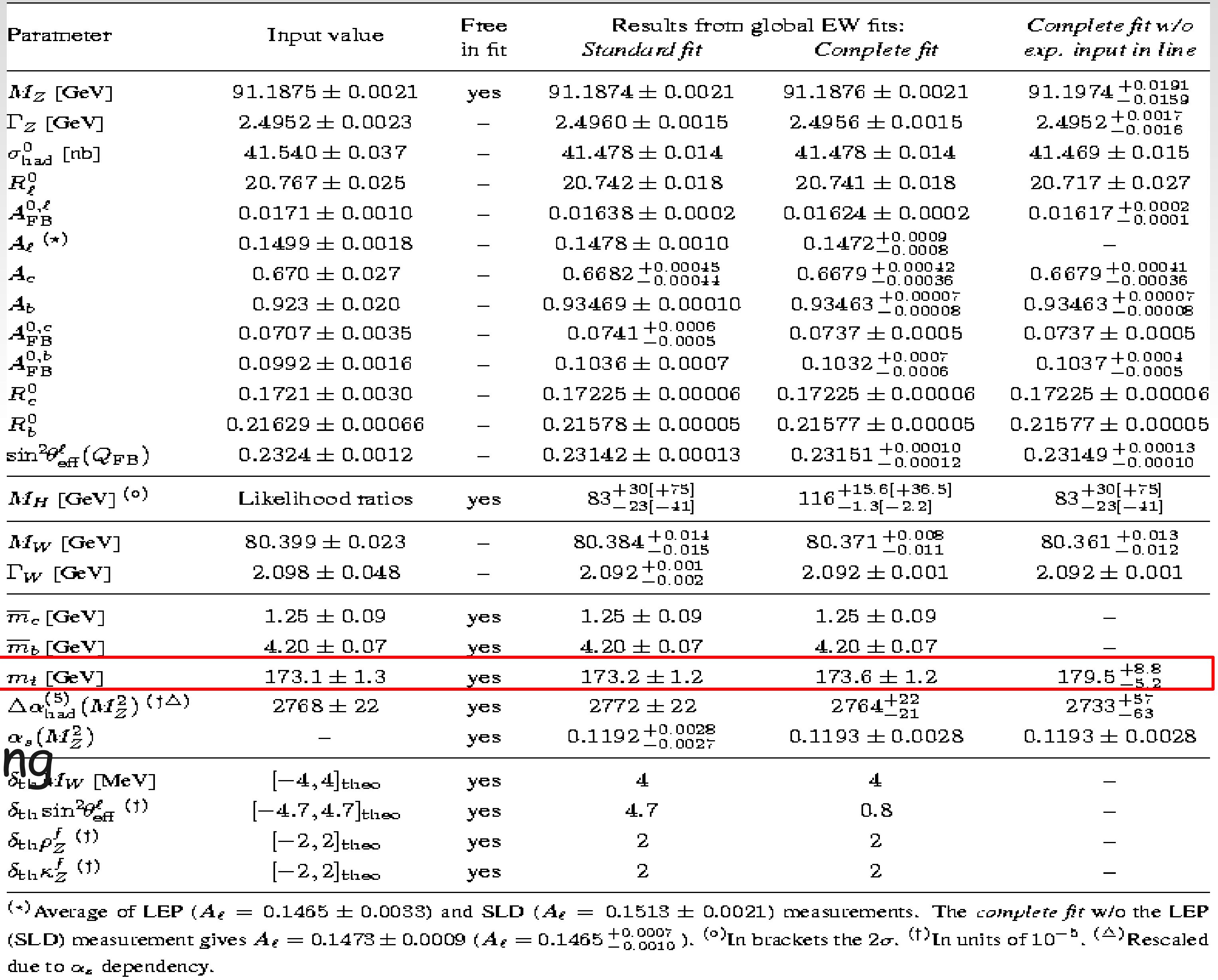


Figure 4. Converting the Tevatron top mass into the $\overline{\text{MS}}$ scheme using the MSR scheme.

# Top Quark Mass and the Standard Model Fit

List of parameters that enter the Standard Model fit. Last column gives the fit results for each parameter without using the corresponding experimental constraint in the fit



| Parameter | Input value | Free in fit | Results from global EW fits: | | *Complete fit w/o* |
|---|---|---|---|---|---|
| | | | *Standard fit* | *Complete fit* | *exp. input in line* |
| $M_Z$ [GeV] | $91.1875 \pm 0.0021$ | yes | $91.1874 \pm 0.0021$ | $91.1876 \pm 0.0021$ | $91.1974^{+0.0191}_{-0.0159}$ |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | – | $2.4960 \pm 0.0015$ | $2.4956 \pm 0.0015$ | $2.4952^{+0.0017}_{-0.0016}$ |
| $\sigma^0_{\rm had}$ [nb] | $41.540 \pm 0.037$ | – | $41.478 \pm 0.014$ | $41.478 \pm 0.014$ | $41.469 \pm 0.015$ |
| $R^0_\ell$ | $20.767 \pm 0.025$ | – | $20.742 \pm 0.018$ | $20.741 \pm 0.018$ | $20.717 \pm 0.027$ |
| $A^{0,\ell}_{\rm FB}$ | $0.0171 \pm 0.0010$ | – | $0.01638 \pm 0.0002$ | $0.01624 \pm 0.0002$ | $0.01617^{+0.0002}_{-0.0001}$ |
| $A_\ell$ (*) | $0.1499 \pm 0.0018$ | – | $0.1478 \pm 0.0010$ | $0.1472^{+0.0009}_{-0.0008}$ | – |
| $A_c$ | $0.670 \pm 0.027$ | – | $0.6682^{+0.00045}_{-0.00044}$ | $0.6679^{+0.00042}_{-0.00036}$ | $0.6679^{+0.00041}_{-0.00036}$ |
| $A_b$ | $0.923 \pm 0.020$ | – | $0.93469 \pm 0.00010$ | $0.93463^{+0.00007}_{-0.00008}$ | $0.93463^{+0.00007}_{-0.00008}$ |
| $A^{0,c}_{\rm FB}$ | $0.0707 \pm 0.0035$ | – | $0.0741^{+0.0006}_{-0.0005}$ | $0.0737 \pm 0.0005$ | $0.0737 \pm 0.0005$ |
| $A^{0,b}_{\rm FB}$ | $0.0992 \pm 0.0016$ | – | $0.1036 \pm 0.0007$ | $0.1032^{+0.0007}_{-0.0006}$ | $0.1037^{+0.0004}_{-0.0005}$ |
| $R^0_c$ | $0.1721 \pm 0.0030$ | – | $0.17225 \pm 0.00006$ | $0.17225 \pm 0.00006$ | $0.17225 \pm 0.00006$ |
| $R^0_b$ | $0.21629 \pm 0.00066$ | – | $0.21578 \pm 0.00005$ | $0.21577 \pm 0.00005$ | $0.21577 \pm 0.00005$ |
| $\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$ | $0.2324 \pm 0.0012$ | – | $0.23142 \pm 0.00013$ | $0.23151^{+0.00010}_{-0.00012}$ | $0.23149^{+0.00013}_{-0.00010}$ |
| $M_H$ [GeV] (∘) | Likelihood ratios | yes | $83^{+30[+75]}_{-23[-41]}$ | $116^{+15.6[+36.5]}_{-1.3[-2.2]}$ | $83^{+30[+75]}_{-23[-41]}$ |
| $M_W$ [GeV] | $80.399 \pm 0.023$ | – | $80.384^{+0.014}_{-0.015}$ | $80.371^{+0.008}_{-0.011}$ | $80.361^{+0.013}_{-0.012}$ |
| $\Gamma_W$ [GeV] | $2.098 \pm 0.048$ | – | $2.092^{+0.001}_{-0.002}$ | $2.092 \pm 0.001$ | $2.092 \pm 0.001$ |
| $\overline{m}_c$ [GeV] | $1.25 \pm 0.09$ | yes | $1.25 \pm 0.09$ | $1.25 \pm 0.09$ | – |
| $\overline{m}_b$ [GeV] | $4.20 \pm 0.07$ | yes | $4.20 \pm 0.07$ | $4.20 \pm 0.07$ | – |
| $m_t$ [GeV] | $173.1 \pm 1.3$ | yes | $173.2 \pm 1.2$ | $173.6 \pm 1.2$ | $179.5^{+8.8}_{-5.2}$ |
| $\Delta\alpha^{(5)}_{\rm had}(M^2_Z)$ (†△) | $2768 \pm 22$ | yes | $2772 \pm 22$ | $2764^{+22}_{-21}$ | $2733^{+57}_{-63}$ |
| $\alpha_s(M^2_Z)$ | – | yes | $0.1192^{+0.0028}_{-0.0027}$ | $0.1193 \pm 0.0028$ | $0.1193 \pm 0.0028$ |
| $\delta_{\rm th} M_W$ [MeV] | $[-4,4]_{\rm theo}$ | yes | 4 | 4 | – |
| $\delta_{\rm th}\sin^2\theta^\ell_{\rm eff}$ (†) | $[-4.7,4.7]_{\rm theo}$ | yes | 4.7 | 0.8 | – |
| $\delta_{\rm th}\rho^f_Z$ (†) | $[-2,2]_{\rm theo}$ | yes | 2 | 2 | – |
| $\delta_{\rm th}\kappa^f_Z$ (†) | $[-2,2]_{\rm theo}$ | yes | 2 | 2 | – |

(*) Average of LEP ($A_\ell = 0.1465 \pm 0.0033$) and SLD ($A_\ell = 0.1513 \pm 0.0021$) measurements. The *complete fit* w/o the LEP (SLD) measurement gives $A_\ell = 0.1473 \pm 0.0009$ ($A_\ell = 0.1465^{+0.0007}_{-0.0010}$). (∘)In brackets the $2\sigma$. (†)In units of $10^{-5}$. (△)Rescaled due to $\alpha_s$ dependency.

# Top Quark Mass and the H

Contours of 68%, 95% and 99% CL obtained from scans of fits with fixed variable pairs mt vs. MH .