Measurement of α_s

Key element of QCD is the running coupling $\alpha_s(\mu^2) = \frac{1}{b_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)}$

This has been measured in several processes like fragmentation Functions, jets production rate, global fit at LEP and SLAC (Z^0)
At hadron collider can be measured using the inclusive jet cross section.

$$\frac{d\sigma}{dE_T} = \alpha_s^2(\mu_R)\hat{X}^{(0)}(\mu_F, E_T)[1 + \alpha_s(\mu_R)k_1(\mu_R, \mu_F, E_T)]$$

$$\alpha_s^3(\mu_R)\hat{X}^{(0)}(\mu_F, E_T)k_1(\mu_R, \mu_F, E_T)$$

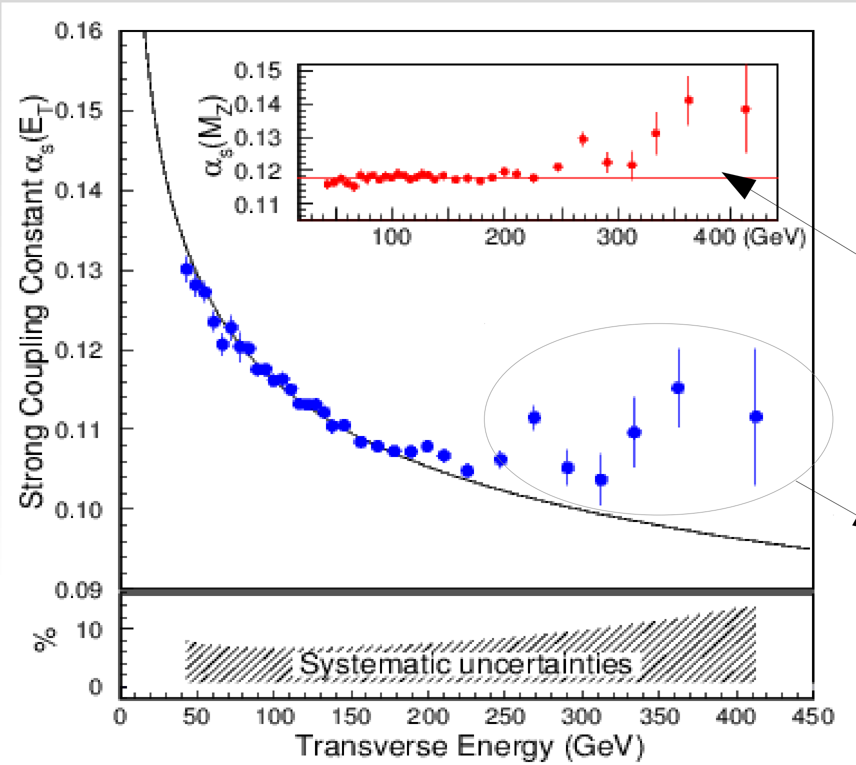
Jets transverse energy distribution

Leading order prediction

Nest to leading order contribution

Jet data are divided into several bins of Et and in each bin α_s is measured.

Measurement of α_s

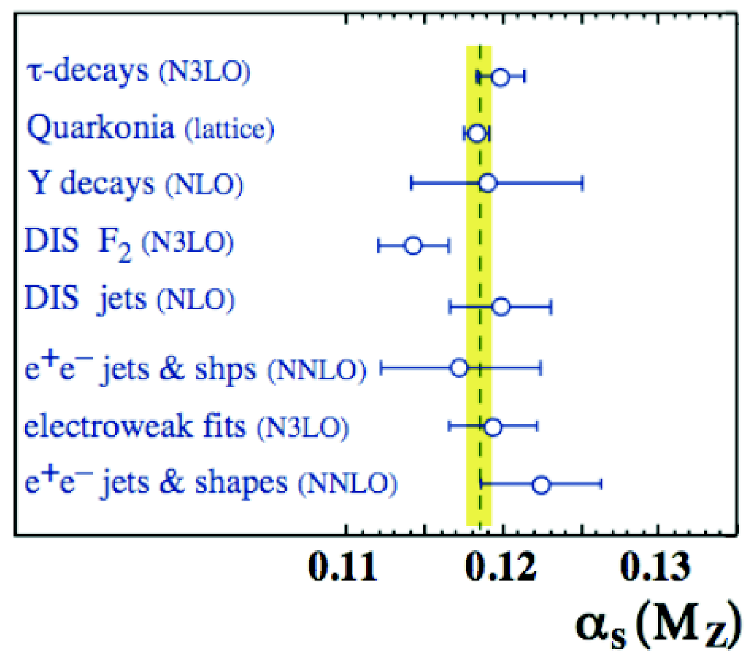


Good agreement between data and and predictions

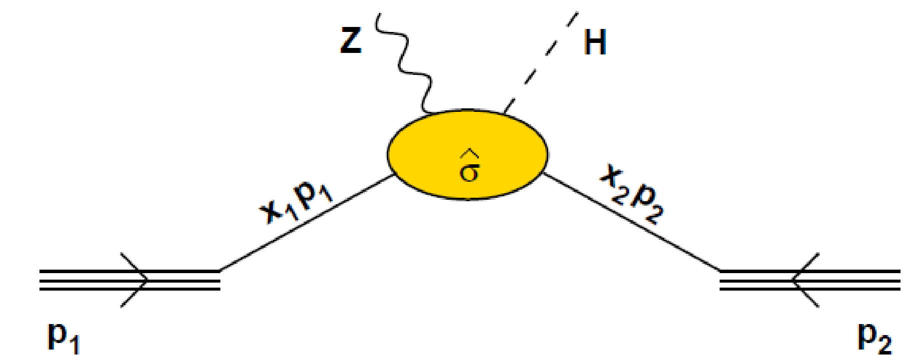
α_s is evolved to M_Z for all the measurements. Then the values are averaged:

$$\alpha_s(M_Z) = 0.1178 \pm 0.0001(\text{stat}).$$

Problem due to the gluon PDF



Parton Distribution Function

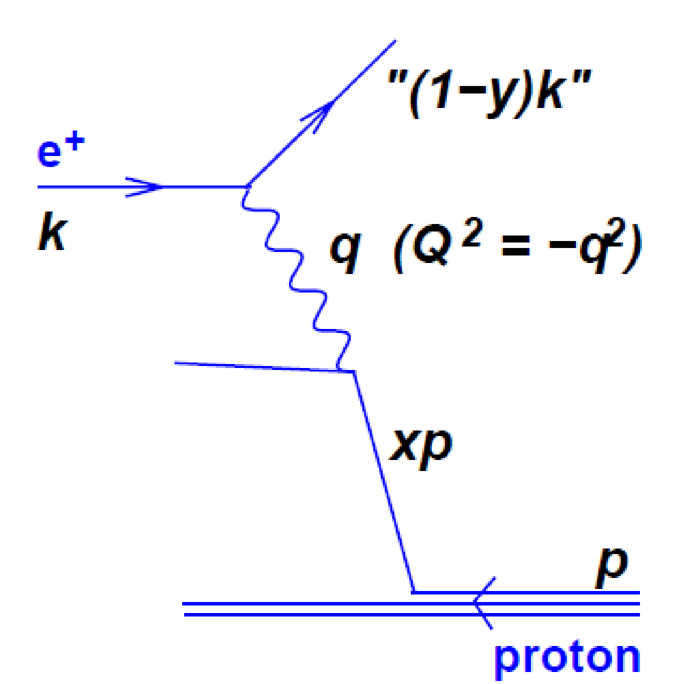


In order to evaluated the cross section of any process involving hadrons in the initial state we must know the parton distribution inside the hadron

PDF are non-perturbative properties, in principle they can be calculated using Lattice QCD but the precision is not enough yet respect to perturbative QCD+experiment measurements

A lot of progress on PDF has come from Deep Inelastic Scattering at HERA, ep collider with 2 experiments: H1 and ZEUS

Deep Inelastic Scattering: description



$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

$\sqrt{s}$ the center of mass energy

x is the fractional momentum of the parton

y is momentum fraction lost by e

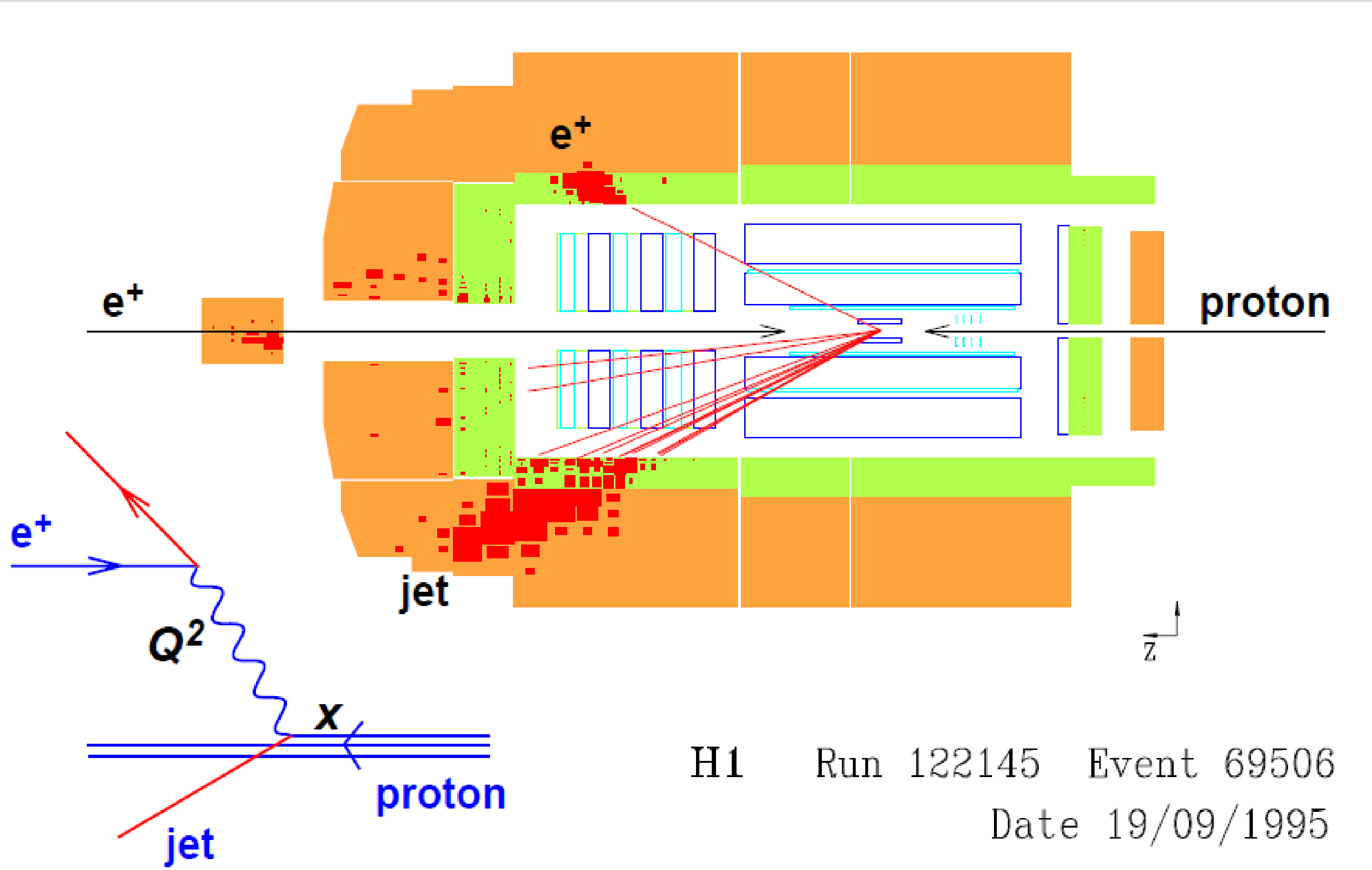
$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

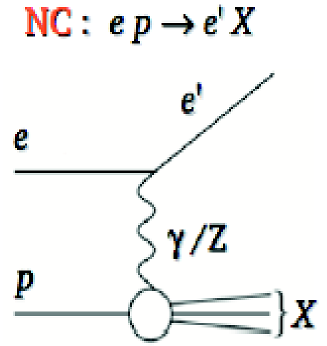
Differential Cross section (what is measured)

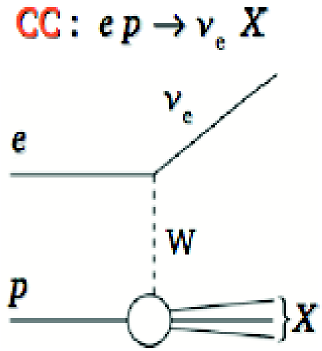
$$F_2^{\text{proton}} = x(e_u^2 u_p(x) + e_d^2 d_p(x)) = x\left(\frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)\right)$$

Not enough to extract u_p and d_p

Experimental measurements





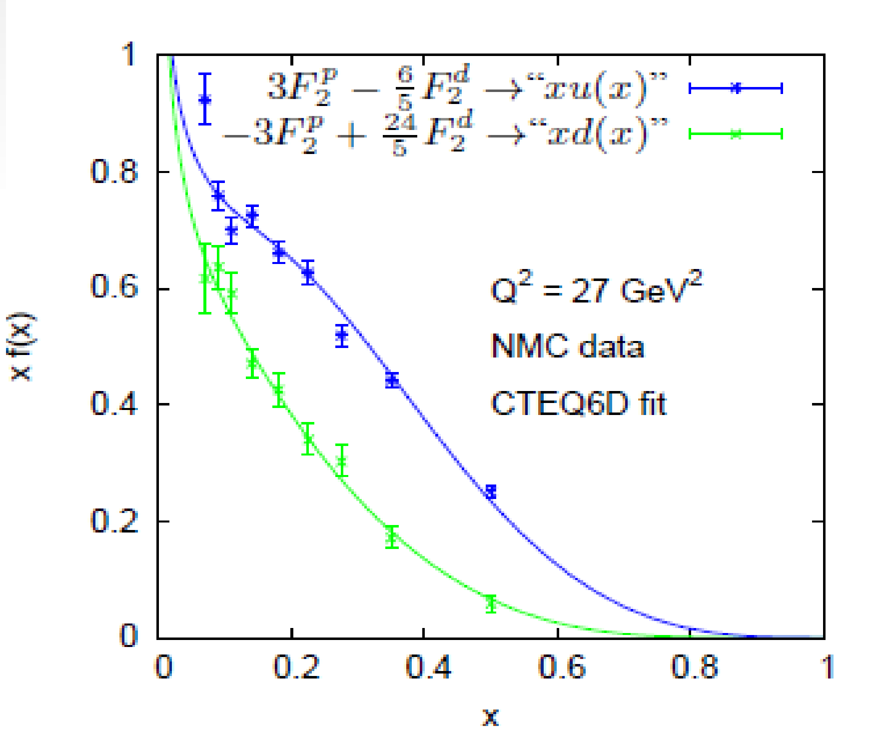


Deep Inelastic Scattering: PDF

We need other measurements, the neutron = proton with u ◄──► d

$$\frac{1}{x}F_2^{\text{neutron}} = \frac{4}{9}u_n(x) + \frac{1}{9}u_n(x) \simeq \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

With a linear combination of proton and deuteron data → xu(x) and xd(x)



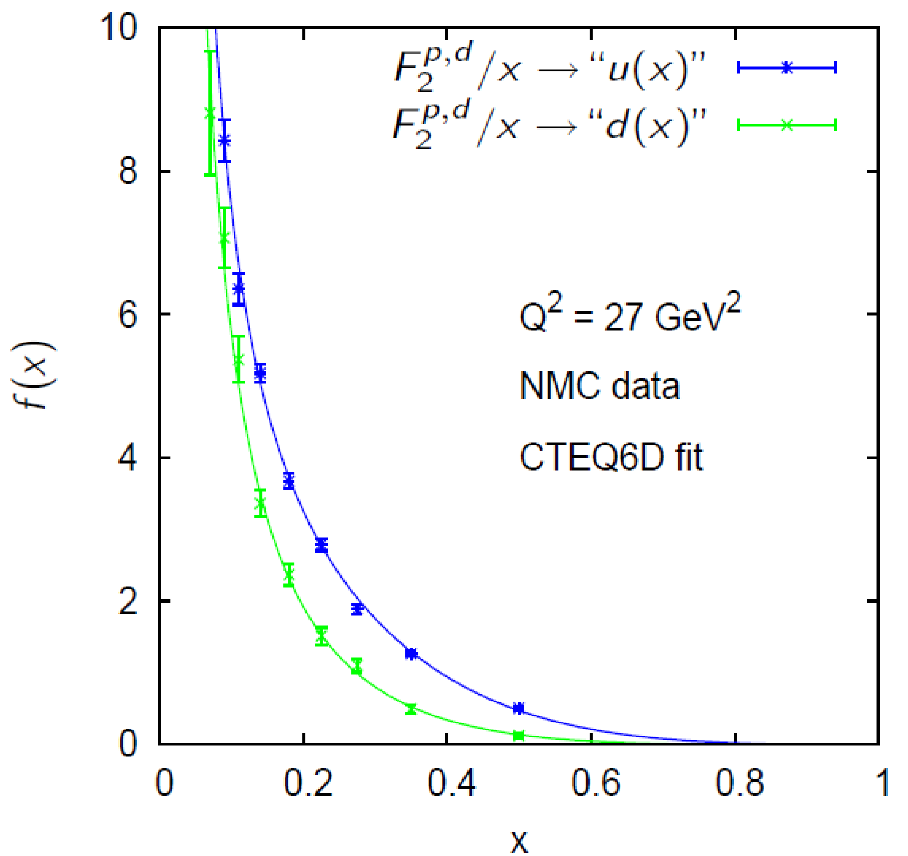
How many u and d quarks are present? Integrate u(x) or d(x) to find the total number of u or d quark.

Deep Inelastic Scattering: PDF - 2

We need other measurements, the neutron = proton with u ◄—► d

$$\frac{1}{x}F_2^{\text{neutron}} = \frac{4}{9}u_n(x) + \frac{1}{9}u_n(x) \simeq \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

With a linear combination of proton and deuteron data → xu(x) and xd(x)



Integrate u(x) or d(x) to find the total number of u or d quark.

PDFs seem to diverge for x → 0 .

In the model we did not include the "sea" quarks but only valence quarks.

In particular $\bar{u}(x)$ and $\bar{d}(x)$ are missing.:

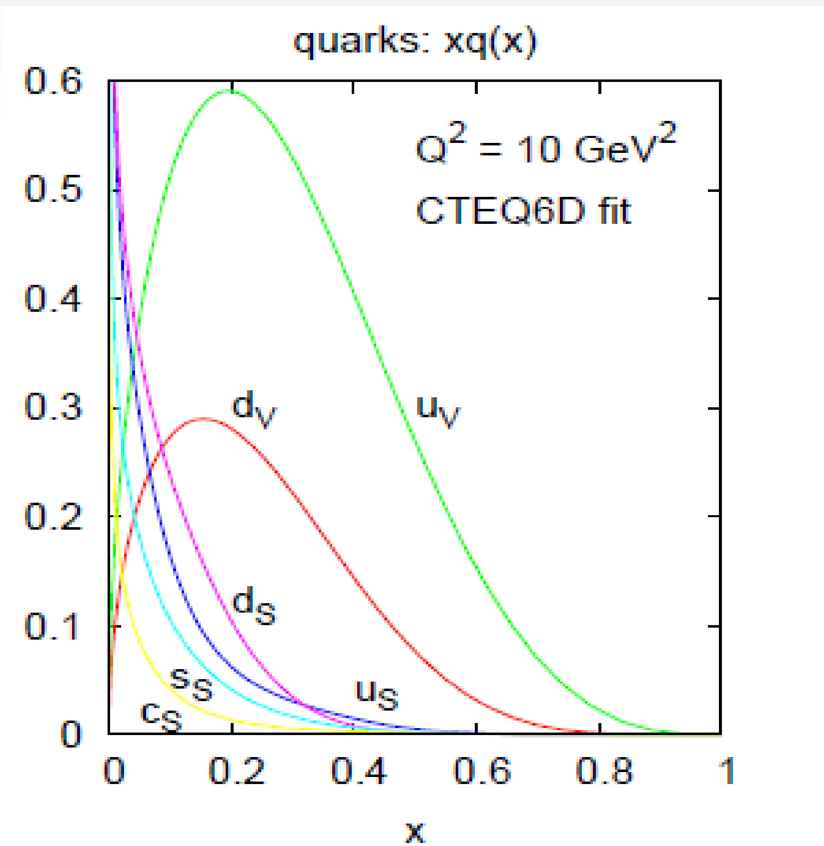
$$xu(x)+x\bar{u}(x) \qquad xd(x)+x\bar{d}(x)$$

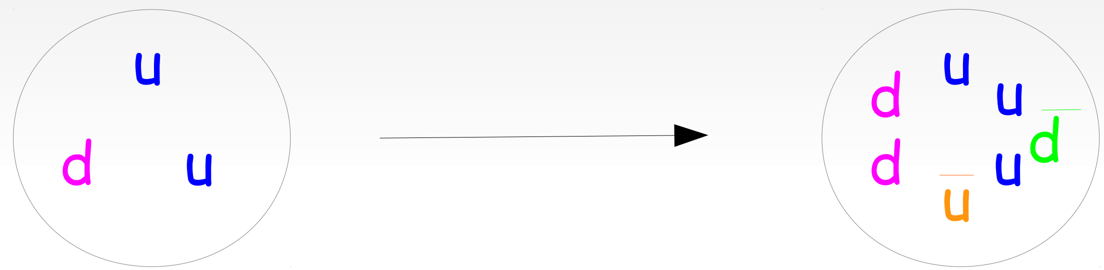
Deep Inelastic Scattering: PDF - 3

The new proton PDF:

$$F_2^{\text{proton}} = \frac{4}{9}\left(xu_p(x) + x\bar{u}_p(x)\right) + \frac{1}{9}\left(d_p(x) + \bar{d}_p(x)\right)$$

Saying p=uud → $\int_0^1 dx(u(x) - \bar{u}(x)) = 2\,, \qquad \int_0^1 dx(d(x) - \bar{d}(x)) = 1$





$u - \bar{u} = u_v$ is valence quark distribution

Valence quark have hard distribution

Sea quark have fairly soft distribution

Deep Inelastic Scattering: PDF - 4

Check sum-rule $$\sum_i \int dx\, x q_i(x) = 1$$

$$\sum_q \int_0^1 dx\, x q(x)$$ ≈0.5

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

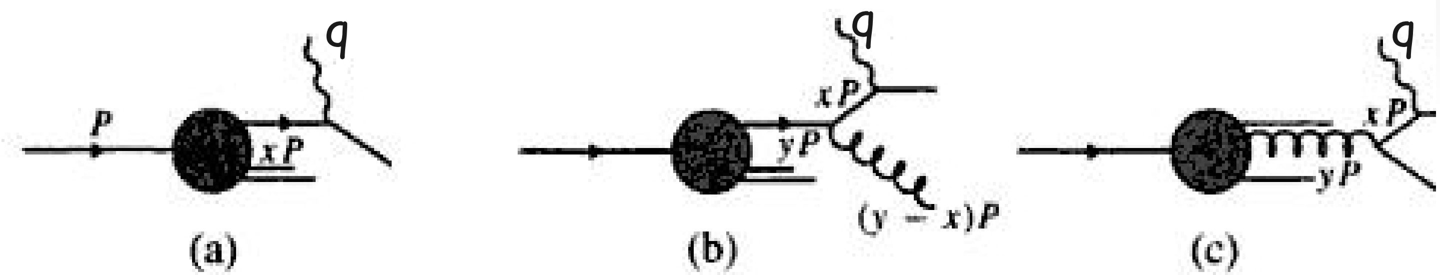
Where is the missing momentum?
There is one missing parton: gluon
which indeed is very important!

Deep Inelastic Scattering: DGLAP - 1

The PDFs depend on q^2. Let's assume $u(x, q^2)dx$ is the density of u with momentum fraction x → x+dx in a nucleon.

$$\frac{du(x, q^2)}{d\ln q^2} = \frac{\alpha_s(q^2)}{2\pi}\int_x^1 u(y, q^2) P_{QQ}\left(\frac{y}{x}\right)\frac{dy}{y}$$

(known as Altarelli-Parisi function) Let's try to understand it



a) quantum of momentum q absorption by quark with momentum fraction x at low q^2

b) quantum of momentum q absorption by quark with momentum fraction x which has radiated a gluon and which had a momentum fraction y

c) quantum of momentum q absorption by quark with momentum fraction x created by a gluon with momentum fraction greater than x

Deep Inelastic Scattering: DGLAP - 2

The events b) + c) that happen at high q^2 are described by the AP equation.

The gluon emission probability is proportional to α_s the probability that the quark retains a fraction z=x/y of its momentum is given by the so called splitting function:

$$P_{QQ}(z) = \frac{4}{3}\frac{(1+z^2)}{(1-z)}$$

The AP equation states that the increase du in u is proportional to α_s and to the integrated number of quarks with y>x that can radiate a gluon in a such way they fall in the interval x → x+dx

$$\frac{du(x,q^2)}{d\ln q^2} = \frac{\alpha_s(q^2)}{2\pi}\int_x^1 u(y,q^2)\,P_{QQ}\left(\frac{y}{x}\right)\frac{dy}{y}$$

This for the valence quark

Deep Inelastic Scattering: DGLAP - 3

If we include the sea quarks (case c for example) and the gluon we have a full PDF description, namely the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation:

$$\frac{d}{d\ln q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \frac{\alpha_s(q^2)}{2\pi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{pg} \end{pmatrix} \times \begin{pmatrix} q \\ g \end{pmatrix}$$

Only one flavor, in general that matrix has to span over all flavor

$$P_{QQ}(z) = \frac{4}{3}\frac{(1+z^2)}{(1-z)}$$

In analogy to P_{qq} the other splitting functions are defined

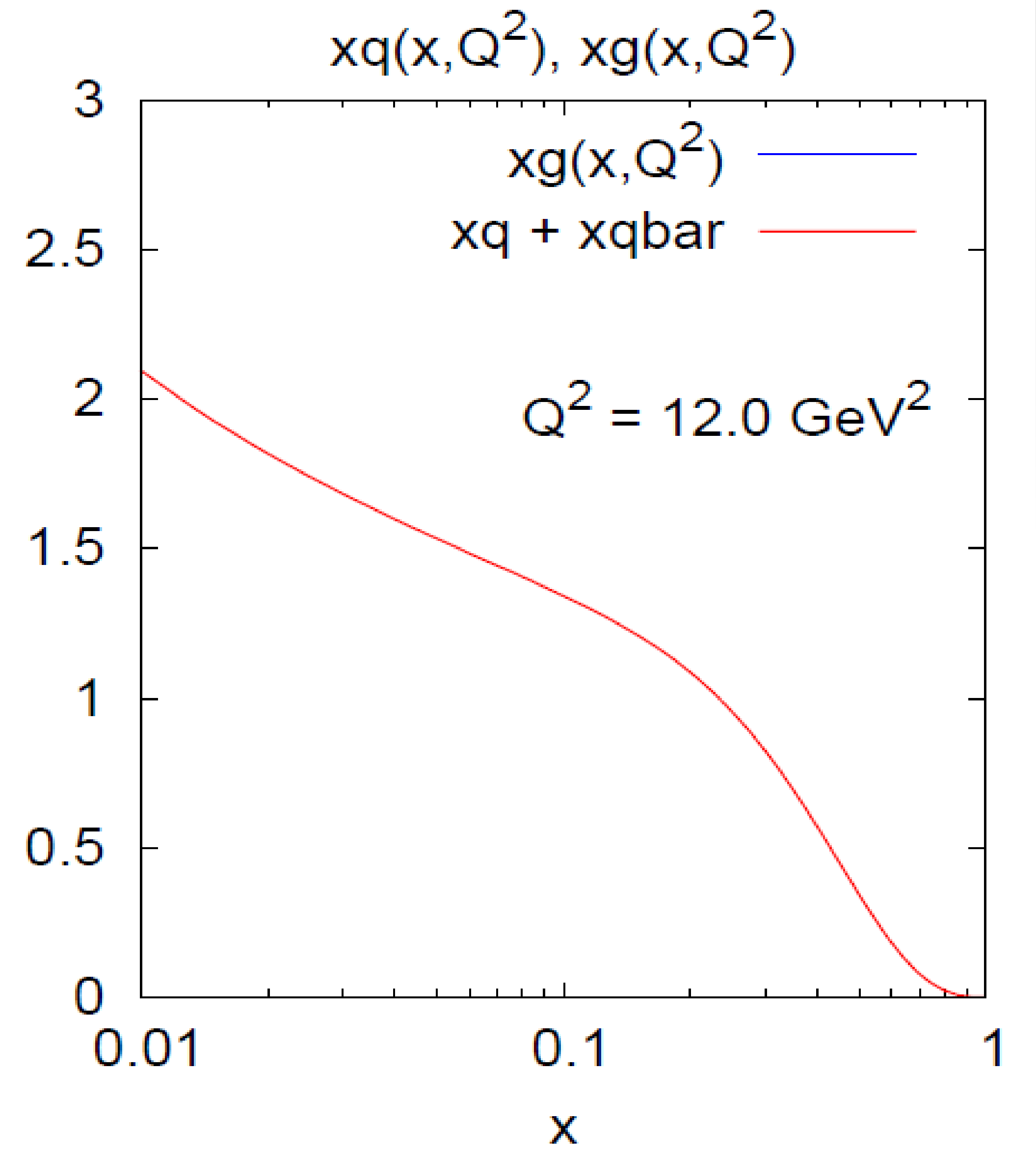
Significant properties:

P_{qg}, P_{gg}: symmetric $z \leftrightarrow 1-z$

P_{qq}, P_{gg}: diverge for $z \to 1$ — Soft gluon emission

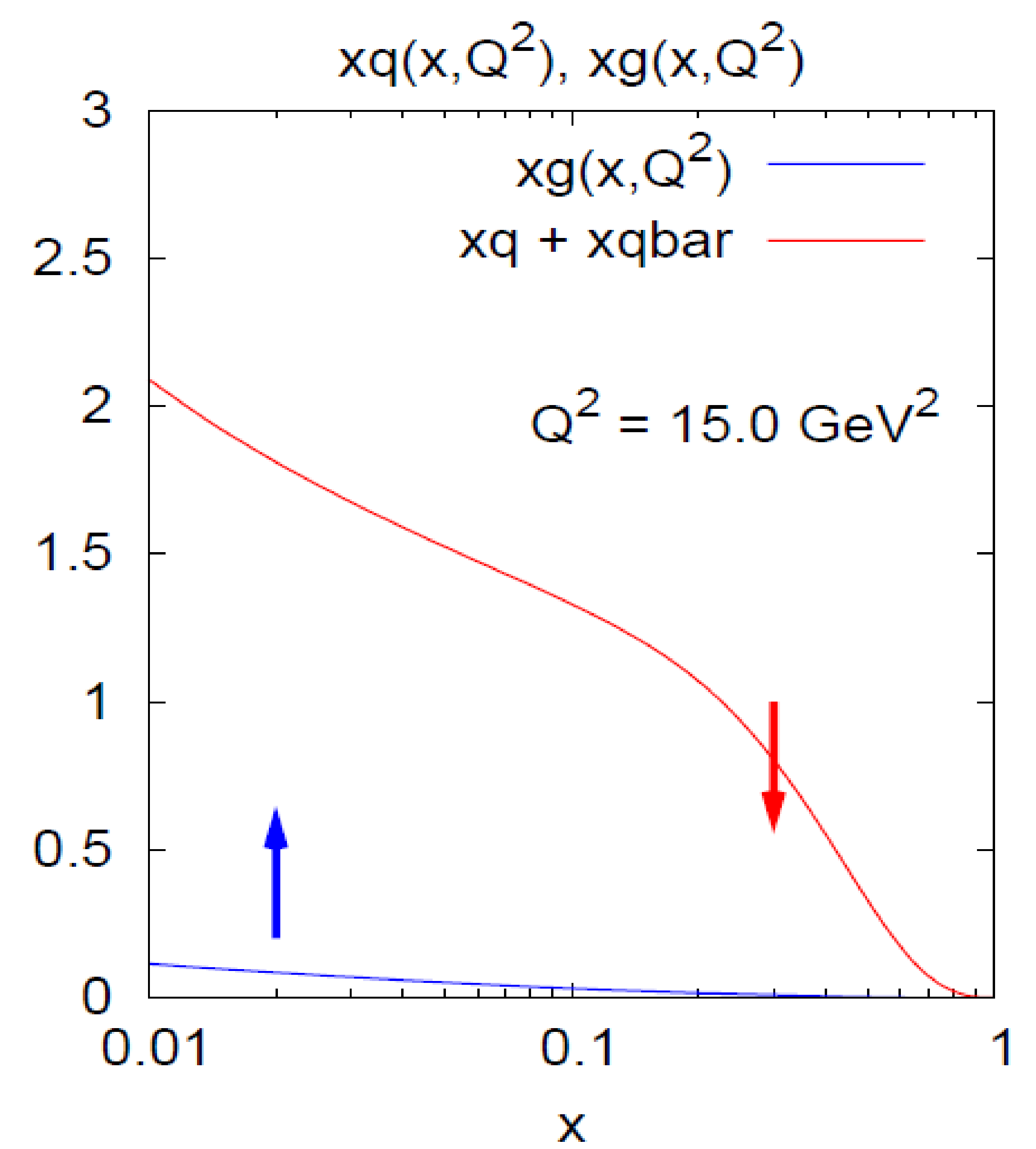
P_{gg}, P_{gq}: diverge for $z \to 0$ — PDF grow at low x

Effect of DGLAP



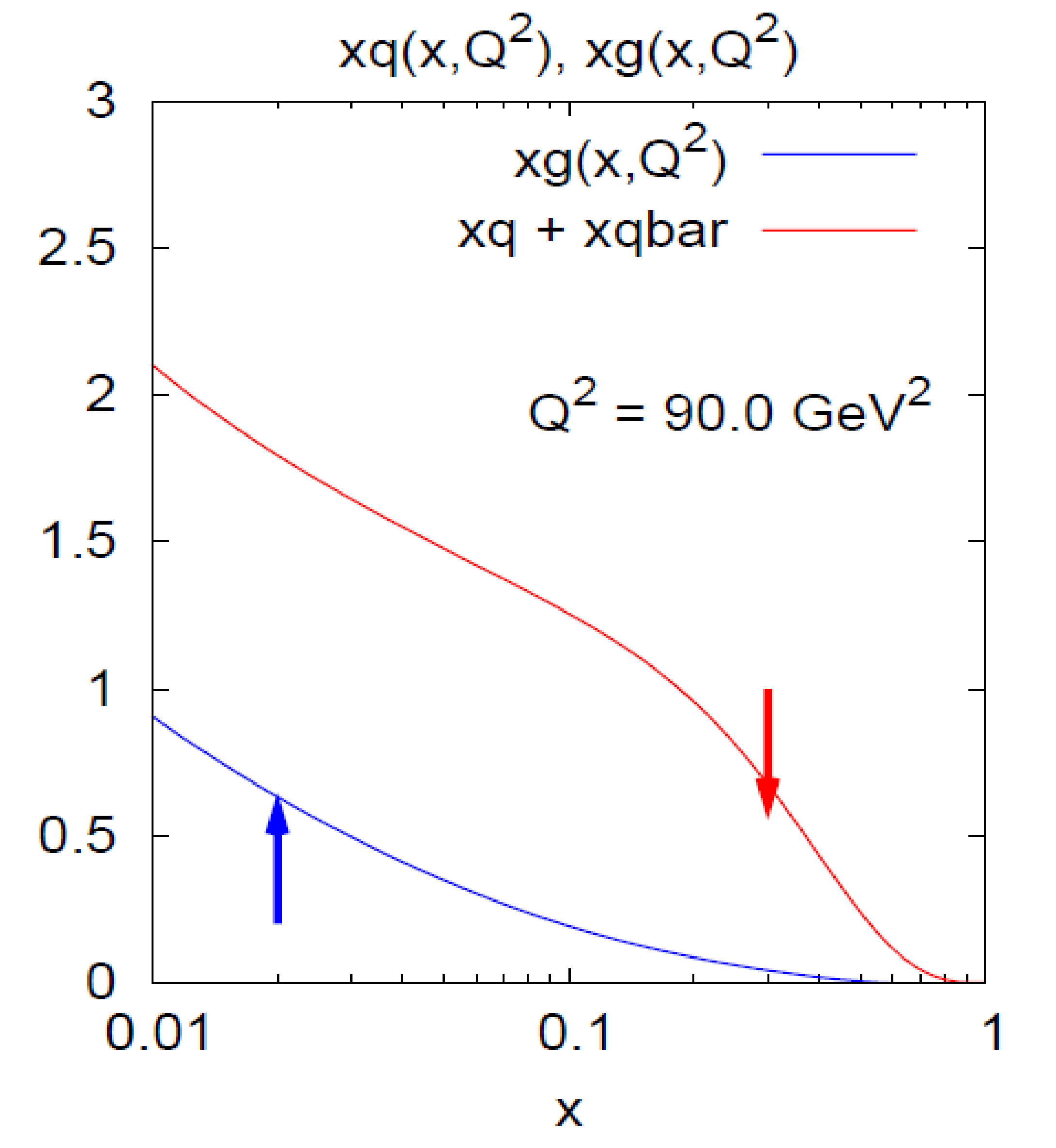
Start with only quark
depleted at large x

Effect of DGLAP



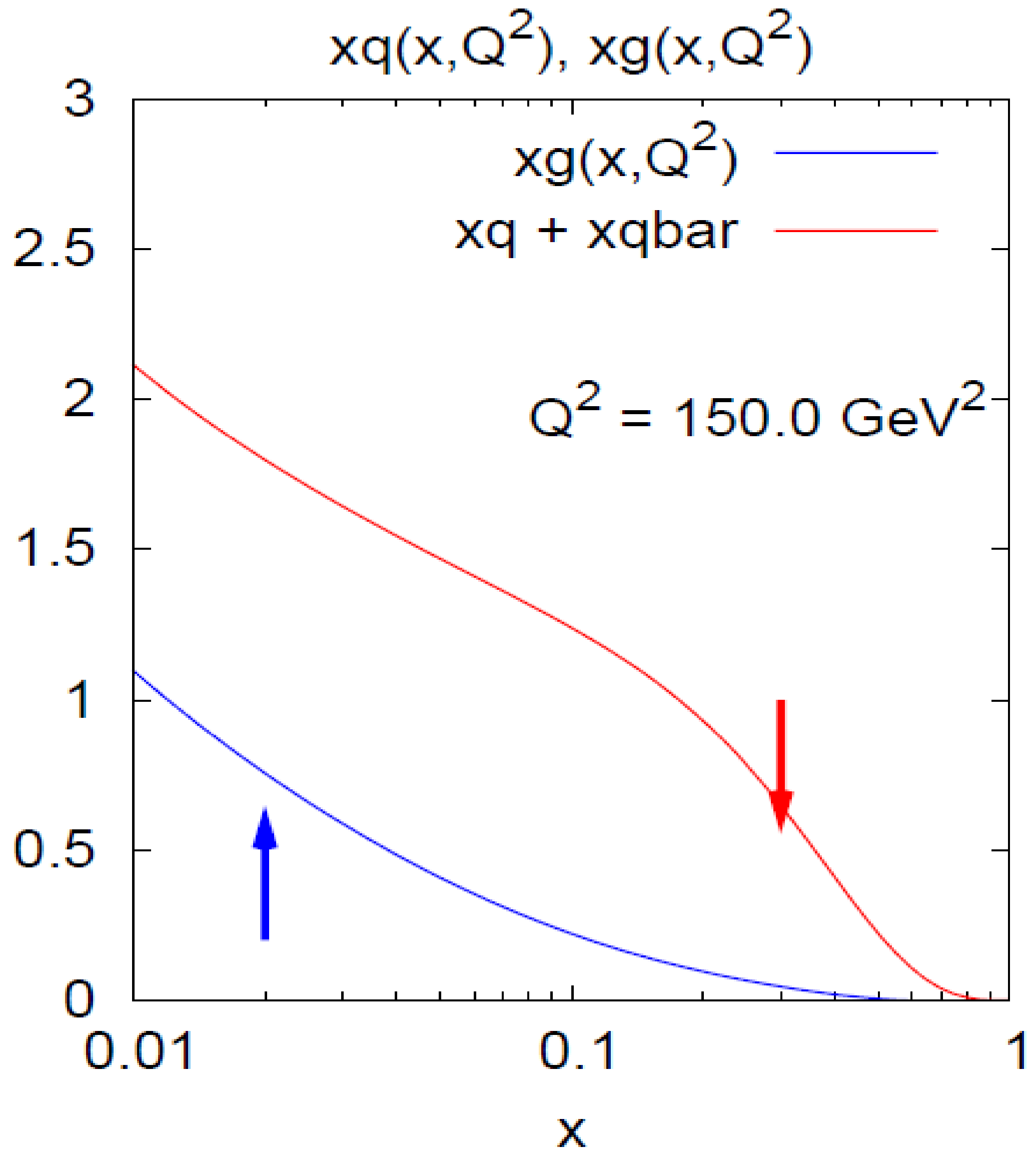
Depleted quark at large x
Gluon increase at small x

Effect of DGLAP



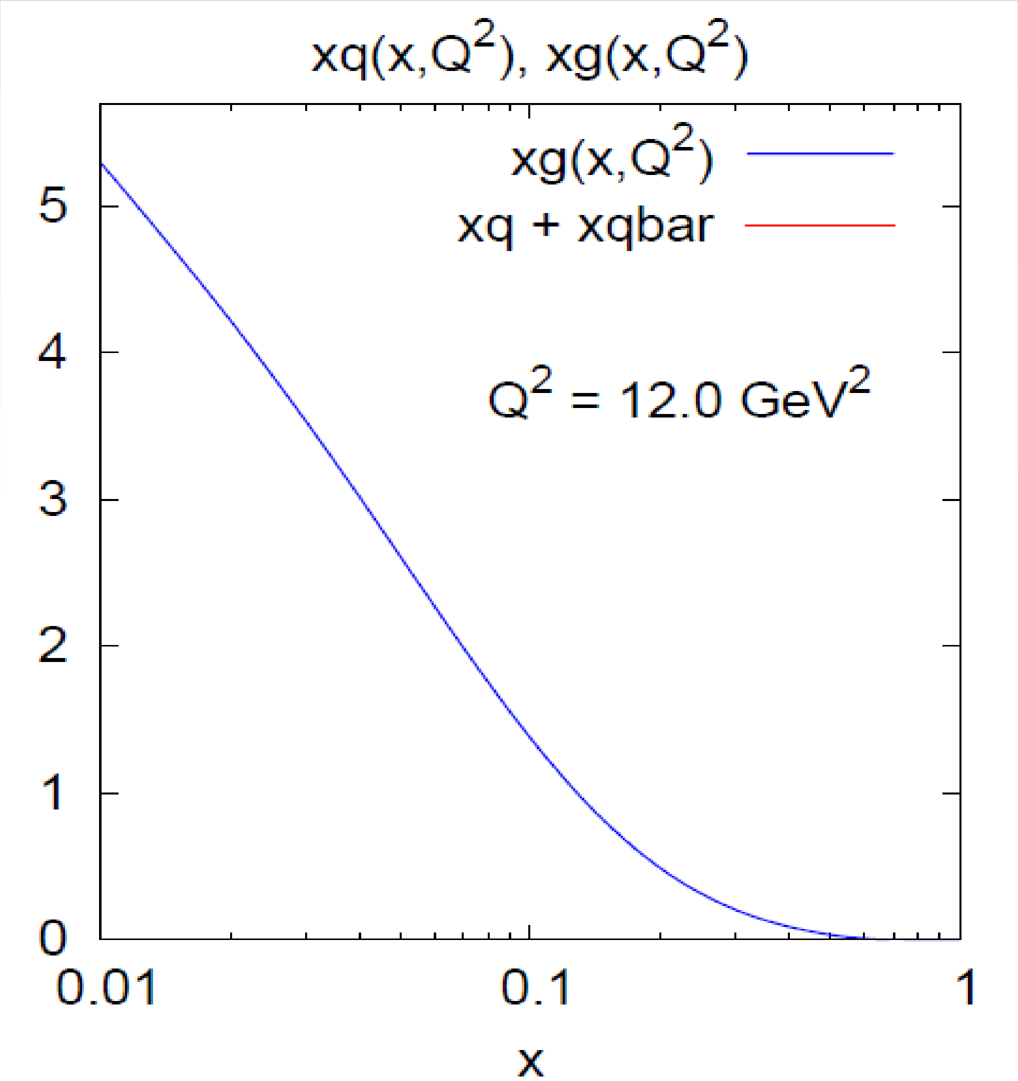
Depleted quark at large x
Gluon increase at small x

Effect of DGLAP



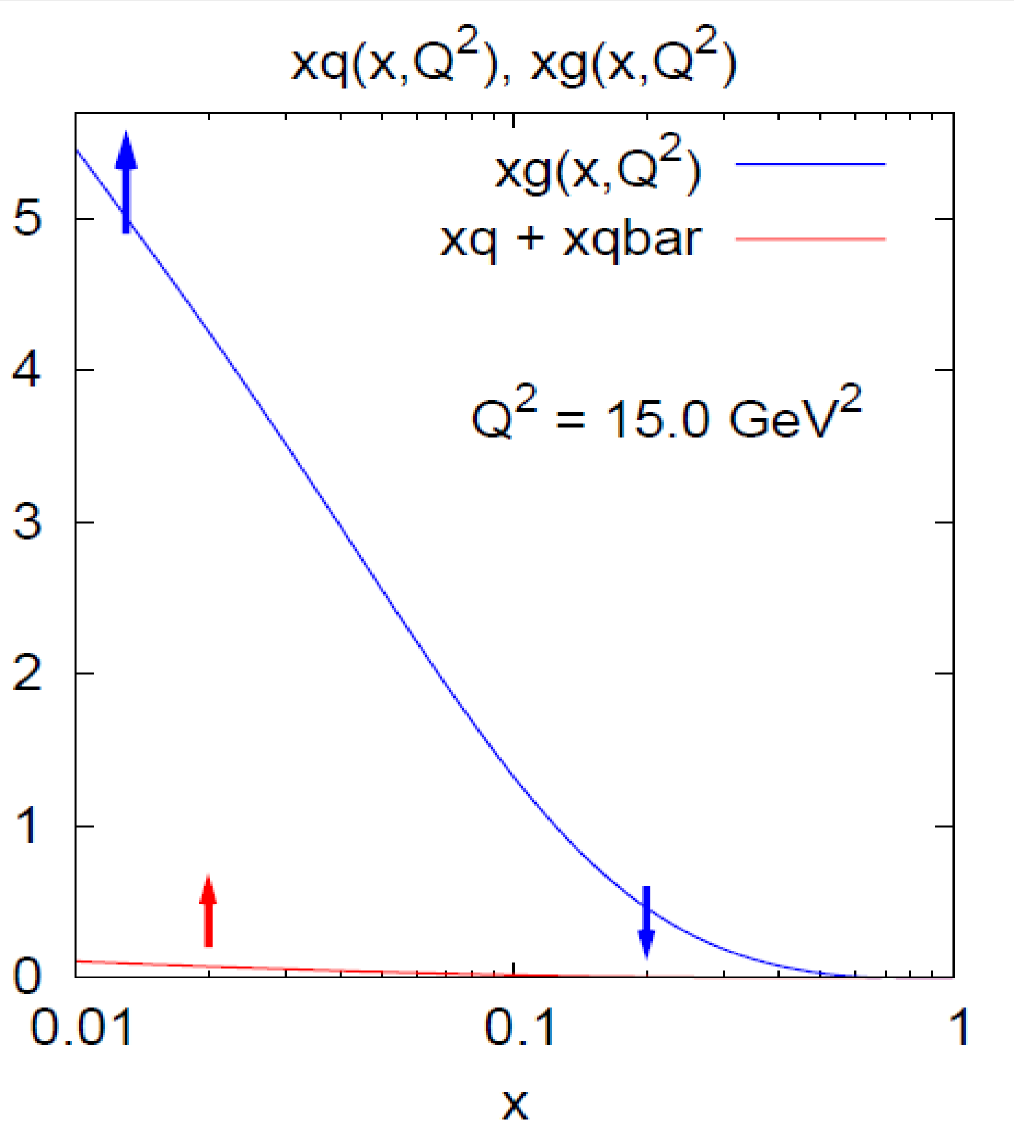
Depleted quark at large x
Gluon increase at small x

Effect of DGLAP



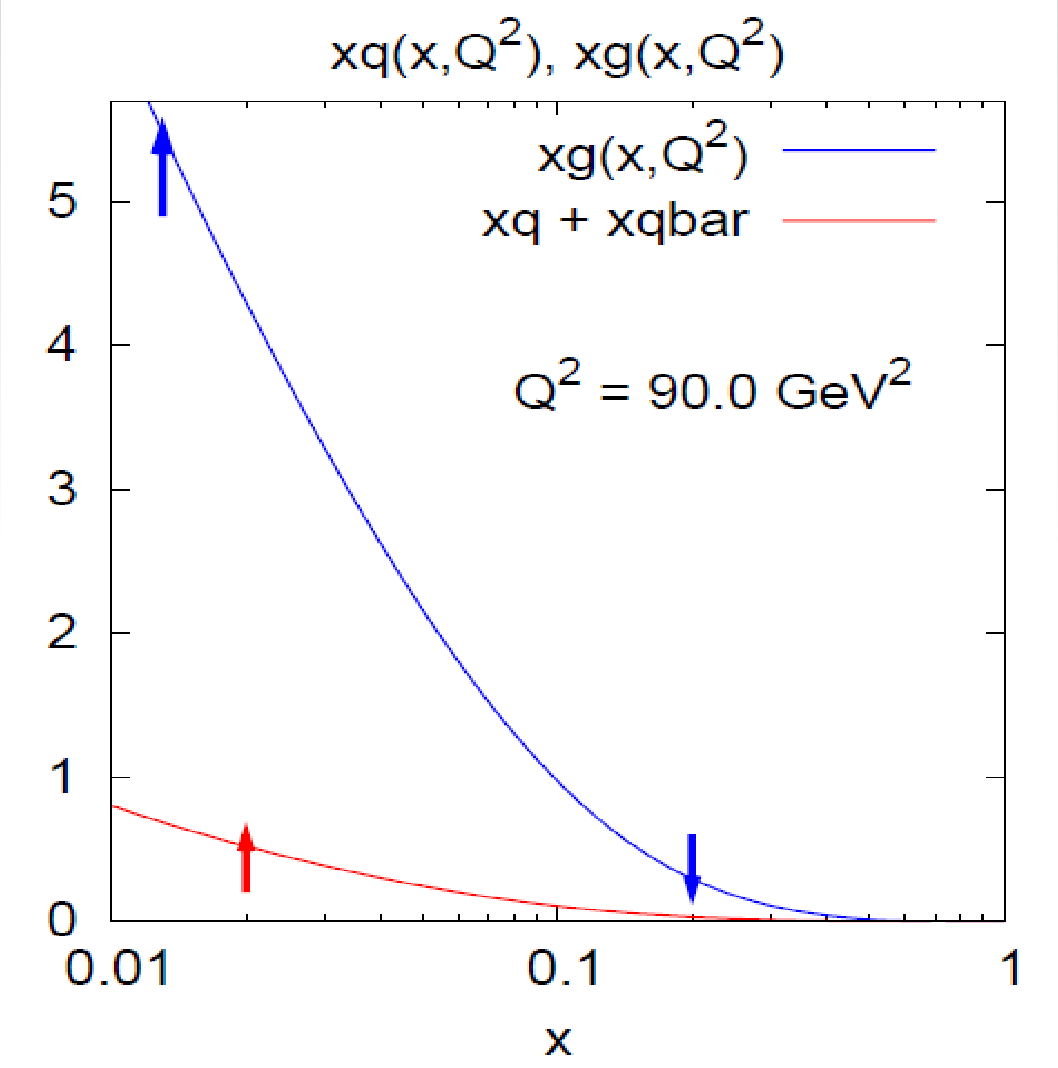
Start with gluon only
depleted at large x

Effect of DGLAP



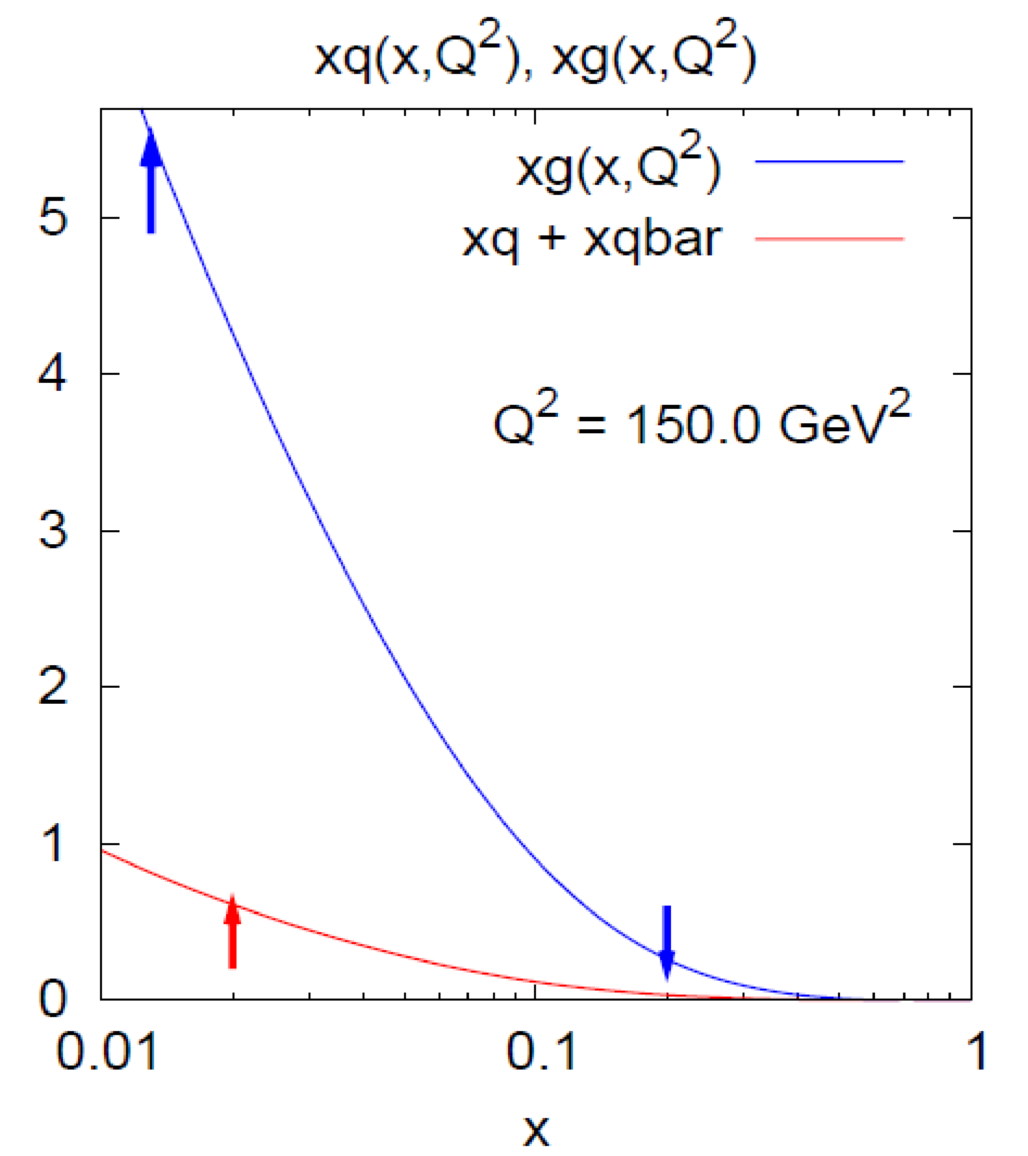
Gluon decreases at large x but increase at low x as the quark

Effect of DGLAP



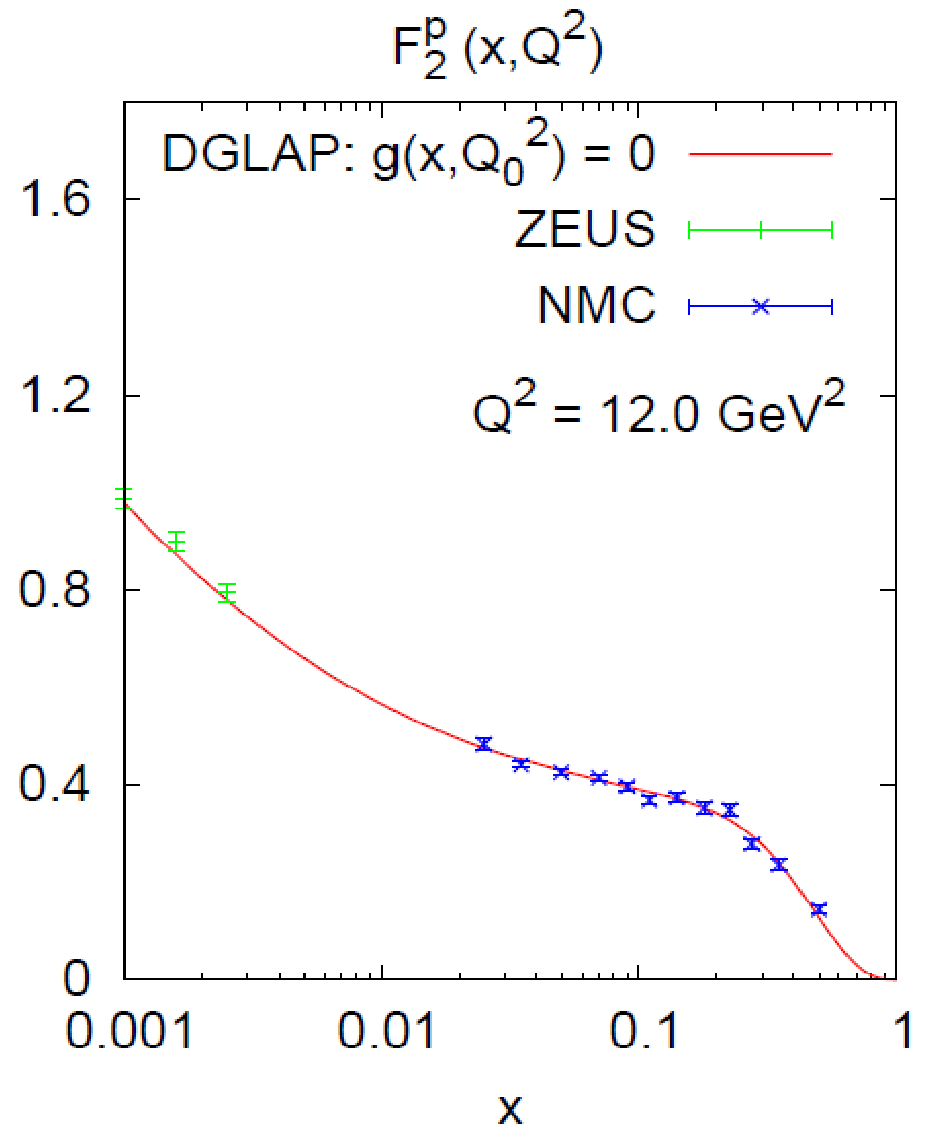
Gluon decreases at large x but increase at low x as the quark

Effect of DGLAP



Gluon decreases at large x but increase at low x as the quark

DGLAP on data

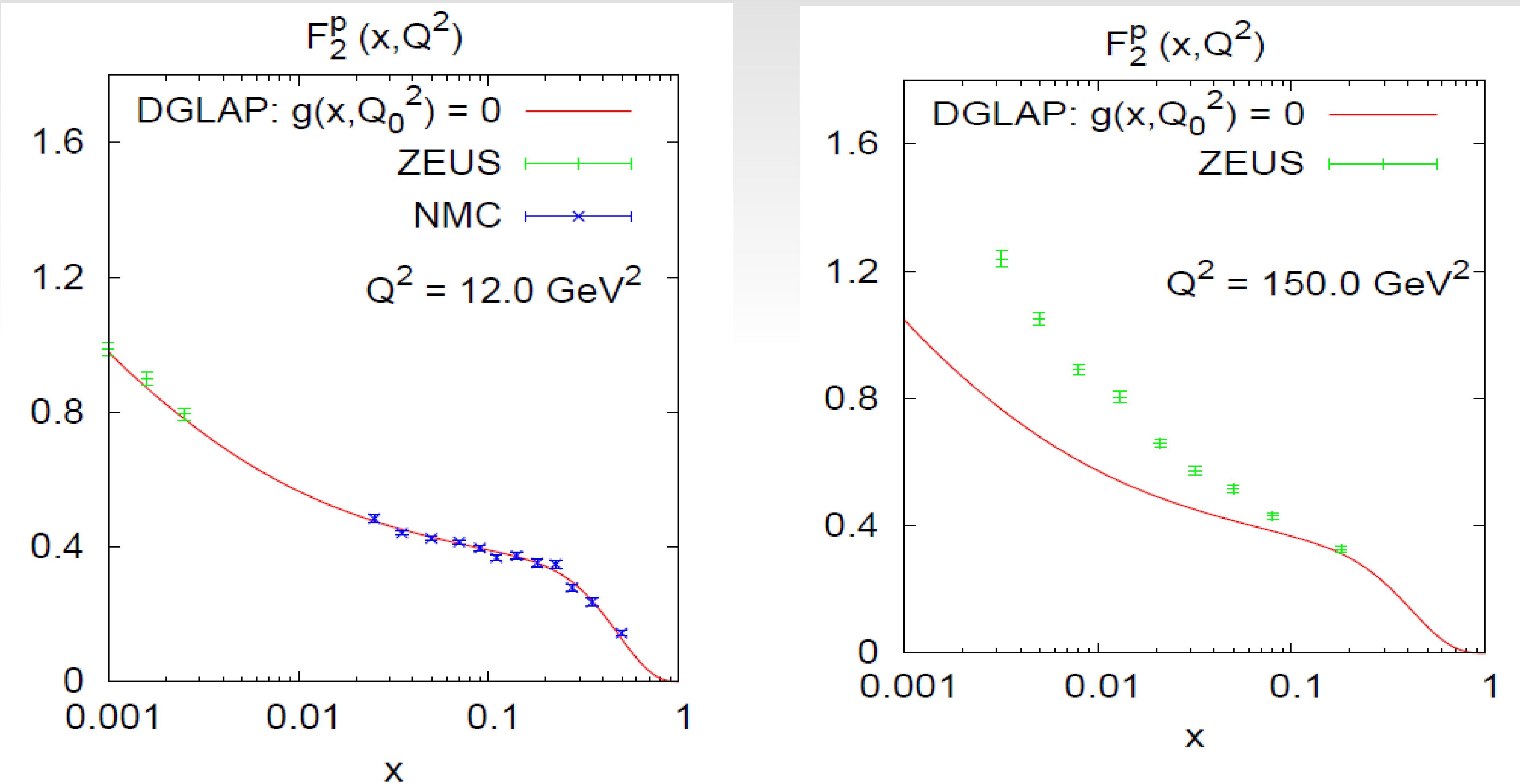


Fit F_2 at low q^2 assuming the gluon = 0

Evolve F_2 to high q^2 using DGLAP

DGLAP on data

Fit F_2 at low q^2 assuming gluon = 0 → Evolve F_2 to high q^2 using DGLAP



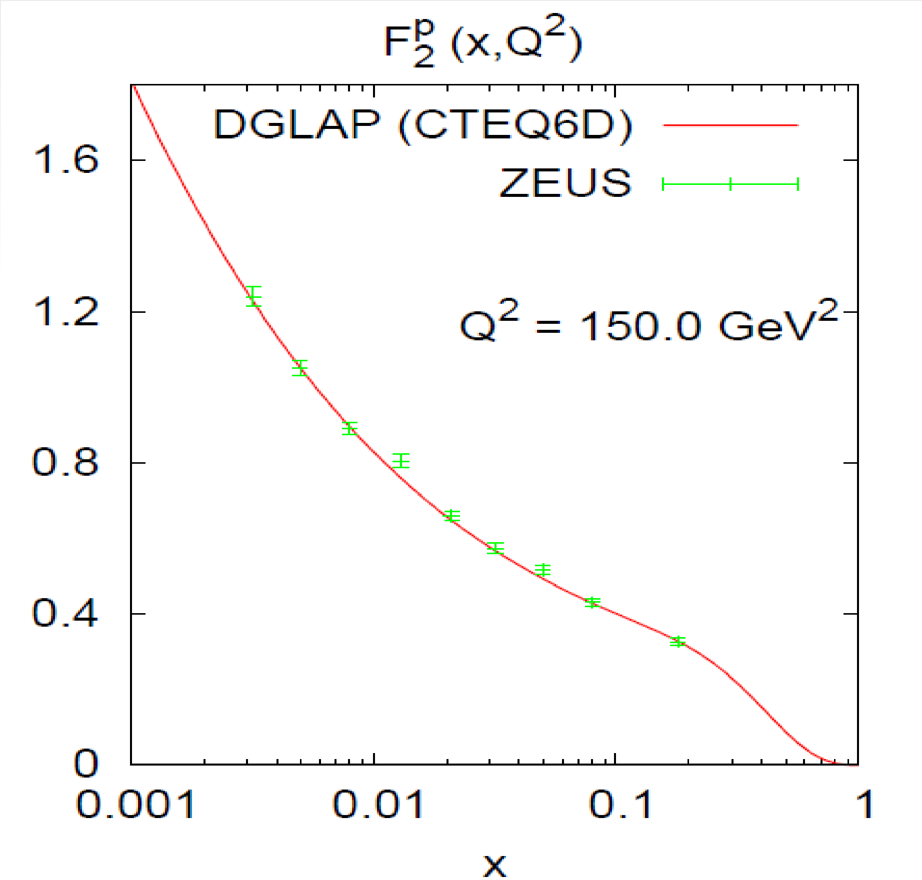
It does not work!

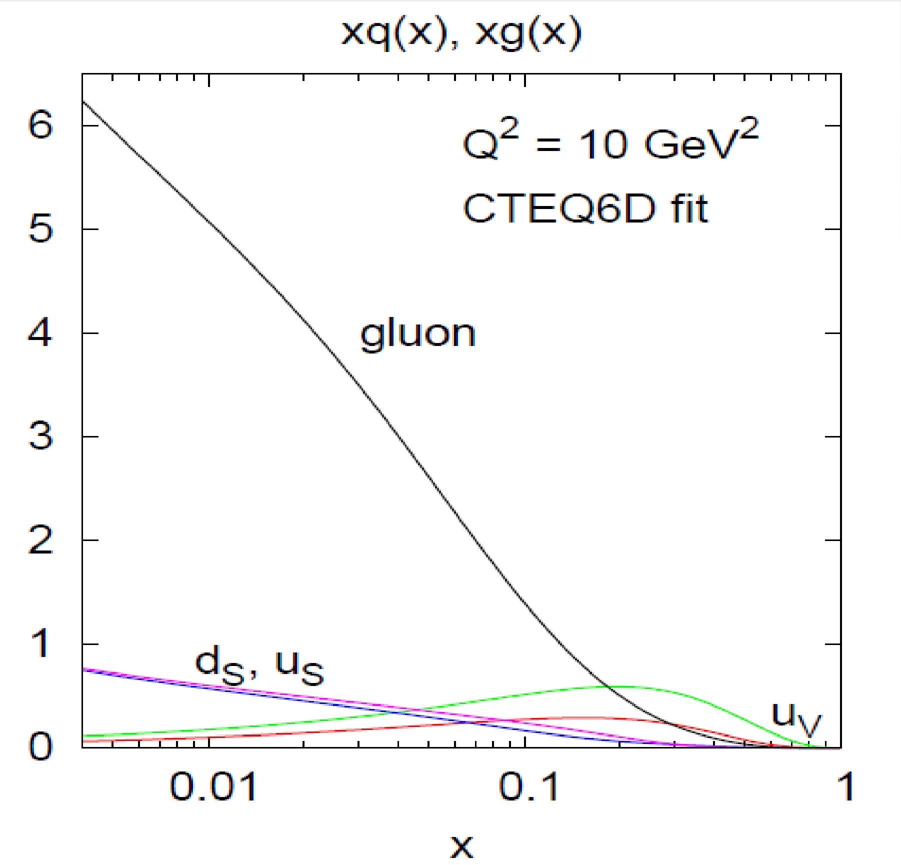
DGLAP on data

Fit F_2 at low q^2 with gluon → Evolve F_2 to high q^2 using DGLAP

g → qq generate extra quark at large q^2 → faster rise of F_2

Gluon distribution is huge

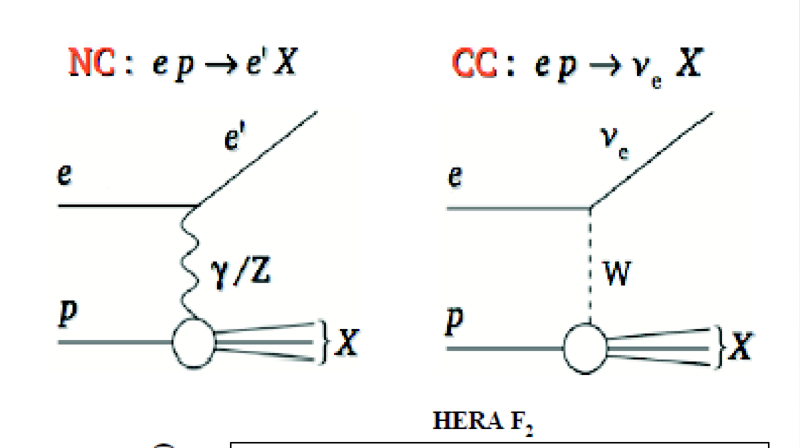




PDF Measurements

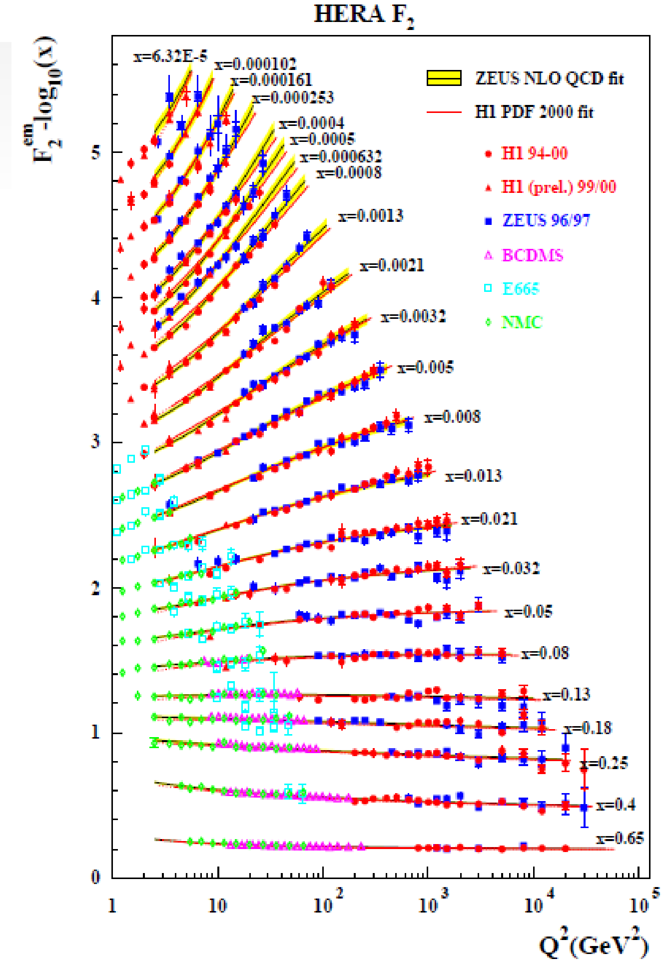
At HERA exploit these interactions. By selecting the final states it is measured the cross section:

$$\frac{d^2\sigma^{em}}{dxdQ^2} \simeq \frac{4\pi\alpha^2}{xQ^4}\left(\frac{1+(1-y)^2}{2}F_2^{em} + \mathcal{O}(\alpha_s)\right)$$

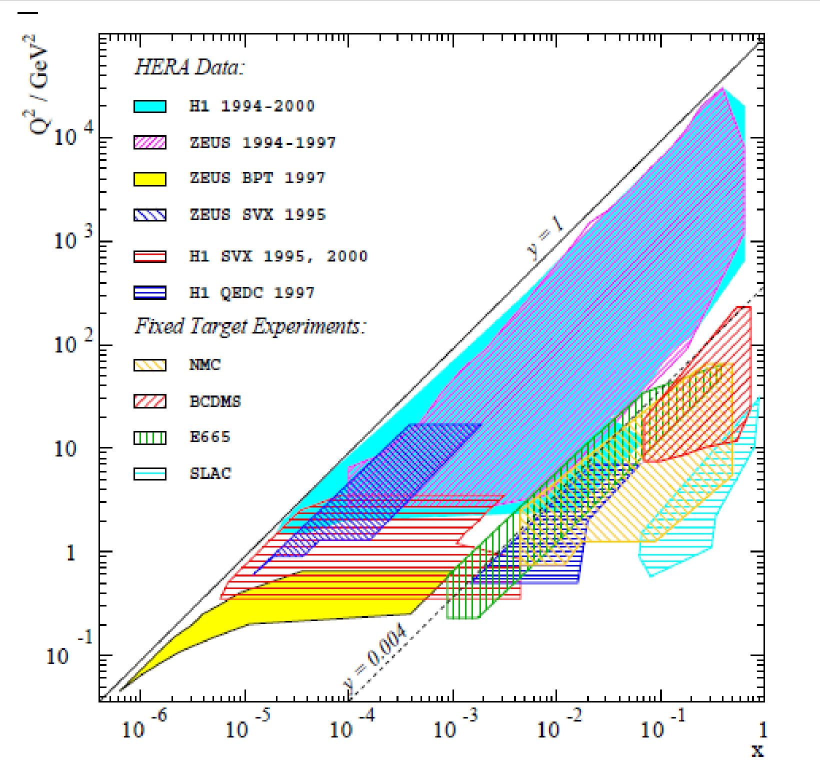


Different final states give access to different PDF

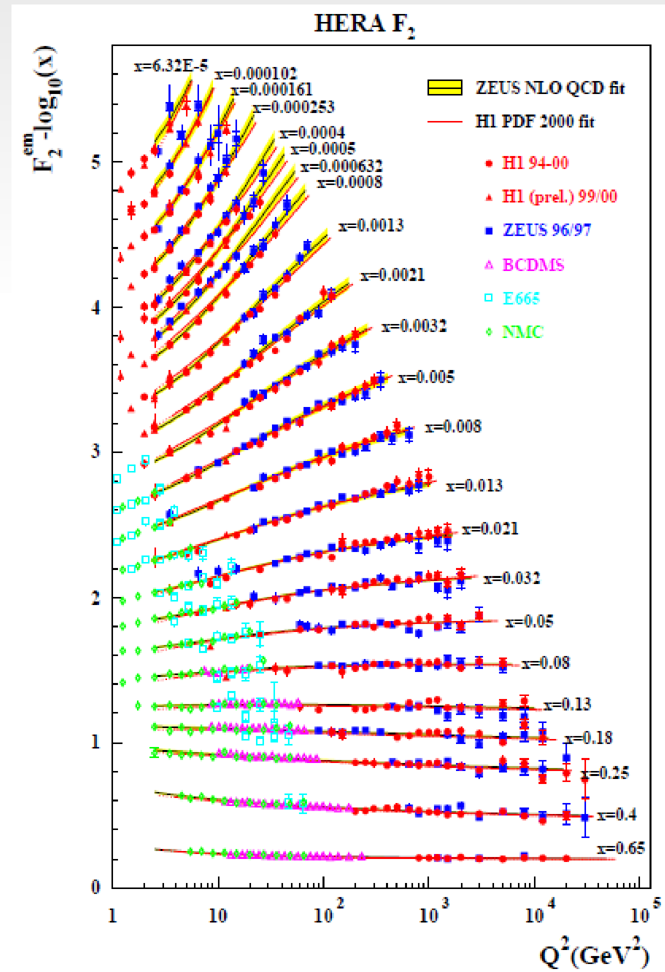
$$\sigma^{+}_{CC} \sim x(\bar{u}+\bar{c}) + x(1-y)^2(d+s)$$
$$\sigma^{-}_{CC} \sim x(u+c) + x(1-y)^2(\bar{d}+\bar{s})$$



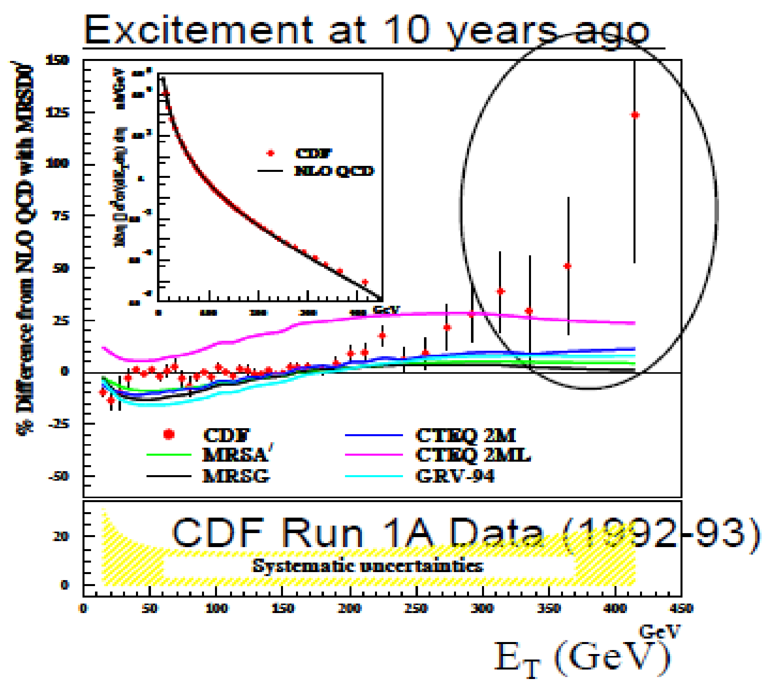
PDF Measurements



Kinematic region and data used for the fit



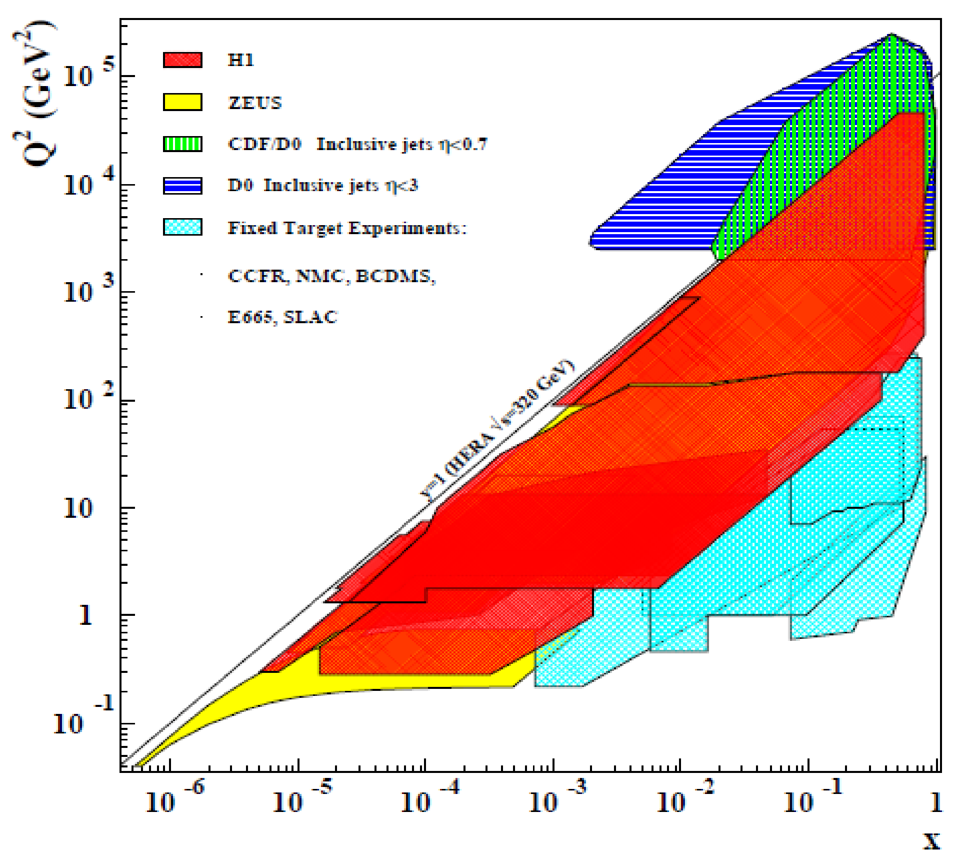
PDF Measurements



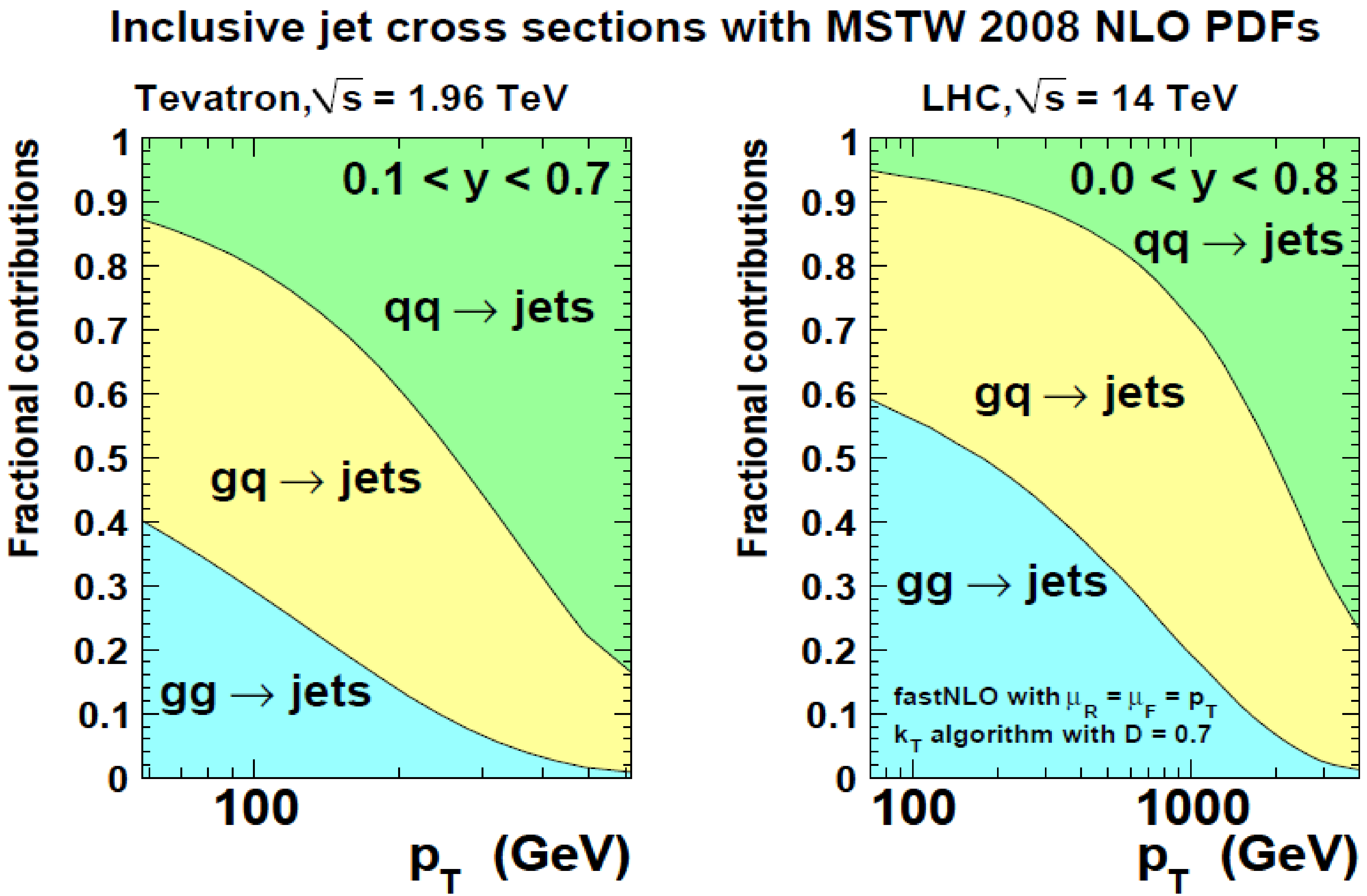
Phys. Rev. Lett. 77, 438 (1996)

The gluon PDF not very well known

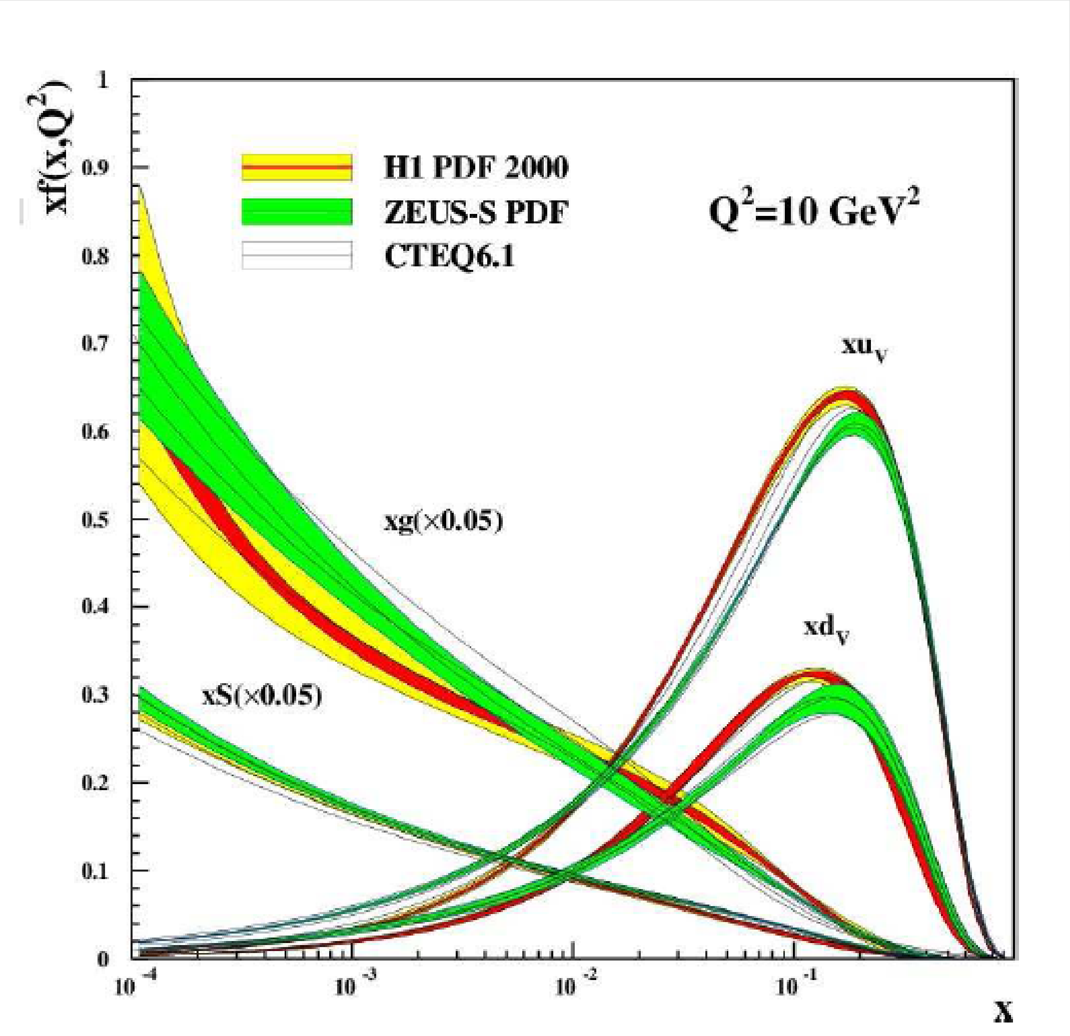
If the CDF/D0 data are included



PDF Channel Contribution



PDF Precision



Translate the experimental errors and theoretical uncertainties into uncertainty band on extracted PDF.

Use these bands to evaluate the reliability of the Monte Carlo predictions include or should include these uncertainties

PDF Measurements