Heavy Flavour Cross Sections Measurements

$$m_u \approx 3 \quad MeV$$
$$m_d \approx 5 \quad MeV$$
$$m_s \approx 100 \quad MeV$$

very light $m < \Lambda_{QCD}$

$$m_c \approx 1300 \quad MeV$$
$$m_b \approx 4200 \quad MeV$$

Flavours considered

$$m_t \approx 170000 \quad MeV$$

→ very heavy

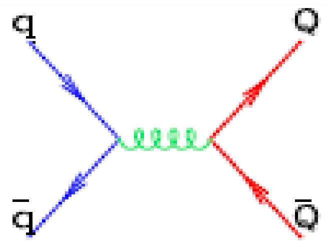
Heavy Flavour Cross Sections at LO

The leading-order process for the production of heavy quark Q of mass m in hadron collisions:

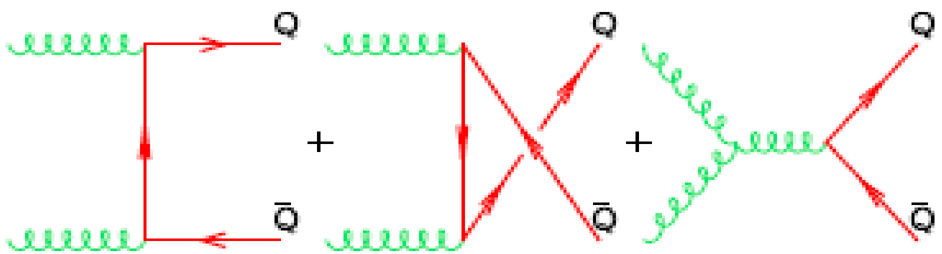
$$
\begin{array}{ll}
(a) & q(p_1) + \overline{q}(p_2) \to Q(p_3) + \overline{Q}(p_4) \\
(b) & g(p_1) + g(p_2) \to Q(p_3) + \overline{Q}(p_4)
\end{array}
$$

Where the four momenta of the partons are given in brackets.

The Feynman diagrams are:



(a)



(b)

Heavy Flavour Cross Sections at LO

The invariant matrix elements squared averaged over initial and final color and spin

Process	$\overline{\sum}\|\mathcal{M}\|^2/g^4$
$q\,\overline{q} \rightarrow Q\,\overline{Q}$	$\frac{4}{9}\left(\tau_1^2+\tau_2^2+\frac{\rho}{2}\right)$
$g\,g \rightarrow Q\,\overline{Q}$	$\left(\frac{1}{6\tau_1\tau_2}-\frac{3}{8}\right)\left(\tau_1^2+\tau_2^2+\rho-\frac{\rho^2}{4\tau_1\tau_2}\right)$

Where it has been introduced the notation:

$$\tau_1 = \frac{2p_1.p_3}{\hat{s}}, \quad \tau_2 = \frac{2p_2.p_3}{\hat{s}}, \quad \rho = \frac{4m^2}{\hat{s}}, \quad \hat{s} = (p_1+p_2)^2$$

The short-distance cross section is obtained from the invariant matrix element:

$$d\hat{\sigma}_{ij} = \frac{1}{2\hat{s}} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

Heavy Flavour Cross Sections at LO

In terms of rapidity $y = \frac{1}{2}\ln((E+p_z)/(E-p_z))$ and of transverse momentum P_T the relativistically invariant space volume element of the final state heavy quark is :

$$\frac{d^3p}{E} = dy\, d^2p_T$$

The invariant cross section may be written at LO:

$$\frac{d\sigma}{dy_3 dy_4 d^2p_T} = \frac{1}{16\pi^2\hat{s}^2}\sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2)\overline{\sum}|\mathcal{M}_{ij}|^2$$

x_1 and x_2 are fixed if transverse momenta and rapidity of the outgoing heavy quarks are known. In the CM of the incoming hadrons we can write

$$\begin{aligned}
p_1 &= \frac{1}{2}\sqrt{s}(x_1,0,0,x_1)\\
p_2 &= \frac{1}{2}\sqrt{s}(x_2,0,0,-x_2)\\
p_3 &= (m_T\cosh y_3, p_T, 0, m_T\sinh y_3)\\
p_4 &= (m_T\cosh y_4, -p_T, 0, m_T\sinh y_4)
\end{aligned}$$

Heavy Flavour Cross Sections at LO

Applying the energy and momentum conservation

$$x_1 = \frac{m_T}{\sqrt{s}}\left(e^{y_3} + e^{y_4}\right)$$

$$x_2 = \frac{m_T}{\sqrt{s}}\left(e^{-y_3} + e^{-y_4}\right)$$

$$\hat{s} = 2m_T^2(1 + \cosh \Delta y).$$

$m_T = \sqrt{m^2 + p_T^2}$ transverse mass of the heavy quarks

$\Delta y = y_3 - y_4$ rapidity difference between heavy quark

With this notation the matrix elements

$$\overline{\sum}|\mathcal{M}_{q\bar{q}}|^2 = \frac{4g^4}{9}\left(\frac{1}{1+\cosh(\Delta y)}\right)\left(\cosh(\Delta y) + \frac{m^2}{m_T^2}\right), \quad \sim \text{costant}$$

$$\overline{\sum}|\mathcal{M}_{gg}|^2 = \frac{g^4}{24}\left(\frac{8\cosh(\Delta y) - 1}{1+\cosh(\Delta y)}\right)\left(\cosh(\Delta y) + 2\frac{m^2}{m_T^2} - 2\frac{m^4}{m_T^4}\right) \quad \sim \exp(-\Delta y)$$

Low contribution at high Δy and dominant contribution for $\Delta y<1$
Heavy quarks produced by light quark are more correlated in rapidity respect to those produced by gluon-gluon fusion.

Applicability of Perturbation Theory

The propagators in the diagrams:

$$
\begin{aligned}
(p_1+p_2)^2 = &\quad 2p_1.p_2 &= 2m_T^2\left(1+\cosh\Delta y\right)\,, \\
(p_1-p_3)^2-m^2 = &\quad -2p_1.p_3 &= -m_T^2\left(1+e^{-\Delta y}\right)\,, \\
(p_2-p_3)^2-m^2 = &\quad -2p_2.p_3 &= -m_T^2\left(1+e^{\Delta y}\right)\,.
\end{aligned}
$$

are off-shell by a quantity of the order of m^2 so the perturbation theory should be applicable.

This is valid until the mass m is larger of Λ_{QCD}.

The question is if the bottom and charm mass are large enough.

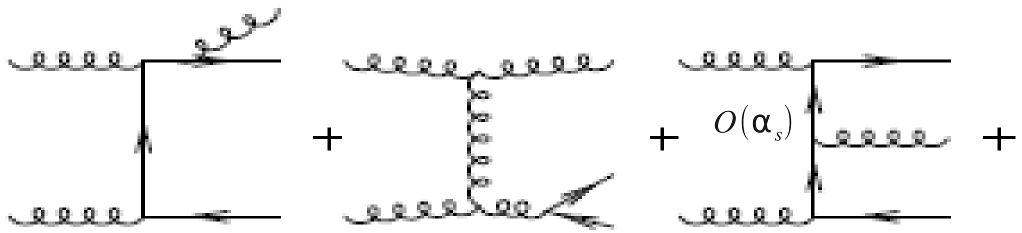
Heavy Flavour Cross Sections at higher-order

At NLO, $O(\alpha_s^3)$, the production cross section of the heavy flavor quark of mass m:

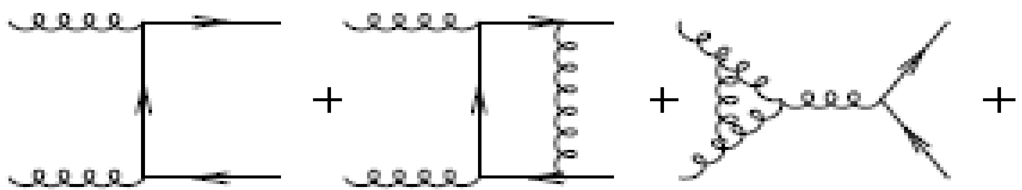
$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2\, \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

Where $\hat{\sigma}_{i,j}(\hat{s}, m^2, \mu^2) = \sigma_0 c_{ij}(\hat{\rho}, \mu^2)$ $\hat{\rho} = 4m^2/\hat{s}, \bar{\mu}^2 = \mu^2/m^2, \sigma_0 = \alpha_S^2(\mu^2)/m^2$

Examples of higher order diagrams



Real emission diagrams



Virtual emission diagrams

Heavy Flavour Cross Sections at higher-order

There are several dependences.

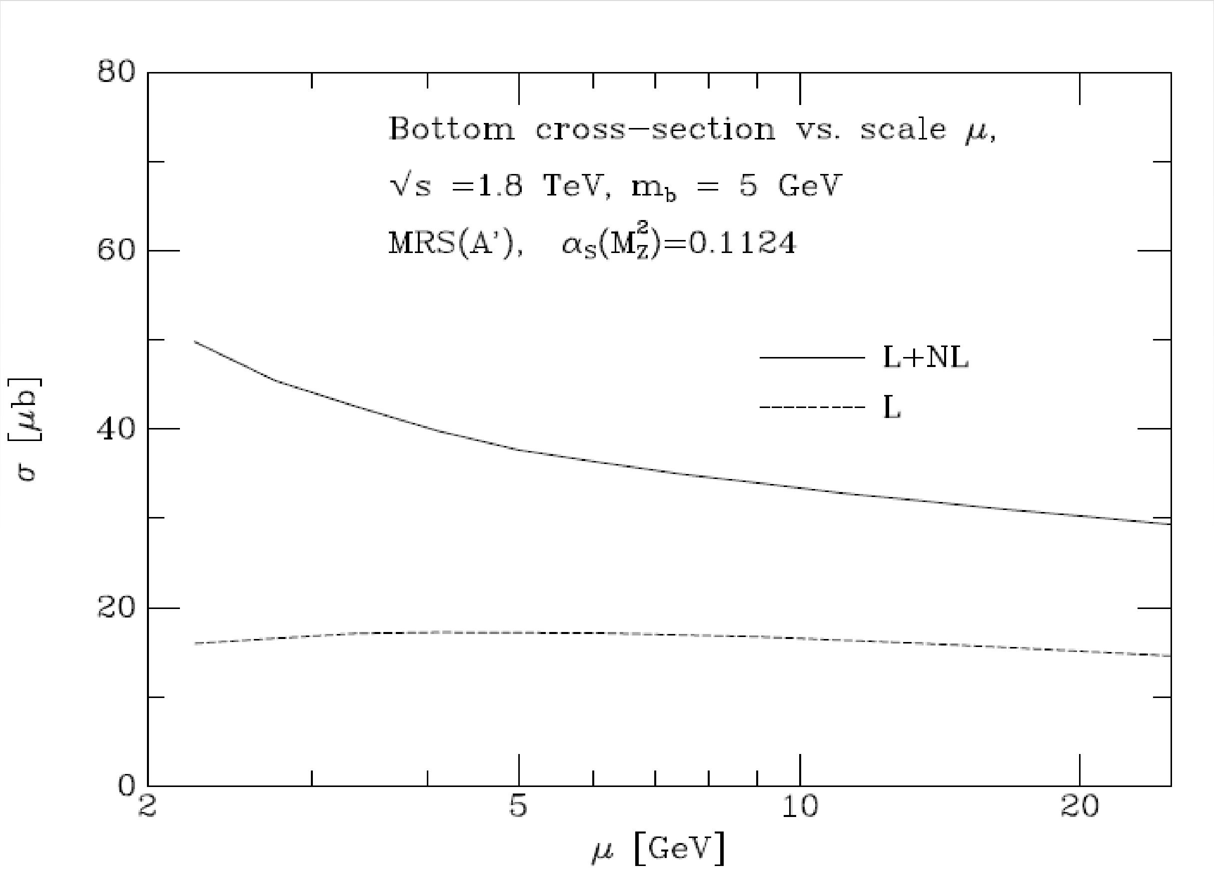
1) Scale, μ:

- PDF following the DGLAP equations
- running coupling constant
- short-distance cross section: if we perform a calculation to $O(\alpha_s^3)$, the variation of the scale contributes: $\mu^2 \frac{d}{d\mu^2}\sigma = O(\alpha_S^4)$.

The above variations combine in a such a way that the scale dependence is formally small because of higher order in α_s (α_s^4)

This does not guarantee that the numerical value of the cross section is smaller for higher series when varying the scale.

Heavy Flavour Cross Sections at higher-order



LO cross section is almost scale independent because of α_s behavior and increased gluon distribution with μ

NLO is almost 2xLO → large uncertainties on the cross section

Heavy Flavour Cross Sections at higher-order

2) Heavy quark mass m :

$1.2 < m_c < 1.8$ GeV

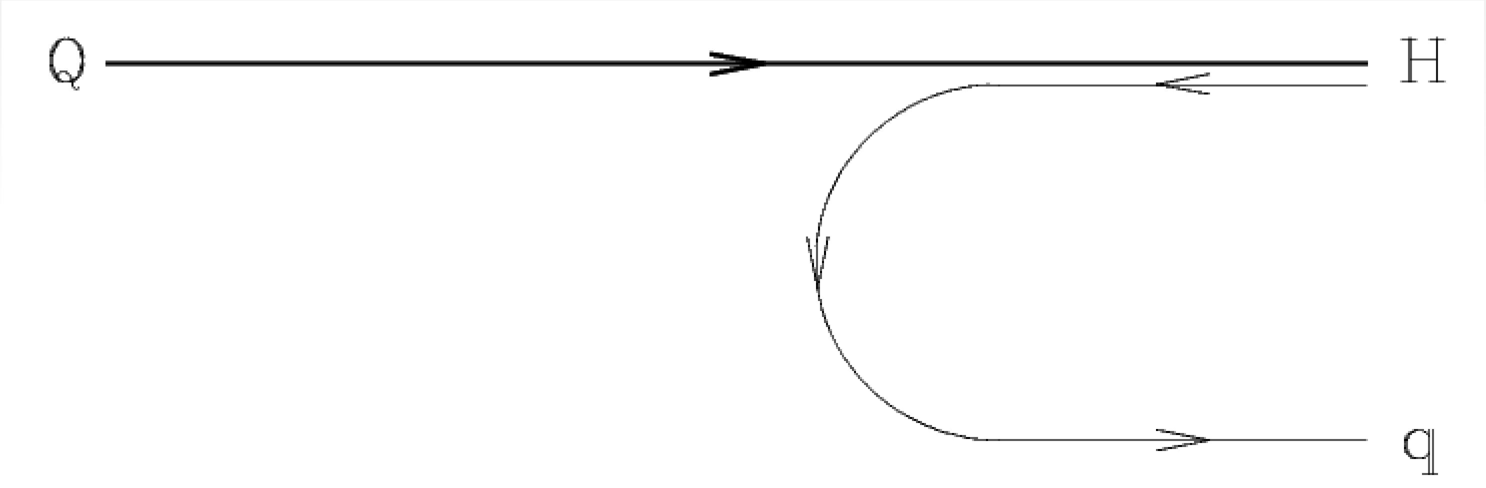
$4.5 < m_b < 5.0$ GeV

- explicit dependence on $1/m^2$ in the short-distance cross section
- PDF, as m decreases the x value at which the PDF are calculated become smaller and the cross section increases because the parton flux increase
- α_s depends on the the scale μ and Λ if we take $m/2 < \mu < 2m$ we have problems with the charm because we arrive at $\mu < 1$ GeV where the perturbation theory is not valid → for charm the lower limit for $\mu = 2m_c$

$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln(\frac{\mu^2}{\Lambda^2})}$$

Heavy Quarks Fragmentation

Heavy quarks after the production fragment in hadrons.
The model is different of those used for light quarks
the attachment of a light quark to and heavy one Q produce a small deceleration of the heavy quark Q.



Heavy Quarks Fragmentation

An heavy quark Q of momentum P generates a hadron H=Qq of momentum zP. To model this process the energy difference before and after the fragmentation is needed

$$\Delta E = E_Q - E_H - E_q = \sqrt{m_Q^2 + P^2} - \sqrt{m_H^2 + z^2 P^2} - \sqrt{m_q^2 + (1-z)^2 P^2}$$

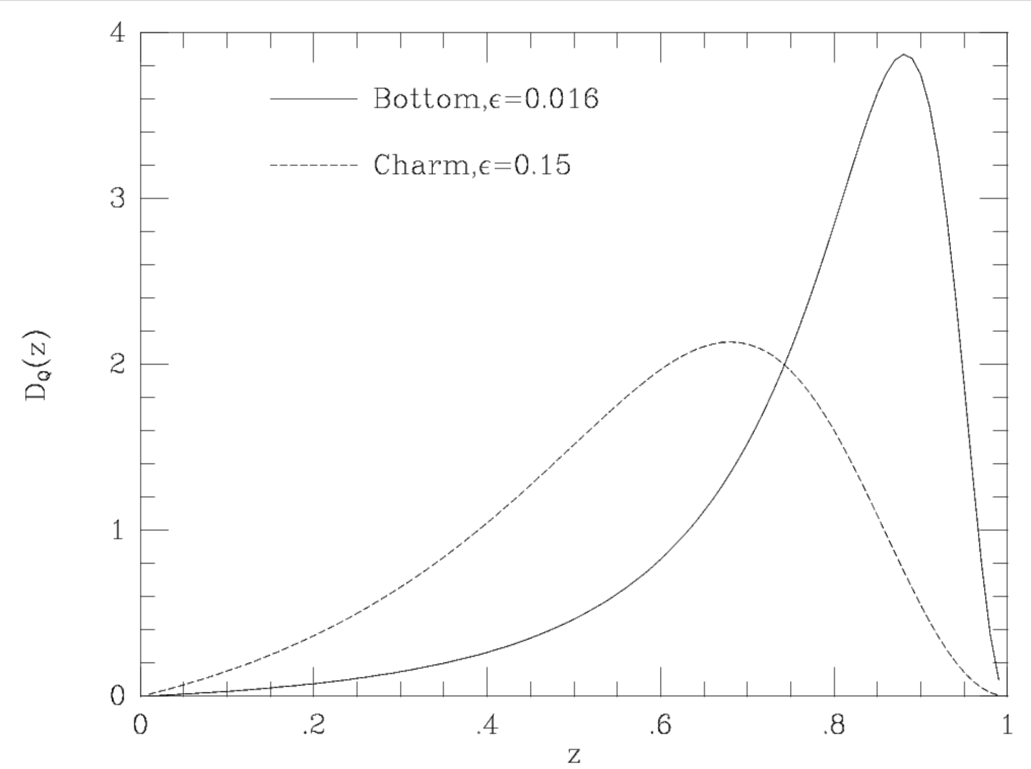
$$\approx \frac{m_Q^2}{2P}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right] \quad \text{with} \quad \epsilon_Q = \frac{m_q^2}{m_Q^2} \quad \text{and} \quad m_H = m_Q$$

The transition amplitude is $T \sim \frac{1}{\Delta E}$, squaring the amplitude and including a factor 1/z for phase space we obtain the Peterson function for the heavy quark fragmentation

$$D_Q^H(z) = \frac{N_H}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}$$

N_H is a normalization factor $\sum \int dz\, D_Q^H(z) = 1$

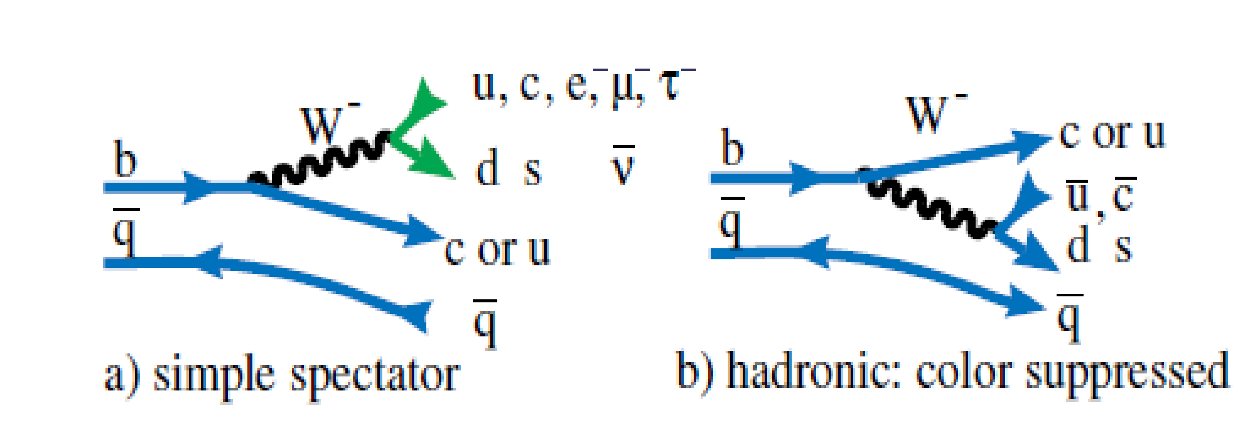
Heavy Quarks Fragmentation



$$\epsilon_Q = \frac{m_q^2}{m_Q^2}$$

is determined by the ratio of quark masses but it is treated as parameter and the best value is obtained from data.

b and c Meson Decay

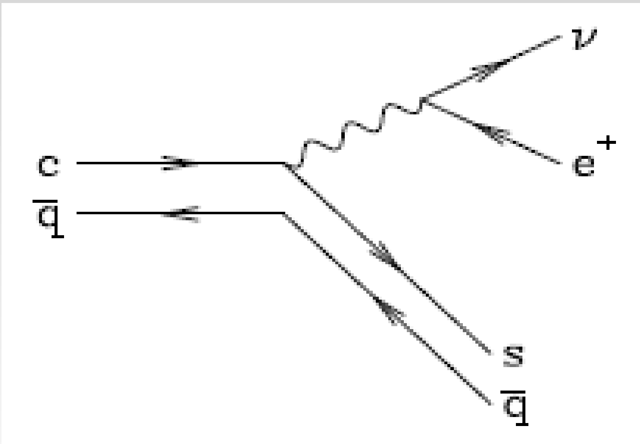


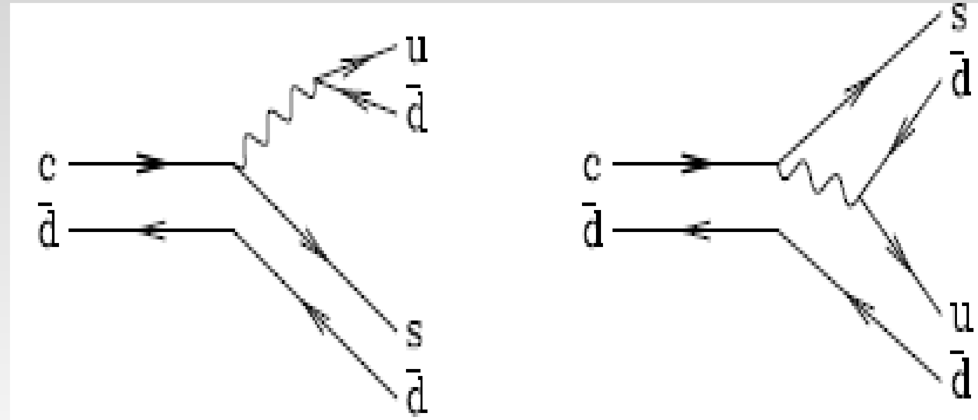
Several different decay modes of the b meson based on the spectator Model where the light quark does not participate.

B can be selected based on:

- lepton (e,μ): $B \to l\nu D$
- D meson: $B \to D\pi$
- J/ψ: $B \to J/\psi\ K$

c Meson Decay





Similar diagram governs the charm decay.

The procedure to identify charm meson are:

- lepton (e,μ): $D \rightarrow l\nu K(\pi)$
- K meson: $D \rightarrow K(n)\pi$

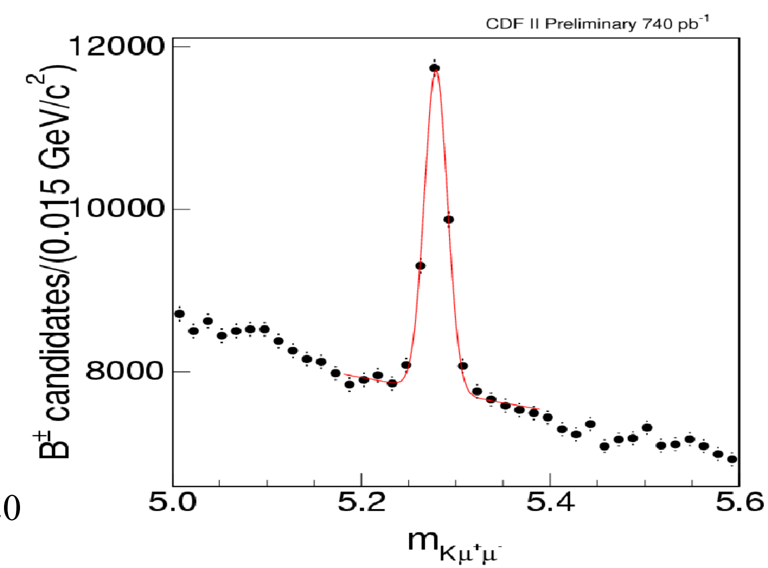
b Meson Cross Sections Measurements

The procedure to measure the cross section is simple.

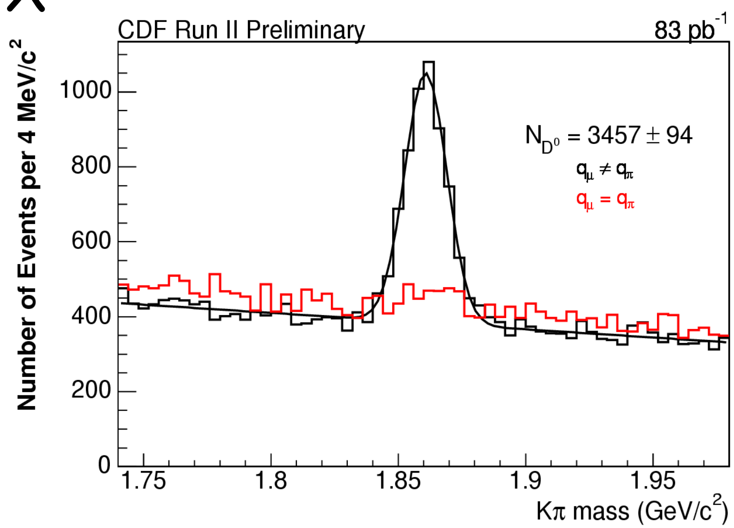
$$\sigma = \frac{N_{Data} - N_{Background}}{Acc \int Ldt}$$

B-mesons are selected exploiting the decay channels.
The selection procedure depends on the decay channel as the background evaluation. Examples:

$B \rightarrow J/\psi\ K$



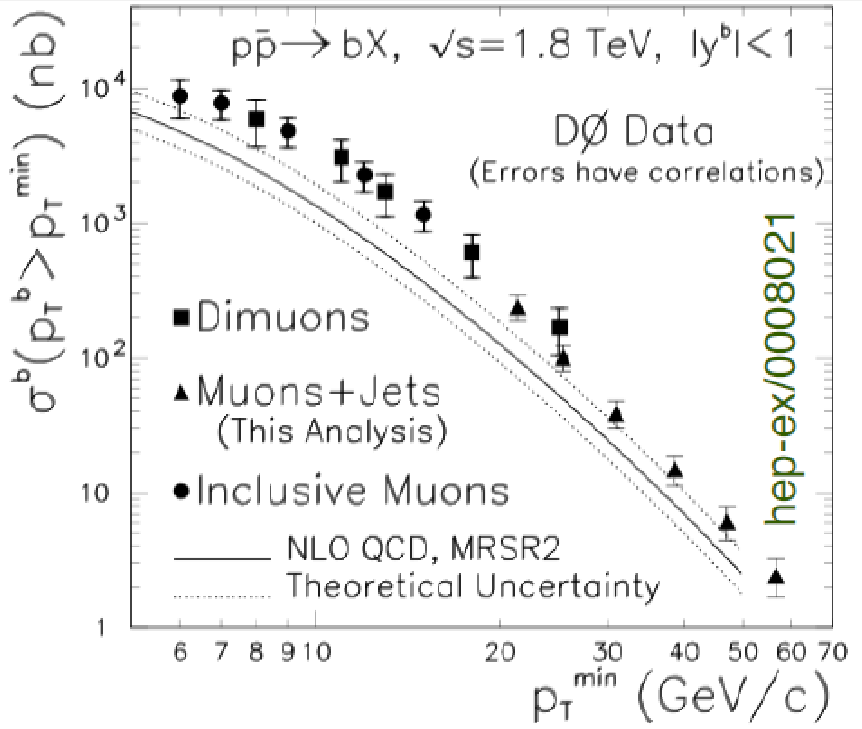
$Hb \rightarrow \mu D^0 X$

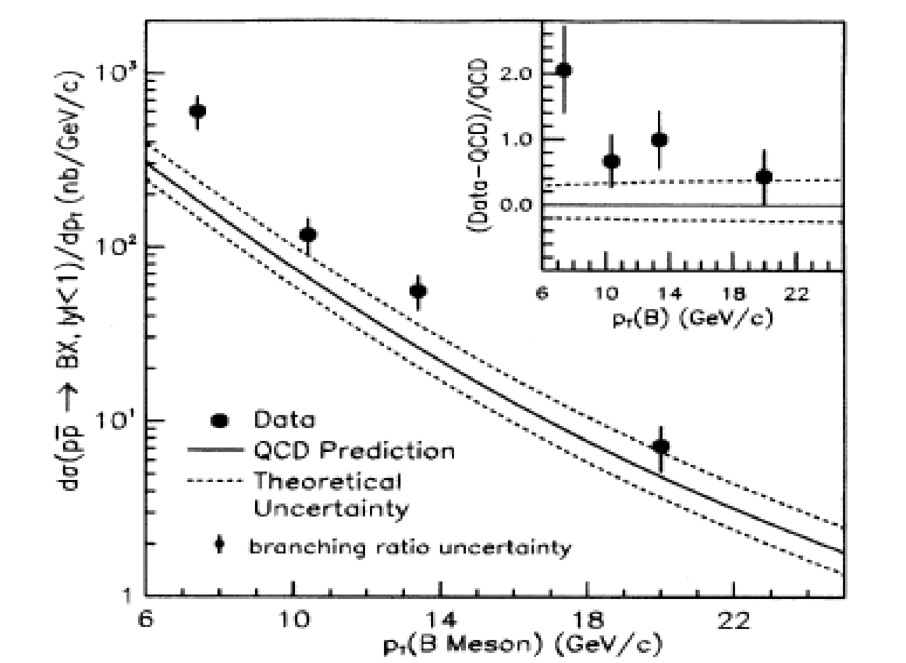


b Meson Cross Sections History

Until 2002 data and theory had a discrepancy, the measured cross section was higher of about factor 3

New Physics?





b Meson Cross Sections Measurements

A lot of work done by the experiments to improve the measurements.

A lot of work done by theoreticians to improve the theory:

M.Cacciari and P.Nason, PRL 89, 122003 (2002)

- new calculation of the cross section to include corrections (instead of NLO NLL) at high order
- new tuning of the fragmentation function D(z)
- new PDF

The purpose of this Letter is precisely to implement correctly the effect of heavy quark fragmentation in the QCD calculation. Several ingredients are necessary in order to do this: (i) A calculation with resummation of large transverse momentum logarithms at the next-to-leading level (NLL) should be used for heavy quark production [21], in order to correctly account for scaling violation in the fragmentation function. (ii) A formalism for merging the NLL resummed results with the NLO fixed order calculation (FO) should be used, in order to account properly for mass effects [22]. This calculation will be called FONLL in the following. (iii) A NLL formalism should be used to extract the nonperturbative fragmentation effects from e^+e^- data [23–29].

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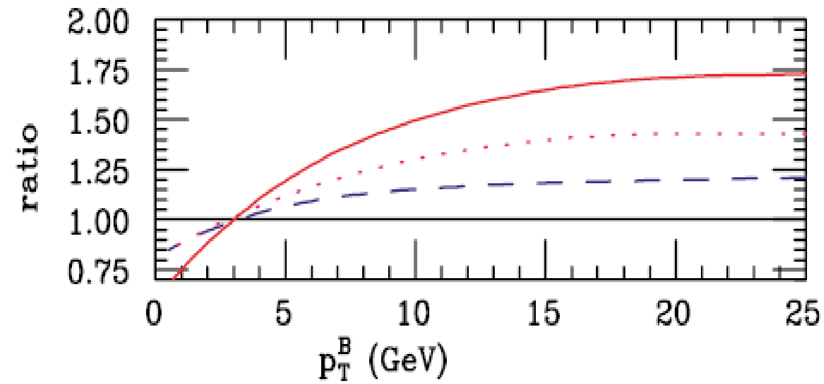
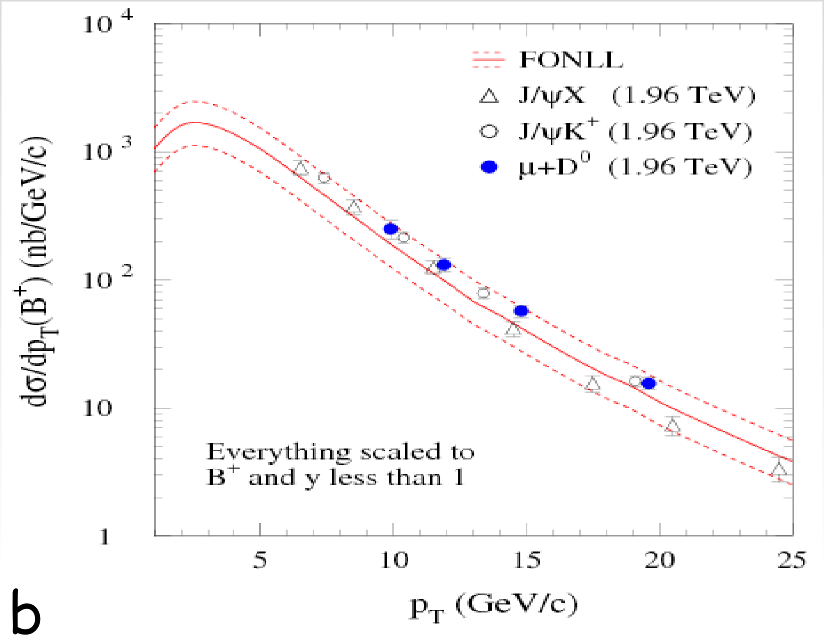


FIG. 4 (color online). The effect of the different ingredients in the calculation presented in this work, normalized to a fixed order calculation with Peterson fragmentation and $\epsilon = 0.006$. Dashed line: FO, $\epsilon = 0.002$; dotted line: FONLL, $\epsilon = 0.002$; solid line: FONLL, $N = 2$ fit.

b Meson Cross Sections Measurements

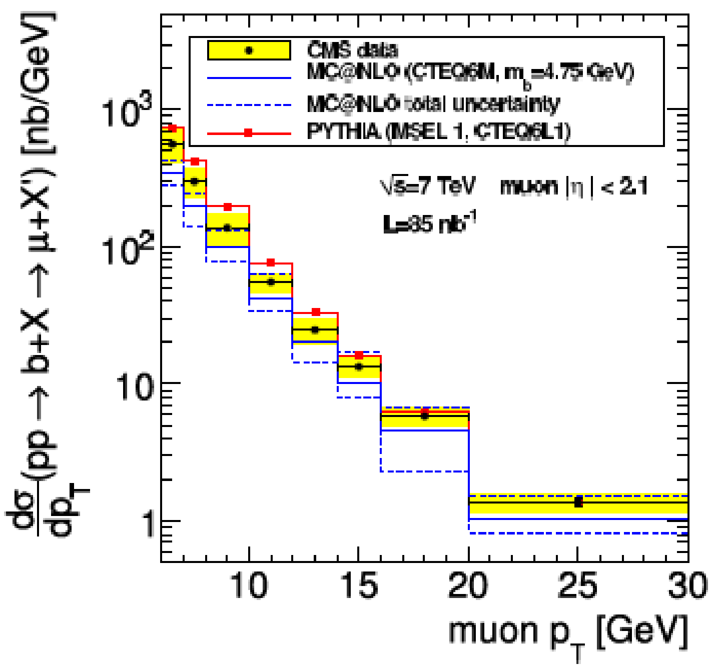
Tevatron:

- 3 separate measurements
- improved precision down to ~10%
- allow a reliable test of theory
- consistent with theory



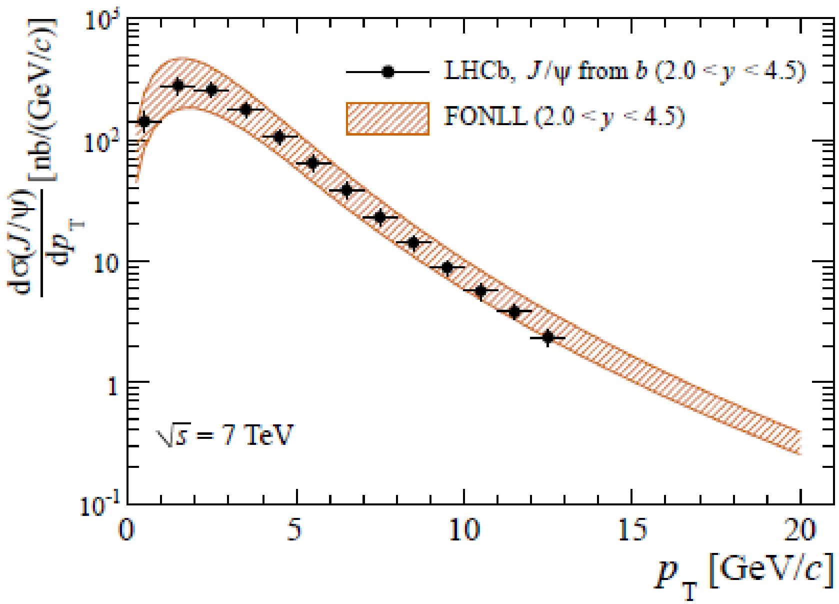
CMS:

- inclusive μ decays



LHCb:

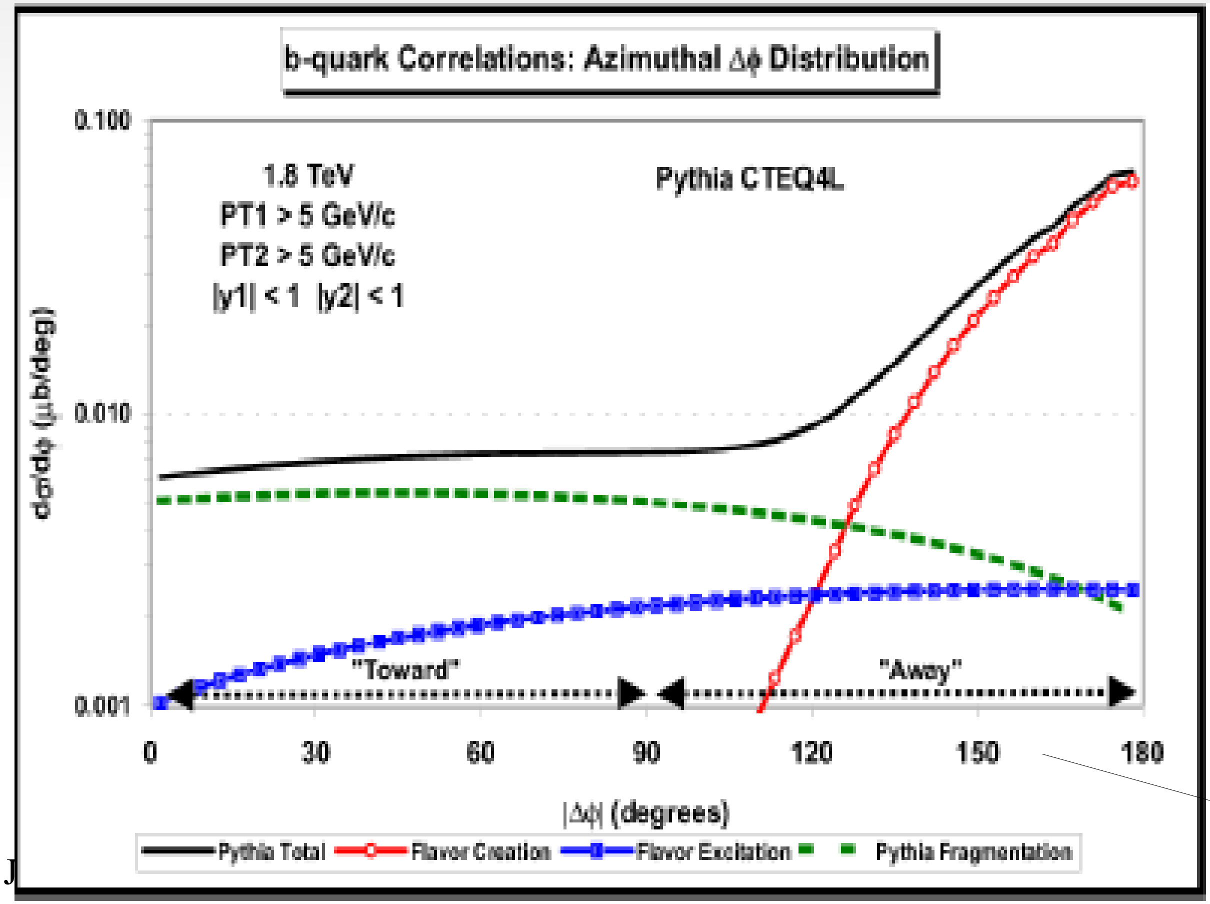
- J/ψ from b



Correlated $b\text{-}\bar{b}$ Cross Section Measurements

b-quark can be identified in jets by using inclusive selections. This allow to reconstruct events with 2 b-jets.

$b\text{-}\bar{b}$ cross section measurement allow to test high order theoretical contributions



LO produced almost back-to-back

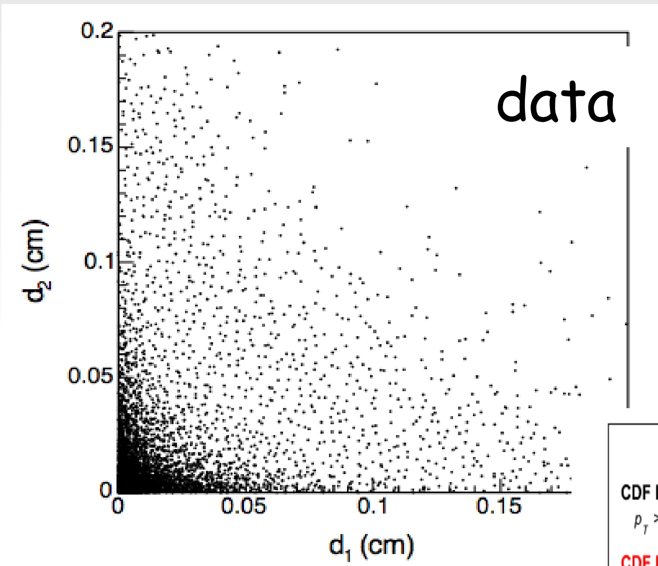
Fitting the cross section as function of the angular separation allow to determine the relative contribution of each process

$\Delta\Phi$ between 2 b-jets

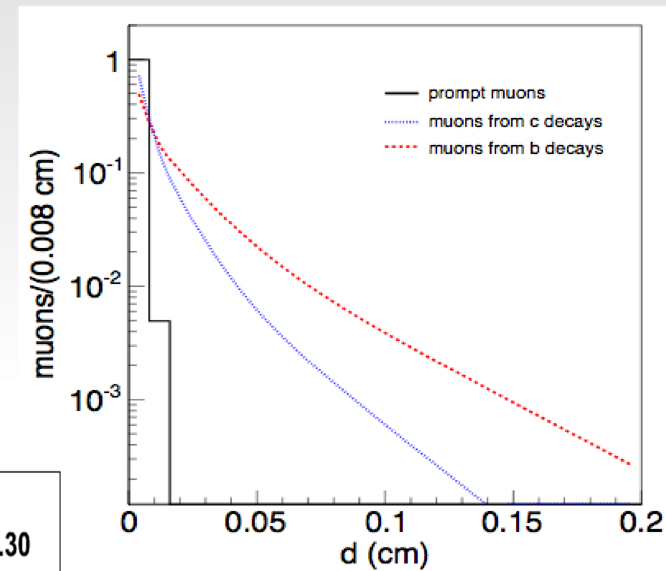
Correlated b-$\bar{b}$ Cross Section Measurements

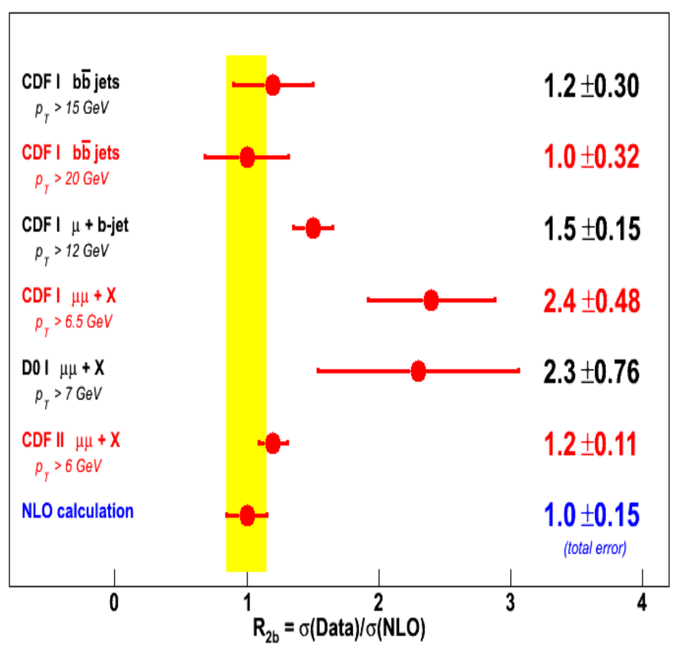
Tevatron:

2 μ are required then their impact parameter distribution is fitted to extract c and b components



Fit component



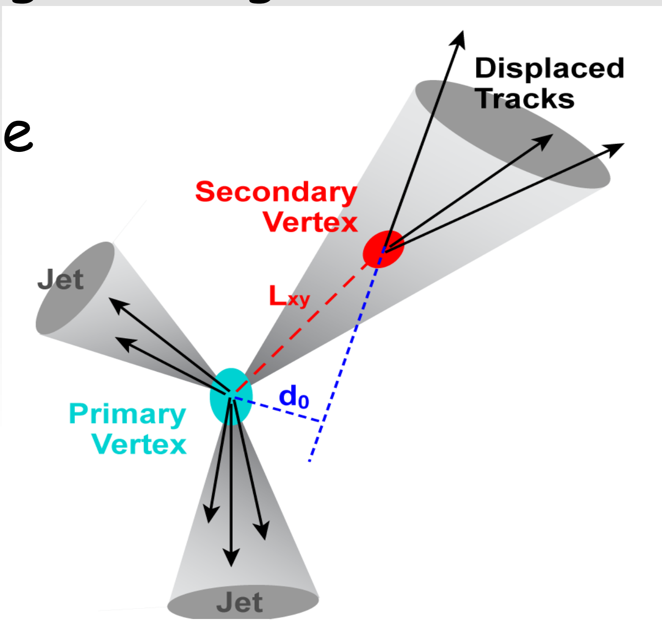


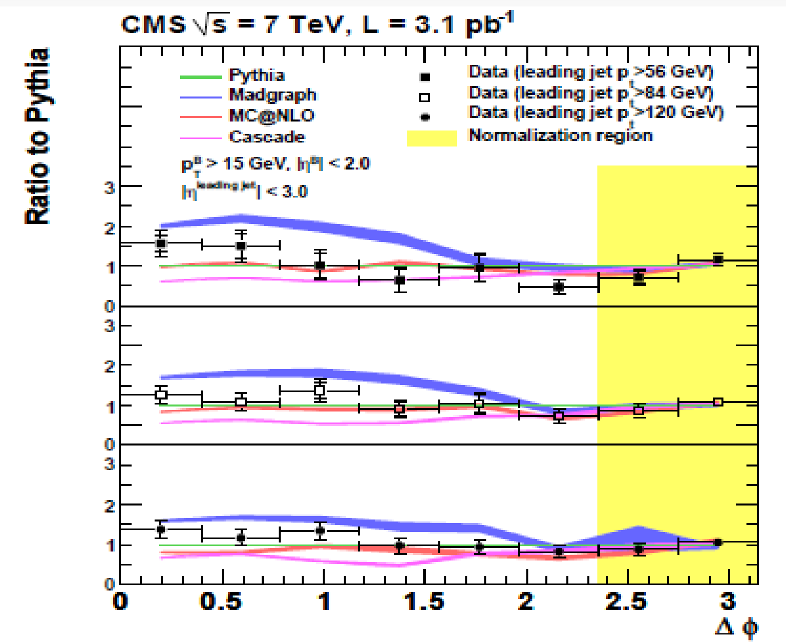
Correlated b-$\bar{b}$ Cross Section Measurements

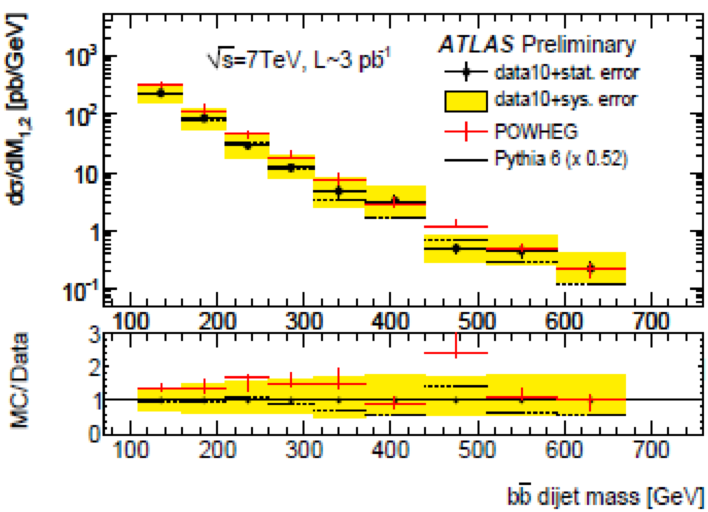
LHC:

ATLAS and CMS identify jets with b exploiting the long b lifetime

B-tagging: 2 or 3 tracks displaced from the primary vertex the decay length L_{xy} compatible with the distance traveled by the b-hadron.

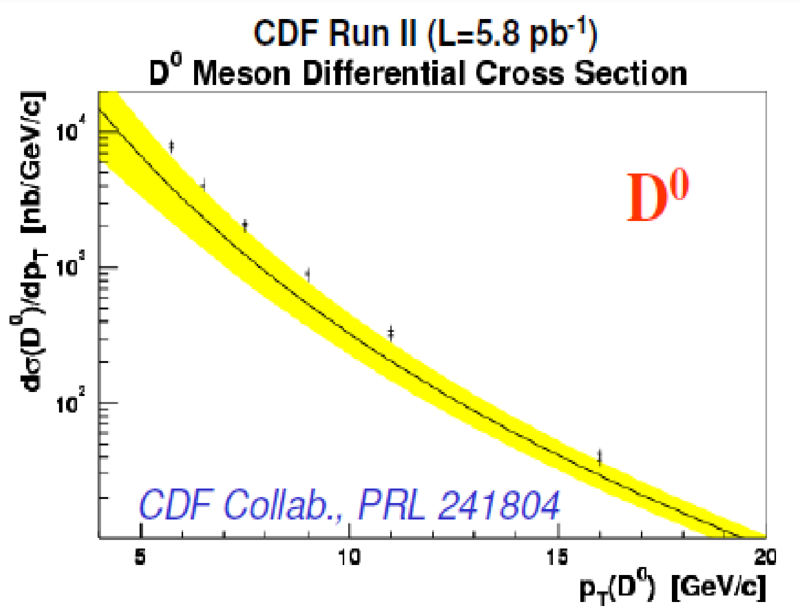


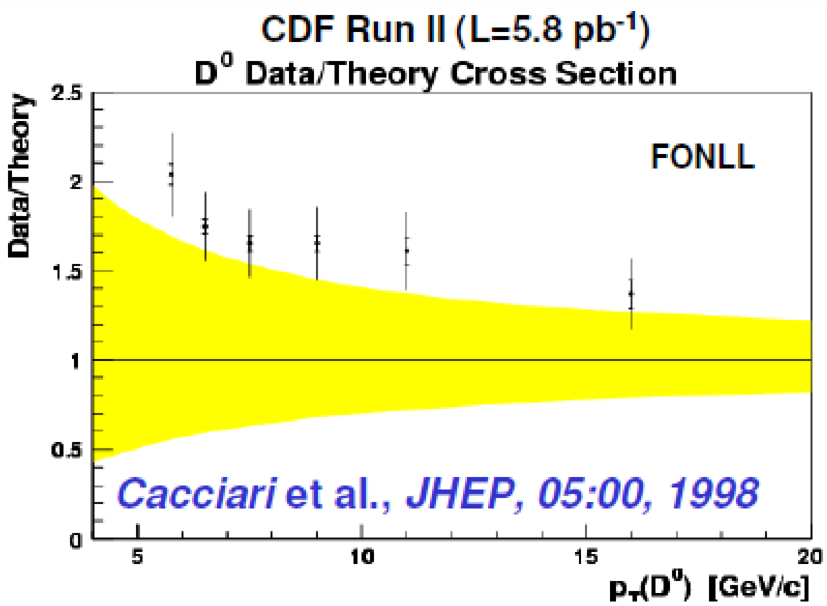




Inclusive charm cross section

Identify only one c-meson and measure the cross section with the same procedure described for b-mesons.
Theory predictions have larger uncertainties than the b-meson cross Section.
Charm meson cross section measured in several experiments, fixed target and at collider.





At Tevatron data/theory~2

Inclusive charm cross section

Also in this case a lot of work done by theoreticians

Charm cross sections for the Tevatron Run II
Journal of High Energy Physics Volume 2003 JHEP09(2003)

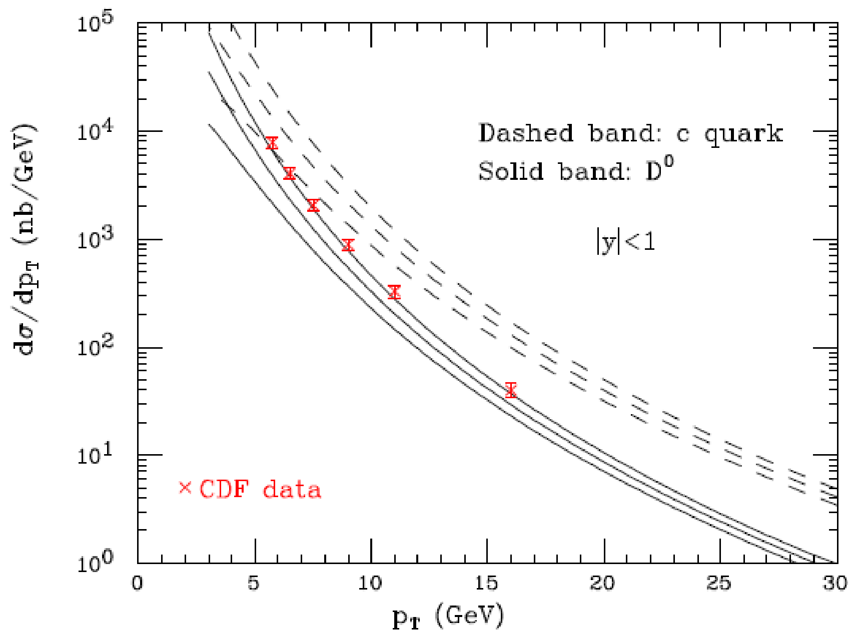
- Use the same framework of the b-meson
- Build new fragmentation functions non-perturbative using data;

Ex. $F(c \to D^0) = F_p(c \to D^0) + F(c \to D^{*+}) \otimes F(D^{*+} \to D^0) + F(c \to D^{*0}) \otimes F(D^{*+} \to D^0)$

$F(c \to D^0)$= directly produced D^0 fragmentation function

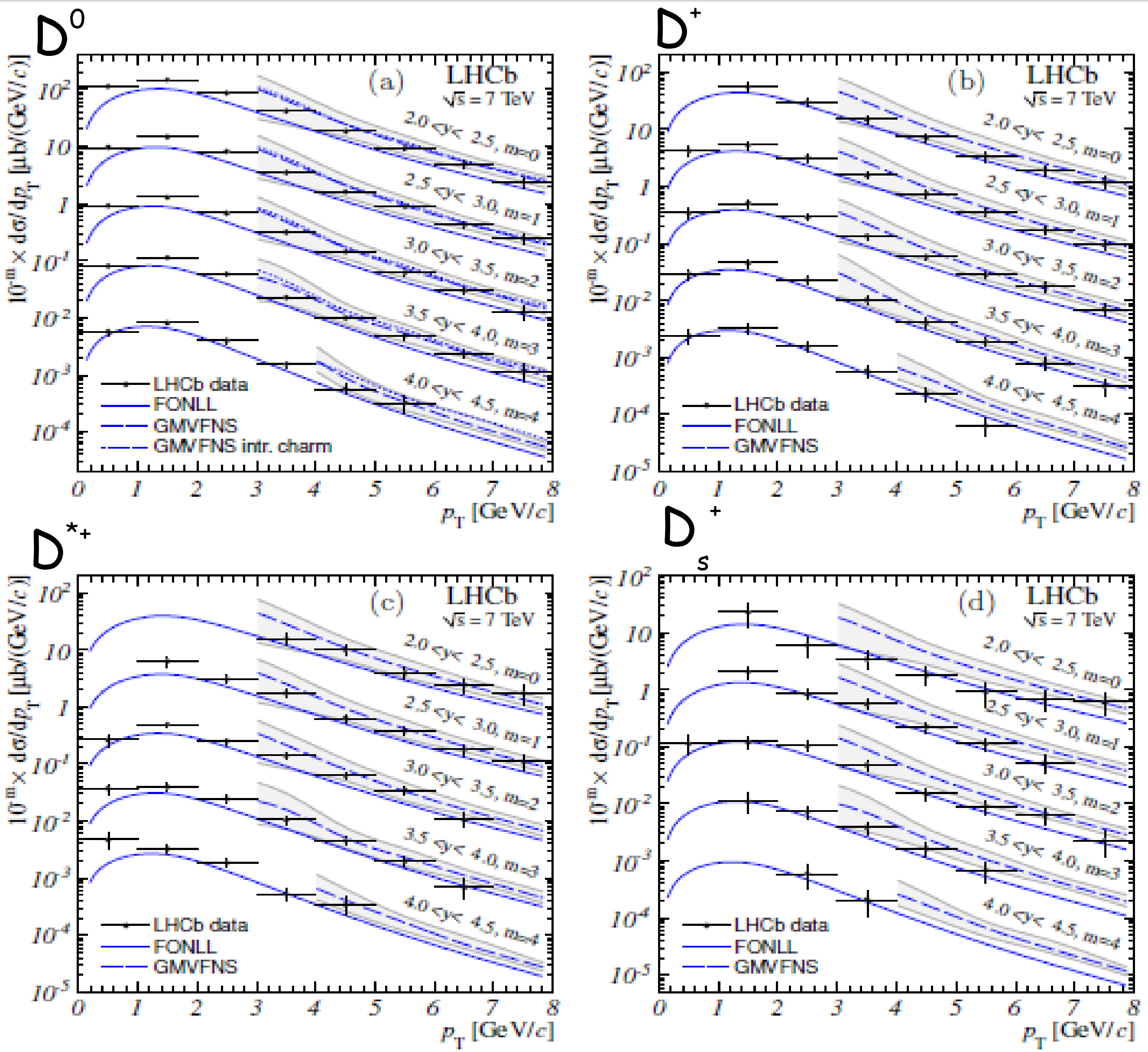
$F(c \to D^{0*})$= fragmentation function of D^{0*} that then decay to D^0

$F(c \to D^{+*})$= fragmentation function of D^{+*} that then decay to D^0

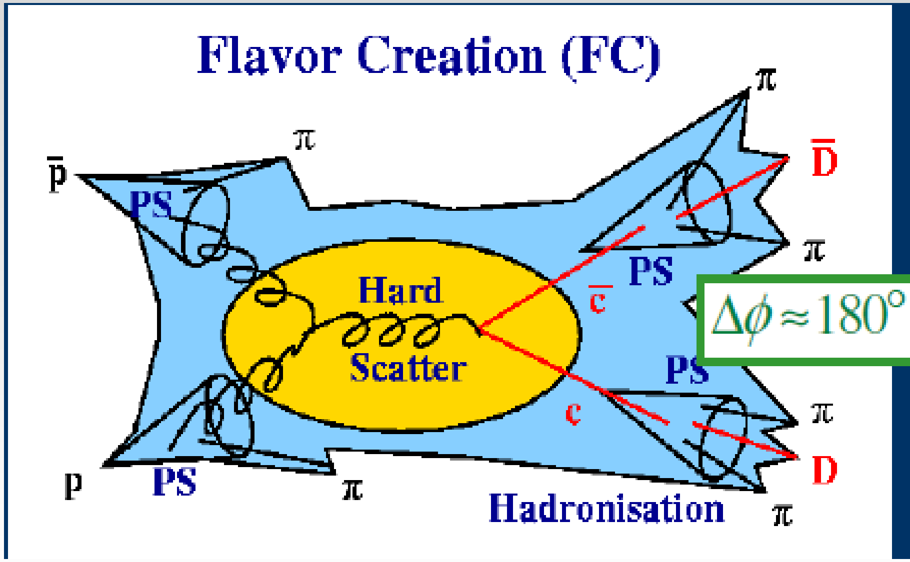


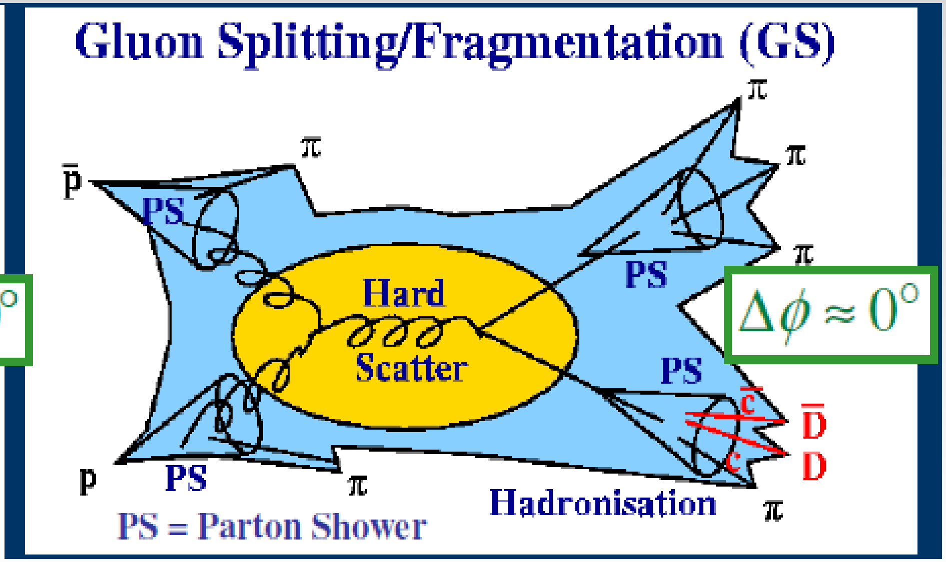
Inclusive charm cross section

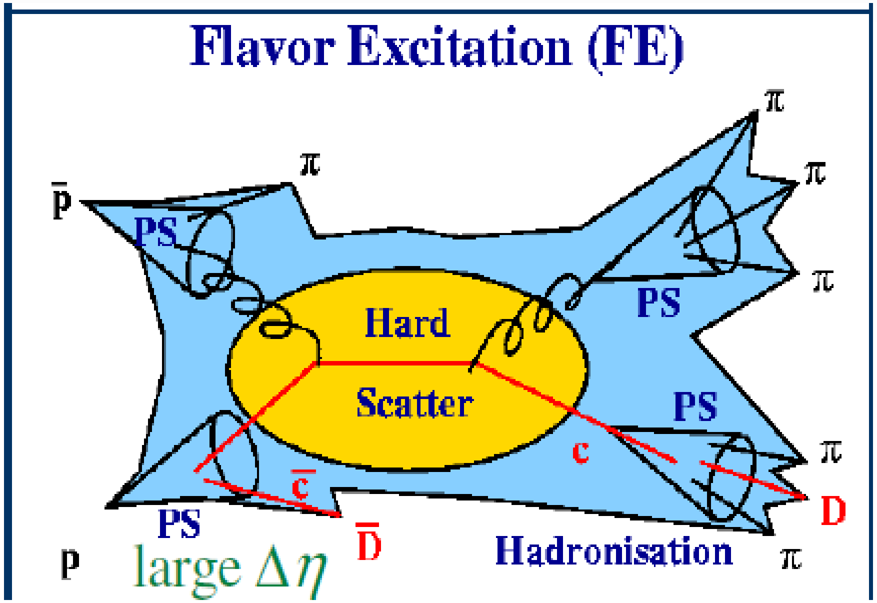
Now at LHCb

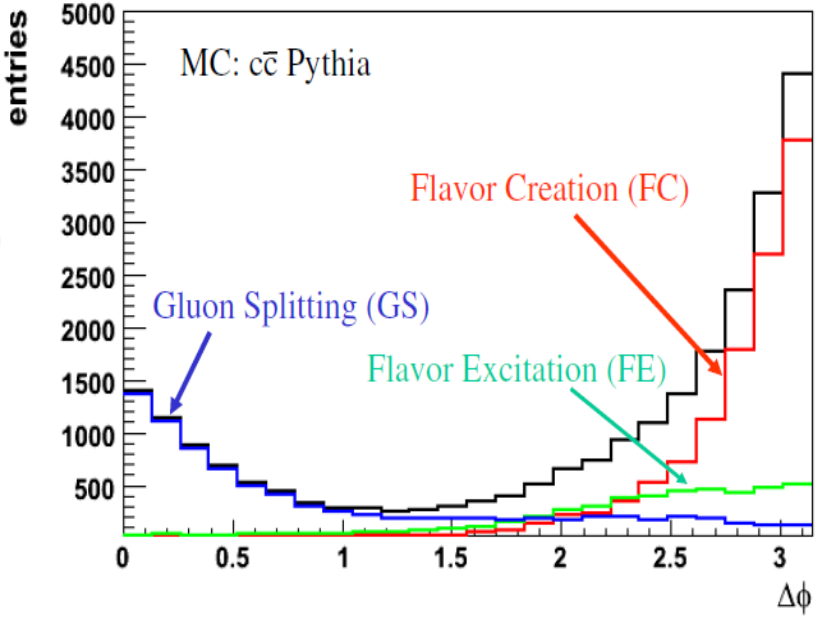


Charm Correlated Cross Sections Measurements

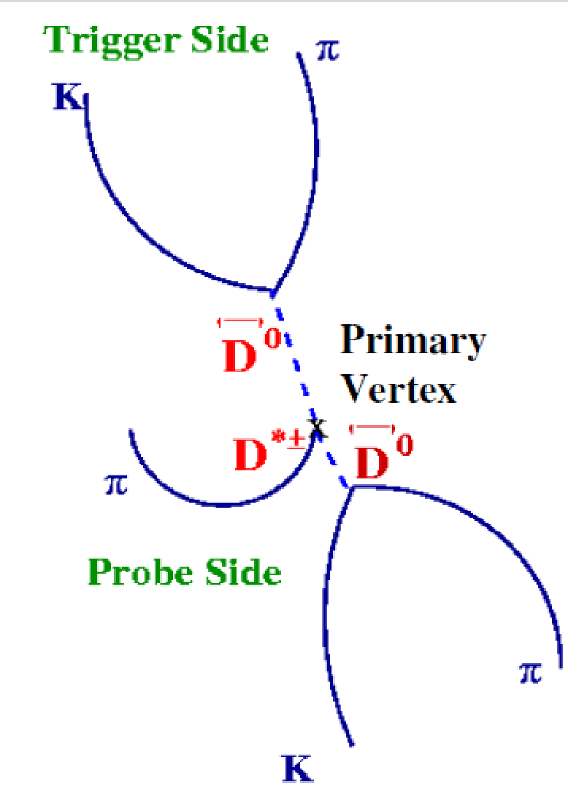


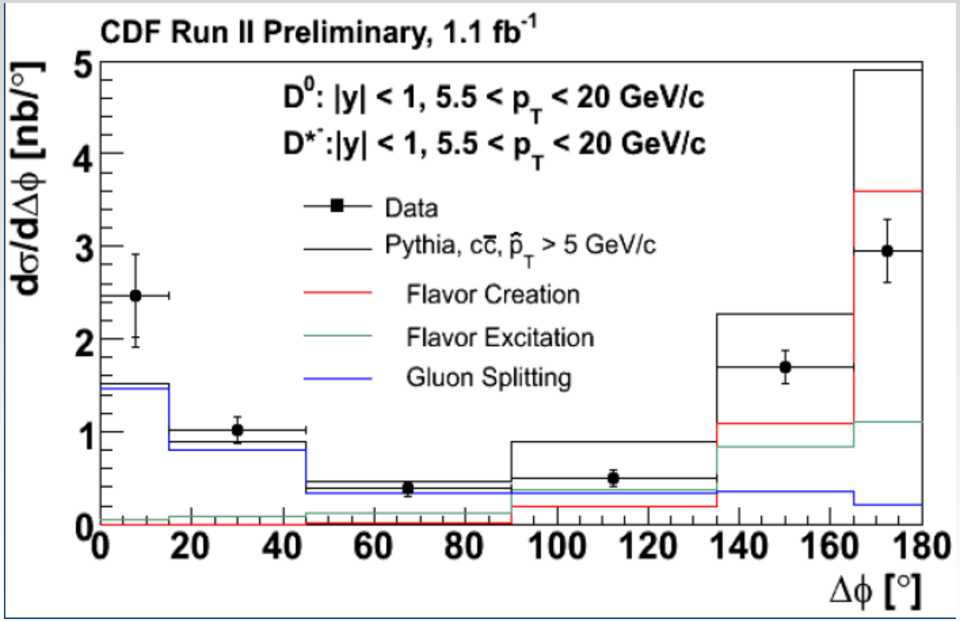


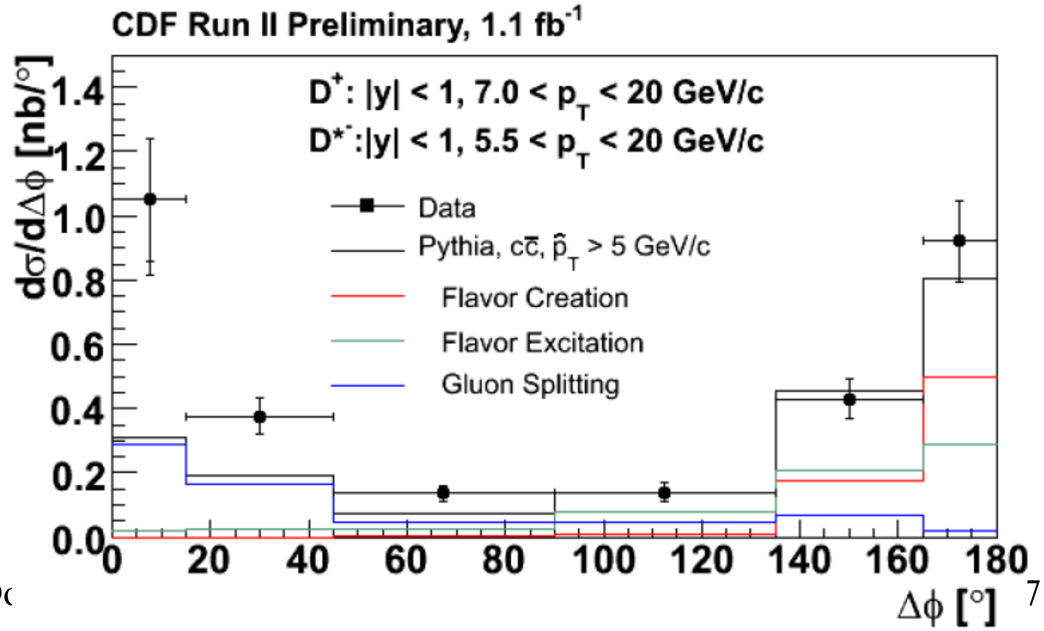




Charm Correlated Cross Sections Measurements







- Identify the 2 D meson
- Correct for D coming from b-hadron
- Evaluated the efficiencies