Elements of QCD: Introduction

The material of these lessons is taken from:

Elements of QCD for hadron colliders, G. P. Salam
arXiv:1011.5131v2 [hep-ph] 10 Jan 2011

Introduction to High Energy Physics, D. H. Perkins

QCD and Collider Physics, Ellis, Stirling, Webber

Elements of QCD: Introduction

The QCD (Quantum Chromo-Dynamics) is the theory of quarks, gluons and their interactions.
It has similarities and differences with QED:

- electrons have electric charge, quarks have colour charge
- there is only 1 electric charge but 3 colours: red, green and blue
- photons are neutral, gluons carry colour charge and there are 8 possible combinations of colours and anti-colours charge
- the coupling α_s tend to zero at high momentum scales (opposite in QED) and it is large at small scale. In between its evolution with scale is quite fast: at the LHC its value will range from α_s =0.08 at a scale of 5TeV, .to $\alpha_s \sim 1$ at a scale of 0.5GeV

The Lagrangian of QCD is similar to QED one, it can be written with terms that are quark-quark, quark-gluon and gluon-gluon interactions.

Elements of QCD: tools - 1

How do you use this theory to make predictions?

1. Lattice QCD:

-The most complete approach.

- Build a 4-dimensional lattice with imaginary time by discretizing space-time and considering quark and gluon fields values at all the vertices.
- Monte Carlo sampling over all possible field configurations determines the relative likelihood of different field configurations providing solution to QCD.
- This is very useful for "static" predictions like particle masses
- Not suitable for processes at very high energy like LHC where the number of particles to follow is very high and a huge number of nodes is needed.

Elements of QCD:tools - 2

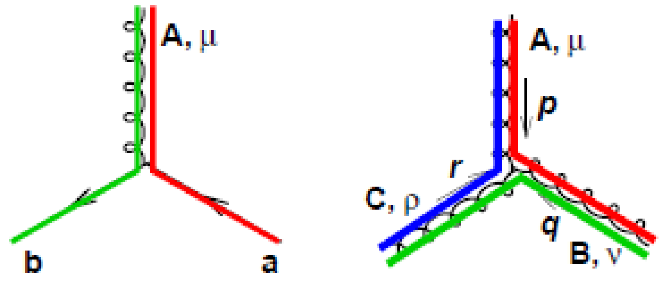
2. Perturbative QCD:

- Order-by-order expansion in α_s with α_s<<1 An observable f can be written:

$$f = f_1 \alpha_s + f2 \alpha_s^2 + f_3 \alpha_s^3 + \ldots$$

Where only the first two (or three) terms are calculated and the other are assumed to be negligible.

- the f factors are calculated via Feynman diagrams



Note: colour currents

Elements of QCD: α_s

The value of $\alpha_s = g_s/4\pi$ depends on the scale μ at which it is calculated
μ renormalization scale, used to keep consistent dimensions (units) for all quantities.
With some approximations we can write:

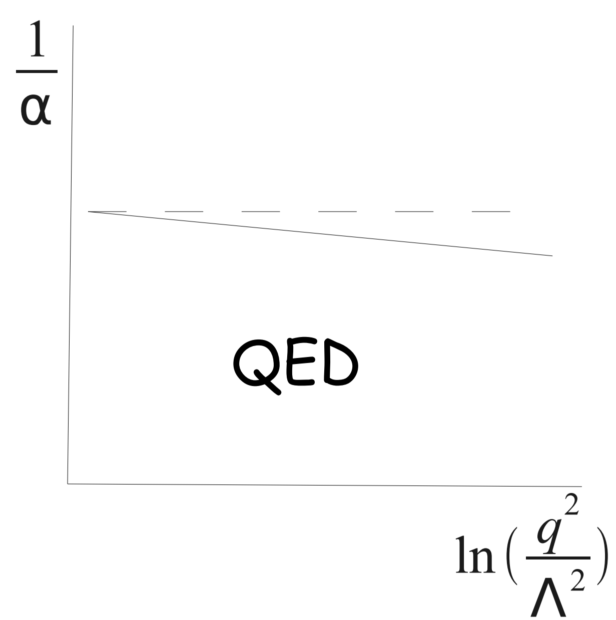
$$\alpha_s(q^2) = \frac{1}{b_0 \ln\left(\frac{q^2}{\Lambda^2}\right)}$$

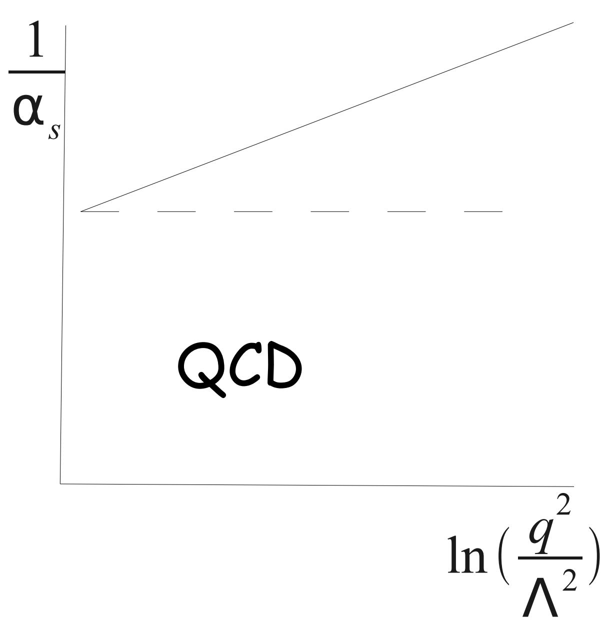
where Λ (or Λ_{QCD}) is the scale at which the coupling diverges.

Only $q \gg \Lambda \rightarrow \alpha_s(q^2) \ll 1$ the perturbation theory is valid.
Λ is not well defined since it is non-perturbative quantity, it depends on the approximations made in the α_s definition. The order of magnitude used is ~200 MeV

Elements of QCD: running coupling

At high momentum scale α_s the coupling become weaker, quarks and gluons do not interact, the theory is almost a free theory.
At low momentum scale the coupling grows forcing the quarks and gluons to be bound in the hadrons.
This is exactly the opposite of QED. Several explanations more or less intuitive have been tried. The gluon colour charge is the basis.
Measurements of α_s will be discussed.





Elements of QCD: e+e- → hadrons

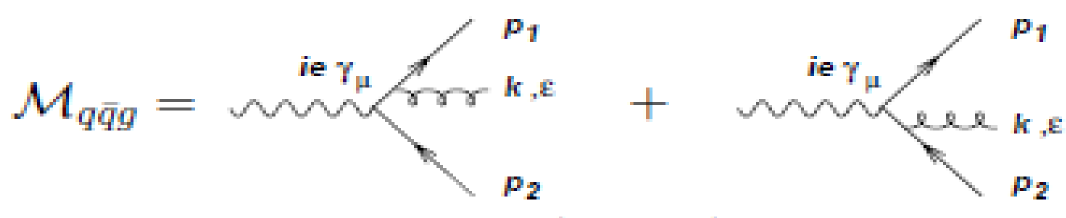
These process have allowed to study and verify QCD predictions, in fact:

- there are only hadrons only in the final state
- we have a plethora of data from LEP and SLC

In the calculations very important approximations soft and collinear are used:

- soft: the gluon emitted by partons (quarks or gluons) has very low energy compare to the parton that emitted it
- collinear: the gluon emitted by partons is very close in angle to the emitting partons

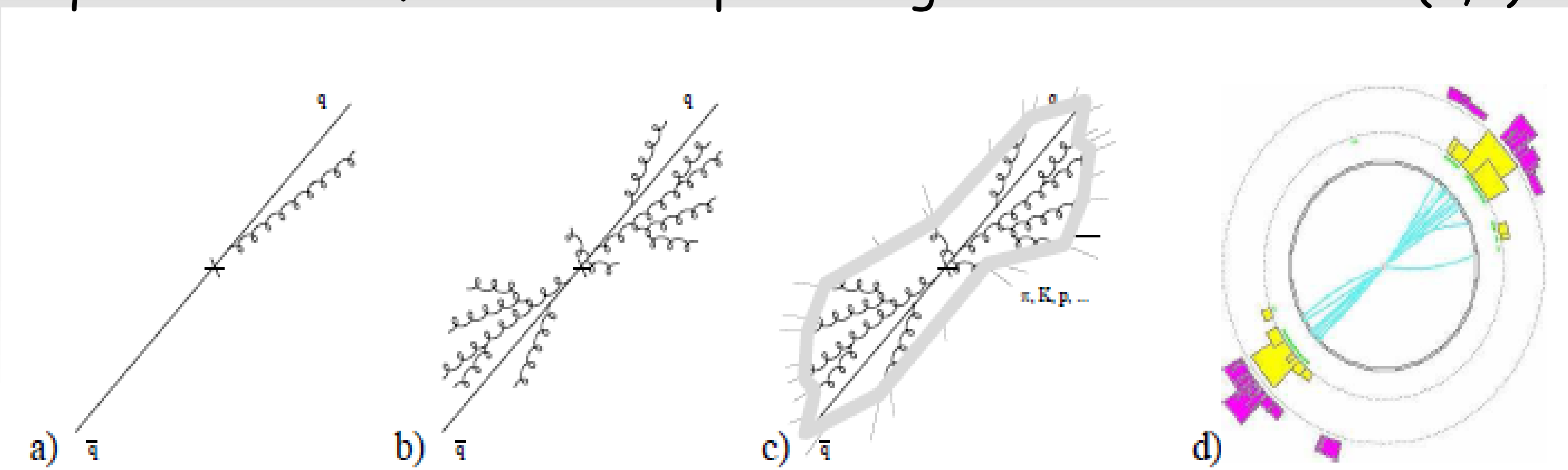
The amplitude



can be calculated easily.

e+e- → hadrons: particles in the final state

The emitted gluon can emit itself other gluons. The gluons can split in quarks and so forth. These process go on till the Λ scale (a,b).



This process describes a complex final state, still in term of partons. Hadrons are formed in the so called "hadronization" phase, a non-perturbative process (c).

Hadrons properties:

- directions and momenta closely related to the parton's ones
- multiplicity very similar to the parton's

The event (d) (OPAL) shows the same angular distribution of partons.

Hadronization Models - 1

- Hadronization: low momentum-transfer, long-distance regime → non-perturbative effects dominate.
- General approach: local parton-hadron duality, ie. the transition from partons to hadrons is essentially local in phase space
- The flow of momentum and quantum numbers at hadron level tends to follow the flow established at parton level.

  Ex.: the flavour of the quark initiating the shower should be found in a hadron near the jet axis.

Mainly three models of fragmentation are used.

Independent fragmentation

- original approach of Field and Feynman
- each parton fragments independently
- designed to reproduce the limited transverse momenta of jets in e+e-
- fragmenting quark is combined with an anti-quark from a $q\bar{q}$ pair created out of vacuum to give the “first meson” with energy fraction z.

Hadronization Models - 2

The leftover quark (1-z energy fraction) is fragmented in the same way and so on until the energy is below the cut-off.

- gluon first splits in $q\bar{q}$ pair then the above schema is followed
- z is the fragmentation function

This model describes quite well e+e- → two-jets and three-jets.

Main issues:

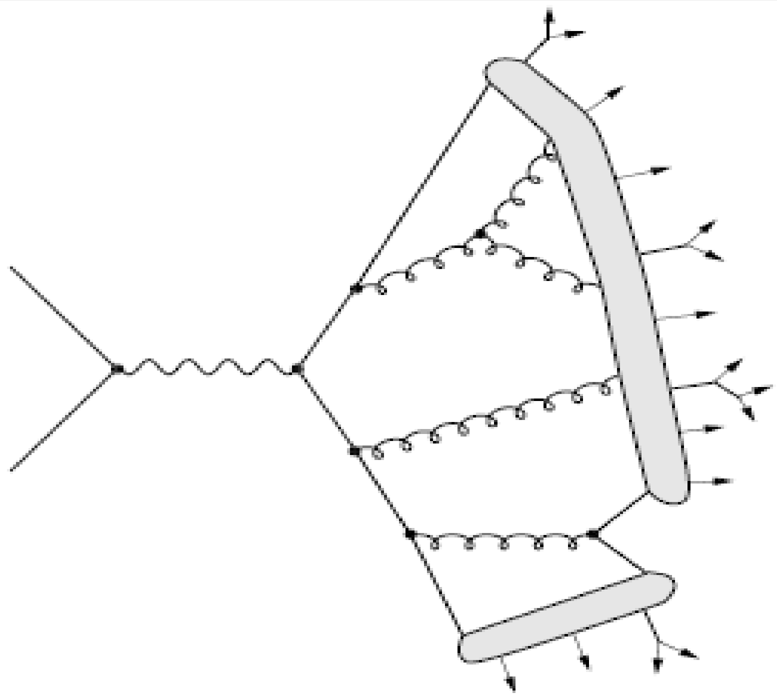
- the fragmentation depends on parton energy this leads to a violation of momentum conservation that has to be adjusted at the end of the fragmentation.
- Even when two jets are very close in angle they remain two distinguishable jets and not merged.

String Model

Quark and anti-quark produced in e+e- move out in opposite direction losing energy to colour field which collapses into a string-like configuration between them.

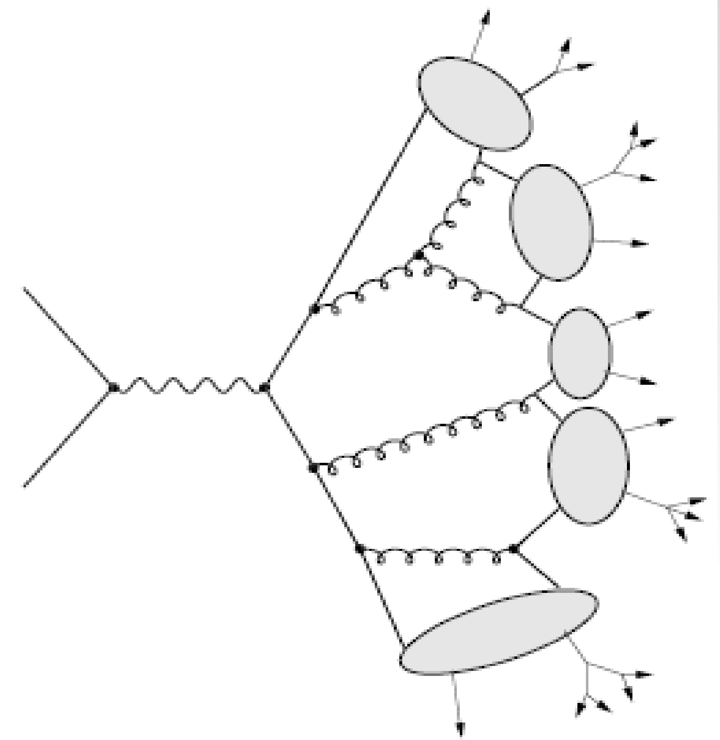
Hadronization Models - 3

- the string breaks into hadron pieces producing $q\bar{q}$ pair.
- gluons produce kinks on the string
- gluon splitting into $q\bar{q}$ produces an other string segment



- kinked strings fragmentation leads to an angular distribution of hadrons in e+e- events in a better agreement with data than the independent fragmentation model.

Hadronization Models - 4



Cluster Models

- during the parton branching process we have the so-called "preconfinement of colour" → colour connected neighbouring partons have an asymptotic mass independent from the momentum and universal.
- colour-singlet clusters of partons are formed through non-perturbative splitting of gluon after the perturbative phase and later decay into particles
- gluons after the parton shower are splitted in $q\bar{q}$ pairs

Hadronization Models: MC generator

Monte Carlo programs use the three hadronization models to generate full events from parton shower.

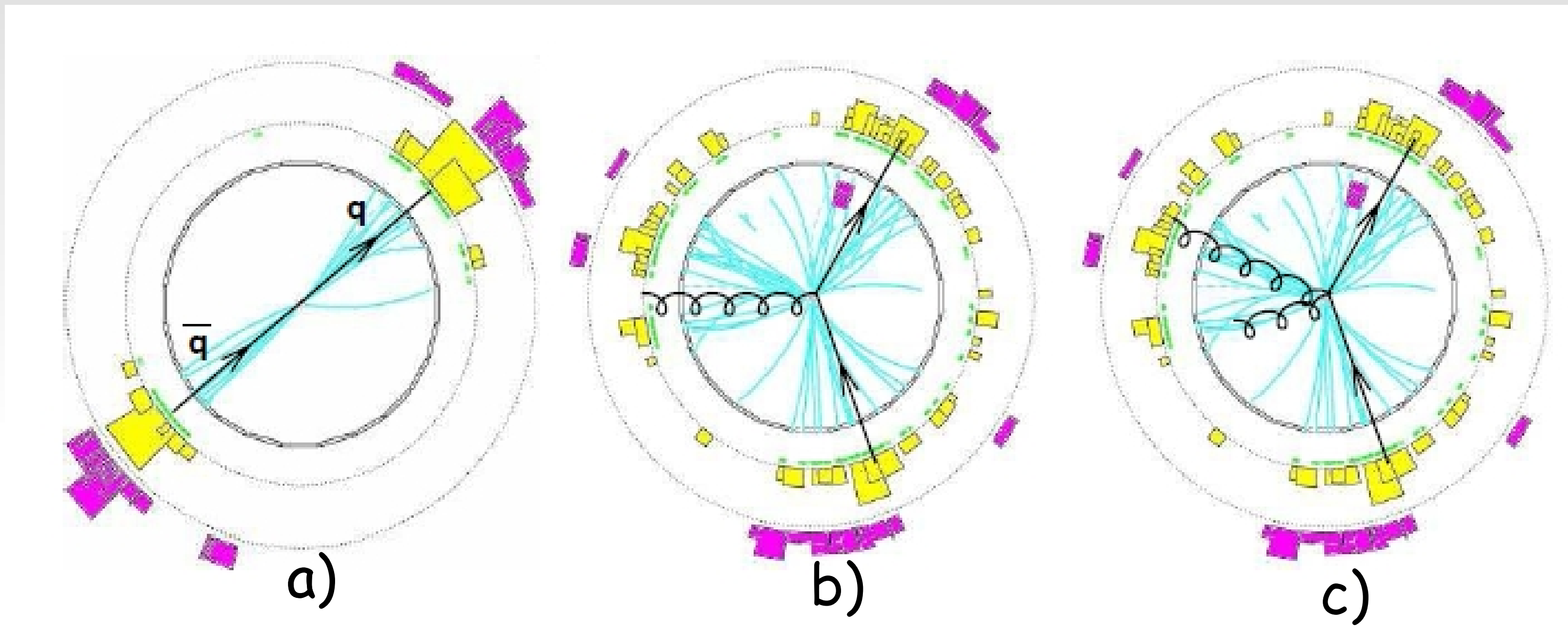
Independent fragmentation
-ISAJET

String hadronization:
- JETSET which is used by PYTHIA

Cluster hadronization:
- HERWIG
- SHERPA

Jets: Introduction

Jet definition: examine a e+e- → jets



Event in figure a) shows the classic two-jets events.
Event in figure b) is more complicate: it can be interpreted as in b), with $q\bar{q}$ pair that emits an hard gluon or as in c) with an additional soft gluon
Every time the events are reconstructed we should make this kind of inspection!

Jets Algorithms

Jet definition starts solving two main questions:

- which particles are grouped together. How to group particles define the jet algorithm
- how to combine the momenta of particles inside the jet. This define the kinematics properties of the jet. Usually the 4-momenta of the particles are summed up.

Cone Algorithm

- Iterative process:

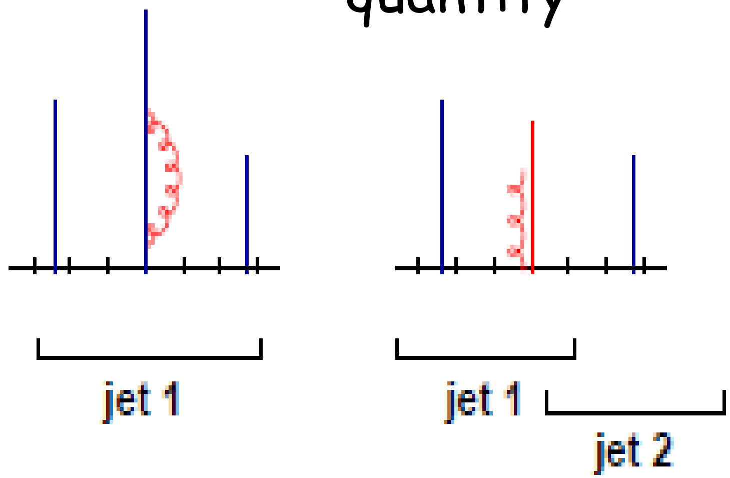
1. make a list of all particles and order them in transverse momentum
2. the highest in transverse momentum is the seed particle
3. draw a cone of radius R around the seed particle
4. calculate the sum of the momenta of all the particles in the cone and find the axis of the jet.

Jets Algorithms: cone - 1

5. if the direction of the jet coincides with the initial one – done! If not go back to 3, draw a new cone around the jet-axis and proceed to 4.
6. Repeat the process until the sum of the cone content coincides with the previous one.

The particles in the stable cone are removed from the list of particles in the event and the procedure starts again to search for a new jet.

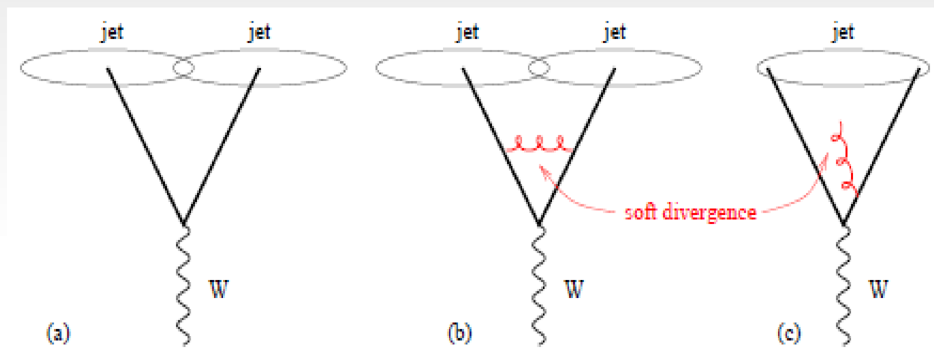
Major issue: the Pt ordering for seed particle, Pt is not collinear safe quantity

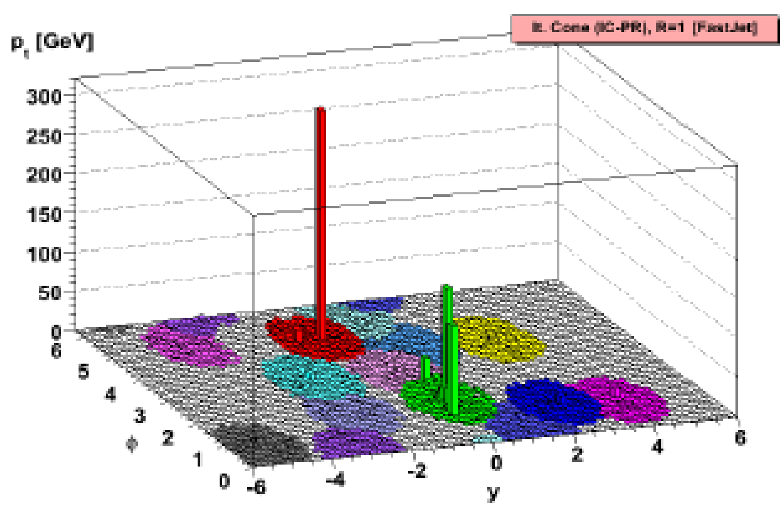


Highest Pt particle emits a gluon and it is not the highest Pt particle and it can be reconstructed as 2 jets event

Jets Algorithms:cone - 2

Take all the particles above a given threshold as seed solve the problem of collinear safe but introduces the infrared safety issue: soft emitted particle can generate new stable cone close to two primary that then are merged





Jets reconstructed with cone algorithm

Jets Algorithms:Kt

Sequential recombination algorithm

For each pair of particle define:

- distance d_{ij} $d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2}$, where R is a parameter similar to the cone radius
- beam distance: $d_{iB} = p_{ti}^2$

The algorithm proceeds searching for the smallest d_{ij} d_{iB}

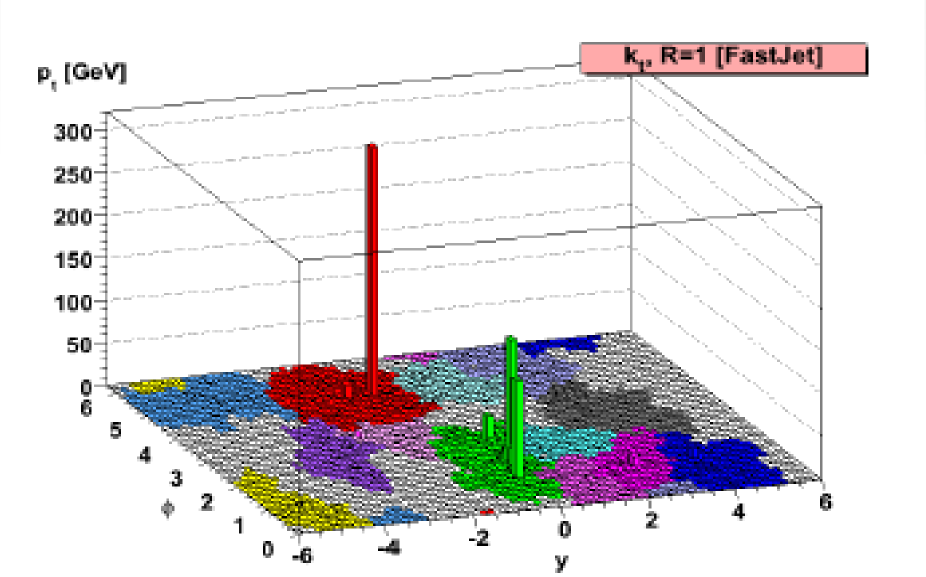
If it is d_{ij} the particles i and j are recombined in one particle, if it is d_{iB} the particle i is called jet and removed from the list of particles.

The procedure continues until the list of particles is empty.

Jets from Kt algorithm can be inspected inside and study the substructure of jet since the clustering sequence it is known.

Jets Algorithms:Kt

Jets from Kt algorithm have irregular edges because many soft particle cluster together at early stage, in pattern that are almost random. The irregular edges is the reason why Kt is not so used, it is difficult to evaluate the acceptance.



Same jets as before reconstructed with the Kt algorithm

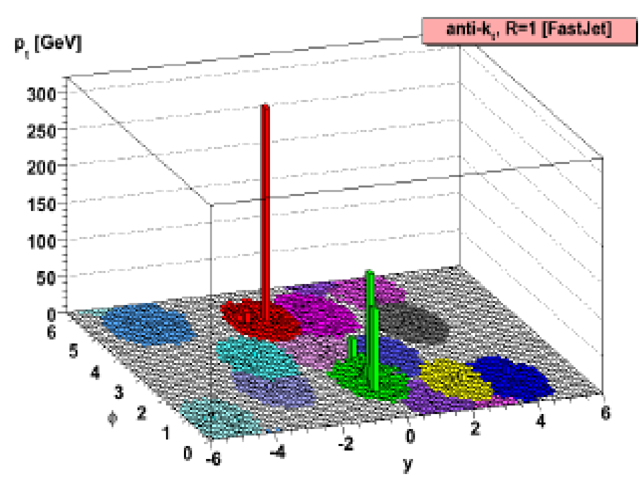
Jets Algorithms:anti-Kt

Sequential recombination algorithm with the "nice" cone properties by modifying the Kt algorithm:

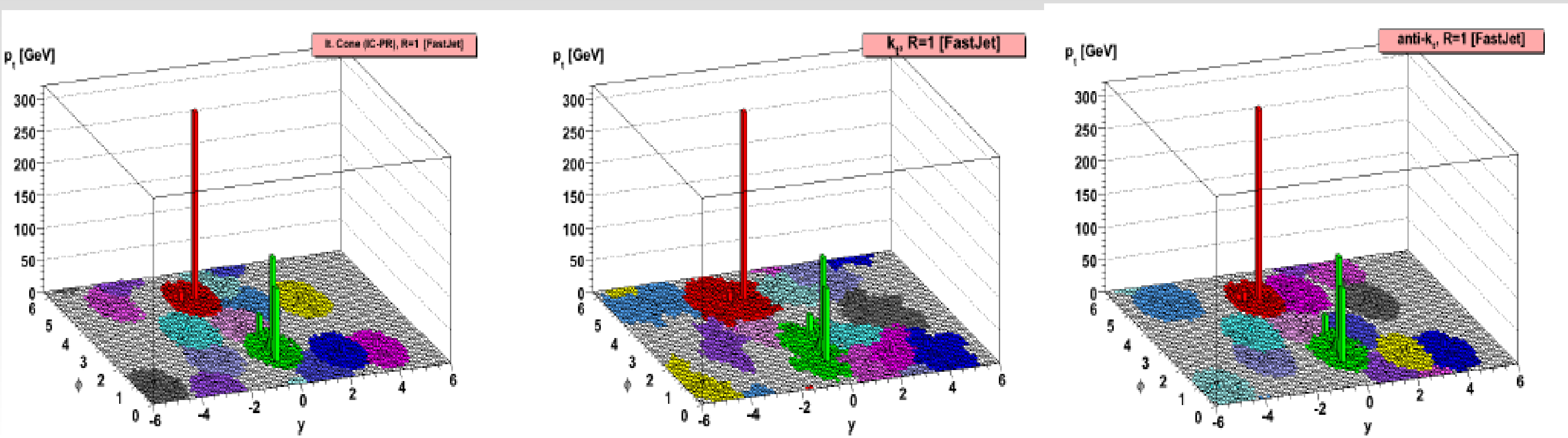
$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2} ,$$

$$d_{iB} = \frac{1}{p_{ti}^2} .$$

Jets grow in concentric circle from the "core" until they reach the cone size R. It is collinear and infrared-safe. It does not provide information on sub-structure of jets.



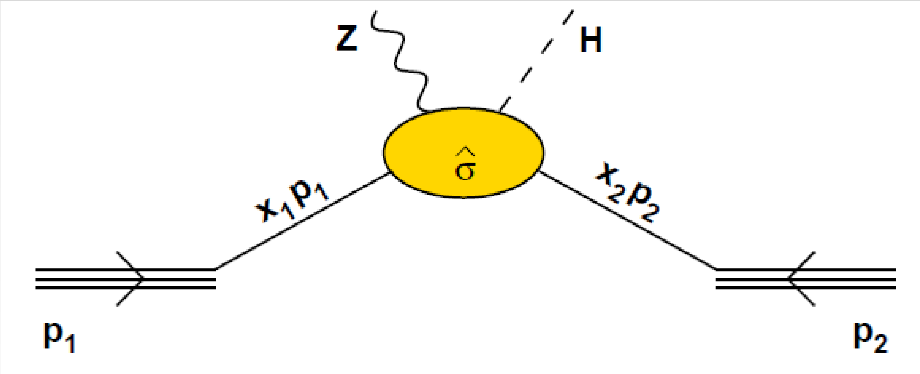
Jets Algorithms Comparison



Which jet algorithm use , which R depends on the physics measurement.

Hadron Interaction

Cross section of an hard process (ZH) in proton-proton collision



$$\sigma = \int dx_1 f_{q/p}(x_1) \int dx_2 f_{\bar{q}/p}(x_2) \hat{\sigma}_{q\bar{q}\to ZH}(x_1 x_2 s)$$

Hard X-section

Parton distribution function

$s = (p_1 + p_2)^2$ Center of mass energy

xi= fraction of the proton momentum carried by the parton-i

We will discuss how to measure the PDF

Jet Production - 1

Cross section can be written as sum of terms

$$\frac{d^3\sigma}{dy_3 dy_4 dp_T^2} = \frac{1}{16\pi s^2} \sum_{i,j,k,l=q,\bar{q},g} \frac{f_i(x_1,\mu^2)}{x_1} \frac{f_j(x_2,\mu^2)}{x_2} \times \overline{\sum} |\mathcal{M}(ij \to kl)|^2 \frac{1}{1+\delta_{kl}}$$

Where:

i.j are the two incoming partons and k,l the outcoming ones.

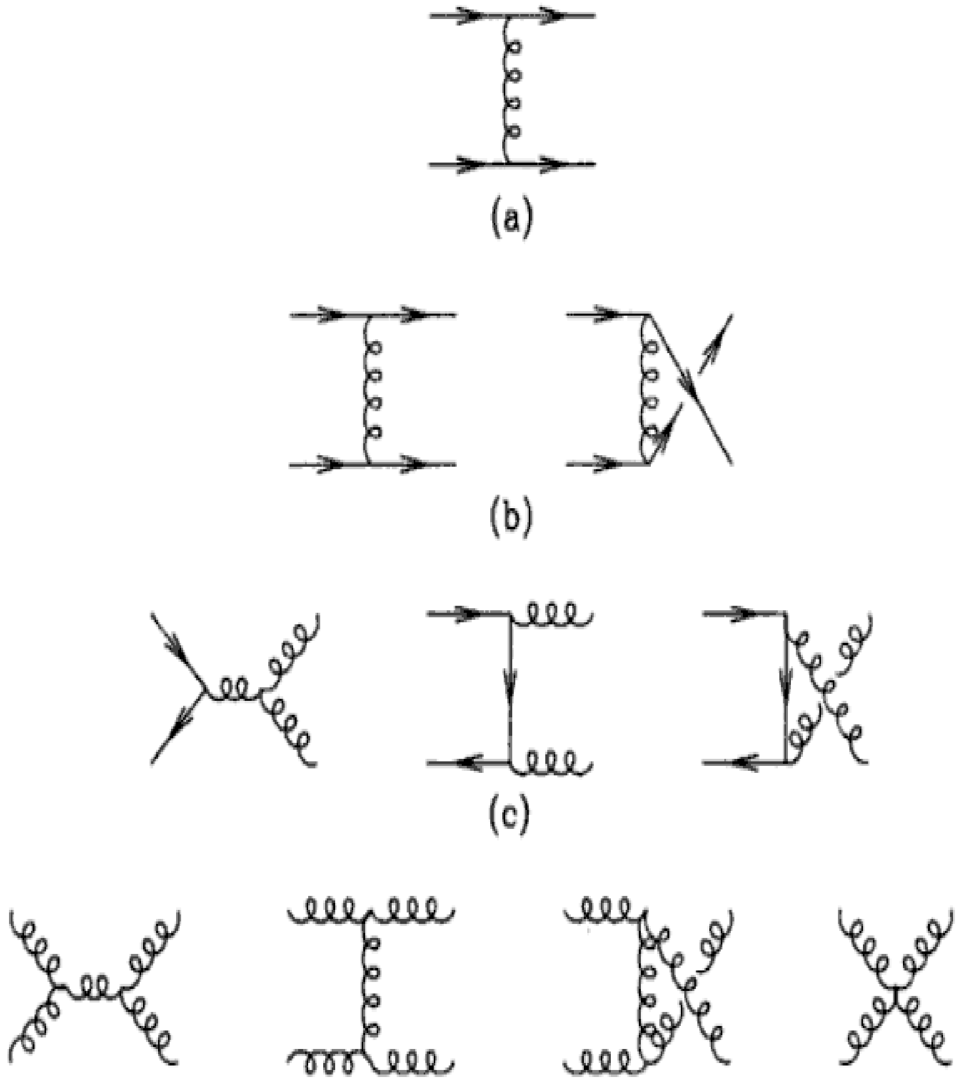
$Y_{3,4}$ are the rapidity of the outcoming partons in the laboratory system.

with rapidity $y = \frac{1}{2} \ln\left(\frac{E+P_z}{E-P_z}\right)$

μ is the scale

M(ij → kl) the matrix element

Jet Production - 2



(d)

Process	$\overline{\sum}\lvert\mathcal{M}\rvert^2/g^4$	$\theta^* = \pi/2$
$q\,q' \to q\,q'$	$\frac{4}{9}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.22
$q\,\overline{q}' \to q\,\overline{q}'$	$\frac{4}{9}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.22
$q\,q \to q\,q$	$\frac{4}{9}\,(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}) - \frac{8}{27}\,\frac{\hat{s}^2}{\hat{u}\hat{t}}$	3.26
$q\,\overline{q} \to q'\,\overline{q}'$	$\frac{4}{9}\,\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.22
$q\,\overline{q} \to q\,\overline{q}$	$\frac{4}{9}\,(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}) - \frac{8}{27}\,\frac{\hat{u}^2}{\hat{s}\hat{t}}$	2.59
$q\,\overline{q} \to g\,g$	$\frac{32}{27}\,\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3}\,\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	1.04
$g\,g \to q\,\overline{q}$	$\frac{1}{6}\,\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8}\,\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.15
$g\,q \to g\,q$	$-\frac{4}{9}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}$	6.11
$g\,g \to g\,g$	$\frac{9}{2}\,(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2})$	30.4

Parton process at lowest order can be derived from the above diagrams

LO matrix elements squared are in the table

Jets Cross Section vs. Et - 1

It is interesting to compare data and theory in :

-Et distribution

-jets angular distribution

$$\frac{d\sigma}{dE_T} = \left[\alpha_s^2(\mu_R)\,\mathcal{A} + \alpha_s^3(\mu_R)\left(\mathcal{B} + 2b_0\log(\mu_R/E_T)\,\mathcal{A} - 2P_{qq}\log(\mu_F/E_T)\,\mathcal{A}\right)\right] \otimes f_q(\mu_F) \otimes f_{\bar{q}}(\mu_F).$$

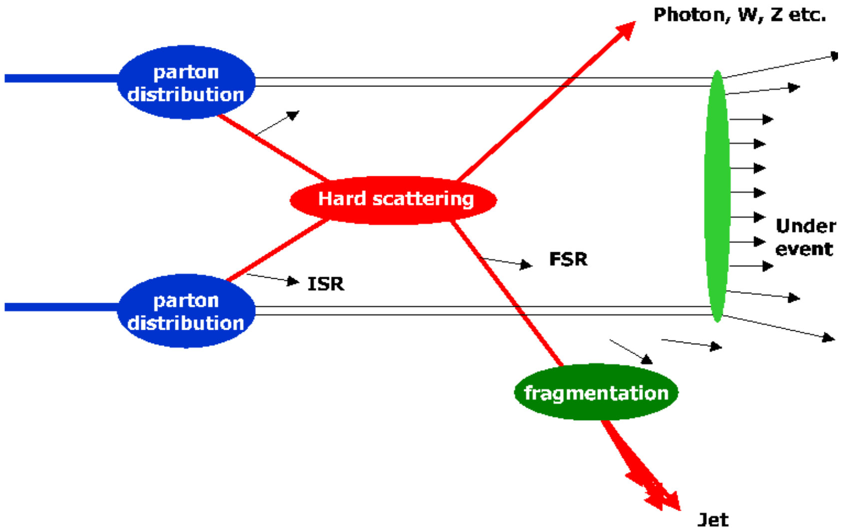
shorthand for convolution with PDF

Leading Order term

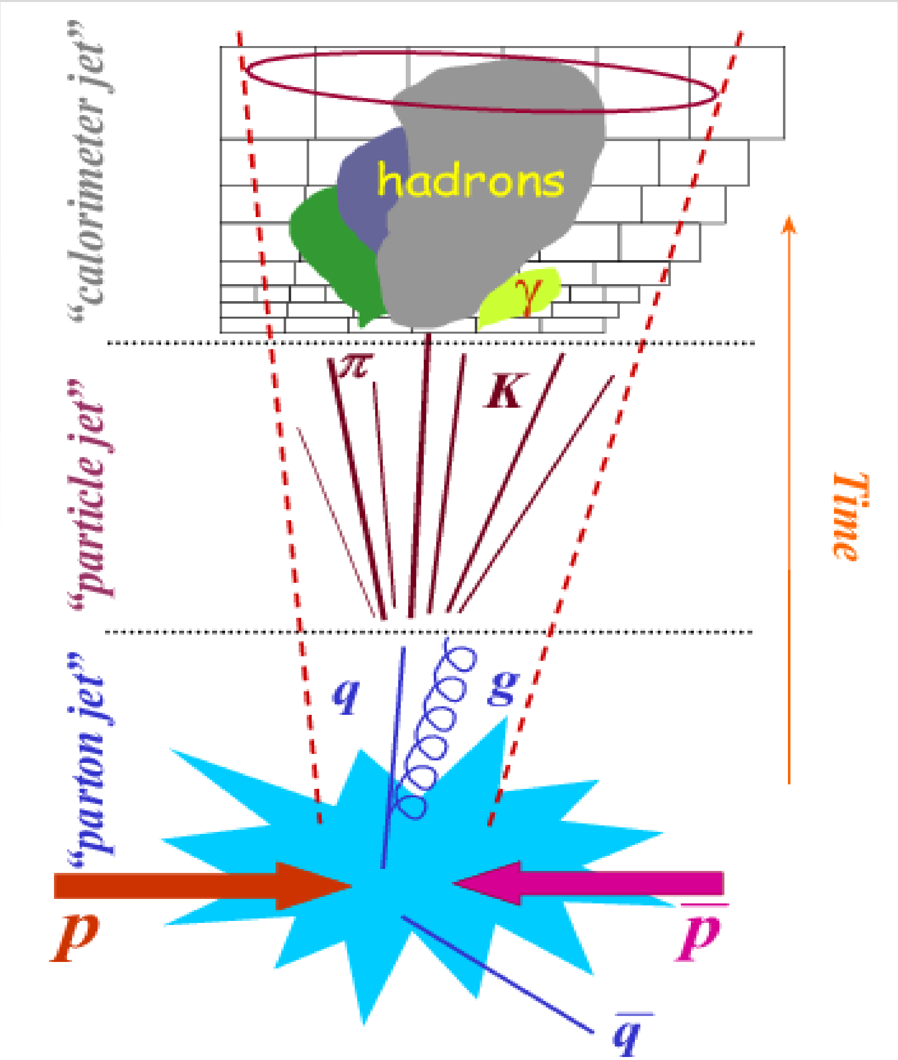
Next to Leading Order term

How to measure it on data?

On real data we have:



Jets Cross Section vs. Et - 2



- Final state partons are revealed through collimated flows of hadrons called jets
- Measurements are performed at hadron level & theory is parton level (hadron → parton transition will depend on parton shower modeling)
- Precise jet search algorithms necessary to compare with theory and to define hard physics

Jets Reconstruction

1) Reconstruct jets using one of the "jet algorithms"
 Required features in a jet finding algorithm:
 Detector independence: the performance of the jet algorithm should not be dependent on detector segmentation, energy resolution, …
 Stability with luminosity: jet finding should not be strongly affected by multiple hard scatterings at high beam luminosities.
 Fast
 Efficient: the jet algorithm should find as many physically interesting jets as possible
2) Need to correct for all the effects that do not guarantee the described criteria

Jets Energy Correction

Jets energy must be corrected for:

Level 0 - "Online/Offline calibrations"

Covert the ADC counts in energy

Level 1 – "Eta-dependent"

Usually experiments cover regions in eta with different calorimetry
The flow of particles changes as eta increases.

Level 2 - "Multiple Interactions"

Energy from different interactions falls inside the jet cluster increasing the measured energy. This additional energy evaluated using minimum bias events is subtracted

Level 3 – "Absolute"

The jet energy measured in the calorimeter needs to be corrected for any non-linearity and energy loss in the un-instrumented regions of each calorimeter.

Jets Energy Correction

Level 4 - "Underlying Event"

The underlying event is defined as the energy associated with the spectator partons in a hard collision event. Depending on the details of the particular analysis, this energy needs to be subtracted from the particle-level jet energy.

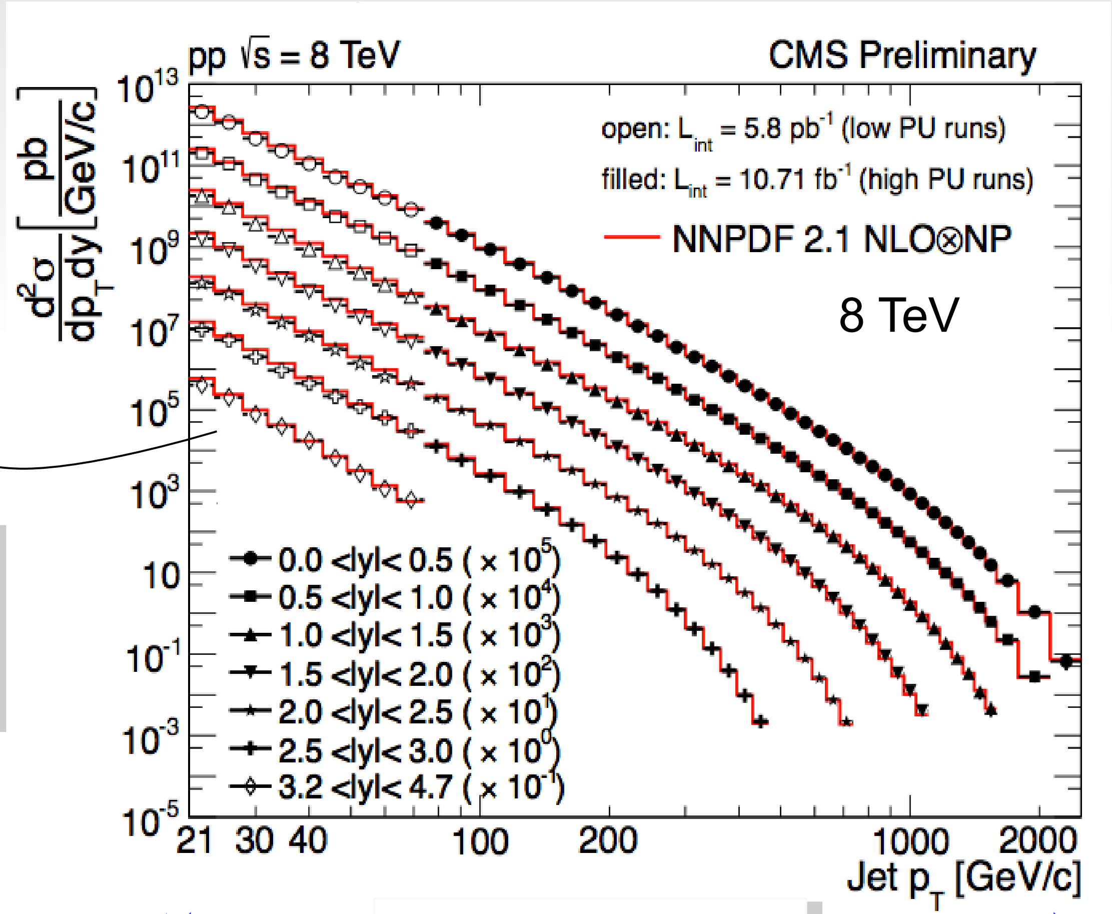
The UE energy was measured from minimum bias data requiring events only one vertex.

Level 5 – "Out-of-cone"

It corrects the particle-level energy for leakage of radiation outside the clustering cone used for jet definition, taking the "jet energy" back to "parent parton energy".

Results on Jets Cross Section vs. Et

After selecting events with at least two jets by mean of an unfolding procedure we can compare the inclusive jet cross section with the QCD expectations



Low pile-up data to extend to the low p_T range down to 20 GeV and $|y|<4.7$

11 orders of magnitude

20 Gev – 2 TeV

Jets angular distribution - 1

Jets angular distribution is sensitive to the form of 2 → 2 matrix element in the parton-parton center of mass and to quark sub-structure. The point-like quarks produce:

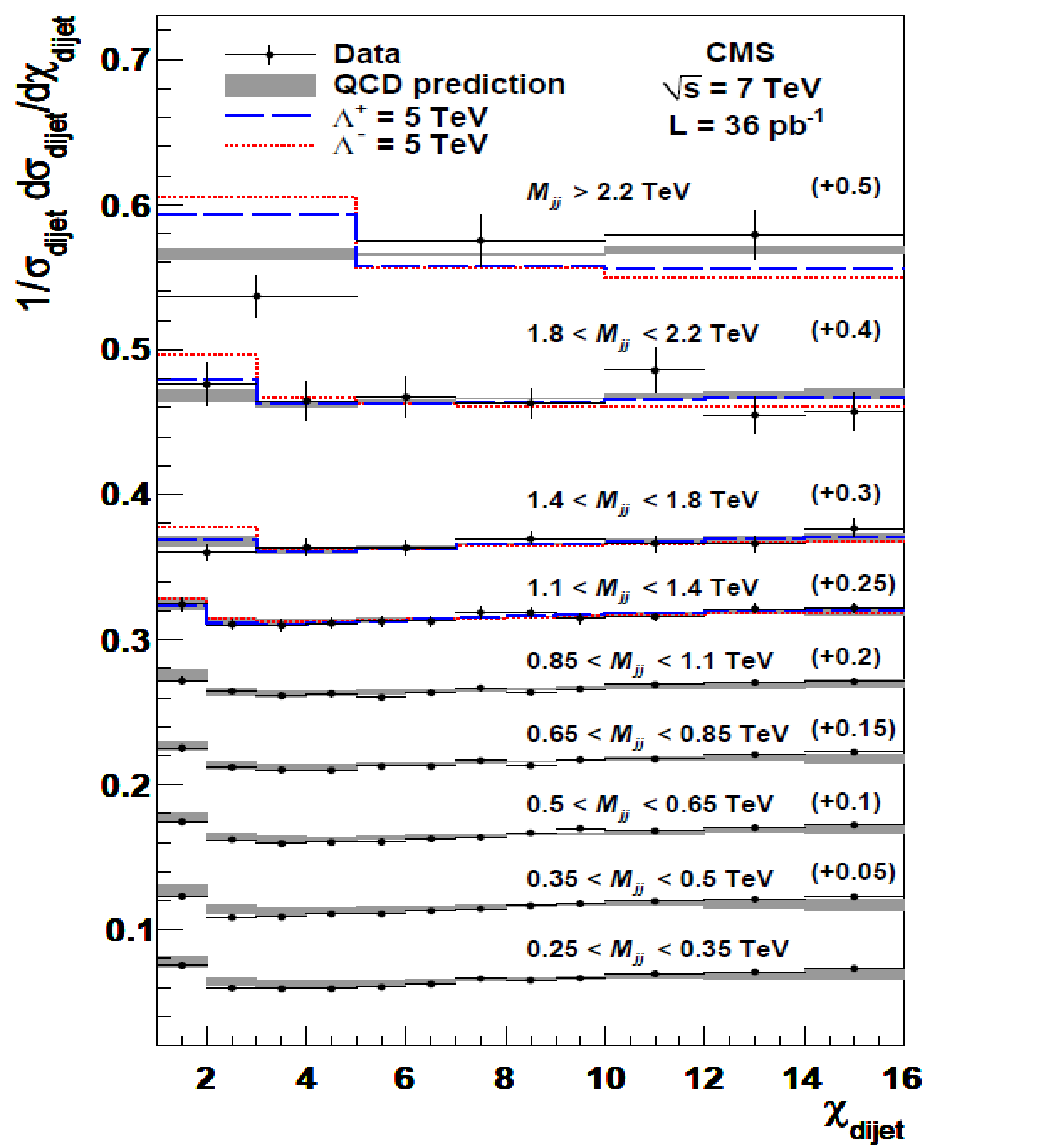
$$\frac{d\sigma}{d\cos\theta^*} \approx \frac{1}{\sin^4\theta^*/2} \qquad \chi = \frac{1+\cos\theta^*}{1-\cos\theta^*} \qquad \frac{d\sigma}{d\chi} \approx constant$$

If quarks have a sub-structure the Lagrangian has to include:

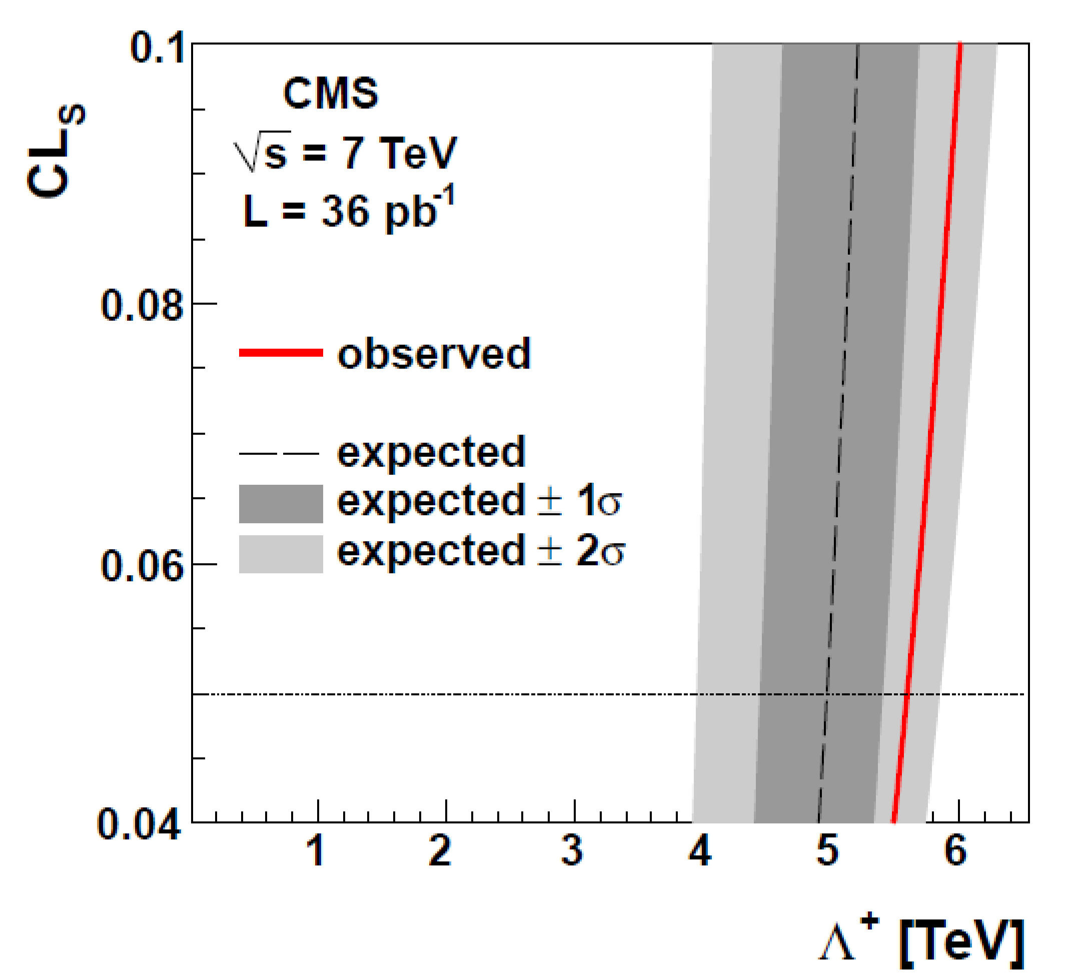
$$L = \pm(g^2/2\Lambda^2(\bar{q}_L\gamma_\mu q_L)(\bar{q}_L\gamma^\mu q_L)),$$ a contact color singlet term.

The coupling constant $g^2/4\pi = 1$ and $1/\Lambda$ is a measure of the quark size. This additional term will enhance the di-jets cross section around 90° in the di-jets center of mass. The amplitude is proportional to $\hat{s}/\Lambda$ which increase with the jet-jet invariant mass.
The experiment is sensitive to $(\hat{s}/\Lambda)^2$

Jets angular distribution - 2



Jets angular distribution - 3



Data in agreement with MC with quark without sub-structure

We can set a limit on the sub-structure coupling:
$\Lambda > 2.4$ TeV

Jets with different size

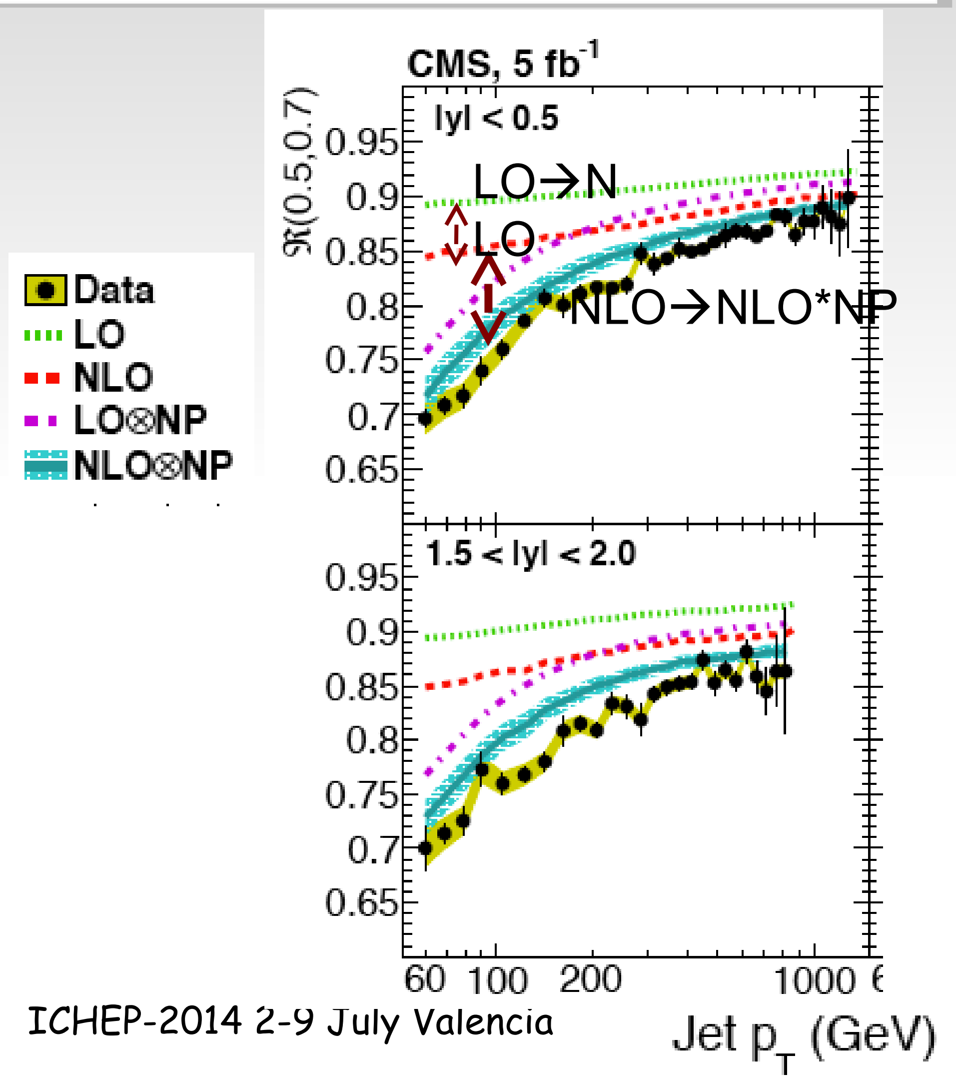
We evaluate the inclusive cross-section ratio measured with two different cone size:

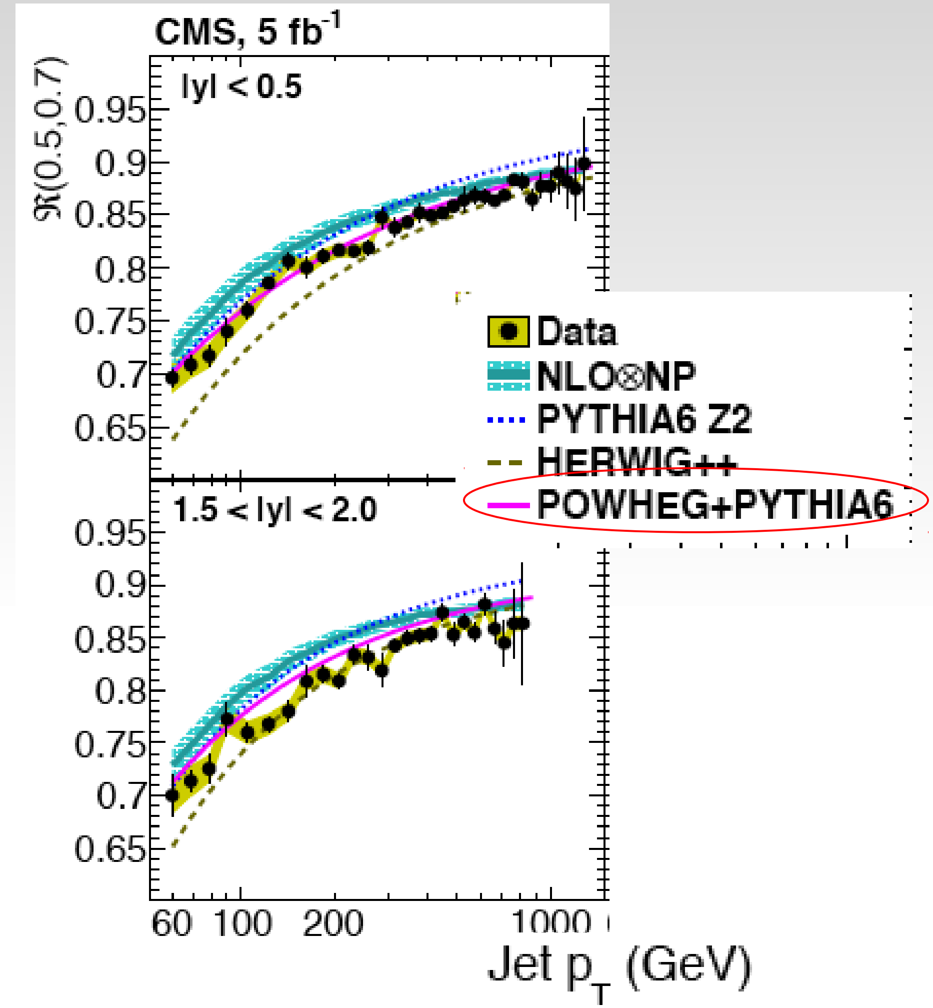
$R(0.5,0.7) = [d\sigma(0.5)/dp_T]/[d\sigma(0.7)/dp_T]$

Variable sensitive to pertubative radiation, hadronization, underlying-event

Jets with different size

$R(0.5,0.7) = [d\sigma(0.5)/dp_T]/[d\sigma(0.7)/dp_T]$





Best description need NLO and parton shower are needed to describe correctly the ratio