



TABLE C.4  
 $\chi^2$  distribution. Values of the reduced chi-square  $\chi^2_\nu = \chi^2 / \nu$  corresponding to the probability  $P_\chi(\chi^2; \nu)$  of exceeding  $\chi^2$  vs. the number of degrees of freedom  $\nu$

| $\nu$ | $P$     |         |         |        |        |       |       |       |
|-------|---------|---------|---------|--------|--------|-------|-------|-------|
|       | 0.99    | 0.98    | 0.95    | 0.90   | 0.80   | 0.70  | 0.60  | 0.50  |
| 1     | 0.00016 | 0.00063 | 0.00393 | 0.0158 | 0.0642 | 0.148 | 0.275 | 0.455 |
| 2     | 0.0100  | 0.0202  | 0.0515  | 0.105  | 0.223  | 0.357 | 0.511 | 0.693 |
| 3     | 0.0383  | 0.0617  | 0.117   | 0.195  | 0.335  | 0.475 | 0.623 | 0.789 |
| 4     | 0.0742  | 0.107   | 0.178   | 0.266  | 0.412  | 0.549 | 0.688 | 0.839 |
| 5     | 0.111   | 0.150   | 0.229   | 0.322  | 0.469  | 0.600 | 0.731 | 0.870 |
| 6     | 0.145   | 0.189   | 0.273   | 0.367  | 0.512  | 0.638 | 0.762 | 0.891 |
| 7     | 0.177   | 0.223   | 0.310   | 0.405  | 0.546  | 0.667 | 0.785 | 0.907 |
| 8     | 0.206   | 0.254   | 0.342   | 0.436  | 0.574  | 0.691 | 0.803 | 0.918 |
| 9     | 0.232   | 0.281   | 0.369   | 0.463  | 0.598  | 0.710 | 0.817 | 0.927 |
| 10    | 0.256   | 0.306   | 0.394   | 0.487  | 0.618  | 0.727 | 0.830 | 0.934 |
| 11    | 0.278   | 0.328   | 0.416   | 0.507  | 0.635  | 0.741 | 0.840 | 0.940 |
| 12    | 0.298   | 0.348   | 0.436   | 0.525  | 0.651  | 0.753 | 0.848 | 0.945 |
| 13    | 0.316   | 0.367   | 0.453   | 0.542  | 0.664  | 0.764 | 0.856 | 0.949 |
| 14    | 0.333   | 0.383   | 0.469   | 0.556  | 0.676  | 0.773 | 0.863 | 0.953 |
| 15    | 0.349   | 0.399   | 0.484   | 0.570  | 0.687  | 0.781 | 0.869 | 0.956 |
| 16    | 0.363   | 0.413   | 0.498   | 0.582  | 0.697  | 0.789 | 0.874 | 0.959 |
| 17    | 0.377   | 0.427   | 0.510   | 0.593  | 0.706  | 0.796 | 0.879 | 0.961 |
| 18    | 0.390   | 0.439   | 0.522   | 0.604  | 0.714  | 0.802 | 0.883 | 0.963 |
| 19    | 0.402   | 0.451   | 0.532   | 0.613  | 0.722  | 0.808 | 0.887 | 0.965 |
| 20    | 0.413   | 0.462   | 0.543   | 0.622  | 0.729  | 0.813 | 0.890 | 0.967 |
| 22    | 0.434   | 0.482   | 0.561   | 0.638  | 0.742  | 0.823 | 0.897 | 0.970 |
| 24    | 0.452   | 0.500   | 0.577   | 0.652  | 0.753  | 0.831 | 0.902 | 0.972 |
| 26    | 0.469   | 0.516   | 0.592   | 0.665  | 0.762  | 0.838 | 0.907 | 0.974 |
| 28    | 0.484   | 0.530   | 0.605   | 0.676  | 0.771  | 0.845 | 0.911 | 0.976 |
| 30    | 0.498   | 0.544   | 0.616   | 0.687  | 0.779  | 0.850 | 0.915 | 0.978 |
| 32    | 0.511   | 0.556   | 0.627   | 0.696  | 0.786  | 0.855 | 0.918 | 0.979 |
| 34    | 0.523   | 0.567   | 0.637   | 0.704  | 0.792  | 0.860 | 0.921 | 0.980 |
| 36    | 0.534   | 0.577   | 0.646   | 0.712  | 0.798  | 0.864 | 0.924 | 0.982 |
| 38    | 0.545   | 0.587   | 0.655   | 0.720  | 0.804  | 0.868 | 0.926 | 0.983 |
| 40    | 0.554   | 0.596   | 0.663   | 0.726  | 0.809  | 0.872 | 0.928 | 0.983 |
| 42    | 0.563   | 0.604   | 0.670   | 0.733  | 0.813  | 0.875 | 0.930 | 0.984 |
| 44    | 0.572   | 0.612   | 0.677   | 0.738  | 0.818  | 0.878 | 0.932 | 0.985 |
| 46    | 0.580   | 0.620   | 0.683   | 0.744  | 0.822  | 0.881 | 0.934 | 0.986 |
| 48    | 0.587   | 0.627   | 0.690   | 0.749  | 0.825  | 0.884 | 0.936 | 0.986 |
| 50    | 0.594   | 0.633   | 0.695   | 0.754  | 0.829  | 0.886 | 0.937 | 0.987 |
| 60    | 0.625   | 0.662   | 0.720   | 0.774  | 0.844  | 0.897 | 0.944 | 0.989 |
| 70    | 0.649   | 0.684   | 0.739   | 0.790  | 0.856  | 0.905 | 0.949 | 0.990 |
| 80    | 0.669   | 0.703   | 0.755   | 0.803  | 0.865  | 0.911 | 0.952 | 0.992 |
| 90    | 0.686   | 0.718   | 0.768   | 0.814  | 0.873  | 0.917 | 0.955 | 0.993 |
| 100   | 0.701   | 0.731   | 0.779   | 0.824  | 0.879  | 0.921 | 0.958 | 0.993 |
| 120   | 0.724   | 0.753   | 0.798   | 0.839  | 0.890  | 0.928 | 0.962 | 0.994 |
| 140   | 0.743   | 0.770   | 0.812   | 0.850  | 0.898  | 0.934 | 0.965 | 0.995 |
| 160   | 0.758   | 0.784   | 0.823   | 0.860  | 0.905  | 0.938 | 0.968 | 0.996 |
| 180   | 0.771   | 0.796   | 0.833   | 0.868  | 0.910  | 0.942 | 0.970 | 0.996 |
| 200   | 0.782   | 0.806   | 0.841   | 0.874  | 0.915  | 0.945 | 0.972 | 0.997 |

**TABLE C.4**  
 **$\chi^2$  distribution** (*continued*)

| $\nu$ | $P$   |       |       |       |       |       |       |        |
|-------|-------|-------|-------|-------|-------|-------|-------|--------|
|       | 0.40  | 0.30  | 0.20  | 0.10  | 0.05  | 0.02  | 0.01  | 0.001  |
| 1     | 0.708 | 1.074 | 1.642 | 2.706 | 3.841 | 5.412 | 6.635 | 10.827 |
| 2     | 0.916 | 1.204 | 1.609 | 2.303 | 2.996 | 3.912 | 4.605 | 6.908  |
| 3     | 0.982 | 1.222 | 1.547 | 2.084 | 2.605 | 3.279 | 3.780 | 5.423  |
| 4     | 1.011 | 1.220 | 1.497 | 1.945 | 2.372 | 2.917 | 3.319 | 4.617  |
| 5     | 1.026 | 1.213 | 1.458 | 1.847 | 2.214 | 2.678 | 3.017 | 4.102  |
| 6     | 1.035 | 1.205 | 1.426 | 1.774 | 2.099 | 2.506 | 2.802 | 3.743  |
| 7     | 1.040 | 1.198 | 1.400 | 1.717 | 2.010 | 2.375 | 2.639 | 3.475  |
| 8     | 1.044 | 1.191 | 1.379 | 1.670 | 1.938 | 2.271 | 2.511 | 3.266  |
| 9     | 1.046 | 1.184 | 1.360 | 1.632 | 1.880 | 2.187 | 2.407 | 3.097  |
| 10    | 1.047 | 1.178 | 1.344 | 1.599 | 1.831 | 2.116 | 2.321 | 2.959  |
| 11    | 1.048 | 1.173 | 1.330 | 1.570 | 1.789 | 2.056 | 2.248 | 2.842  |
| 12    | 1.049 | 1.168 | 1.318 | 1.546 | 1.752 | 2.004 | 2.185 | 2.742  |
| 13    | 1.049 | 1.163 | 1.307 | 1.524 | 1.720 | 1.959 | 2.130 | 2.656  |
| 14    | 1.049 | 1.159 | 1.296 | 1.505 | 1.692 | 1.919 | 2.082 | 2.580  |
| 15    | 1.049 | 1.155 | 1.287 | 1.487 | 1.666 | 1.884 | 2.039 | 2.513  |
| 16    | 1.049 | 1.151 | 1.279 | 1.471 | 1.644 | 1.852 | 2.000 | 2.453  |
| 17    | 1.048 | 1.148 | 1.271 | 1.457 | 1.623 | 1.823 | 1.965 | 2.399  |
| 18    | 1.048 | 1.145 | 1.264 | 1.444 | 1.604 | 1.797 | 1.934 | 2.351  |
| 19    | 1.048 | 1.142 | 1.258 | 1.432 | 1.586 | 1.773 | 1.905 | 2.307  |
| 20    | 1.048 | 1.139 | 1.252 | 1.421 | 1.571 | 1.751 | 1.878 | 2.266  |
| 22    | 1.047 | 1.134 | 1.241 | 1.401 | 1.542 | 1.712 | 1.831 | 2.194  |
| 24    | 1.046 | 1.129 | 1.231 | 1.383 | 1.517 | 1.678 | 1.791 | 2.132  |
| 26    | 1.045 | 1.125 | 1.223 | 1.368 | 1.496 | 1.648 | 1.755 | 2.079  |
| 28    | 1.045 | 1.121 | 1.215 | 1.354 | 1.476 | 1.622 | 1.724 | 2.032  |
| 30    | 1.044 | 1.118 | 1.208 | 1.342 | 1.459 | 1.599 | 1.696 | 1.990  |
| 32    | 1.043 | 1.115 | 1.202 | 1.331 | 1.444 | 1.578 | 1.671 | 1.953  |
| 34    | 1.042 | 1.112 | 1.196 | 1.321 | 1.429 | 1.559 | 1.649 | 1.919  |
| 36    | 1.042 | 1.109 | 1.191 | 1.311 | 1.417 | 1.541 | 1.628 | 1.888  |
| 38    | 1.041 | 1.106 | 1.186 | 1.303 | 1.405 | 1.525 | 1.610 | 1.861  |
| 40    | 1.041 | 1.104 | 1.182 | 1.295 | 1.394 | 1.511 | 1.592 | 1.835  |
| 42    | 1.040 | 1.102 | 1.178 | 1.288 | 1.384 | 1.497 | 1.576 | 1.812  |
| 44    | 1.039 | 1.100 | 1.174 | 1.281 | 1.375 | 1.485 | 1.562 | 1.790  |
| 46    | 1.039 | 1.098 | 1.170 | 1.275 | 1.366 | 1.473 | 1.548 | 1.770  |
| 48    | 1.038 | 1.096 | 1.167 | 1.269 | 1.358 | 1.462 | 1.535 | 1.751  |
| 50    | 1.038 | 1.094 | 1.163 | 1.263 | 1.350 | 1.452 | 1.523 | 1.733  |
| 60    | 1.036 | 1.087 | 1.150 | 1.240 | 1.318 | 1.410 | 1.473 | 1.660  |
| 70    | 1.034 | 1.081 | 1.139 | 1.222 | 1.293 | 1.377 | 1.435 | 1.605  |
| 80    | 1.032 | 1.076 | 1.130 | 1.207 | 1.273 | 1.351 | 1.404 | 1.560  |
| 90    | 1.031 | 1.072 | 1.123 | 1.195 | 1.257 | 1.329 | 1.379 | 1.525  |
| 100   | 1.029 | 1.069 | 1.117 | 1.185 | 1.243 | 1.311 | 1.358 | 1.494  |
| 120   | 1.027 | 1.063 | 1.107 | 1.169 | 1.221 | 1.283 | 1.325 | 1.446  |
| 140   | 1.026 | 1.059 | 1.099 | 1.156 | 1.204 | 1.261 | 1.299 | 1.410  |
| 160   | 1.024 | 1.055 | 1.093 | 1.146 | 1.191 | 1.243 | 1.278 | 1.381  |
| 180   | 1.023 | 1.052 | 1.087 | 1.137 | 1.179 | 1.228 | 1.261 | 1.358  |
| 200   | 1.022 | 1.050 | 1.083 | 1.130 | 1.170 | 1.216 | 1.247 | 1.338  |