Selected Topics in Flavor Physics

- CKM Matrix & The Unitarity Triangle
  - Measurement of the Sides: $|V_{cb}|$, $|V_{ub}|$
- B^0_q Mixing
  - Δm_q and A^{sl}_q
- CPV & Measurement of the Angles
  - β, Φ_s & $\Delta\Gamma_s$, α, γ
  - Constraints on Unitarity Triangle

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CKM Matrix

See for example
G. Isidori arXiv:1302.0661
Y. Grossman arXiv:1006.3534

- In the Standard Model each quark generation consists of three multiplets:

$$Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, \quad u_{Ri}, \quad d_{Ri}$$

where the first is a doublet of SU(2), the others are singlets, L and R is the helicity, and i=1,3 is the generation.

- The masses and mixings of quarks arise from the Yukawa interactions of quarks with the Higgs:

$$L_Y = -h^u_{ij} \bar{Q}_{Li} \tilde{\Phi} u_{Rj} - h^d_{ij} \bar{Q}_{Li} \Phi d_{Rj} + h.c. \quad (1)$$

- where h^u_{ij} and h^d_{ij} are 3 X 3 complex matrices, i and j are generation labels, and Φ is the Higgs single scalar doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}; \quad \tilde{\Phi} = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$$

- The complex nature of $h^{u(d)}_{ij}$ is the source of CP violation in the SM (see later).
- The quarks mass terms arise when Φ acquires a vacuum expectation value: $\langle \Phi^0 \rangle_0 = \eta$.

CKM Matrix

- From the first term in (1), with the usual symmetry breaking transformation

$$\tilde{\Phi}=\begin{pmatrix}\phi^0\\ \phi^-\end{pmatrix}\rightarrow\langle\tilde{\Phi}\rangle=\begin{pmatrix}\eta+\frac{\sigma}{\sqrt{2}}\\ 0\end{pmatrix}$$ one gets:

Particle σ ~ field oscillation around the vacuum expectation value η

$$h_{ij}^{u}(\bar{u}_{Li}\ \ \bar{d}_{Li})\begin{pmatrix}\eta+\frac{\sigma}{\sqrt{2}}\\ 0\end{pmatrix}u_{Rj}\rightarrow h_{ij}^{u}\bar{u}_{Li}\left(\eta+\frac{\sigma}{\sqrt{2}}\right)u_{Rj}=h_{ij}^{u}\bar{u}_{Li}\eta u_{Rj}+h_{ij}^{u}\bar{u}_{Li}\frac{\sigma}{\sqrt{2}}u_{Rj} \qquad (2)$$

  - Mixted terms like $\bar{d}\,\phi\,u$ disappear
  - Not Hermitian (and not diagonal) mass matrices for the up quarks are obtained from the first term in (2):

$$M_{ij}^{u}=h_{ij}^{u}\eta$$

  - The couplings between fermions and the Higgs come from the second term in (2):

$$\frac{h_{ij}^{u}}{\sqrt{2}}=\frac{M_{ij}^{u}}{\eta\sqrt{2}}=\frac{g\,M_{ij}^{u}}{2\,m_W};\quad\left(m_W=\frac{g\,\eta}{\sqrt{2}}\right)$$

- Analogously, for the down quarks, (second term in (1)): $M_{ij}^{d}=h_{ij}^{d}\eta;\quad\frac{h_{ij}^{d}}{\sqrt{2}}=\frac{g\,M_{ij}^{d}}{2\,m_W}$

CKM Matrix

- The mass eigenstates $u_L^0, \; u_R^0$ are obtained by diagonalizing the mass matrices via the biunitary transformation:

$$U_L^+ \, M^u U_R = M^u_{diag} ; U_L \neq U_R$$

- The new mass basis is obtained via

$$u_R = U_R u_R^0 ; \quad \bar{u}_R = \bar{u}_R^0 U_R^+$$
$$u_L = U_L u_L^0 ; \quad \bar{u}_L = \bar{u}_L^0 U_L^+ \qquad (3a)$$

- Therefore the first term in (2) transforms as:

$$h_{ij}^u \eta \bar{u}_{Li} u_{Rj} = \bar{u}_{Li} M_{ij}^u u_{Rj} = \bar{u}_{Li}^0 U_L^+ \, M_{ij}^u U_R u_{Rj}^0 = \bar{u}_{Li}^0 M^u_{diag} u_{Rj}^0$$

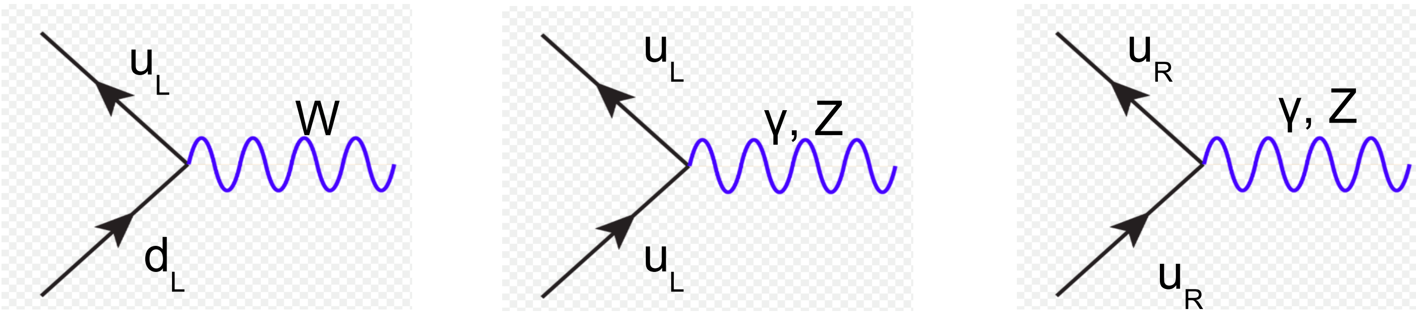
- And for the down quarks analogously:

$$V_L^+ \, M^d V_R = M^d_{diag} ; V_L \neq V_R$$

$$d_R = V_R d_R^0 ; \quad \bar{d}_R = \bar{d}_R^0 V_R^+$$
$$d_L = V_L d_L^0 ; \quad \bar{d}_L = \bar{d}_L^0 V_L^+ \qquad (3b)$$
$$\rightarrow h_{ij}^d \eta \bar{d}_{Li} d_{Rj} = \bar{d}_{Li}^0 M^d_{diag} d_{Rj}^0$$

CKM Matrix

- Before the diagonalization of the mass matrices, interactions are written in terms of "current eigenstates" in the "interaction basis": no change in the particle generation



W does not couple with right handed quarks (SU(2) singlets)

$$\bar{u}_L \gamma^\mu d_L W_\mu^+ = \left(\bar{u}, \bar{c}, \bar{t}\right)_L \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ = \left(\bar{u}_L \gamma^\mu d_L + \bar{c}_L \gamma^\mu s_L + \bar{t}_L \gamma^\mu b_L\right) W_\mu^+$$

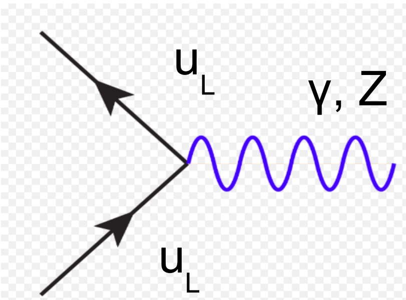
- Using (3a) and (3b), charge current in the "mass eigenstates" basis becomes:

$$\bar{u}_L \gamma^\mu d_L W_\mu^+ = \bar{u}_{Li}^0 \gamma^\mu \left(U_L^+ V_L\right)_{ij} d_{Lj}^0 W_\mu^+$$

  - The $\left(U_L^+ V_L\right)$ matrix (not diagonal) is the Cabibbo Kobayashi Maskawa (CKM) matrix: interactions eigenstates are not mass eigenstates
  - The degree of mixing between two generations is given by the $\left(U_L^+ V_L\right)_{ij}$ matrix element
  - In the "mass basis", generation change is allowed via charge current W exchange

CKM Matrix

- For the neutral current γ, Z exchange:

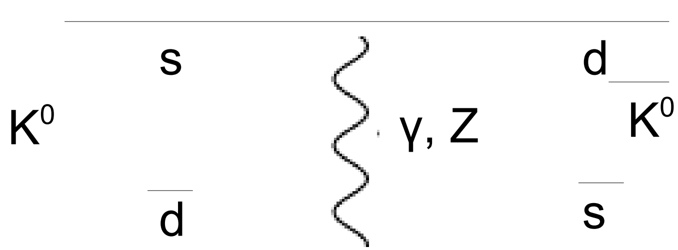


$$\bar{u}_L \gamma^\mu u_L Z(A)_\mu = \bar{u}^0_{Li} \left(U_L^+ \ \gamma^\mu U_L\right)_{ij} u^0_{Lj} Z(A)_\mu = \bar{u}^0_{Li} \gamma^\mu (1)_{ij} u^0_{Lj} Z(A)_\mu$$

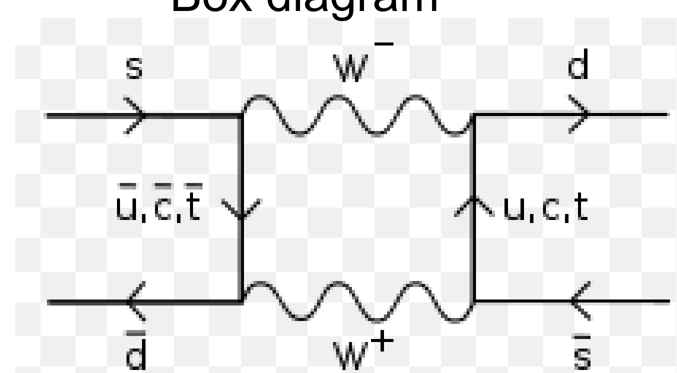
where $(1)_{ij}$ is the unit matrix which is diagonal, therefore Flavor Changing Neutral Currents (FCNC) are not allowed at tree level.

- If FCNC would be allowed it could be possible for instance to have K^0 oscillations at tree level, implying a too large value for the mass difference between the two mass eigenstates, $\Delta m_K = 3.484 \pm 0.006 \ 10^{-12}$ MeV

FCNC at tree level: not possible

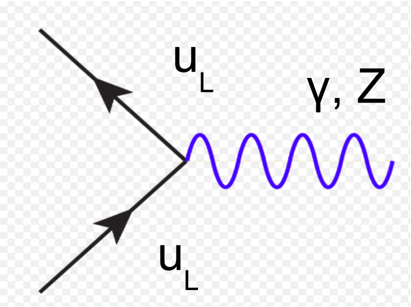


Box diagram



CKM Matrix

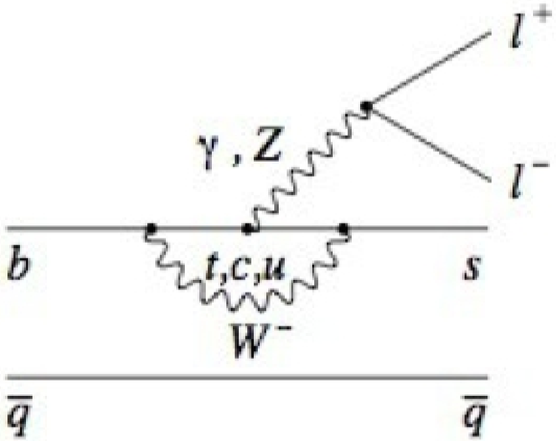
- For the neutral current γ, Z exchange:



$$\bar{u}_L \gamma^\mu u_L Z(A)_\mu = \bar{u}^0_{Li} \left(U_L^+ \ \gamma^\mu U_L\right)_{ij} u^0_{Lj} Z(A)_\mu = \bar{u}^0_{Li} \gamma^\mu (1)_{ij} u^0_{Lj} Z(A)_\mu$$

where $(1)_{ij}$ is the unit matrix which is diagonal, therefore Flavor Changing Neutral Currents (FCNC) are not allowed at tree level.

- Assuming only three quarks: u, d, s, FCNC would be still not enough suppressed to match the experimental results. This was the motivation for the theoretical prediction of the c quark (3rd generation, 1970).



- “Penguin” diagram. Contributions coming from the u, c, t quarks exchange cancel (GIM mechanism)
- $BR(B^0 \to K^* \mu\mu) \sim 10^{-6}$

CKM Matrix

- CKM Matrix determines the rotation of the current eigenstates in mass eigenstates. It is complex and unitary by construction:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad V^{+}_{CKM} V_{CKM} = \left(U^{+}_{L} V_{L}\right)^{+} \left(U^{+}_{L} V_{L}\right) = 1$$

- V_{ij} are the couplings of quark mixing transitions from an up-type quark i = u, c, t to a down-type j = d, s, b

- It depends on $2n^2 - n^2 = n^2$ independent parameters (complex parameters - unitarity conditions)

- For n generations there are 2n fields: $\begin{pmatrix} u_L & d_L \\ c_L & s_L \\ t_L & b_L \end{pmatrix}$

- Each field is defined with a phase, but only one is independent and meaningful. By fixing one phase, the total number of phases which can be eliminated by a redefinition is 2n -1

- Number of meaningful independent physical parameters: $n^2 – (2n - 1) = (n – 1)^2$

CKM Matrix

- Number of rotations in n dimensions equals the number of independent angles between n axis:

$$n_\theta = \binom{n}{2} = \frac{1}{2}(n-1)n$$

- Resulting number of meaningful phases:

$$n_\delta = (n-1)^2 - \frac{1}{2}(n-1)n = \frac{1}{2}(n-1)(n-2)$$

  - Two generations: one independent parameter (1 angle (Cabibbo), 0 phases)

$$n_\theta = 1\,;\quad n_\delta = 0$$

  - Three generations: four independent parameters (3 angles, 1 phase)

$$n_\theta = 3\,;\quad n_\delta = 1$$

- The CKM matrix for 3 families may be represented by three rotations and a matrix generating a irreducible single phase responsible for all CP Violation phenomena in flavor-changing processes in the SM (see later)

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_{13}} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}\,;\quad c_{ij} = \cos\theta_{ij}\,;\quad s_{ij} > 0\,;\quad c_{ij} > 0\,[\theta_{ij} \in \textit{first quadrant}]$$

CKM Matrix

- Particle Data Group representation
  [e.g. Chan and Keung, Phys. Rev. Lett. 53, 1802 (1984)]:

$$V_{\rm CKM} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- The matrix elements exhibit a pronounced hierarchy
  - Diagonal elements ~ 1
  - Experimentally $\theta_{12} \gg \theta_{23} \gg \theta_{13}$
  - Hierarchy introduced explicitly in the Wolfenstein parameterization [Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983)] which is an expansion in $\lambda=V_{us}=s_{12}$ ($\sin\theta_{Cabibbo}$)=0.22 up to the order λ^3:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Small off diagonal elements: $V_{ud} \gg V_{us} \gg V_{ub}$

CKM Matrix

- Four parameters: A, ρ, η (O(1)), and λ; Unitarity satisfied up to order λ^4
- Modern definition [Charles et al., Eur. Phys. J C41, 1-131 (2005)] is given in terms of the four prameters $A, \lambda, \bar{\rho}, \bar{\eta}$ where:

$$\rho + i\eta = (\bar{\rho} + i\bar{\eta}) \frac{\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]} \qquad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

at all orders

- In this definition the CKM matrix is unitary to all orders in λ, and the difference with the Wolfenstein parameterization appears only at higher orders.

- The CKM parameters are fundamental parameter of the SM. The unitarity conditions $V_{CKM} \cdot V^{+}_{CKM} = 1; \; V^{+}_{CKM} \cdot V_{CKM} = 1$ yield six vanishing combinations (two by two nearly degenerate) which can be represented as "Unitarity triangles" in a complex plane with equal areas, reflecting the fact that there is a single CP violating phase

- The common area is equal to half the Jarlskog invariant $J = c_{12} c_{23} c^2_{13} s_{12} s_{23} s_{13} \sin\delta$ [Jarlskog, Phys. Rev. Lett. 55, 1039 (2005)] which is a measure of the CP Violation.

- The unicity of the phase makes all the possible CP violating effects in the SM very closely related. The pattern of CP Violation in B decays is strongly constrained in this model.

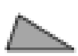
CKM Matrix

- Out of the six triangles, three are not degenerate:

$$V_{ud}V_{us}^{*}+V_{cd}V_{cs}^{*}+V_{td}V_{ts}^{*}=0\rightarrow K$$

$$V_{us}V_{ub}^{*}+V_{cs}V_{cb}^{*}+V_{ts}V_{tb}^{*}=0\rightarrow B_s$$

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0\rightarrow B_d$$



- Knowing the experimental values of the various matrix elements one finds that in the first two triangles one side is much shorter than the other two: they almost collapse to a line. Intuitive understanding of why CPV is small in the leading K decays and in the leading B_s decays. Decays related to the short sides $(K_L\rightarrow\pi\nu\bar{\nu})$ are rare but could exhibit significant CPV.

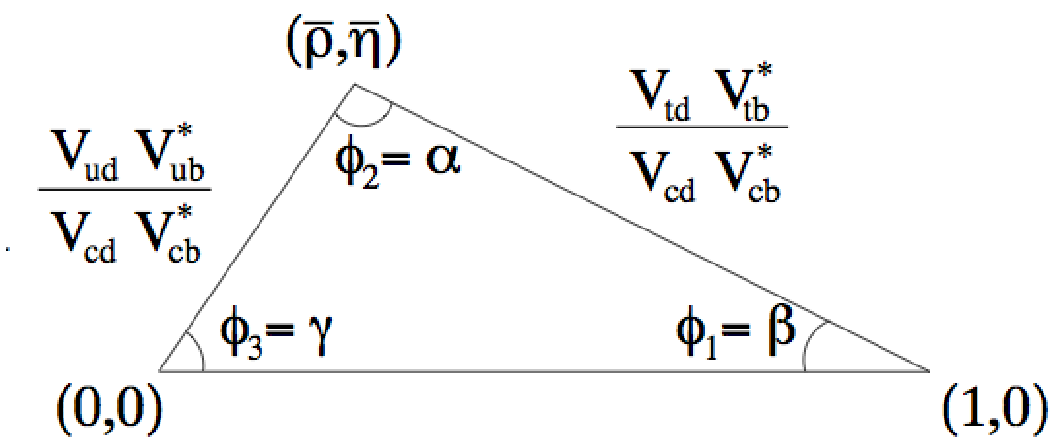
- CPV is large in leading B_d decays: the third one is called “The Unitarity Triangle”

CKM Matrix

- "The Unitarity Triangle"

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0$$

- Choosing a phase convention in which $V_{cd}V_{cb}^{*}$ is real and dividing the sides by $V_{cd}V_{cb}^{*}$ one gets the apex at $(\bar{\rho},\bar{\eta})$ and aligns the side with lenght 1 with the real axis:



- Lenght of sides:

$$R_b=\sqrt{\bar{\rho}^2+\bar{\eta}^2}=\frac{1-\lambda^2/2}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

$$R_t=\sqrt{(1-\bar{\rho})^2+\bar{\eta}^2}=\frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$

- Three angles:

$$\phi_1=\beta\equiv\arg\left[-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*}\right]$$
$$\phi_2=\alpha\equiv\arg\left[-V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*}\right]$$
$$\phi_3=\gamma\equiv\arg\left[-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*}\right]$$

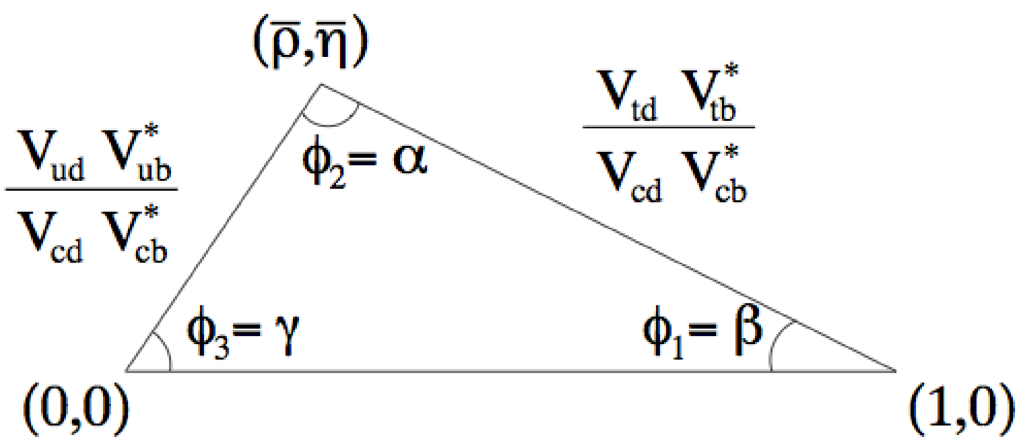
- Measured from CP asymmetries in various B decays
- Consistency of different measurements provide tests of the SM

CKM Matrix

- "The Unitarity Triangle"

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

- Choosing a phase convention in which $V_{cd}V_{cb}^*$ is real and dividing the sides by $V_{cd}V_{cb}^*$ one gets the apex at $(\bar{\rho}, \bar{\eta})$ and aligns the side with lenght 1 with the real axis:



- Lenght of sides:

$$R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{1-\lambda^2/2}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

$$R_t = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$

- β gives to a good approximation the phase between the B^0_d mixing amplitude and its leading decay amplitude (see later).
- Analogously, for the B^0_s meson: $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

$$\phi_s = \beta_s = arg\left[-V_{ts}V_{tb}^* / V_{cs}V_{cb}^*\right]$$

CKM Matrix

Magnitude of the CKM Matrix elements

- $|V_{ud}|$: from superallowed $0^+ \rightarrow 0^+$ nuclear beta decays, pure vector transitions
  $|V_{ud}|=0.97425\pm0.00022$

- $|V_{us}|$: from $K_L^0 \rightarrow \pi e(\mu)\nu, K^+ \rightarrow \pi^0 e^+(\mu^+)\nu, K_s \rightarrow \pi e \nu$
  $|V_{us}|=0.2253\pm0.0008$

- $|V_{cd}|$: from semileptonic charm decays $D \rightarrow \pi(K) l \nu$
  $|V_{cd}|=0.225\pm0.008$

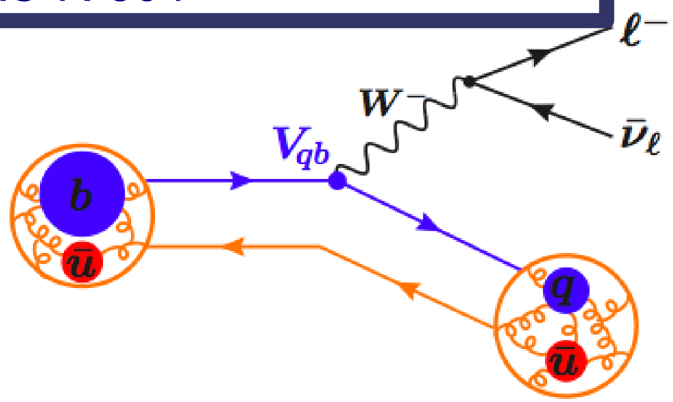
- $|V_{cs}|$: from semileptonic D and leptonic $D_s^+ \rightarrow \mu^+(\tau^+)\nu$ decays
  $|V_{cs}|=0.986\pm0.016$

- $|V_{tb}|$: from $R = \mathcal{B}(t \rightarrow Wb)/\mathcal{B}(t \rightarrow Wq) = |V_{tb}|^2/(\sum_q |V_{tq}|^2) = |V_{tb}|^2$
  $|V_{tb}|=1.021\pm0.032$

- $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$ & $|V_{ts}|$ in the following...

Measurement of $|V_{cb}|$ & $|V_{ub}|$

See for example
The Physics of the B Factories
The Eur. Phys. Journ. C
Springer
The BaBar Physics Book
SLAC-R-504



- Measurements performed exploiting semileptonic (SL) B^+ and B^0 decays, free from non-SM contributions
- $|V_{cb}|$ normalizes the Unitarity Triangle, and the ratio $|V_{ub}| / |V_{cb}|$ determines the side opposite to β.
- Their values impact several measurements of Flavor Physics and CPV.

- Two experimental methods both for V_{cb} and V_{ub}:
  - Exclusive semileptonic decays to D, D*, D** (V_{cb}) or π, ρ (V_{ub})
  - Inclusive decays $B \to X_{c(u)} l \nu$
- Complementary methods, rely on theoretical description of the QCD contributions to the different processes. Comparison of the independent results provide a crucial cross check.

Measurement of $|V_{cb}|$ & $|V_{ub}|$

Theoretical Overview

- W leptonic decay does not involve any other CKM matrix elements, hence the $B \to X_{c(u)} l \nu$ decay rate can be used to directly measure V_{cb} & V_{ub}
- The Electroweak Effective Hamiltonian describing the transition is:

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}\,(\bar{q}\gamma_\mu P_L b)(\ell\gamma^\mu P_L \nu_\ell) \qquad P_L = (1-\gamma_5)/2$$

- Differential decay rate:

$$d\Gamma \propto G_F^2 |V_{qb}|^2 \left| L^\mu \langle X|\bar{q}\gamma_\mu P_L b|B\rangle \right|^2$$

- The Hadron Matrix Element $\langle X|\bar{q}\gamma_\mu P_L b|B\rangle$ includes QCD corrections depending on the actual initial and final states which are factorized from the leptonic part L^μ (neglecting small effects as photon exchange between quarks and leptons)
  - The dermination of the HME is the challenge in the extraction of the CKM matrix elements.
- Calculations are simplified exploiting the large mass of the b quark

Measurement of $|V_{cb}|$ & $|V_{ub}|$

Hadronic Matrix Elements Determination

Exclusive Decays

- $B \rightarrow X\,;\quad X = D, D^*, \pi, \rho$ HME are parameterized in terms of Form Factors: Non-perturbative functions of the squared 4-momentum transfer $q^2 = (p_B - p_X)^2$ using:
  - Lattice QCD (LQCD): the FF(q^2) integrals are computed numerically on a discretized space-time Lattice using Heavy Quark Effective Theory (HQET) and non-relativistic QCD (NRQCD) with steadily improvable errors.
    - Best estimates at high $q^2 > 10$ GeV^2
  - Light-Cone Sum Rules (LCSR): FF(q^2) computed by using Operator Product Expansion (OPE)
  - Accuracy limited by OPE and quark-hadron duality: final states are replaced by partons
    - Best estimates at low $q^2 < 10$ GeV^2

Measurement of $|V_{cb}|$ & $|V_{ub}|$

Hadronic Matrix Elements Determination

Inclusive Decays

- The sum over all the final states kinematically allowed are considered and replaced by sum over partonic final states using Parton-Hadron duality:
  - Long-distance (hadronization) sensitivity to the final state is reduced
  - Short-distance QCD corrections at the typical scale $\mu \sim m_b$ computed perturbatively in terms of $\alpha_s(m_b) \sim 0.2$
  - Long-distance corrections related to the initial state expanded using the Heavy Quark Expansion (HQE) in powers of:

    $\Lambda_{QCD}/m_b \sim 0.1\,;\, \Lambda_{QCD} \sim m_B - m_b \sim 0.5\, GeV$
    - Decay rates expressed in terms of non-perturbative parameters

Measurement of $|V_{cb}|$ & $|V_{ub}|$

Experimental Strategy

- The current most precise measurements come from Beauty Factories where B mesons are created in pairs at a center of mass energy corresponding to the $\Upsilon(4S)$, $m_{\Upsilon(4S)}=10578.4 \pm 1.2$ MeV
- The production cross section receives sizable contributions other than $B\bar{B}$ events, rate dominated by non-B events

$e^+e^- \rightarrow$	Cross-section (nb)
$b\bar{b}$	1.05
$c\bar{c}$	1.30
$s\bar{s}$	0.35
$u\bar{u}$	1.39
$d\bar{d}$	0.35
$\tau^+\tau^-$	0.94
$\mu^+\mu^-$	1.16
e^+e^-	~ 40

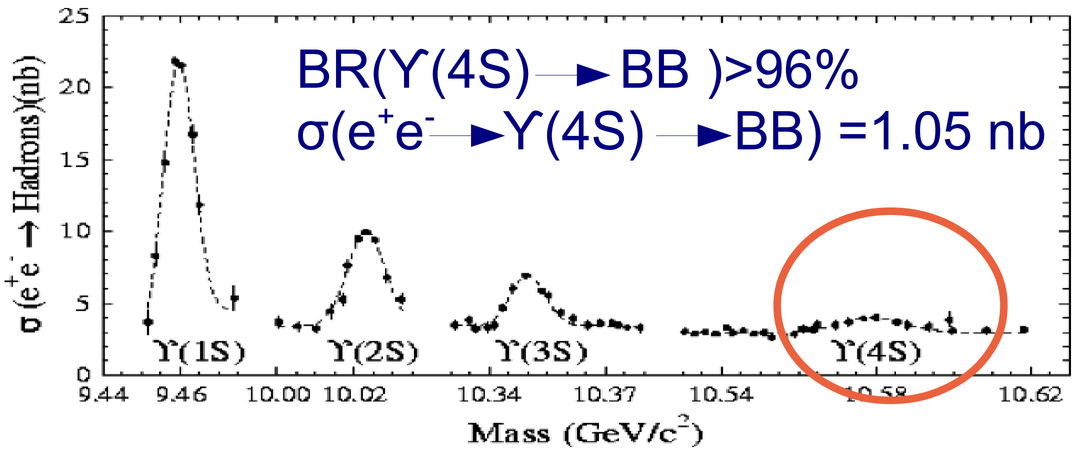
- Semileptonic B decays are affected by two dominant sources of background:
  - Continuum decays

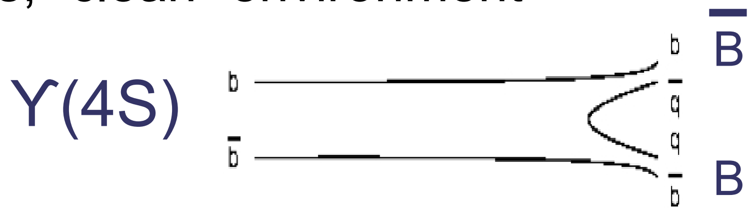
    $$e^+e^- \rightarrow l^+l^-(\gamma),\ \ q\bar{q}(\gamma),\ \ [q=u,d,s,c]$$

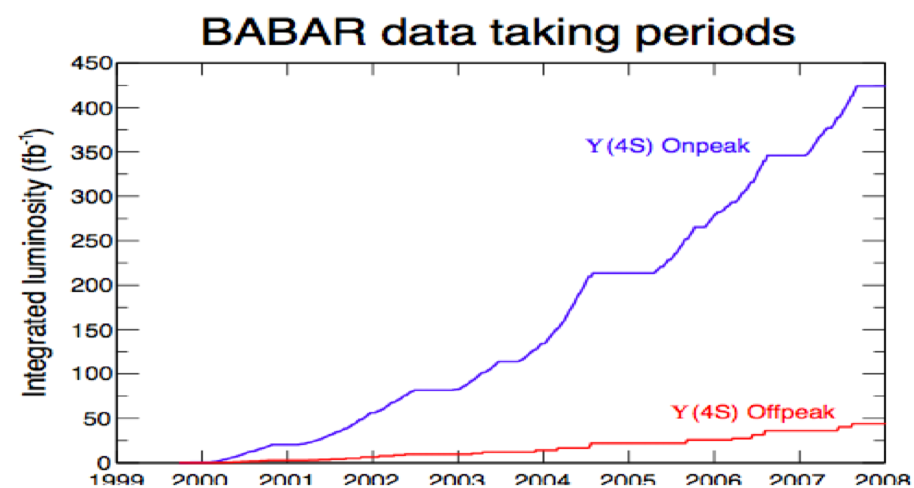
    - Studied on real data by collecting a significant fraction of off-resonance data 40/60 MeV below the $\Upsilon(4S)$.
  - $B\bar{B}$ combinatorial background from random tracks combinations

B-Factories

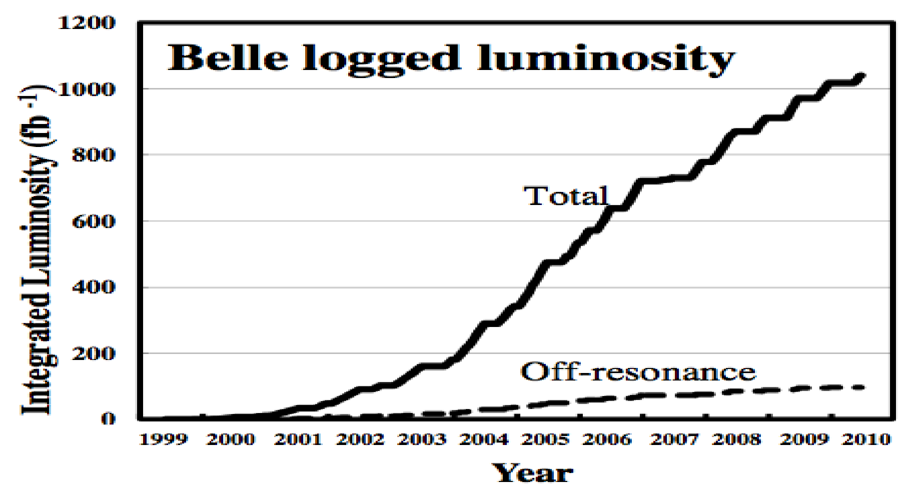
- Beauty-Factories: developed specifically for B physics, e^+e^- collisions at E_{CM} =10.5 GeV with asymmetric energy beams, “clean” environment





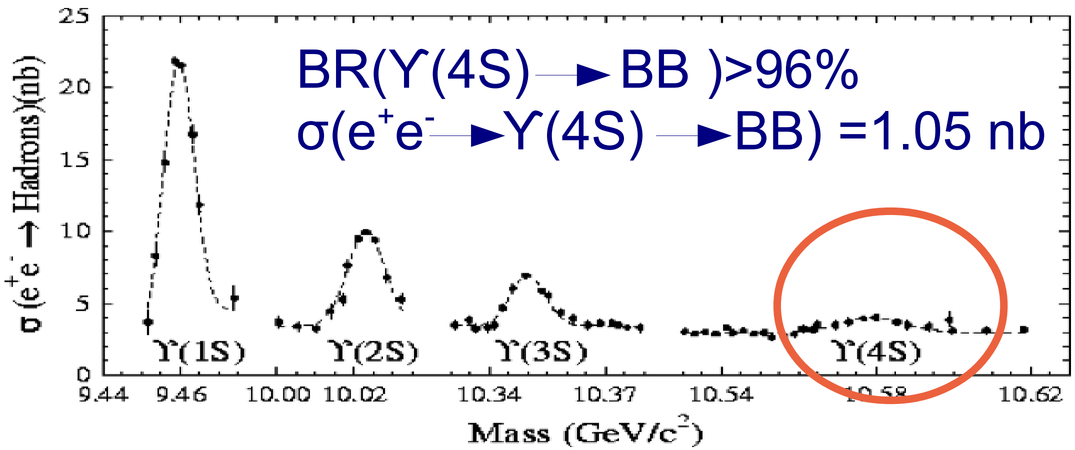


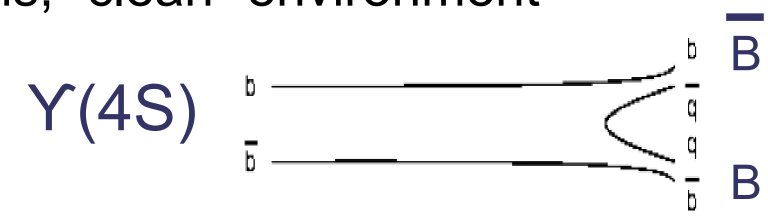
Parameters			PEP-II	KEKB
Beam energy		(GeV)	9.0 (e^-), 3.1 (e^+)	8.0 (e^-), 3.5 (e^+)
Beam current		(A)	1.8 (e^-), 2.7 (e^+)	1.2 (e^-), 1.6 (e^+)
Beam size at IP	x	(μm)	140	80
	y	(μm)	3	1
	z	(mm)	8.5	5
Luminosity		($cm^{-2}s^{-1}$)	1.2×10^{34}	2.1×10^{34}
Number of beam bunches			1732	1584
Bunch spacing		(m)	1.25	1.84
Beam crossing angle		(mrad)	0 (head-on)	±11 (crab-crossing)

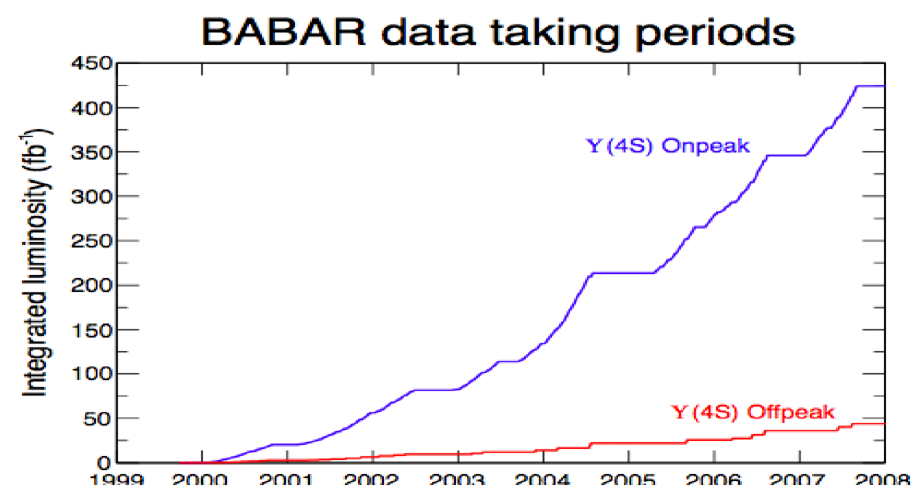


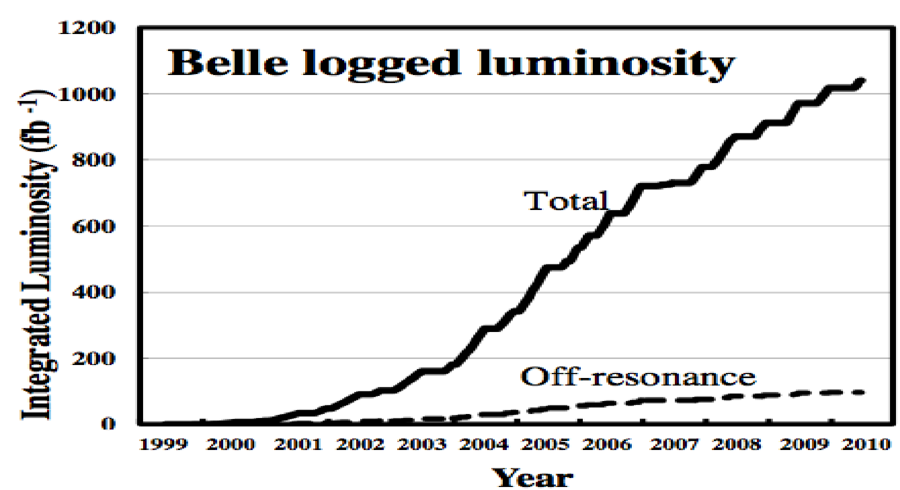
B-Factories

- Beauty-Factories: developed specifically for B physics, e^+e^- collisions at E_{CM} =10.5 GeV with asymmetric energy beams, “clean” environment







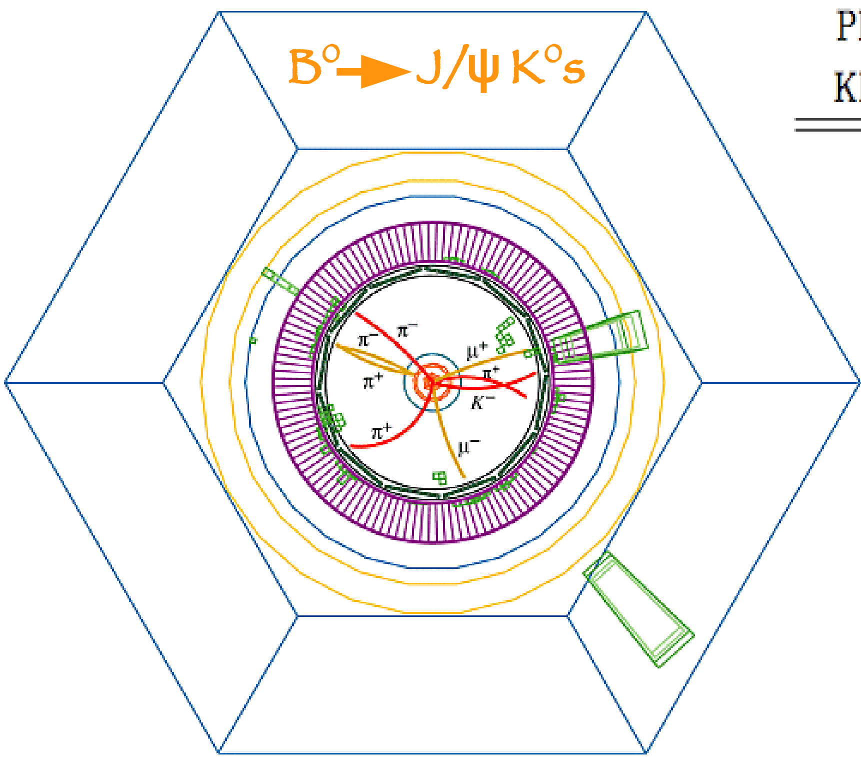


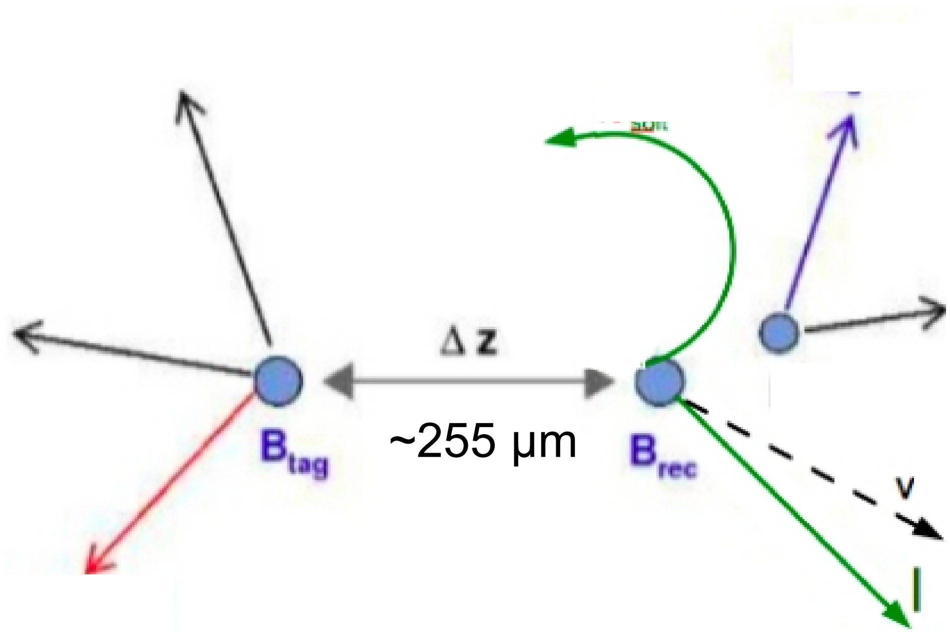
Experiment	Resonance	On-resonance Luminosity (fb^{-1})	Off-resonance Luminosity (fb^{-1})
BABAR	$\Upsilon(4S)$	424.2	43.9
	$\Upsilon(3S)$	28.0	2.6
	$\Upsilon(2S)$	13.6	1.4
	Scan > $\Upsilon(4S)$	n/a	~4
Belle	$\Upsilon(5S)$	121.1	1.7
	$\Upsilon(4S)$ - SVD1	140.7	15.6
	$\Upsilon(4S)$ - SVD2	562.6	73.8
	$\Upsilon(3S)$	2.9	0.2
	$\Upsilon(2S)$	24.9	1.7
	$\Upsilon(1S)$	5.7	1.8
	Scan > $\Upsilon(4S)$	n/a	25.6

B-Factories

- Two B mesons produced almost at rest in the $\Upsilon(4S)$ reference frame (p^*_B~340 MeV)
- No jets from fragmentation: B decays can be fully reconstructed
- Asymmetric beams: the two B mesons decay about 255 μm far one from the other (BaBar)

B Factory	e^- beam energy E_- (GeV)	e^+ beam energy E_+ (GeV)	Lorentz factor $\beta\gamma$	crossing angle φ (mrad)
PEP-II	9.0	3.1	0.56	0
KEK-B	8.0	3.5	0.425	22





B Reconstruction @ B-Factories

Signal Reconstruction

- ϒ(4S) decays in two same-mass particles. If the B meson is correctly reconstructed, its energy in the Center of Mass (CM) is equal to half the available energy, and also equal to the beam energy in the ϒ(4S) rest frame:

$$E^*_{rec} = E^*_{beam} = \sqrt{s}/2, \quad m_{rec} = m_B; \quad [\, * = \Upsilon(4S)\, frame]$$

- B-meson decay selected by means of two variables weakly correlated: Energy difference & Beam-Energy Substituted Mass

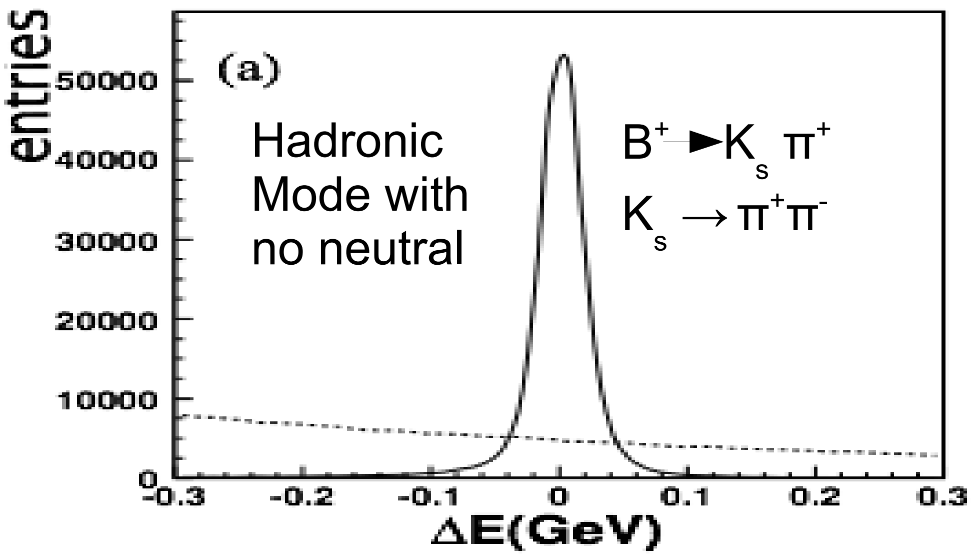
Energy difference

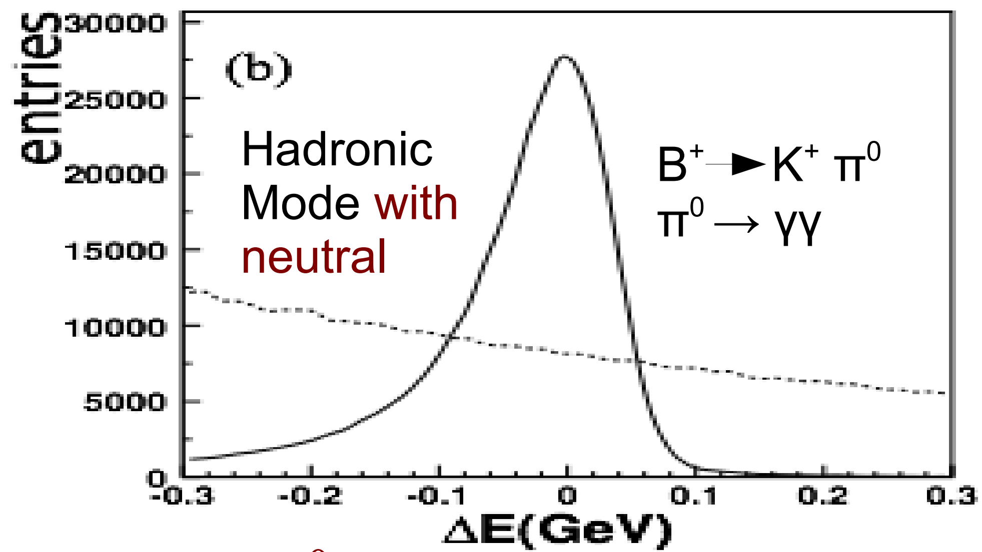
$$\Delta E = E^\star_B - E^\star_{\rm beam} \qquad \sigma^2_{\Delta E} = \sigma^2_{E^\star_B} + \sigma^2_{E^\star_{\rm beam}}$$

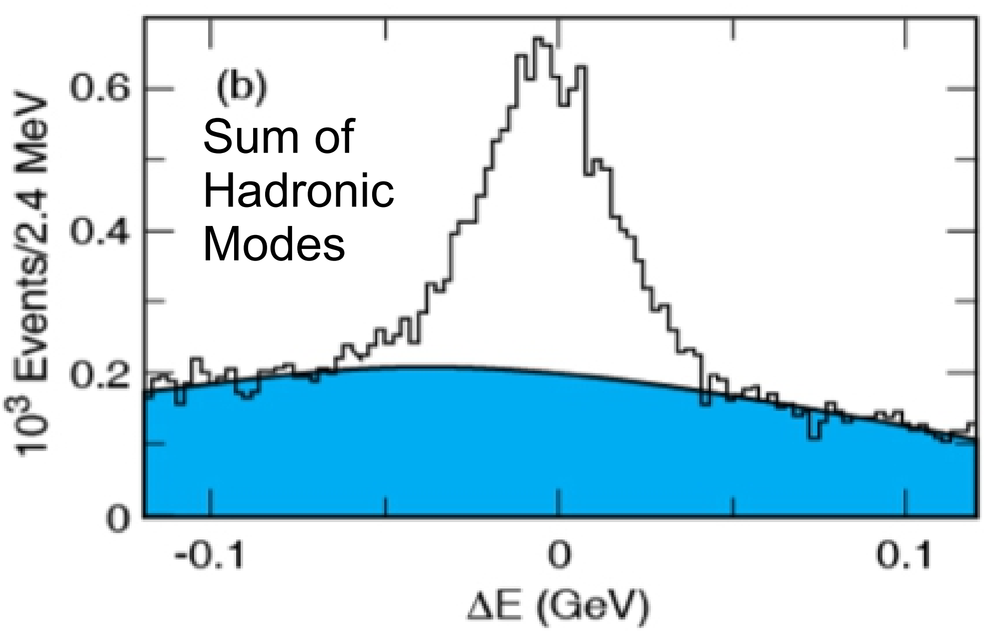
- Peaks at zero for correctly reconstructed candidates
- Depends strongly on the mass hypothesis for each particle (e.g. a K → π misidentification will result in a negative ΔE)
- Error dominated by the detector energy resolution (expecially for modes involving photons) and affected by the measurement of the neutrino momentum and energy in SL decays

B Reconstruction @ B-Factories

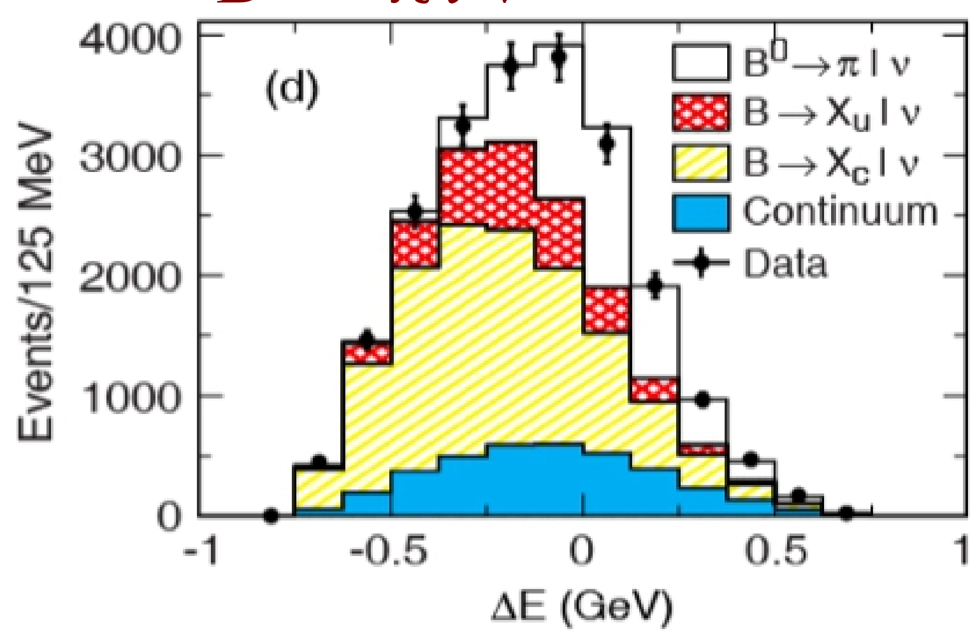
$\sigma(\Delta E)$~6-30 MeV/c^2 (larger for low mass)







$B^0 \to \pi\, l\, \nu$



B Reconstruction @ B-Factories

Beam-energy substituted mass

$$m_{ES} = \sqrt{E^{*\,2}_{beam} - p^{*\,2}_{B}}; \qquad \sigma^2_{m_{\rm ES}} \approx \sigma^2_{E^\star_{\rm beam}} + \left(\frac{p^\star_B}{m_B}\right)^2 \sigma^2_{p^\star_B}$$

- The CM energy of the B candidate is substituted by the beam one. Independent by construction to the mass hypothesis for the various particles in case of symmetric-energy colliders (CLEO).
- At asymmetric colliders the B momentum vector is boosted to the CM frame after mass has been assigned and therefore the result depends weakly on the mass assignment.
- Error dominated by the spread of the beam energy ($p^*_B/m_B \sim 0.06$) and affected by the measurement of the neutrino momentum and energy in SL decays

B Reconstruction @ B-Factories

Beam-energy substituted mass

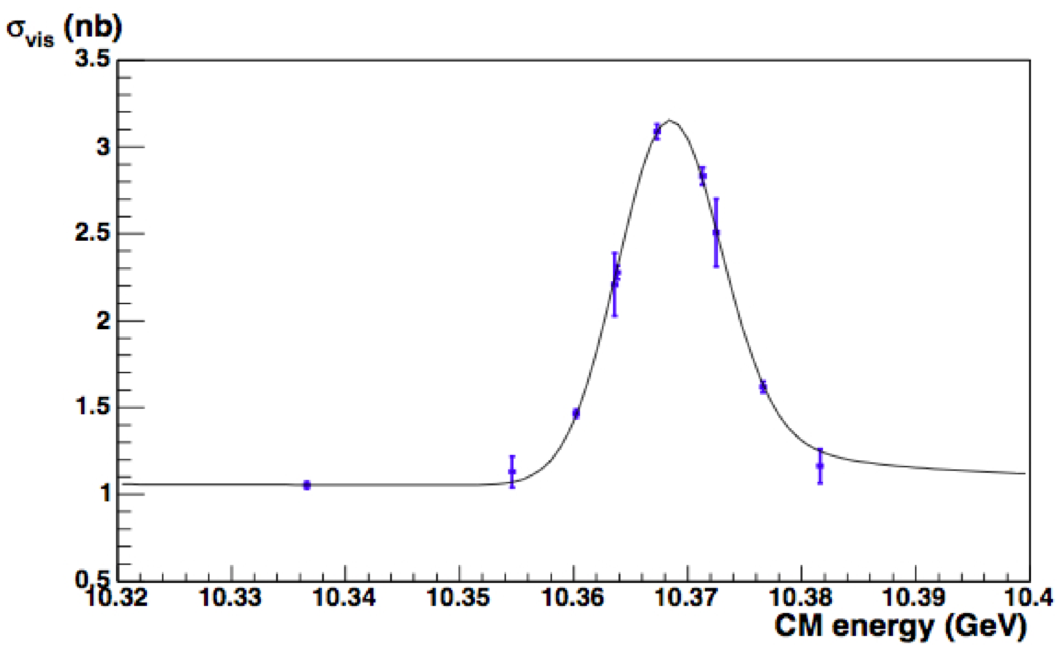
$$m_{ES} = \sqrt{E^{*\,2}_{beam} - p^{*\,2}_{B}};$$

$$\sigma^2_{m_{\rm ES}} \approx \sigma^2_{E^\star_{\rm beam}} + \left(\frac{p^\star_B}{m_B}\right)^2 \sigma^2_{p^\star_B}$$

$(p^*_B << E^*_B) \rightarrow E^{*\,2}_B / m_B^2 \sim 1$

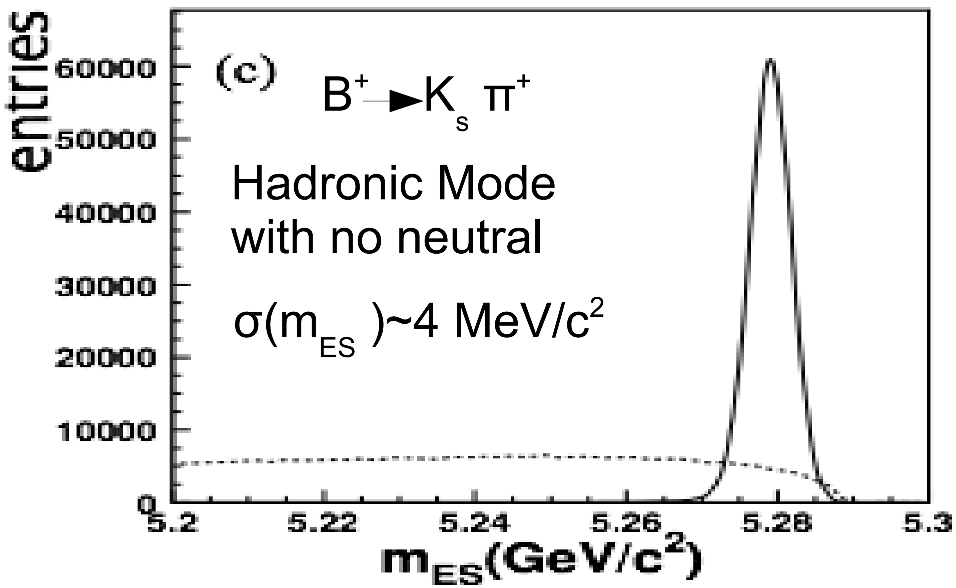
$\sigma_{ebeam} \sim$ 4 MeV depending on horizontal correctors and frequency of the RF system

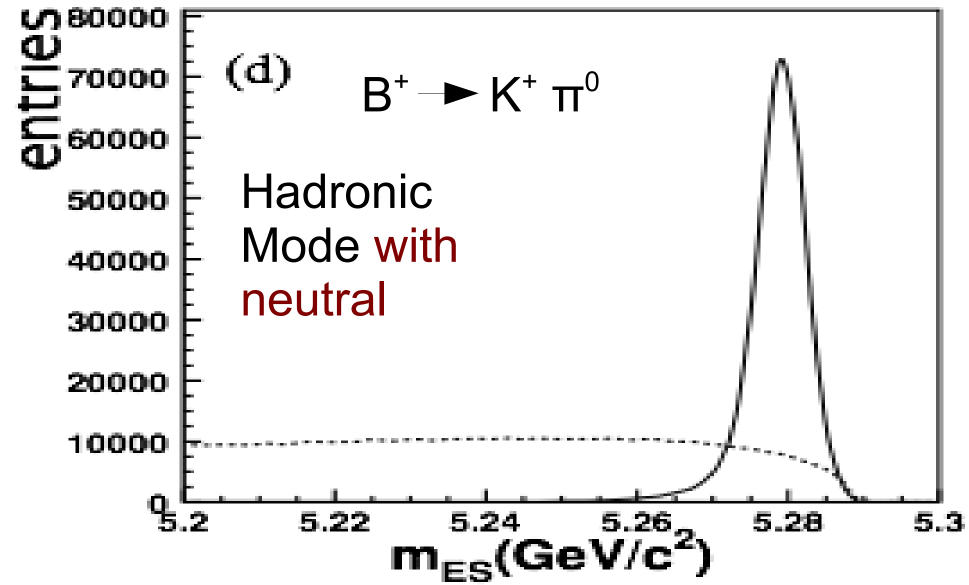
Measured from a scan of the ϒ(3S) resonance (Γ=20 KeV): measured width determined by beam energy spread

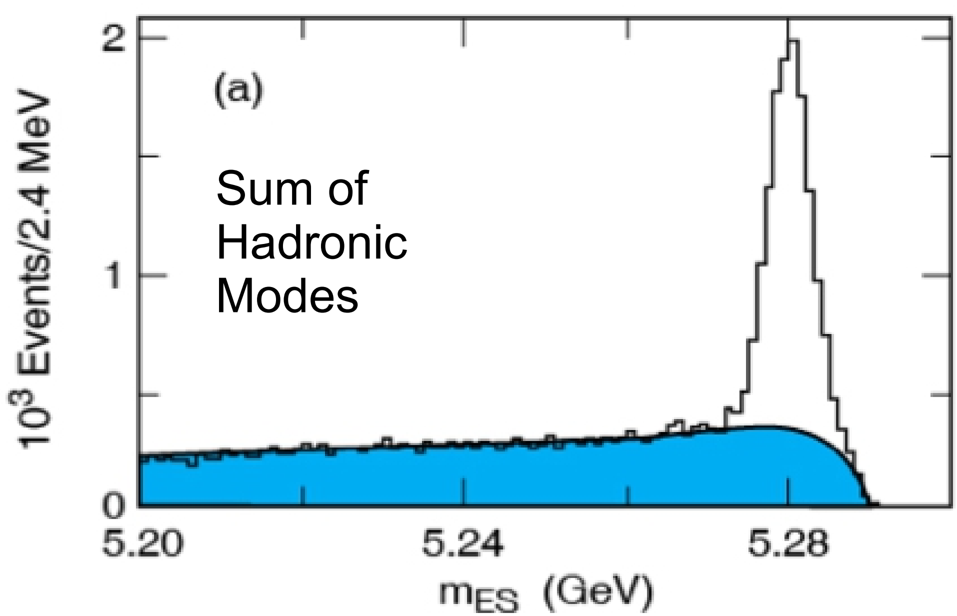


$$\Delta = (4.44 \pm 0.09)\,\mathrm{MeV},$$
$$M^{\rm fit}_{3S} = (10367.98 \pm 0.09)\,\mathrm{MeV}/c^2;$$

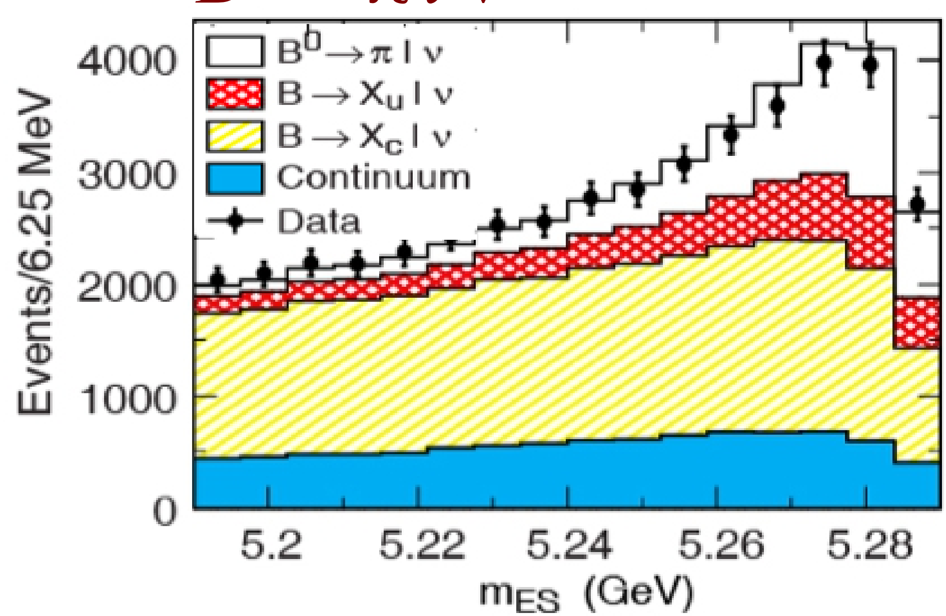
B Reconstruction @ B-Factories



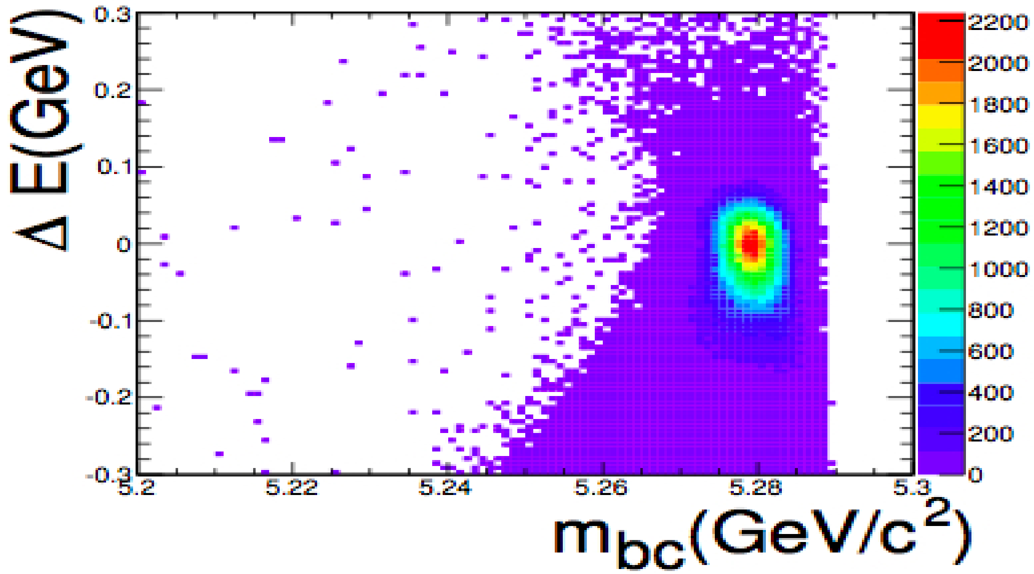


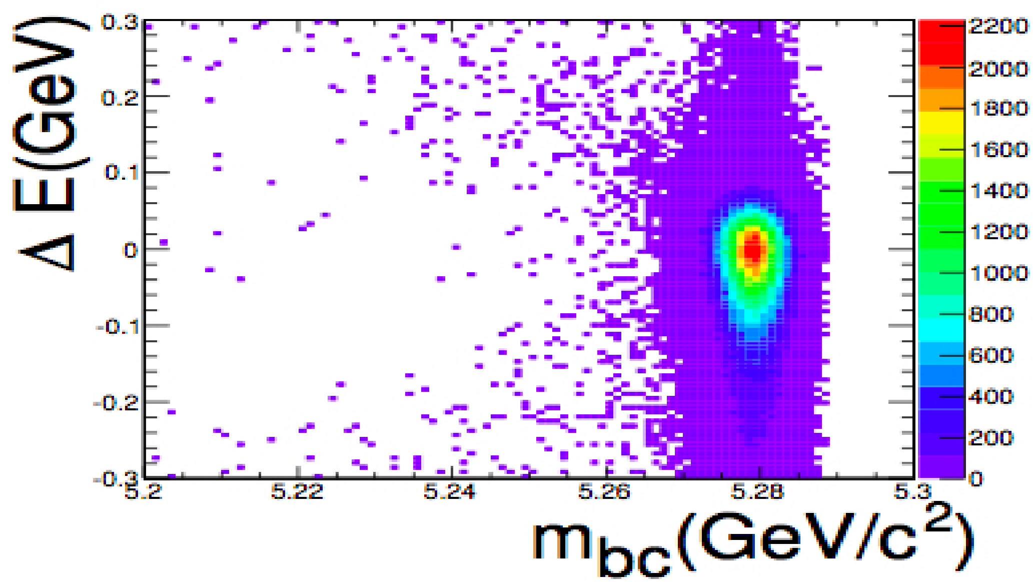


$B^0 \to \pi\, l\, \nu$



B Reconstruction @ B-Factories





$B^+ \rightarrow K^+ \pi^0$

$$m_{ES} = \sqrt{E^{*\,2}_{beam} - p^{*\,2}_{B}};$$

Small correlation due to mass assignment before computing p^*_B

Modified definition in terms of quantities in the laboratory frame removes correlation (E_0,p_0)=CM 4-momentum

$$m_{ES} = \sqrt{(s/2 + \mathbf{p}_B \mathbf{p}_0)^2 / E_0^2 - \mathbf{p}_B^2}.$$

Semileptonic B decays

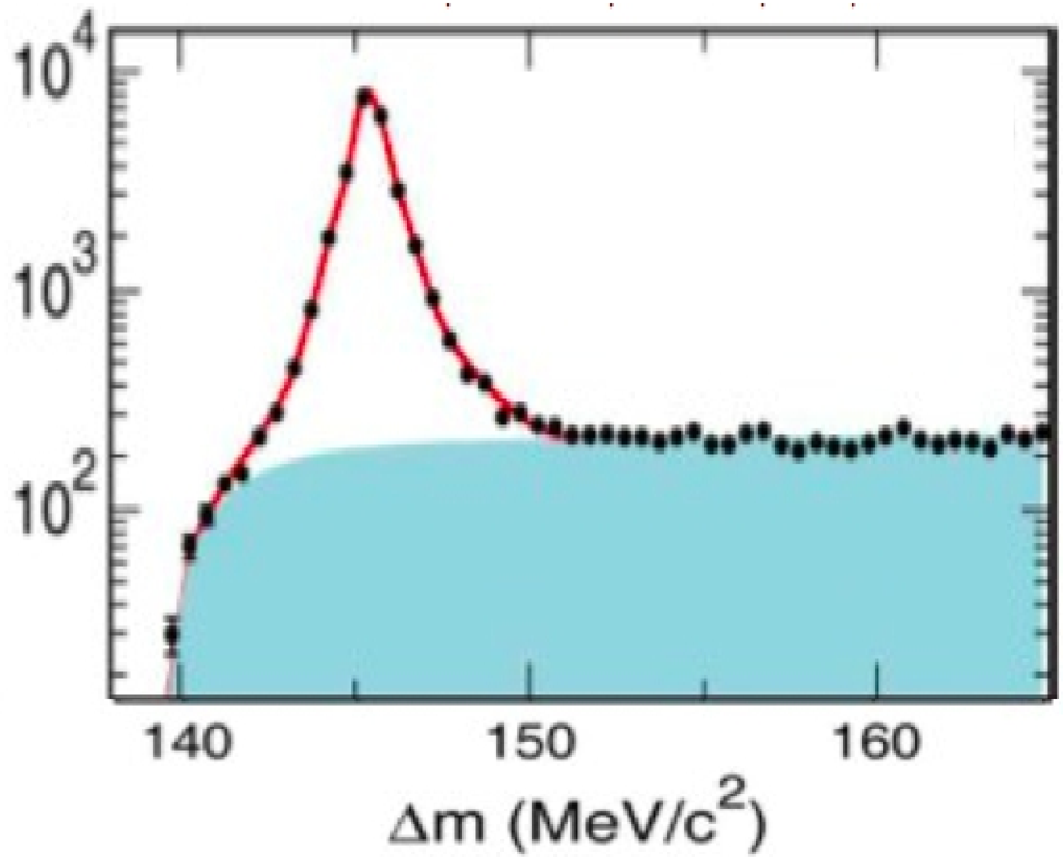
- Semileptonic decays $B \to X_c l \nu$
- For example:

$$B^0 \to D^{*-}\ell^+\nu$$
$$\hookrightarrow \bar{D}^0\pi^-$$
$$\hookrightarrow K^+\pi^-\pi^0$$
$$\hookrightarrow \gamma\gamma$$

- Full event reconstruction proceeds from the identification of the charged lepton

- The reconstructed charmed meson (charged or neutral) may be combined with a soft pion in attempt to form a D*: tight constraint on Δm applied

$$B^0 \to D^{*-} l^+ \nu$$
$$D^{*-} \to \bar{D}^0\pi^- ,\quad \bar{D}^0 \to K^+\pi^-$$
$$\Delta m = m(D^{*-}) - m(\bar{D}^0)$$



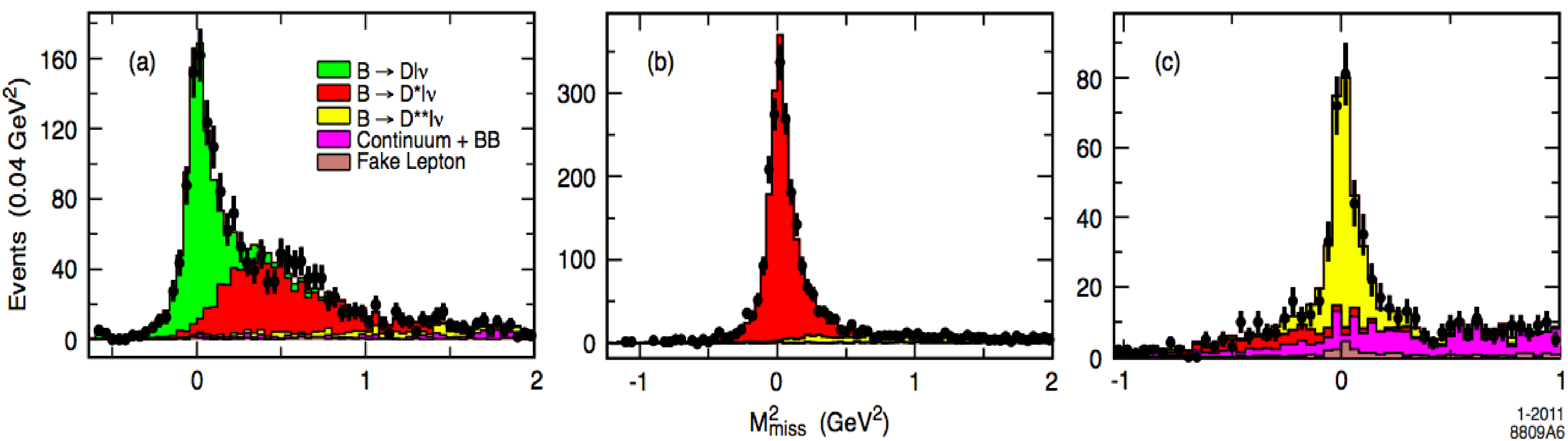
Semileptonic B decays

- Combinatorial rejection very challenging due to one or more undetected neutrinos.
- Missing particles 4-momentum computed from the sum of all the reconstructed particles in terms of the 4-vector of the colliding beams (E_0, p_0):

$$(E_{\text{miss}}, \mathbf{p}_{\text{miss}}) = (E_0, \mathbf{p}_0) - \left(\sum_i E_i, \sum_i \mathbf{p}_i\right) \qquad m_{\text{miss}}^2 = E_{\text{miss}}^2 - |\mathbf{p}_{\text{miss}}|^2$$

- For one single missing neutrino, the missing mass is close to zero
- In case of missing particles there are broad enhancements above the peak (e.g. $B^- \rightarrow D^{*0} l^- \bar{\nu}$; $D^{*0} \rightarrow D^0 \pi^0 (D^0 \gamma)$ with undetected neutral pion or photon):

Reconstructed D | Reconstructed D* | Reconstructed D**

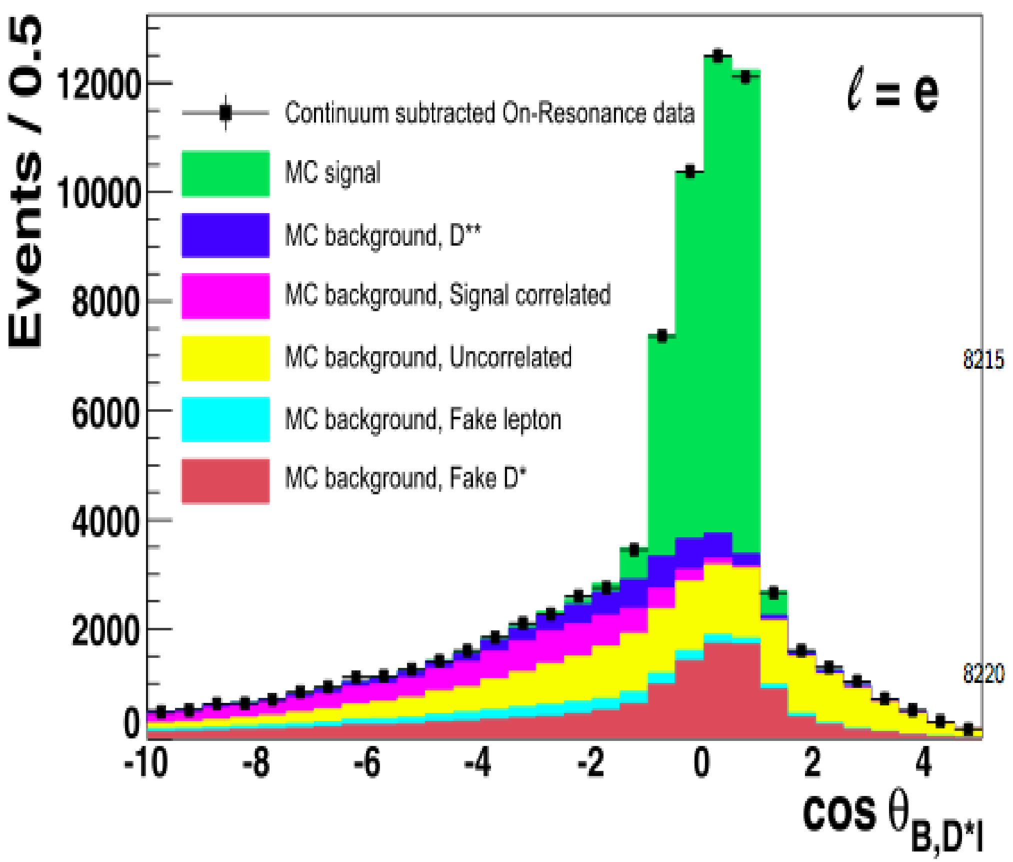


Semileptonic B decays

- Exclusive $B \to Y\nu;\ (Y = X_c + l)$ events selected exploiting angle between B and Y momenta. From $P_\nu = P_B - P_Y$ follows:

$$\cos\theta_{BY} = (2E_B E_Y - m_B^2 - m_Y^2)/2|\mathbf{p}_B||\mathbf{p}_Y|$$

$B^0 \to D^{*-} e^+ \nu$



- Variables in the $\Upsilon(4S)$ reference frame
- $|\cos\theta_{BY}| \leq 1.0$ for correctly reconstructed events
- For Background and incompletely reconstructed events it extends beyond this range

Semileptonic B decays Partial Reco

- In the full event reconstruction approach, all the products of the B decay are identified. For example:

$$B^0 \to D^{*-}\ell^+\nu_\ell$$
$$\hookrightarrow \overline{D}^0\pi_s^-$$

- Several D^0 decays, as Kπ (3.9%), K3π (8%), $K\pi\pi^0$ (14%) are considered.
- Higher efficiency is achieved by performing a Partial Reconstruction:

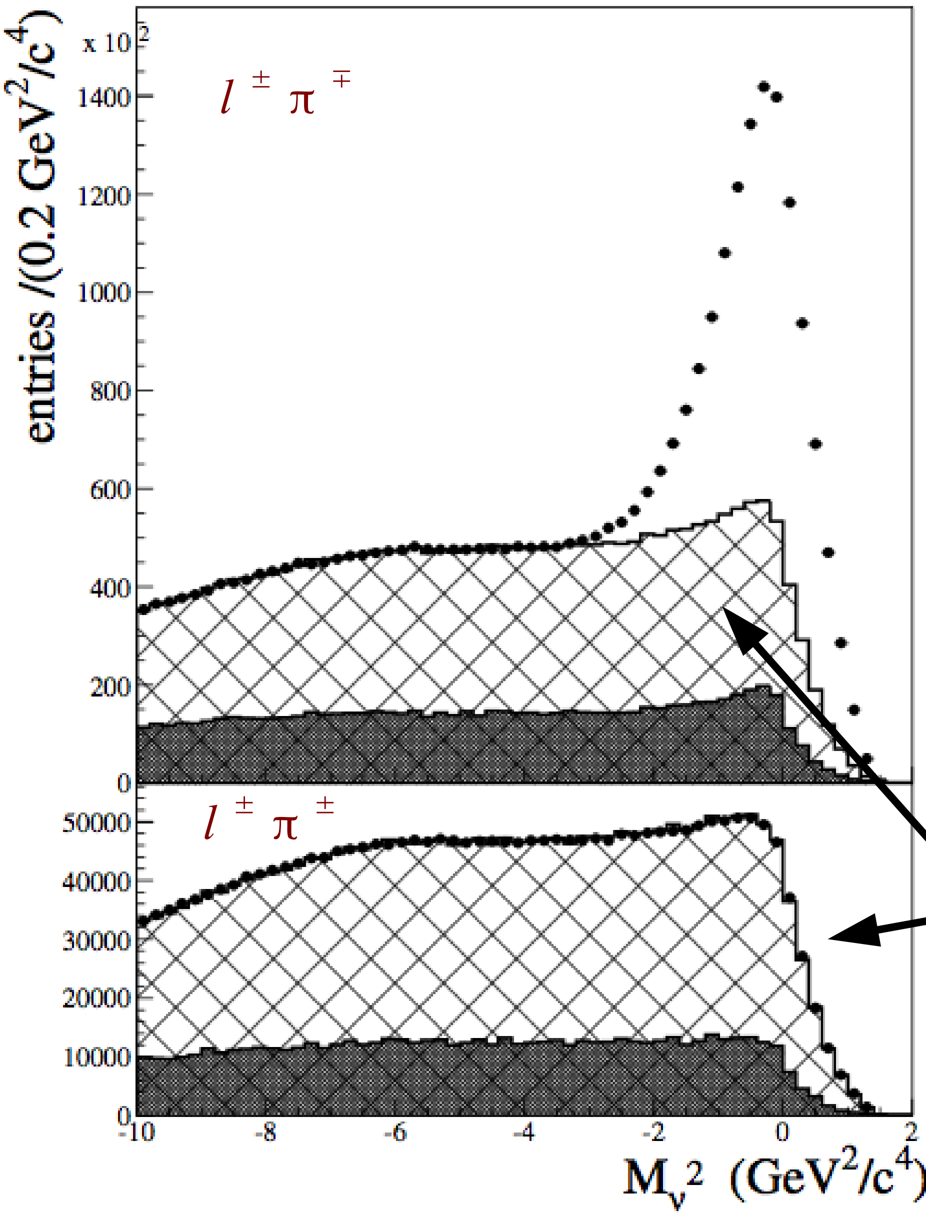
- Only the charged lepton from the B decay and the slow pion from the D* are identified. Due to the limited phase space available (m_{D^*} - m_{D0} ~ 150 MeV), the slow pion is emitted within a one-radian wide cone centered about the D* direction in the ϒ(4S) rest frame.

- D* 4-momentum parameterized as a function of the pion momentum:

$$p_{D^*} = \alpha + \beta p_{\pi_s},$$
$$E_{D^*} = \sqrt{p_{D^*}^2 + m_{D^*}^2},$$

- The B-meson 4-momentum is not known, but the 3-momentum is small as compared to the lepton and D* ones (p_B~0.34 GeV/c) and can be neglected (Boost Approximation)

Semileptonic B decays Partial Reco



$$B^0 \rightarrow D^{*-}\ell^+\nu_\ell$$
$$\hookrightarrow \overline{D}^0\pi_s^-$$

Neutrino invariant mass in the ϒ(4S) frame:

$$M_\nu^2 = \left(\frac{\sqrt{s}}{2} - E_{D^*} - E_\ell\right)^2 - (\boldsymbol{p}_{D^*} + \boldsymbol{p}_\ell)^2$$

B energy is assumed equal to half the CM energy

$$\sigma(M_\nu^2) = 0.85\, GeV^2/c^4$$

BKG combination studied by using the lepton-pion combination with the wrong charge correlation $l^{\pm}\pi^{\pm}$

Semileptonic B decays on Recoil

Background reduction improved using tagging techniques

- $B\bar{B}$ pairs produced without any additional particles
- Detector hermeticity
- Collision energy precisely known

  - Reconstruction of the B semileptonic decays on the recoil of an exclusively or inclusively reconstructed B decay
  - Detection of one B decay produced at the ϒ(4S) identifies the second B decay, determines its momentum, mass, charge and flavor.
  - Kinematics of the final state are constrained such that a neutrino from the second decay can be identified from the missing momentum and missing energy of the rest of the event.
  - Cleanest sample from recoil of fully reco hadronic tag (ε~0.3% / 1%, Purity~50%)

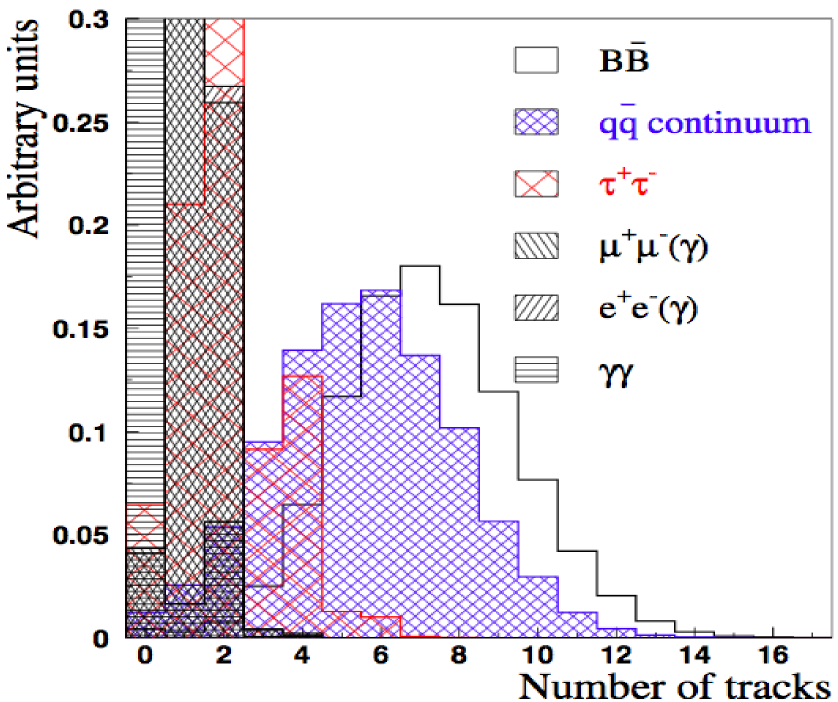
$$\varepsilon_{B_{\rm tag}} = \sum_f \varepsilon_f \mathcal{B}_f$$

  - ε~1-3% using semileptonic tags, looser kinematics constraints on the recoiling B due to the presence of the neutrino

Continuum Rejection

- $B\overline{B}$ events are produced almost at rest in the ϒ(4S) frame as the ϒ(4S) mass is barely above the threshold: the B products are distributed isotropically in the ϒ(4S) rest frame: spherical event
- In continumm events (ϒ(4S) → qq, q≠b) quarks are produced with large initial momentum and yield a back-to-back fragmentation into two jets of light hadrons
- Discrimination obtained exploiting:

  - Number of charged tracks



- Events shape variables :

  - Thrust: sum of the longitudinal momenta of all particles relative to the axis which maximize it:

$$T = \frac{\sum_{i=1}^{N} |\boldsymbol{T} \cdot \boldsymbol{p}_i|}{\sum_{i=1}^{N} |\boldsymbol{p}_i|}$$

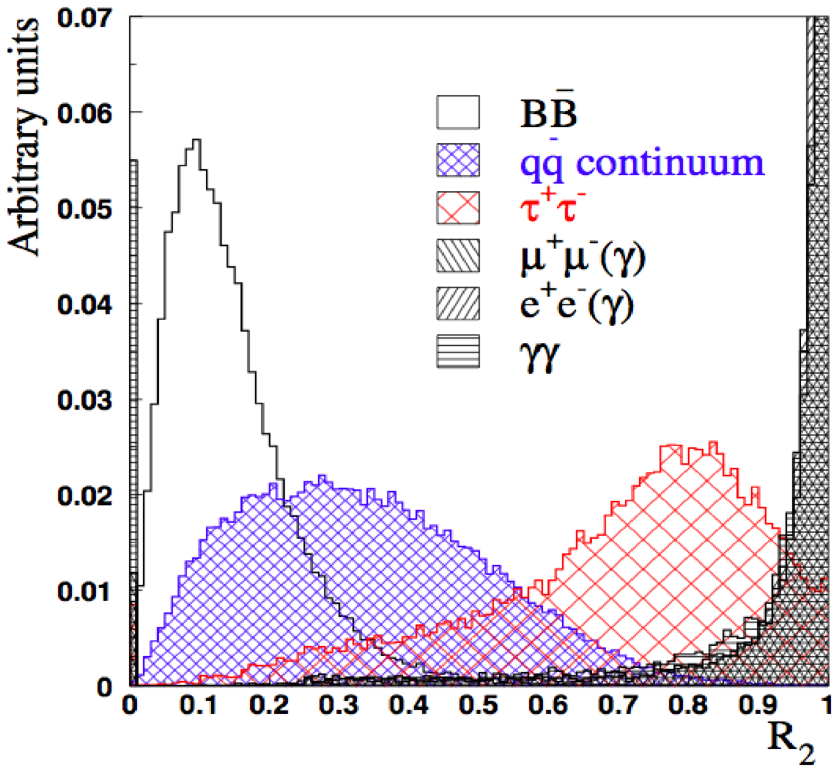
  - Angle between the thrust axis of the particles associated with the signal decay and of the rest of the event

Continuum Rejection

- Fox-Wolfram moments: for N particles, moment H_k is defined in terms of the momenta p_i of the particles and the angles between particle pairs, θ_{ij} :

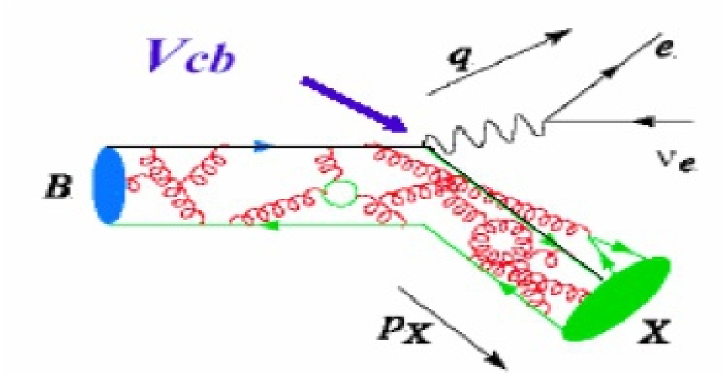
$$H_k = \sum_{i,j}^{N} |\boldsymbol{p}_i| \, |\boldsymbol{p}_j| \, P_k(\cos\theta_{ij})$$

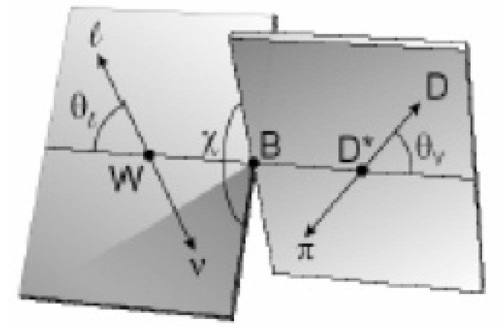
(P_k : k-th order Legendre polynomial)



- In the limit of vanishing particle mass, H_0=1.
- Usually the ratio $R_k=H_k/H_0$ is used: for events with strongly collimated jets, R_k is close to zero (one) for odd (even) values of k

V_{cb} Measurements





Inclusive Measurements VS **Exclusive Measurements**

$$\Gamma_{sl}(b\to c l\nu) = \gamma_{th} |V_{cb}|^2 = \frac{BR(b\to c l\nu)}{\tau_b}$$

➔HQE ($1/m_b$, α_s) to deal with QCD perturbative + non–perturbative (not calculable) interactions

➔Experimentally: measure moments of kinematic variables (E_l, M_X) & BR($b\to X_c l\nu$)

➔Major issue: unfold the true shapes from the measured ones

➔V_{cb} from the measured differential rate of $B\to D^{(*)}l\nu$ + OPE

$$\frac{d\Gamma}{dw} \propto G(w) F(w)^2 |V_{cb}|^2, \quad w = \vec{v}_B \cdot \vec{v}_D$$

➔Probe different FF parameterizations & QCD bounds

$|V_{cb}|$: Exclusive Measurement

Heavy-Quark Symmetry

•The properties of hadronic bound states composed of a heavy quark and other light constituents are characterized by a large separation of lenght scales (b quark / B hadron)

•The heavy quark is surrounded by a strongly interacting cloud of soft gluons and light quarks or antiquarks

•Strong coupling constant decreases with increasing q^2 of the process: the QCD scale Λ_{QCD}~0.5 GeV approximately separates regions with perturbative (short distance) & non-perturbative (long distance) interactions.

- Size of the heavy hadron is determined by Λ_{QCD}
  - Compton wavelenght $\lambda_b \sim 1/m_b << R_{HAD} \sim 1/\Lambda_{QCD} \sim 1$ fm:

    Soft gluons ($p \sim \Lambda_{QCD}$) exchanged between the heavy quark and the light ones do not resolve the heavy quark quantum numbers (mass, flavor, spin)

- Ligth quarks are blind to the quantum number of the heavy quark, they experience only its color field which extends over distances larger than λ_b

$|V_{cb}|$: Exclusive Measurement

Heavy-Quark Symmetry

•The properties of hadronic bound states composed of a heavy quark and other light constituents are characterized by a large separation of lenght scales (b quark / B hadron)

•The heavy quark is surrounded by a strongly interacting cloud of soft gluons and light quarks or antiquarks

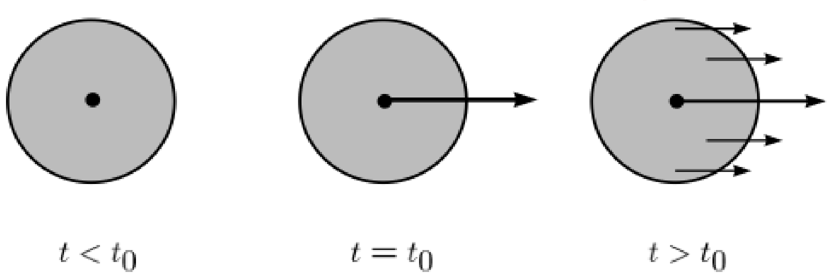
•Strong coupling constant decreases with increasing q^2 of the process: the QCD scale Λ_{QCD}~0.5 GeV approximately separates regions with perturbative (short distance) & non-perturbative (long distance) interactions.

- In the limit $m_b \to \infty$ the hadron quantum numbers are defined by its ligth constituents: hadronic systems which differ only in the flavor or spin quantum numbers of the heavy quark have the same configuration of their light degrees of freedom

$|V_{cb}|$: Exclusive Measurement

Elastic scattering of a B meson induced by a current

- For $t < t_0$: light quark orbit around the heavy quark which acts as a static color source. The velocity v of the b quark is ~ the same of the B hadron one
- At $t = t_0$: due to the scatterning, the color source speed changes from v to v'
- If v = v' nothing happens; if v ≠ v' soft gluons exchange rearrange to form a B with the new velocity.
- For $m_b \to \infty$ the FF depend only on the Lorentz boost connecting the rest frames of the initial and final state mesons $w = v \cdot v' \geq 1$
- Transition amplitude is described by the single elastic FF $\xi(w)$ (Isgur-Wise function) [Isgur, Wise, Phys. Lett. B 232, 113 (1989), Phys. Lett. B 237, 527 (1990)]
- For zero recoil (maximum 4-momentum transfer) and neglecting corrections:



$$\xi(w=1)=1$$

$|V_{cb}|$: Exclusive Measurement

- In the $B \to D^{(*)} l \nu$ decays, the heavy b quark is substituted by a c quark in the final state, indistinguishable from the previous in the heavy quark limit
- With an appropriate basis of FF, the transition matrix elements depend on the 4-velocities v_B and $v_{D(*)}$ of the two heavy mesons:

$$\frac{\langle D|\bar{c}\gamma^\mu b|B\rangle}{\sqrt{m_B m_D}} = h_+(w)\,(v_B + v_D)^\mu + h_-(w)\,(v_B - v_D)^\mu,$$

$$\frac{\langle D^*|\bar{c}\gamma^\mu b|B\rangle}{\sqrt{m_B m_{D^*}}} = h_V(w)\,\varepsilon^{\mu\nu\rho\sigma} v_{B,\nu} v_{D^*,\rho} \epsilon^*_\sigma,$$

$$\frac{\langle D^*|\bar{c}\gamma^\mu \gamma^5 b|B\rangle}{\sqrt{m_B m_{D^*}}} = i h_{A_1}(w)\,(1+w)\epsilon^{*\mu} - i\left[h_{A_2}(w) v_B^\mu + h_{A_3}(w) v_{D^*}^\mu\right] \epsilon^* \cdot v_B$$

$$w = v_B \cdot v_{D^{(*)}};\quad 1 \le w \le 1.5$$

$$q^2 = (p_B - p_{D^{(*)}})^2 = m_B^2 + m_{D^{(*)}}^2 - 2 m_B m_{D^{(*)}} w$$

The relevant FF are normalized at the point of zero recoil w=1 which corresponds to the maximum momentum transfer (B and D have the same 4-velocity):

$$q^2_{max} = (m_B - m_{D^{(*)}})^2 = 10.69\, GeV^2$$

$$h_+(1) = 1 + O(\alpha_s) + O\left((\Lambda_{QCD}/m_q)^2\right)$$

$$h_-(1) = 0 + O(\alpha_s) + O(\Lambda_{QCD}/m_q),$$

$$h_{A_1}(1) = 1 + O(\alpha_s) + O\left((\Lambda_{QCD}/m_q)^2\right)$$

- FF related to the different D* polarization states
- Affected by perturbative and non-perturbative corrections to Heavy-Quark-Symmetry

$|V_{cb}|$: Exclusive Measurement

- V_{cb} from fit to the $B \to D^{(*)} l \nu$ decay rates

$$\frac{d\Gamma_{B^- \to D^0 \ell^- \bar{\nu}}}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2 (w^2 - 1)^{3/2} \times |\eta_{\rm EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2,$$

$$\frac{d\Gamma_{B^- \to D^{0*} \ell^- \bar{\nu}}}{dw} = \frac{G_F^2 m_{D^*}^3}{4\pi^3}(m_B - m_{D^*})^2 (w^2 - 1)^{1/2} \times |\eta_{\rm EW}|^2 |V_{cb}|^2 \chi(w) |\mathcal{F}(w)|^2,$$

- G(w), F(w) are combinations of the FF defined before
- η_{EW}=1.0066 one-loop EW corrections

- For the $|V_{cb}|$ determination, D* is better than D:
  - Lower suppression of the rate at zero recoil $[(w^2-1)]^{1/2}$ vs $[(w^2-1)]^{3/2}$
  - Luke's theorem: at zero recoil:
    - → $F(1) = 1 + O(\alpha_s) + O(\Lambda_{QCD}/m_Q)^2$
    - → $G(1) = 1 + O(\alpha_s) + O(\Lambda_{QCD}/m_Q)$
  - The three polarization states of D* increase the rate

$|V_{cb}|$: Exclusive Measurement

- Using Operator Product Expansion to compute the $(1/m_Q)^n$ corrections (n=2, 3) [Gambino, Mannel, Uraltsev, Phys. Rev. D 81, 113002 (2010)]:

  F(1)=0.86±0.02

- With lattice QCD the action is discretized on an Euclidean space-time lattice and calculations are performed numerically. In principle precision depends only on the computer resources. Most demanding CPU calculation is the treatment of the sea of virtual quark-antiquark pairs [Bailey et al. PoS LATTICE2010, 311 (2010)]:

  F(1)=0.908±0.017

- Error from MC statistics, discretization errors, tuning of the bare quark masses, High $m_b \rightarrow$ low spacing, low $m_{u/d} \rightarrow$ large volume.

- Difference ~ 5% between the two estimations could derive from a breakdown of the OPE which cannot converge at scales lower than 1GeV (long distance contributions).

- For $B \rightarrow D$ decays (analogous behaviour):

G(1)=1.04±0.02 HQE calculations, [Uraltsev, Phys. Lett. B 585, 253-262(2004)]
G(1)=1.074±0.024 LQCD, [Okamoto, Nucl. Phys. Proc. Suppl. 140, 461-463 (2005)]

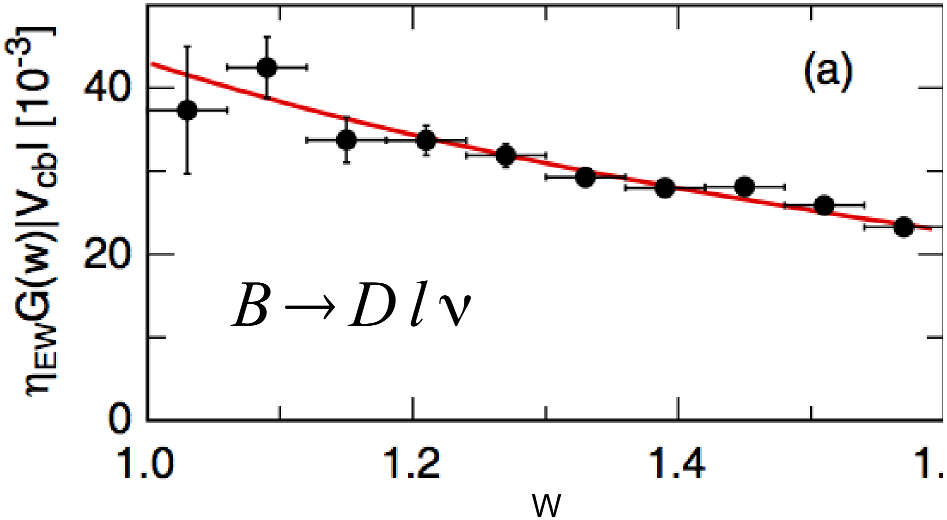
$|V_{cb}|$: Exclusive Measurement

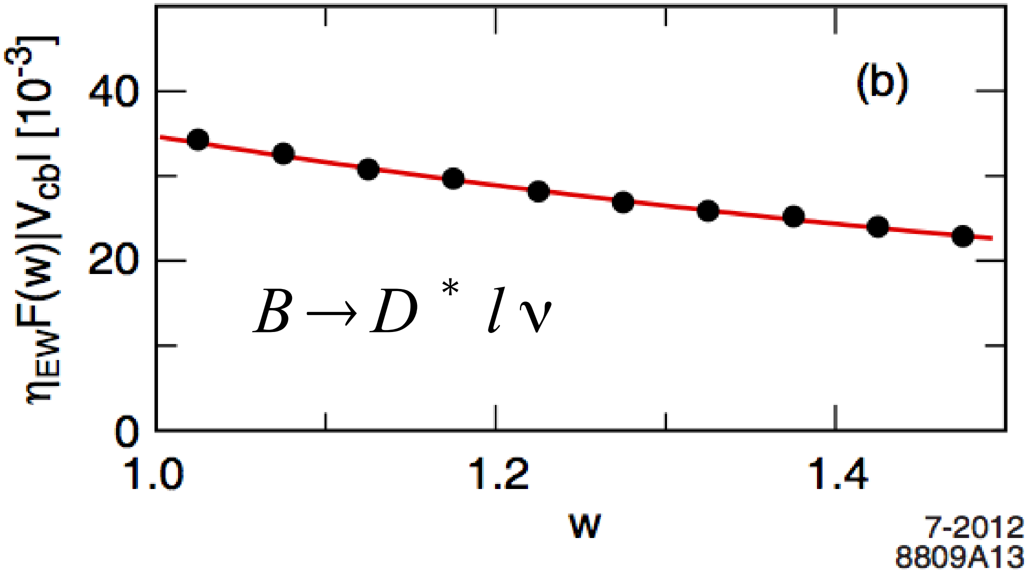
- $B \to D^{(*)} l \nu$ decay measured at Belle and Babar assuming the HQET parameterization of $\eta_{EW}F(w)$ in terms of four parameters: $\eta_{EW}F(1)$, the slope $\rho^2_{D^*}$ and two FF ratios

$$R_1(1) = R^{*2}\, V(1)/A_1(1),\quad R_2 = R^{*2}\, A_2(1)/A_1(1);\quad (R^* = 2\sqrt{m_B m_{D^*}}/(m_B + m_{D^*}))$$

[Caprini, Lellouch, Neubert, Nucl. Phys. B530, 153-181 (1998)]

- Some analyses fit $d\Gamma/dw$ to determine $\eta_{EW}F(1)$ and the slope $\rho^2_{D^*}$, fixing $R_1(1)$ and $R_2(1)$ from other measurements





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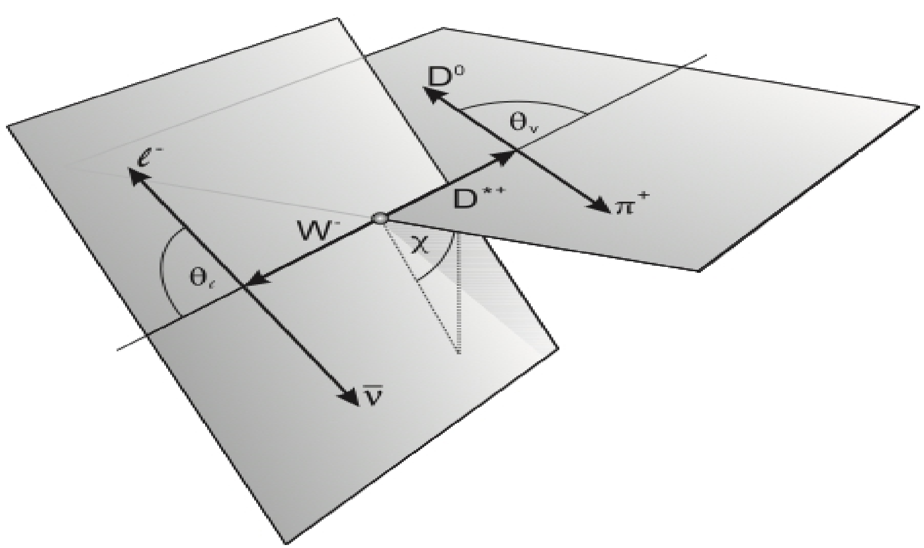
[Caprini, Lellouch, Neubert, Nucl. Phys. B530, 153-181 (1998)]

- Some analyses fit $d\Gamma/dw$ to determine $\eta_{EW}F(1)$ and the slope $\rho^2_{D^*}$, fixing $R_1(1)$ and $R_2(1)$ from other measurements
- More sophisticated analyses determine all the four parameters by fitting the differential rate in terms of w and three angles between:



- Θ_l: angle between lepton and direction opposite to B in the W rest frame
- Θ_v: angle between D and direction opposite to B in the D* rest frame
- X: angle between D* and W decay planes in the B rest frame

$|V_{cb}|$: Exclusive Measurement

- $B \to D^{(*)} l \nu$ decay measured at Belle and Babar assuming the HQET parameterization of $\eta_{EW}F(w)$ in terms of four parameters: $\eta_{EW}F(1)$, the slope $\rho^2_{D^*}$ and two FF ratios

$$R_1(1) = R^{*2}\ V(1)/A_1(1),\quad R_2 = R^{*2}\ A_2(1)/A_1(1);\quad \left(R^* = 2\sqrt{m_B m_{D^*}}/(m_B + m_{D^*})\right)$$

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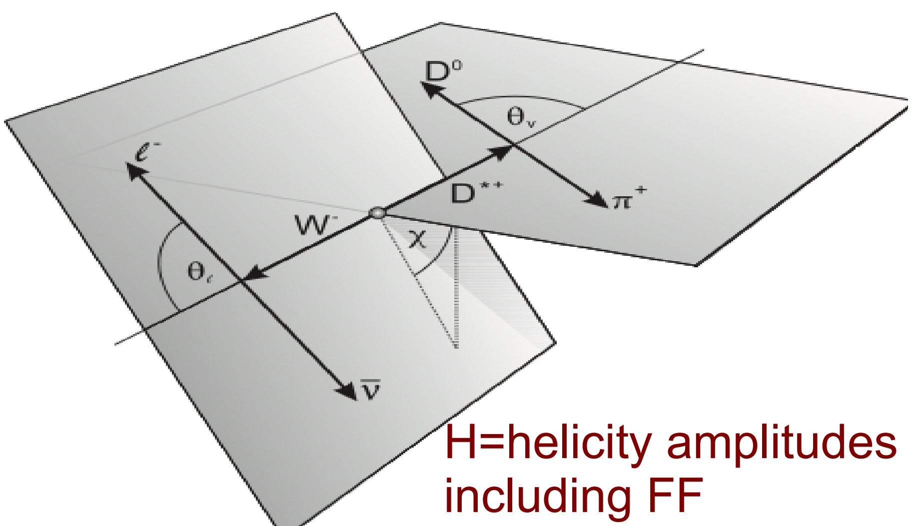


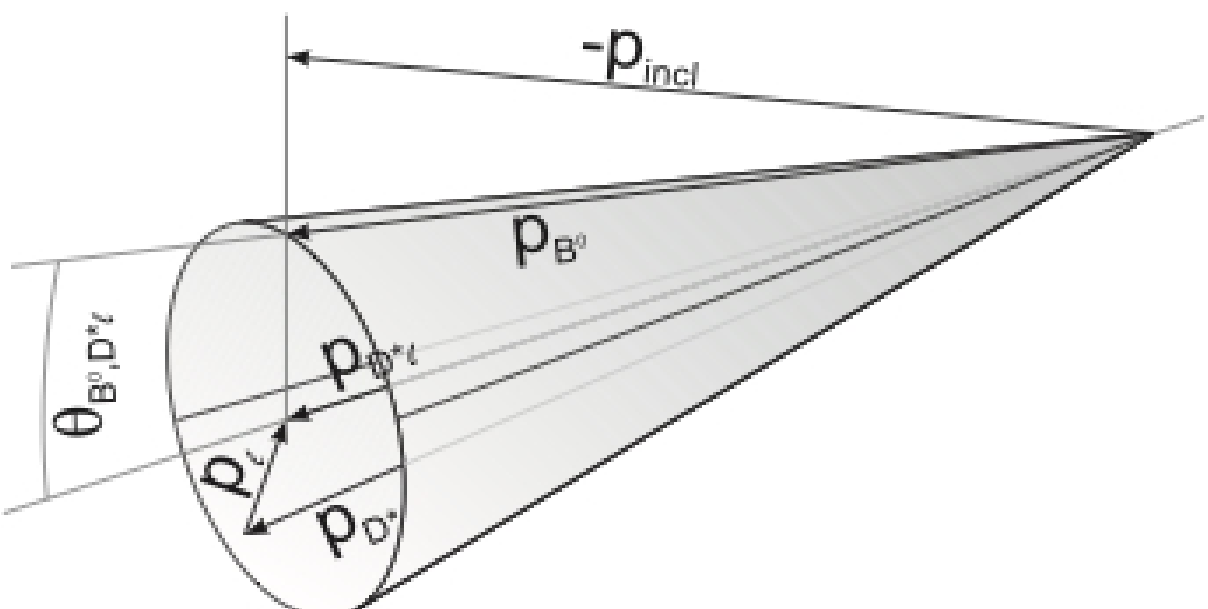
H=helicity amplitudes including FF

$$\frac{d^4\Gamma(B^0 \to D^{*-}\ell^+\nu_\ell)}{dw\, d(\cos\theta_\ell)\, d(\cos\theta_V)\, d\chi} = \frac{6 m_B m_{D^*}^2}{8(4\pi)^4}\sqrt{w^2-1}$$
$$(1 - 2wr + r^2) G_F^2 |V_{cb}|^2 \{(1-\cos\theta_\ell)^2 \sin^2\theta_V H_+^2(w)$$
$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_V H_-^2(w) + 4\sin^2\theta_\ell \cos^2\theta_V H_0^2(w$$
$$- 2\sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+(w) H_-(w)$$
$$- 4\sin\theta_\ell (1-\cos\theta_\ell)\sin\theta_V \cos\theta_V \cos\chi H_+(w) H_0(w)$$
$$+ 4\sin\theta_\ell (1+\cos\theta_\ell)\sin\theta_V \cos\theta_V \cos\chi H_-(w) H_0(w)\}$$

$|V_{cb}|$: Exclusive Measurement

Belle Measurement (L=711 fb^{-1}) [Phys. Rev. D92, 112007 (2010)]

- Fully reconstruct 120k $B \to D^{*-} l^{+} \nu$, $D^{*-} \to \bar{D}^{0} \pi^{-}$; $\bar{D}^{0} \to K^{+} \pi^{-}$ events with p_l=(0.8-2.4) GeV
- Neutrino direction not known, B direction constrained to a cone centered on the Y=D* + l direction using $\cos\theta_{BY}$. The B direction is choosen as the direction on the cone which minimizes the difference with the opposite of the sum of the momenta of all the particles not associated to the B meson, -P_{incl}.



σ(w)=0.025
σ(cosθ$_l$)=0.049
σ(cosθ$_v$)=0.050
σ(χ)=13.5°

$|V_{cb}|$: Exclusive Measurement

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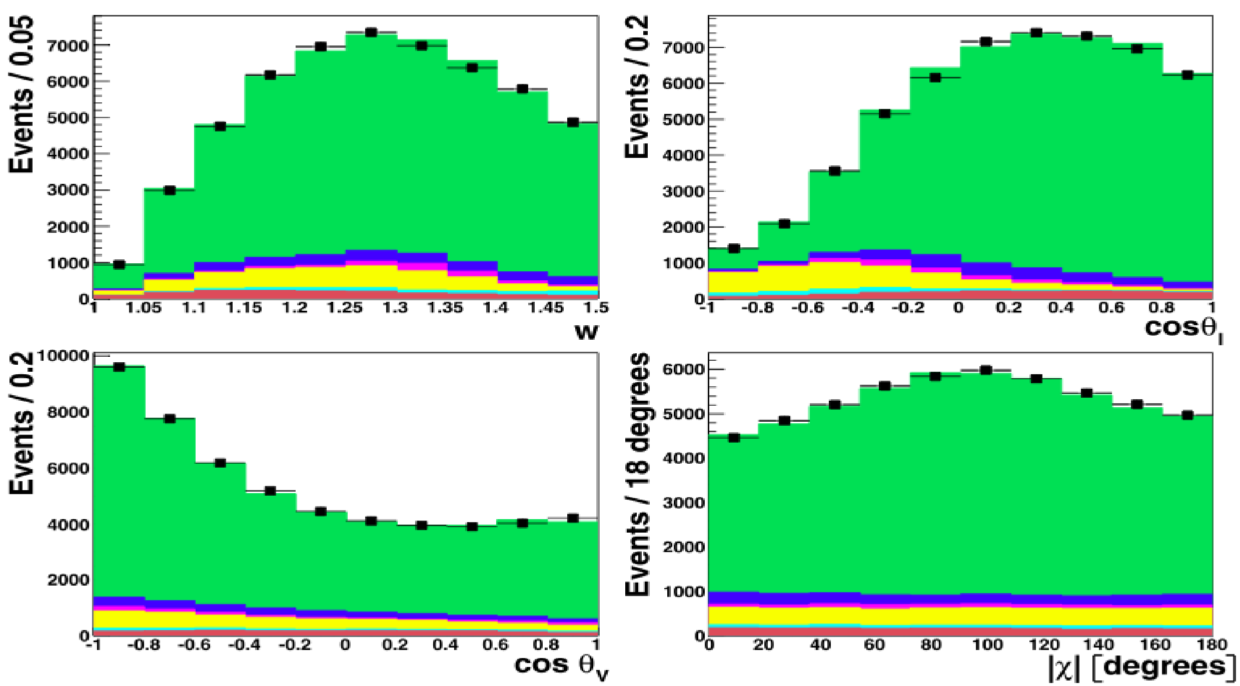
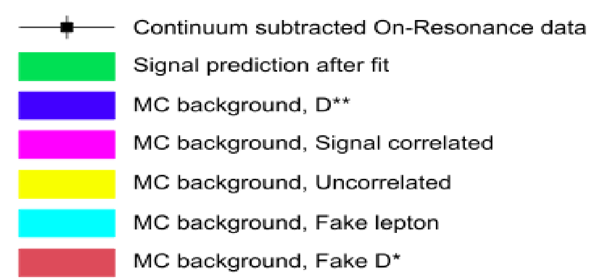


Simultaneous Fit



Correlated BKG: D* & lepton from the same B decay

$|V_{cb}|$: Exclusive Measurement

Belle Measurement (L=711 fb^{-1}) [Phys. Rev. D92, 112007 (2010)]

	ρ^2	$R_1(1)$	$R_2(1)$	$\mathcal{F}(1)\|V_{cb}\| \times 10^3$	$\mathcal{B}(B^0 \to D^*\ell\nu)$ [%]
Value	1.214	1.401	0.864	34.6	4.58
Statistical Error	0.034	0.034	0.024	0.2	0.03
Systematic Error	0.009	0.018	0.008	1.0	0.26
Fast track efficiency				-0.78	-0.206
Slow track efficiency	+0.002	+0.003	-0.004	-0.28	-0.059
ρ_{π_s} stability	+0.001	-0.001	+0.000	-0.03	-0.003
LeptonID	+0.002	+0.006	-0.002	-0.38	-0.100
Norm - D^{**}	+0.001	+0.001	-0.001	-0.03	-0.008
Norm - Signal Corr.	+0.002	-0.003	+0.002	+0.02	+0.006
Norm - Uncorr	+0.002	+0.008	-0.003	-0.02	-0.001
Norm - Fake ℓ	+0.003	-0.003	-0.001	-0.01	-0.003
Norm - Fake D^*	+0.001	-0.001	+0.000	+0.00	+0.003
Norm - Continuum	+0.002	+0.002	-0.001	+0.00	-0.003
D^{**} composition	+0.004	+0.009	-0.003	-0.10	-0.025
D^{**} shape	+0.003	+0.005	-0.002	-0.04	-0.011
$N(\Upsilon(4S))$				-0.24	-0.063
f_{+-}/f_{00}	+0.004	-0.009	+0.003	+0.24	+0.062
B^0 life time				-0.10	-0.027
$\mathcal{B}(D^* \to D^0\pi_s)$				-0.13	-0.034
$\mathcal{B}(D^0 \to K\pi)$				-0.22	-0.059

Systematics on BR from track efficiency

Systematics on FF from D** BKG composition (resonant / non resonant and different states) and D** q^2 distribution shape

$|V_{cb}|$: Exclusive Measurement

Comparison between FF parameters and BRs measurements

- $B \rightarrow D^{*}\, l\, \nu$

Analysis	$\eta_{\rm EW}\mathcal{F}(1)\lvert V_{cb}\rvert$ (10^{-3})	$\rho^2_{D^*}$	$R_1(1)$	$R_2(1)$
Belle (Dungel, 2010)	$34.7 \pm 0.2 \pm 1.0$	$1.21 \pm 0.03 \pm 0.01$	$1.40 \pm 0.03 \pm 0.02$	$0.86 \pm 0.02 \pm 0.01$
BABAR $D^{*-}\ell^+\nu$ (Aubert, 2008h)	$34.1 \pm 0.3 \pm 1.0$	$1.18 \pm 0.05 \pm 0.03$	$1.43 \pm 0.06 \pm 0.04$	$0.83 \pm 0.04 \pm 0.02$
BABAR $\overline{D}^{*0}e^+\nu$ (Aubert, 2008v)	$35.1 \pm 0.6 \pm 1.3$	$1.12 \pm 0.06 \pm 0.06$		
BABAR $DXl\nu$ (Aubert, 2009ab)	$35.8 \pm 0.2 \pm 1.1$	$1.19 \pm 0.02 \pm 0.06$		
Average	$35.5 \pm 0.1 \pm 0.5$	$1.20 \pm 0.02 \pm 0.02$	$1.40 \pm 0.03 \pm 0.01$	$0.86 \pm 0.02 \pm 0.01$

Analysis	$\mathcal{B}(B^0 \rightarrow D^{*-}\ell^+\nu)$ (%)	
Belle (Dungel, 2010)	$4.59 \pm 0.03 \pm 0.26$	15810
BABAR $D^{*-}\ell^+\nu$ (Aubert, 2008h)	$4.58 \pm 0.04 \pm 0.25$	
BABAR $\overline{D}^{*0}e^+\nu$ (Aubert, 2008v)	$4.95 \pm 0.07 \pm 0.34$	
BABAR $DXl\nu$ (Aubert, 2009ab)	$4.96 \pm 0.02 \pm 0.20$	
Average	$4.83 \pm 0.01 \pm 0.12$	15815

$|V_{cb}|$: Exclusive Measurement

Extraction of Vcb

- The different $B \to D^{(*)} l \nu$ measurements at the B-Factories are combined using a four-dimensional fit to the HQET parameters $\eta_{EW} F(1)|V_{cb}|,\ \rho^2_{D^*},\ R_1(1),\ R_2$

$$\eta_{\rm EW}\mathcal{F}(1)|V_{cb}| = (35.45 \pm 0.50) \times 10^{-3}$$
$$\rho^2_{D^*} = 1.199 \pm 0.027,$$
$$R_1(1) = 1.396 \pm 0.033,$$
$$R_2(1) = 0.860 \pm 0.020.$$

$$\eta_{\rm EW}\mathcal{G}(1)|V_{cb}| = (42.68 \pm 1.67) \times 10^{-3}$$
$$\rho^2_D = 1.186 \pm 0.057,$$

- Using LQCD results:

$$\eta_{\rm EW}\mathcal{F}(1) = 0.908 \pm 0.017,$$

[Bailey et al. PoS LATTICE2010, 311 (2010)]

$$\mathcal{G}(1) = 1.074 \pm 0.024$$

[Okamoto, Nucl. Phys. Proc. Suppl. 140, 461-463 (2005)]

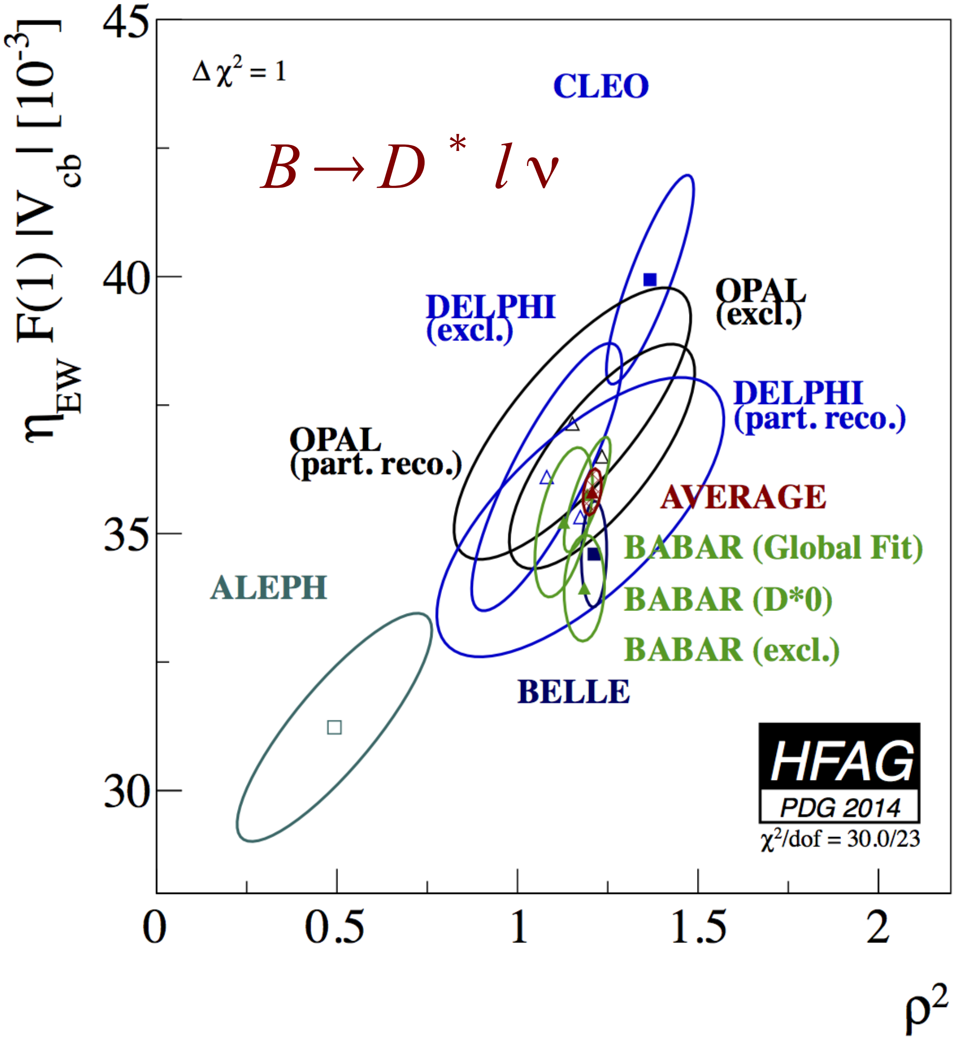
- One gets:

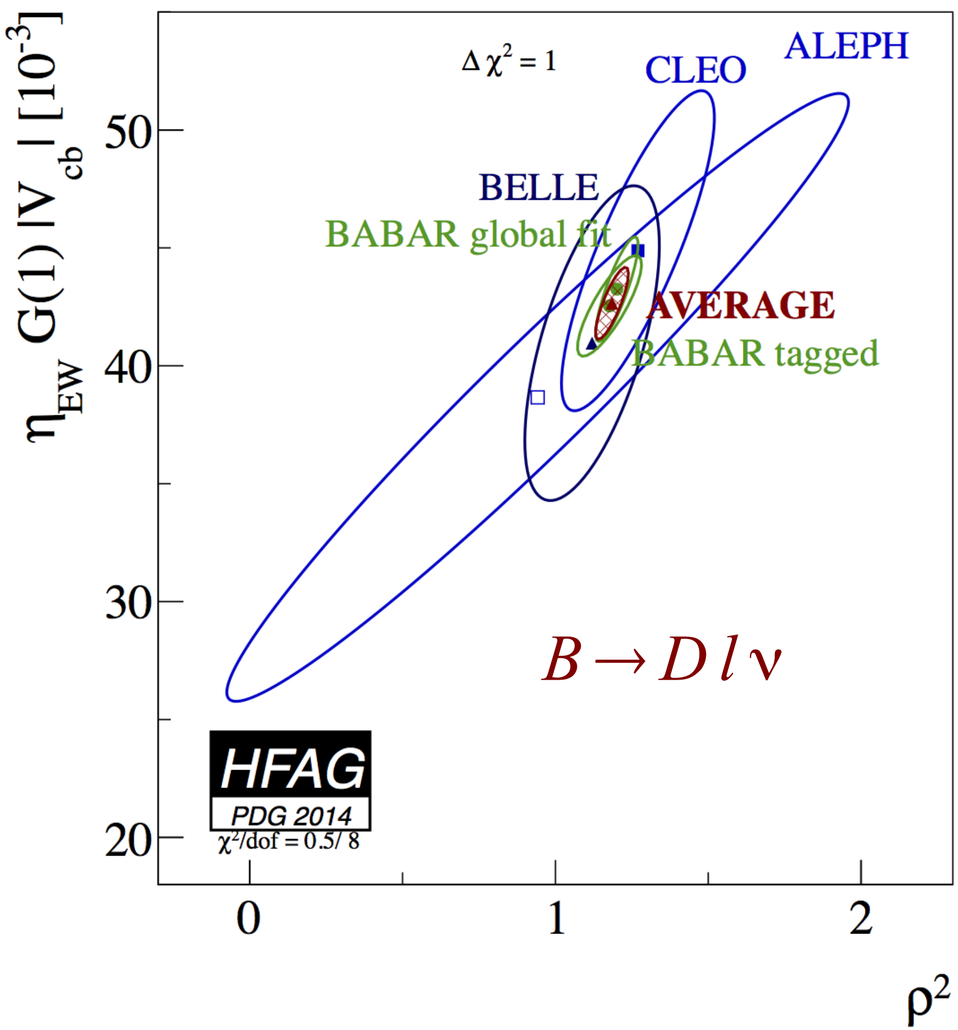
$$|V_{cb}| = (39.04 \pm 0.55_{\rm exp} \pm 0.73_{\rm th}) \times 10^{-3}$$
$$|V_{cb}| = (39.46 \pm 1.54_{\rm exp} \pm 0.88_{\rm th}) \times 10^{-3}$$

In good agreement between the different channels
Error dominated by theory

$|V_{cb}|$: Exclusive Measurement

$|Vcb|=(39.5 \pm 0.8)10^{-3}$ (average of the two LQCD results, PDG'14)





$|V_{cb}|$: Inclusive Measurement

Inclusive Cabibbo-favored decays (most precise $|V_{cb}|$ determination)

- Inclusive decays include all possible hadronic Final States. Heavy quark hadronizes with unit probability and the transition is sensitive only to the long-distance dynamics of the initial B meson (no FF dependence related to final states).

- Operator Product Expansion disentangles the physics associated with soft scales of order Λ_{QCD} (matrix elements) from that associated with hard scales ~ m_b (Wilson coefficients expressed as a perturbative series in α_s): cutoff μ=O(1 GeV) computed in different schemes
  [Bigi et al., Phys. Rev. Lett. 71, 496-499 (1993), Bigi et al., Phys. Lett. B 293, 430-436 (1992), Blok et al., Phys. Rev. D 49, 3356 (1994)]

$|V_{cb}|$: Inclusive Measurement

Observables

- Total rate $\Gamma(B \to X_c l \nu)$
- Lepton energy moment of order n, above a given threshold for the lepton energy $E_l > E_{cut}$:

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E>E_{\rm cut}}} \int_{E>E_{\rm cut}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}$$

- Hadronic mass moment of order n:

$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E>E_{\rm cut}}} \int_{E>E_{\rm cut}} dm_X^2 m_X^{2n} \frac{d\Gamma}{dm_X^2}$$

m^2_X: squared mass of the hadronic system

- Total semileptonic width and moments of the kinematic distributions are double expansions in α_s and Λ_{QCD}/m_b with relevant parameters: m_b, m_c, matrix elements and α_s
- **Total rate allows $|V_{cb}|$ extraction**
- Method limited by higher order contribution, and quark-hadron duality violation [Bigi, Uraltsev, Int. J. Mod. Phys. A16, 5201-5248 (2001)]

$|V_{cb}|$: Inclusive Measurement

Observables

- Total rate $\Gamma(B \to X_c l \nu)$
- Lepton energy moment of order n, above a given threshold for the lepton energy $E_l > E_{cut}$:

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- Hadronic mass moment of order n:

$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E>E_{\rm cut}}} \int_{E>E_{\rm cut}} dm_X^2 m_X^{2n} \frac{d\Gamma}{dm_X^2}$$

m^2_X: squared mass of the hadronic system

- OPE parameters obtained from moments fit and b mass are important inputs also for the $|V_{ub}|$ inclusive determination and $B \to X_s \gamma$ decays.
  - ➔ Moments of the photon energy distribution in $B \to X_s \gamma$ decays, sensitive to m_b, can be included in fits to V_{cb}.

V_{cb} Inclusive Measurements: Strategy

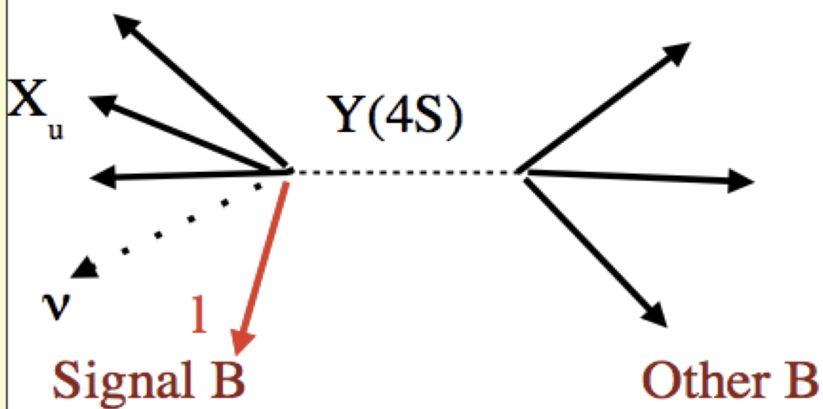
STUDY

Inclusive variables:

➔ $BR(b \rightarrow X_c l\nu)$

➔ Lepton energy spectrum (better experimental precision)

➔ Hadronic mass spectrum (more sensitive to non–perturbative terms)



USING

➔ Recoil of Fully Reconstructed $B \rightarrow D^{(*)}X$ (clean environment, small sample)

➔ High momentum lepton (high efficiency)

➔ V_{cb} & non–perturbative parameters from a global fit to several distributions

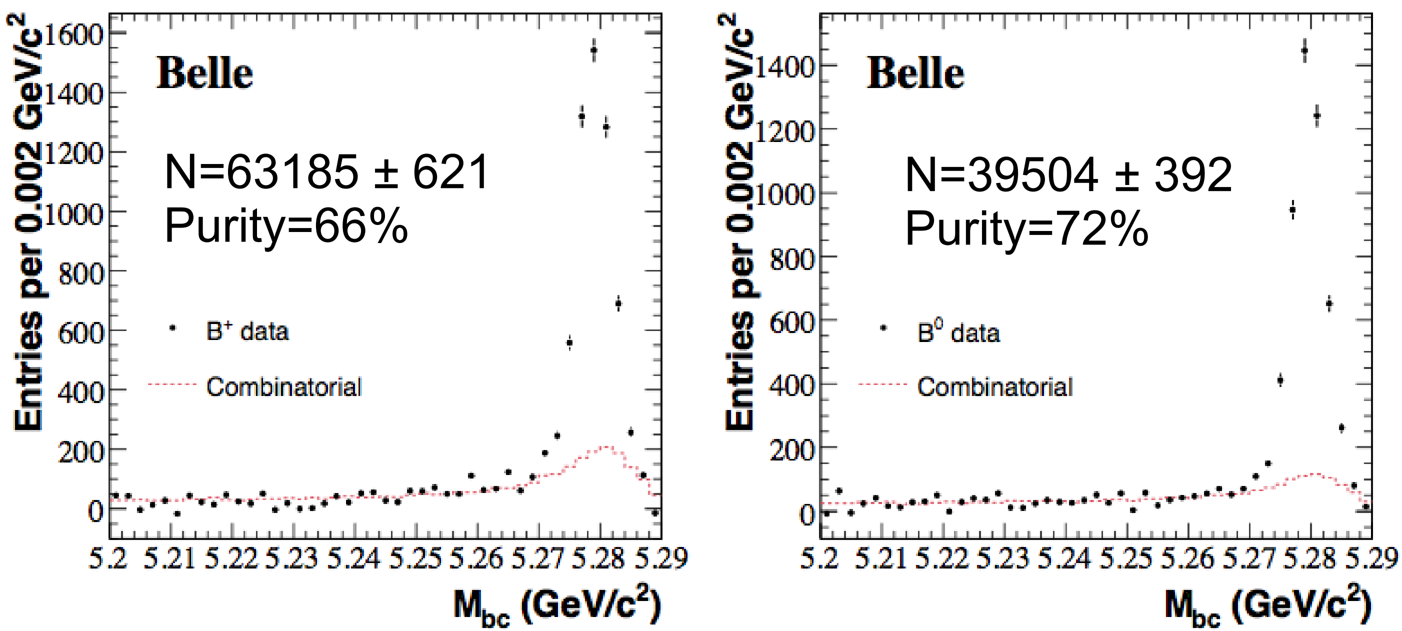
$|V_{cb}|$: Inclusive Measurement

Belle Measurement (L=140 fb^{-1})

[Phys. Rev. D75, 032001 (2007), Phys. Rev. D75, 032005 (2007)]

- Measured spectra of electron energy and hadronic mass in $B \to X_c l \nu$ decays on the recoil of a fully reconstructed B hadronic decay (B_{tag}) in the modes

$$B^+ \to \bar{D}^{(*)0}\pi^+, \bar{D}^{(*)0}\rho^+, \bar{D}^{(*)0}a_1^+ \text{ and } B^0 \to D^{(*)-}\pi^+, D^{(*)-}\rho^+, D^{(*)-}a_1^+$$



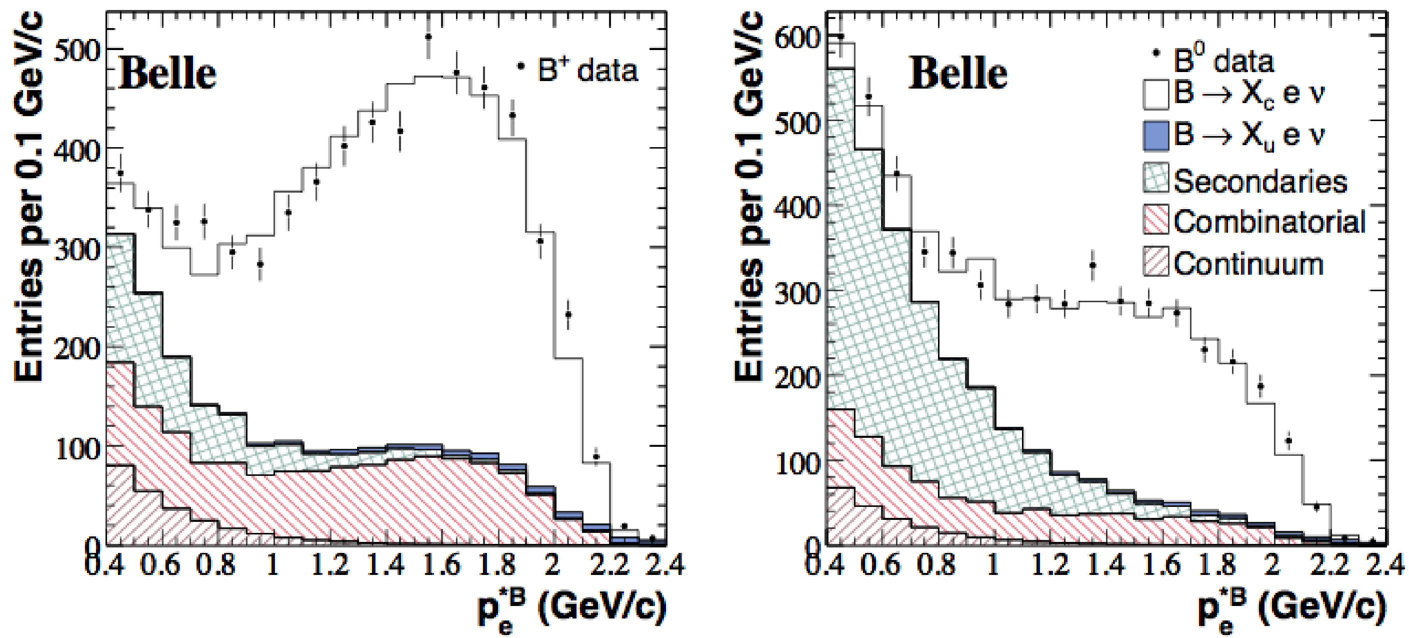
- Continuum from off-peak rescaled events
- $B\bar{B}$ combinatorial shape from MC, normalization from Sideband

$|V_{cb}|$: Inclusive Measurement

Belle Measurement (L=140 fb^{-1})

[Phys. Rev. D75, 032001 (2007), Phys. Rev. D75, 032005 (2007)]

- The B semileptonic decay is identified by searching for a charged electron among the particles not used for B_{tag} reconstruction with momentum $0.4 < p^*_e < 2.4$ GeV in the B rest frame



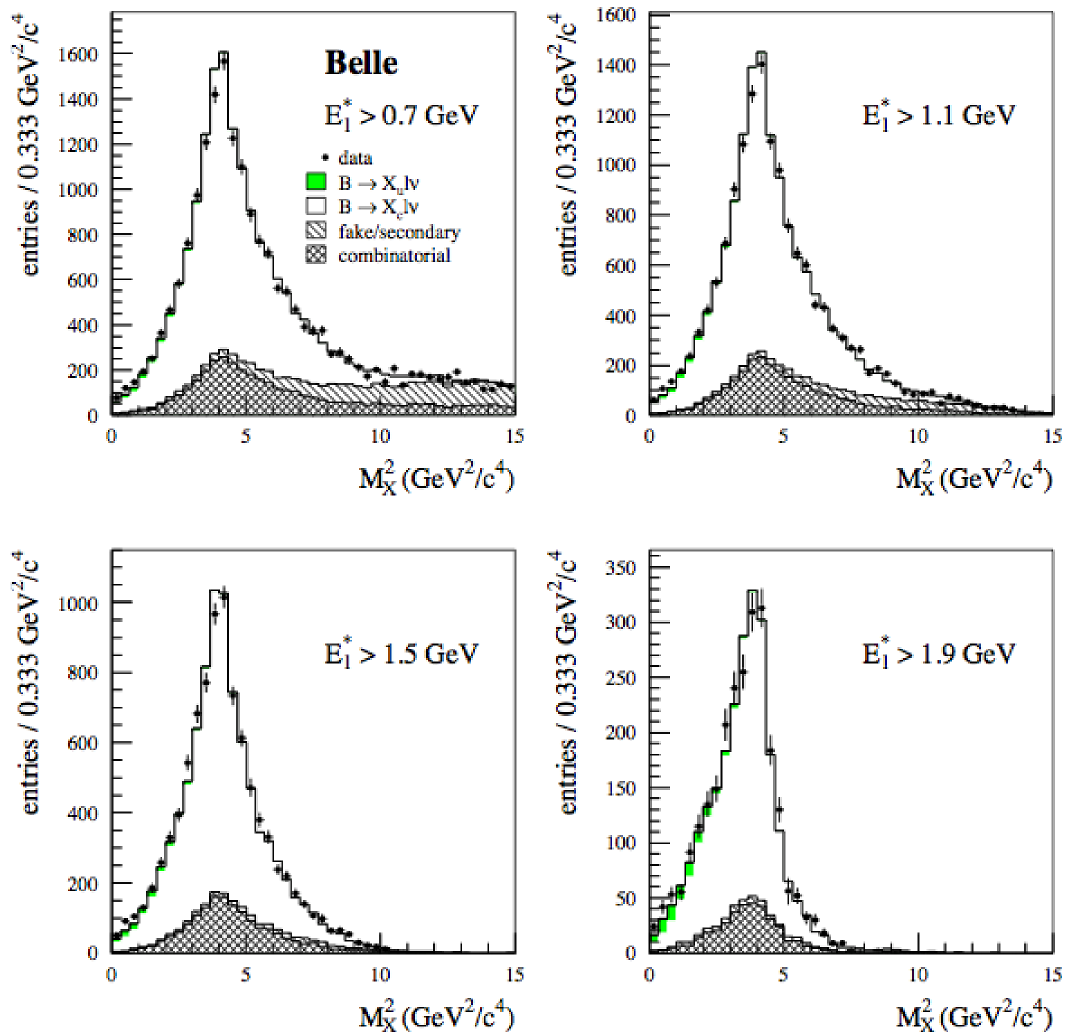
- Cascade SL decays $b \rightarrow c \rightarrow e$ suppressed in the B^+ sample using charge correlation (not possible in the B^0 sample due to mixing: higher BKG)

- $B \rightarrow X_u$ decays subtracted assuming the BR from PDG

$|V_{cb}|$: Inclusive Measurement

Belle Measurement (L=140 fb^{-1})

[Phys. Rev. D75, 032001 (2007), Phys. Rev. D75, 032005 (2007)]

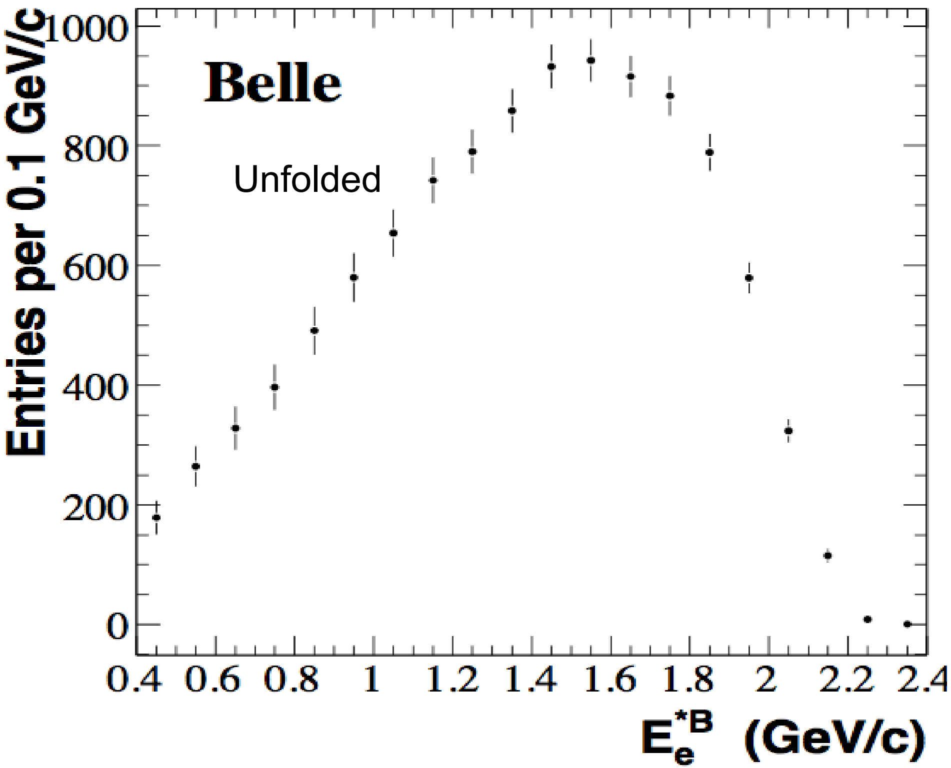


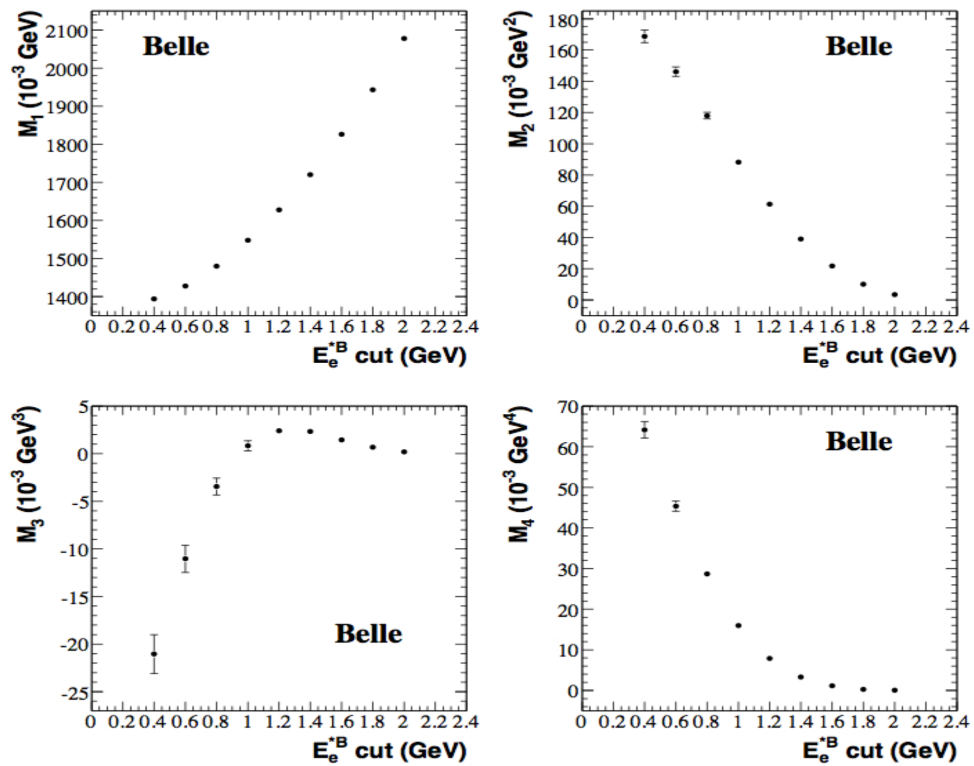
- All the particles in the event, excluding the B_{tag} and the lepton in the signal side, are combined to reconstruct the hadronic X system.
- M_X measured for different lepton energy thresholds in the B rest frame
- Observed spectra distorted by resolution & acceptance corrected by unfolding and QED radiative effects [Hocker, Kartelishvili, Nucl. Instrum. Meth. A372, 469-481 (1996)]

$|V_{cb}|$: Inclusive Measurement

Belle Measurement (L=140 fb^{-1})

[Phys. Rev. D75, 032001 (2007), Phys. Rev. D75, 032005 (2007)]





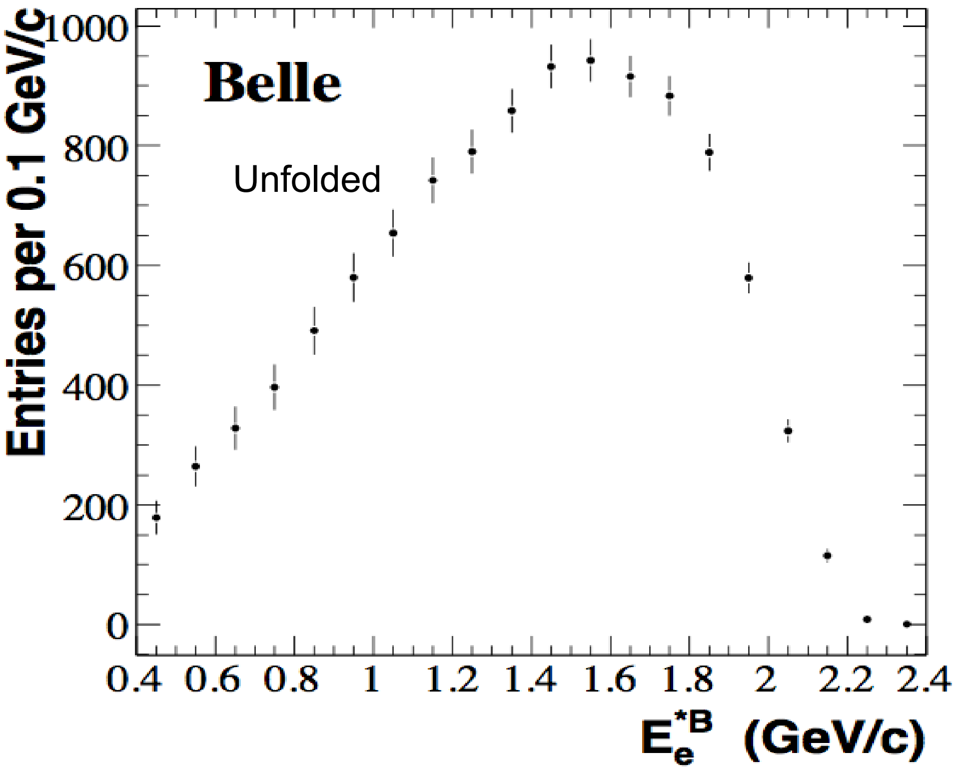
- Lepton Energy Moments up to the 4th order are measured with minimum E_e from 0.4 to 2.0 GeV

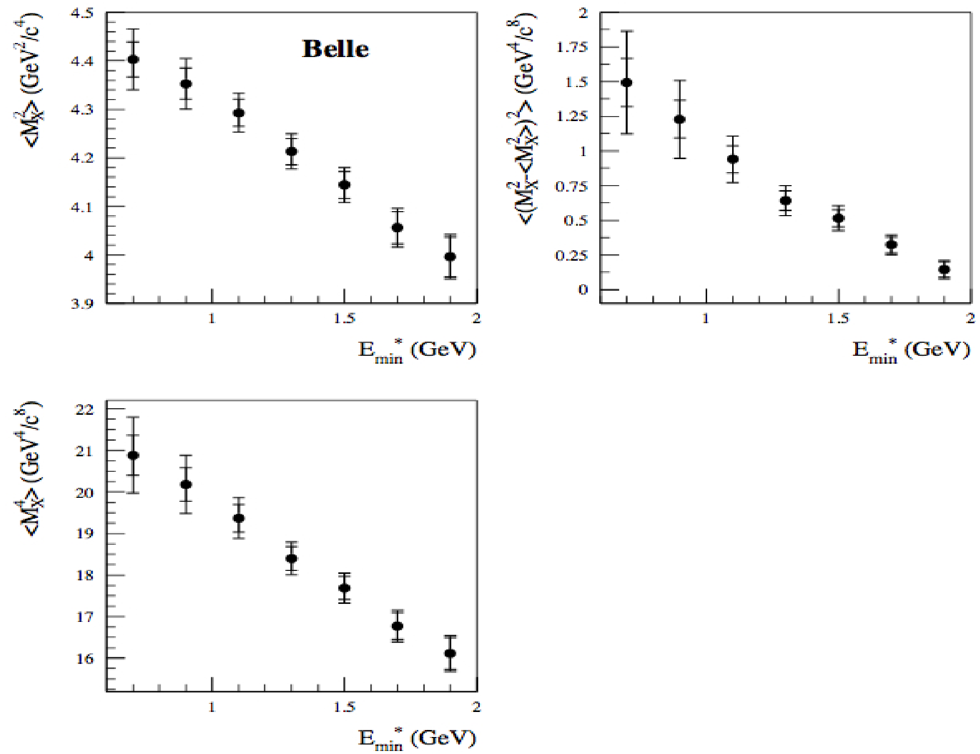
- Systematics from event selection, D** modeling and BB BKG subtraction using MC shapes

$|V_{cb}|$: Inclusive Measurement

Belle Measurement (L=140 fb^{-1})

[Phys. Rev. D75, 032001 (2007), Phys. Rev. D75, 032005 (2007)]

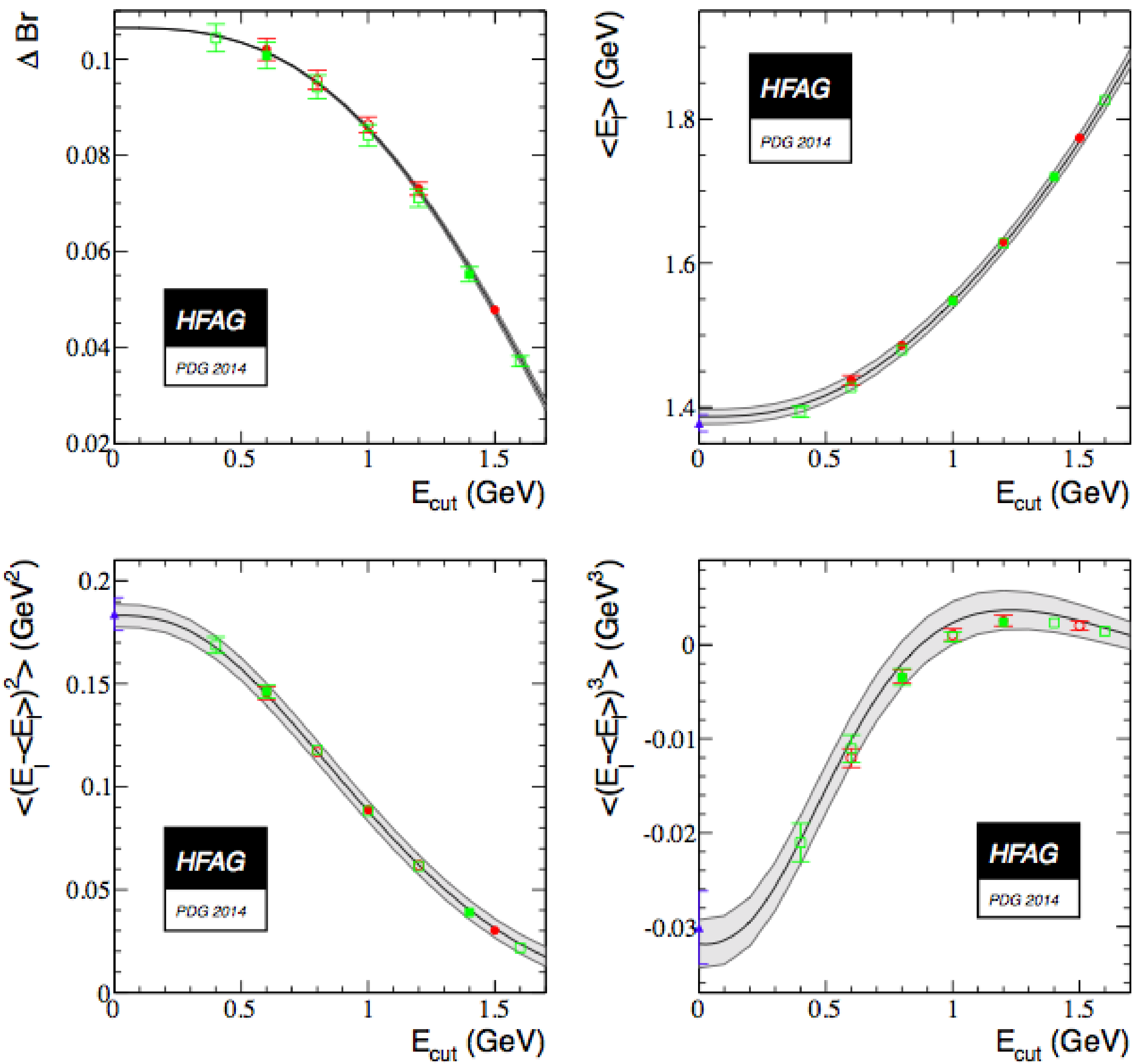




- Hadronic Mass Moments are measured for orders 2-4 and minimum E_e from 0.7 to 1.9 GeV
- Systematics from event selection, D** modeling and BB BKG subtraction using MC shapes

$|V_{cb}|$: Inclusive Measurement

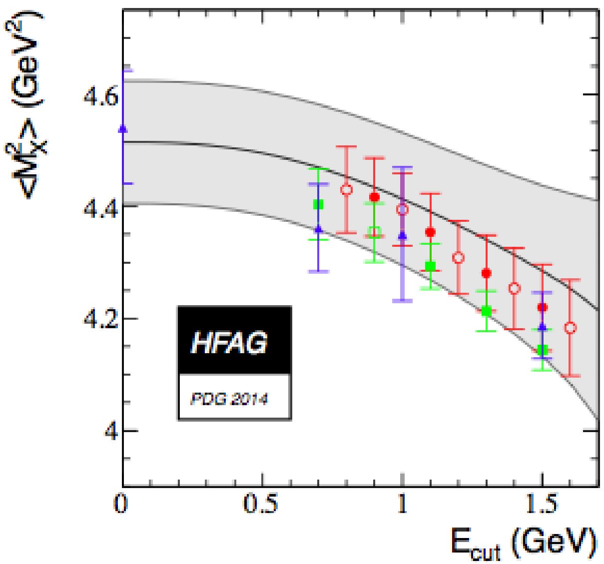
- Leptonic energy spectrum moments fit [HFAG 2014, arXiv:1412.7515]

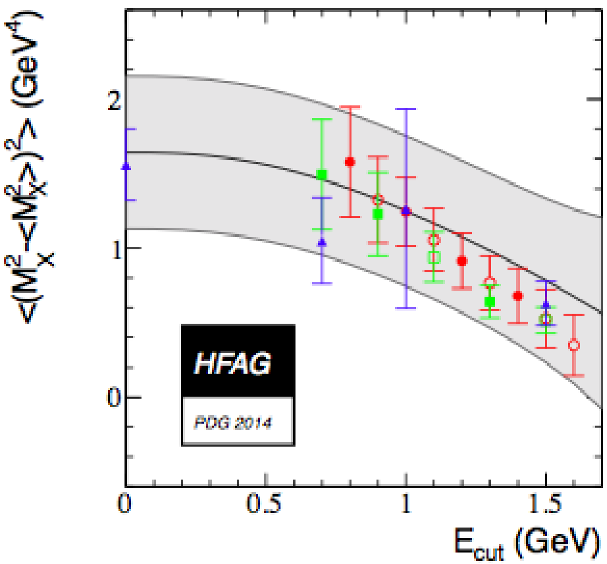


- Grey band: theory prediction with total error (BaBar, Belle, Others)

$|V_{cb}|$: Inclusive Measurement

- Hadronic mass spectrum moments fit [HFAG 2014, arXiv:1412.7515]





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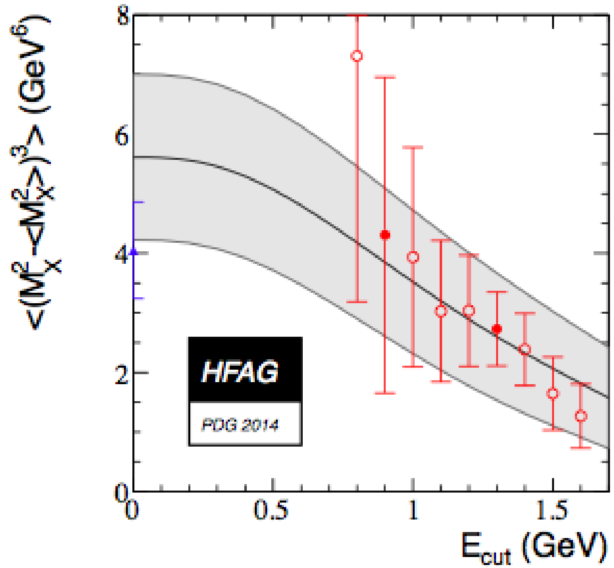
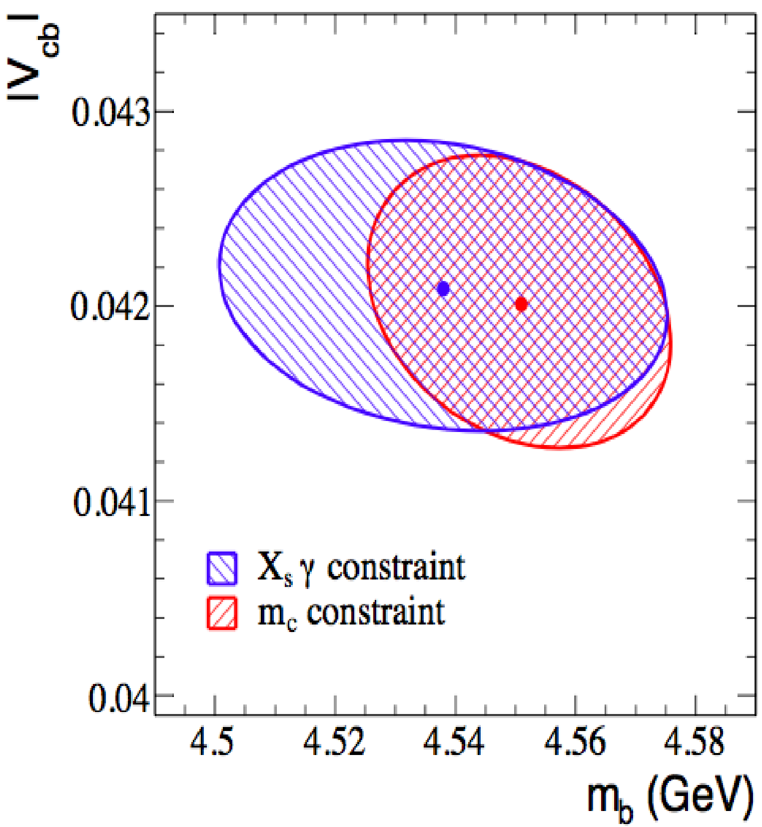


$|V_{cb}|$: Inclusive Measurement

- $|V_{cb}|$, m_b and OPE parameters extracted with a fit to 54 moments from the B-Factories (only external input B meson lifetime)



- Precision increased by using constraint from $B \to X_s\gamma$ or m_c from LCSR [arXiv:1102.2264]:

Constraint	$\lvert V_{cb}\rvert$ (10^{-3})	m_b^{kin} (GeV)	μ_π^2 (GeV^2)
$B \to X_s\gamma$	$42.09 \pm 0.46 \pm 0.59$	4.538 ± 0.038	0.515 ± 0.045
m_c(3 GeV)	$42.01 \pm 0.47 \pm 0.59$	4.551 ± 0.025	0.499 ± 0.044

- Fit gives also:
  $BR(B \to X_c\, l\, \nu) = (10.86 \pm 0.16)\%$

$$|V_{cb}|_{\rm incl} = (42.01 \pm 0.47_{\rm exp} \pm 0.59_{\rm th}) \times 10^{-3}$$

Error dominated by theory

Summary on $|V_{cb}|$

- $|V_{cb}|$:
  - From fits to $B \to X_c\, l\, \nu$ moments total error ~ 1.8%

$$|V_{cb}|_{\rm incl} = [42.01\ (1 \pm 0.011_{\rm exp} \pm 0.014_{\rm th})] \times 10^{-3}$$

  - From exclusive decay $B \to D^*\, l\, \nu$ total error ~ 2.3%

Using LQCD $$|V_{cb}|_{\rm excl} = [39.04\ (1 \pm 0.014_{\rm exp} \pm 0.019_{\rm th})] \times 10^{-3}$$

Using LCSR $$|V_{cb}|_{\rm excl} = [40.93\ (1 \pm 0.014_{\rm exp} \pm 0.023_{\rm th})] \times 10^{-3}$$

    - 2.5 σ difference using LQCD, better agreement with LCSR
    - Discrepancy decreases including $B \to D\, l\, \nu$: $|V_{cb}|=(39.5 \pm 0.8)10^{-3}$

- Discrepancy between exclusive and inclusive measurements is one of the most long standing tensions in the SM and still an open question