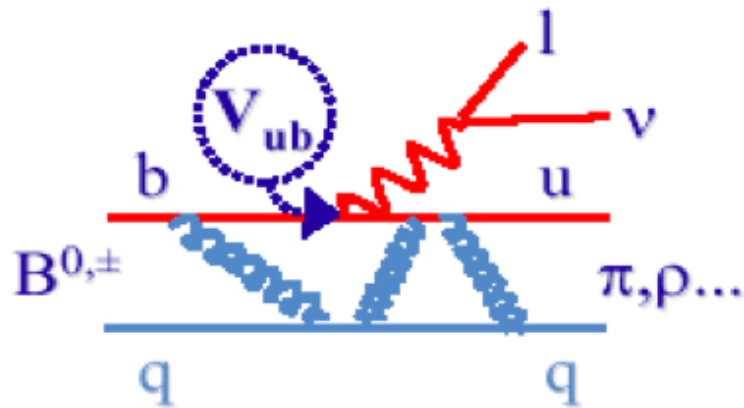


V_{ub} Measurements

Charmless semileptonic B decays, $B \rightarrow X_u l \nu$



Experimentally Challenging
Very High $B \rightarrow X_c l \nu$ BKG

- Tight selection cuts
- Limited phase space
- Extrapolation uncertainties

Inclusive Measurements

VS

Exclusive Measurements

Study inclusive kinematic quantities after charm suppression;

→ V_{ub} from total $BR(b \rightarrow ul\nu)$

→ Major issue: Extrapolation to the full phase space (Shape Function)

Study exclusive decay channels;

→ V_{ub} from partial $BR(B \rightarrow X_u l \nu)$

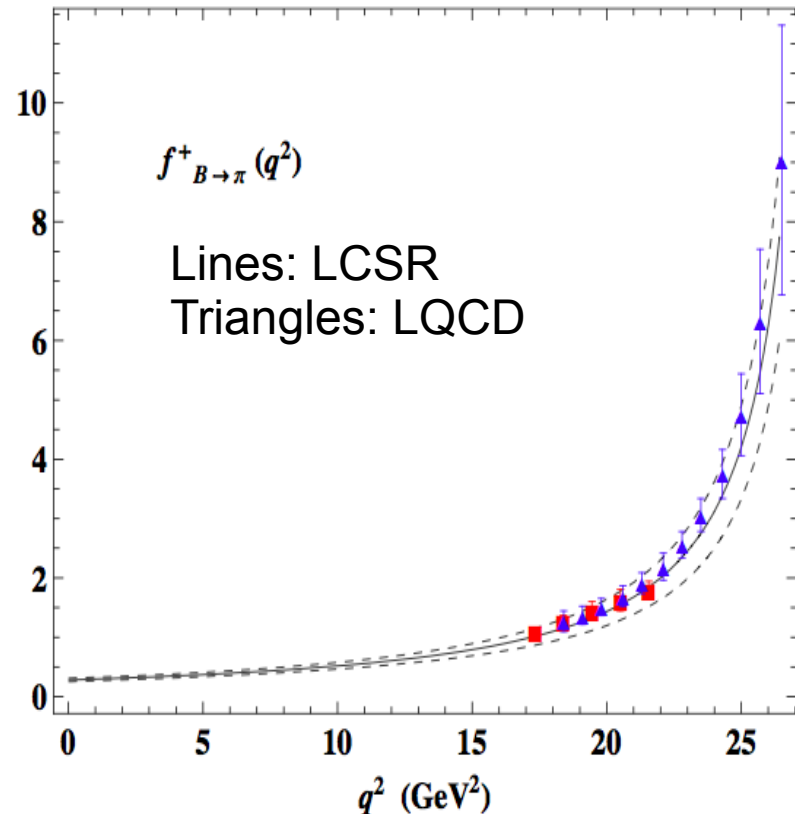
→ Major issue: QCD uncertainties in Form Factor prediction (non perturbative effects)

$|V_{ub}|$: Exclusive Measurement

- In the limit of negligible lepton mass, the differential decay rate for $B \rightarrow \pi l \nu$ decays is:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \mathbf{p}_\pi^3 |f_+^{B\pi}(q^2)|^2$$

- $q = p_B - p_\pi$: 4-momentum transfer
- $\mathbf{p}_\pi = [(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2]^{1/2} / (2m_B)$
- $f_+^{B\pi}(q^2)$ Form Factor for the B transition to a pseudoscalar final state



- Theory Challenge: calculation of FF**
 - LQCD:** High q^2 . Limited computational resources prohibit calculations with real values for the u/d quarks masses. Generate data with a sequence of light-quark masses (down to $m_s/10$) and lattice spacing (down to $a \sim 0.09$ fm) and extrapolate to the physical masses and zero spacing
 - Realistic estimates by HPQCD [Dalgic et al., Phys. Rev. D73, 074502 (2006)] and Fermilab MILC [Aubin et al., Phys. Rev. D70, 114501 (2004)]
 - LCSR:** Low q^2 . Uncertainty from quark-hadron duality approximation, values of the quark masses, renormalization scale

$|V_{ub}|$: Exclusive Measurement

- Experimental goal: measurement of the BR and the q^2 spectrum to determine the $FF(q^2)$ dependence.
- **Experimental Challenge:** background rejection from $B \rightarrow X_c \ell \nu$ decays and separation of $B \rightarrow \pi \ell \nu$ from other $B \rightarrow X_u \ell \nu$ decays

Different strategies:

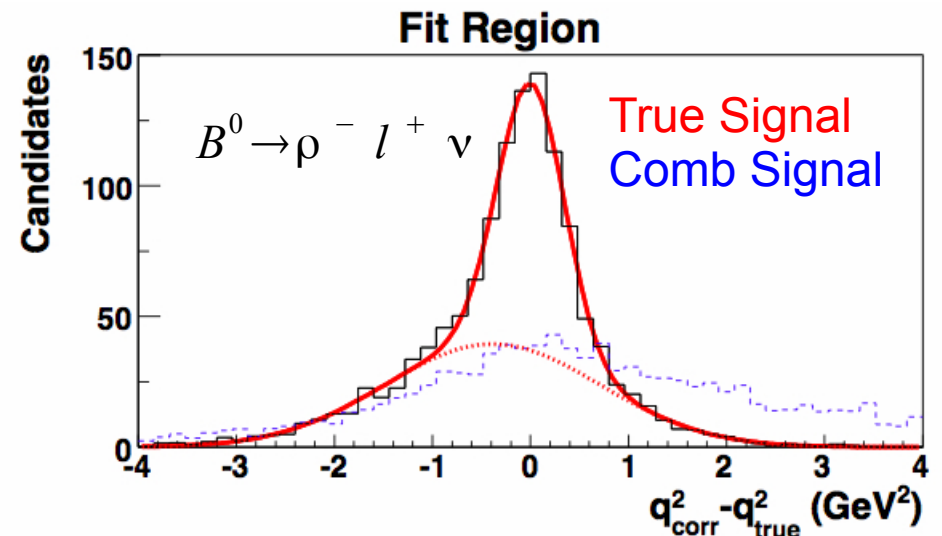
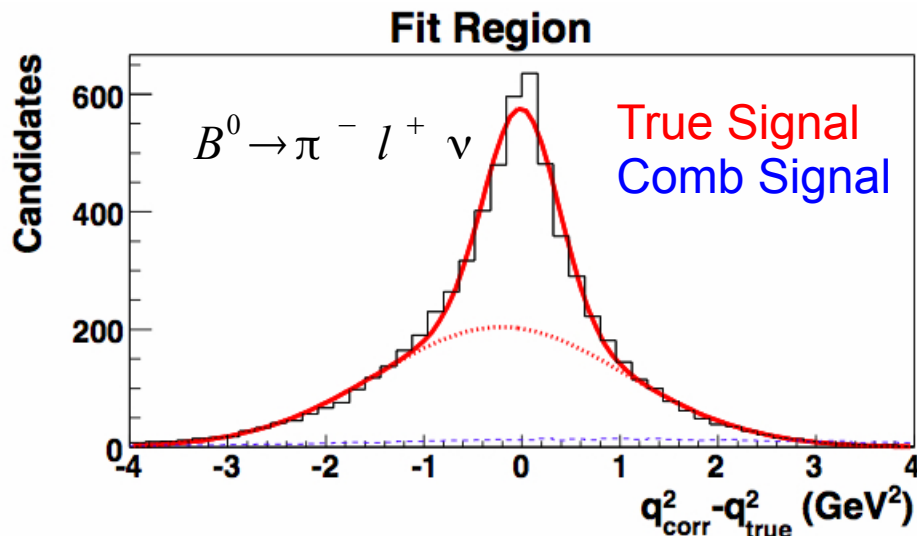
- **Tagged analyses**
Signal identification on the recoil of a fully (partially) reconstructed B hadronic (semileptonic) decay
 - Very clean, precise neutrino reconstruction, but statistically limited
- **Untagged analyses** (most precise results)
Neutrino 4-momentum from missing energy. “Reconstructed” neutrino combined with a lepton
 - Background dominated by continuum (real lepton from hadron decays or misidentified hadrons). $B \rightarrow X_c \ell \nu$ contamination suppressed using squared missing mass.

$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Four signal modes selected: $B^0 \rightarrow \pi^- (\rho^-) l^+ \nu$, $B^+ \rightarrow \pi^0 (\rho^0) l^+ \nu$; ($\rho^- \rightarrow \pi^- \pi^0$; $\rho^0 \rightarrow \pi^+ \pi^-$)
 - Approach reduces uncertainties due to cross-feed between the different modes
- Signal reconstructed with $p_\pi^* > 1.3 \text{ GeV}$, $p_l^* > 2.2 (2.0) \text{ GeV}$, $p_\pi^* + p_l^* > 2.8(2.65) \text{ GeV}$ for π (ρ) reconstruction and $|\cos \theta_{BY}| \leq 1.0$
 - Neutrino momentum obtained from $P_\nu^2 \simeq m_{\text{miss}}^2 = E_{\text{miss}}^2 - |\vec{p}_{\text{miss}}|^2$

$$q_{\text{raw}}^2 = [(E_\ell, \vec{p}_\ell) + (p_{\text{miss}}, \vec{p}_{\text{miss}})]^2 \longrightarrow \vec{p}_\nu = \alpha \vec{p}_{\text{miss}} \quad \text{with } \alpha = 1 - \frac{\Delta E}{E_{\text{miss}}}$$



$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Selected sample composed by:
 - Signal:
 - True signal
 - Combinatorial signal: hadron selected from decay tracks of the other B
 - Isospin-conjugate: $B^0 \rightarrow \pi^- l^+ \nu \leftrightarrow B^+ \rightarrow \pi^0 l^+ \nu$
 - Cross-feed: $B^0 \rightarrow \rho l \nu \rightarrow B \rightarrow \pi l \nu$
 - Background:
 - Continuum (true leptons, fake leptons)
 - Different $B \rightarrow X_u$ decays
 - $B \rightarrow X_c$ decays
 - Secondary leptons from J/ψ , τ , γ
 - Fake leptons
- Background suppression exploiting event shape, neutrino reconstruction and kinematical variables optimized as a function of q^2
- $N_{\pi^+} = 7181$, $N_{\pi^0} = 3446$, $N_{\rho^+} = 1577$, $N_{\rho^0} = 1970$
with a S/N ~ 0.2

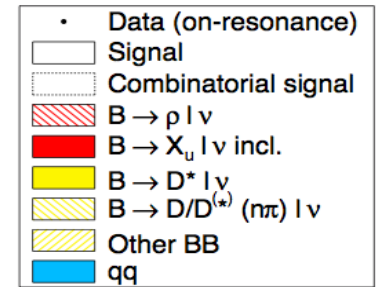
$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

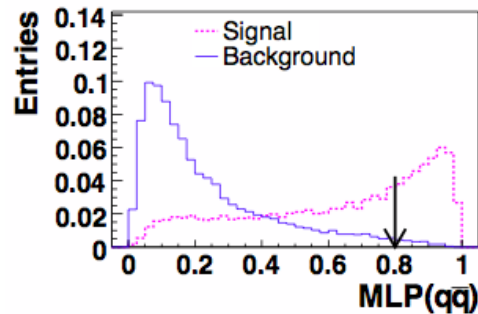
- Variables included in neural-network discriminators, trained for each background class and q^2 range
- Fox-Wolfram moments, thrust, missing mass, $\cos\theta_{BY}, \dots$

Continuum NN discriminator:

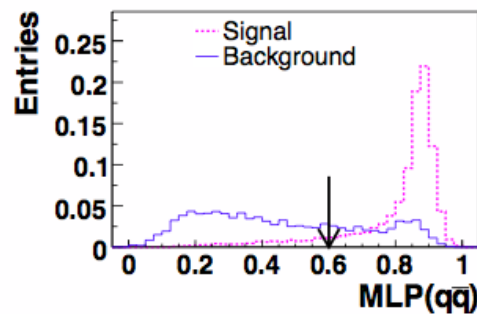
$$B^0 \rightarrow \pi^- l^+ \nu$$



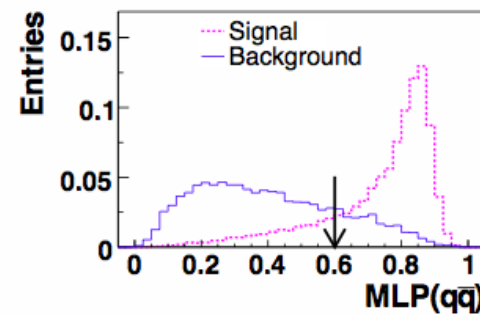
$0 < q^2 < 4 \text{ GeV}^2$



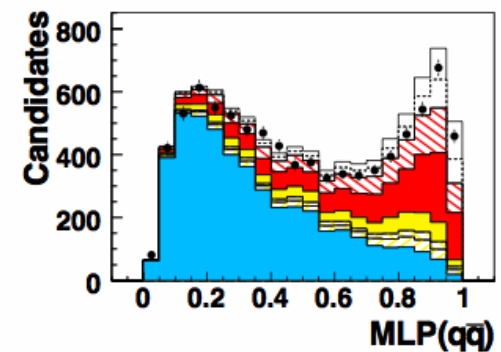
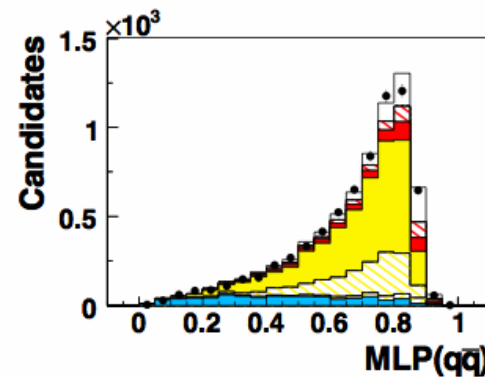
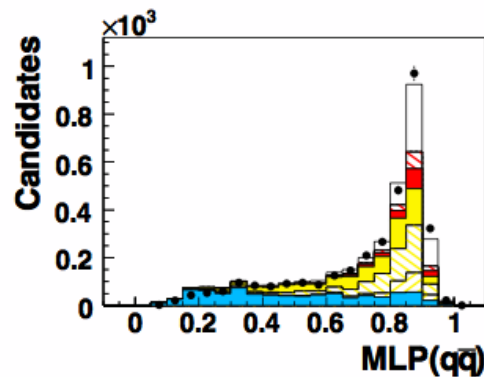
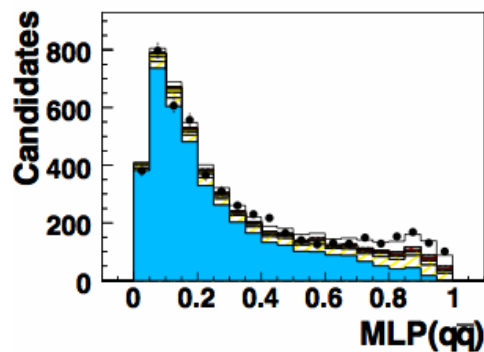
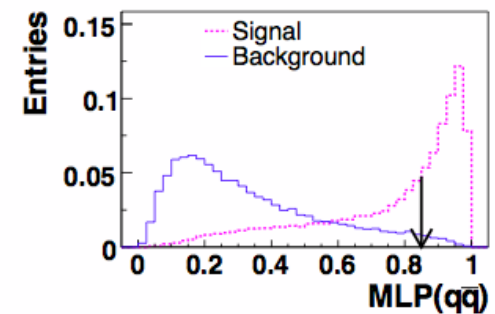
$4 < q^2 < 8 \text{ GeV}^2$



$12 < q^2 < 16 \text{ GeV}^2$



$q^2 > 20 \text{ GeV}^2$



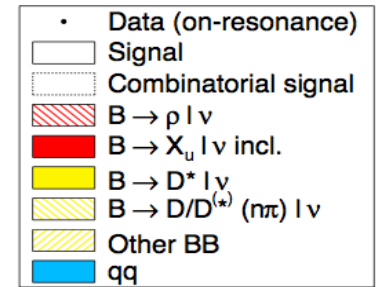
$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Variables included in neural-network discriminators, trained for each background class and q^2 range
- Fox-Wolfram moments, thrust, missing mass, $\cos\theta_{BY}, \dots$

$B \rightarrow Xc l \nu$ NN discriminator:

$$B^0 \rightarrow \pi^- l^+ \nu$$

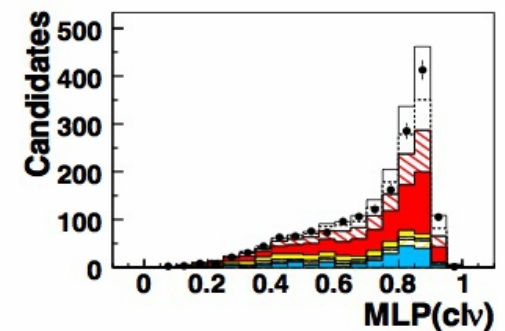
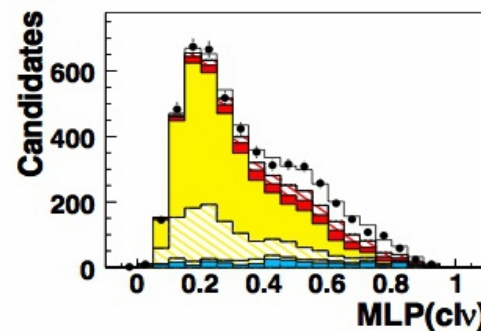
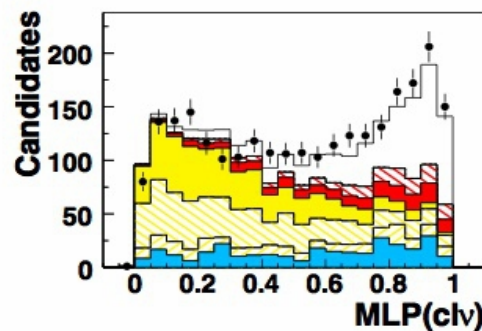
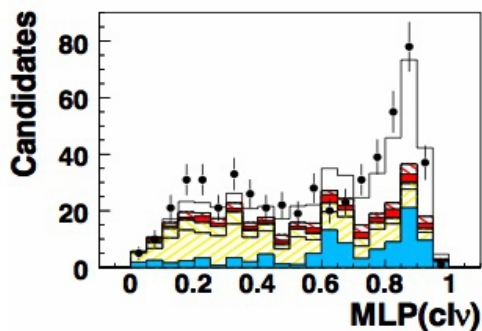
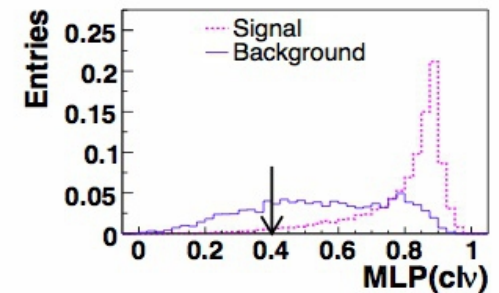
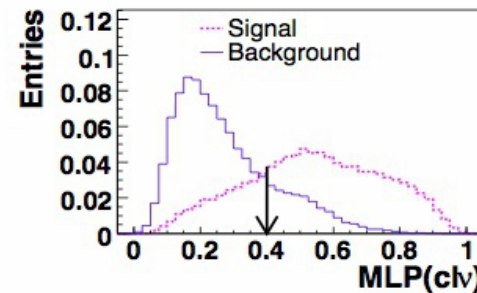
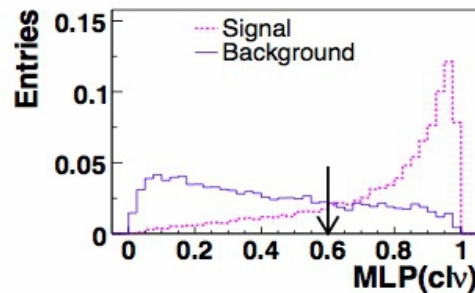
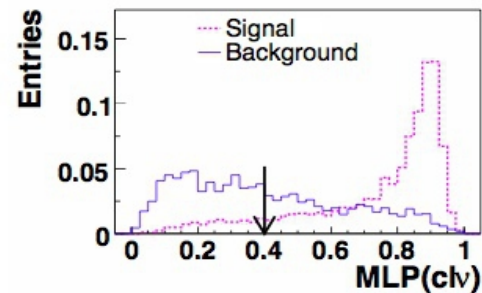


$0 < q^2 < 4 \text{ GeV}^2$

$4 < q^2 < 8 \text{ GeV}^2$

$12 < q^2 < 16 \text{ GeV}^2$

$q^2 > 20 \text{ GeV}^2$



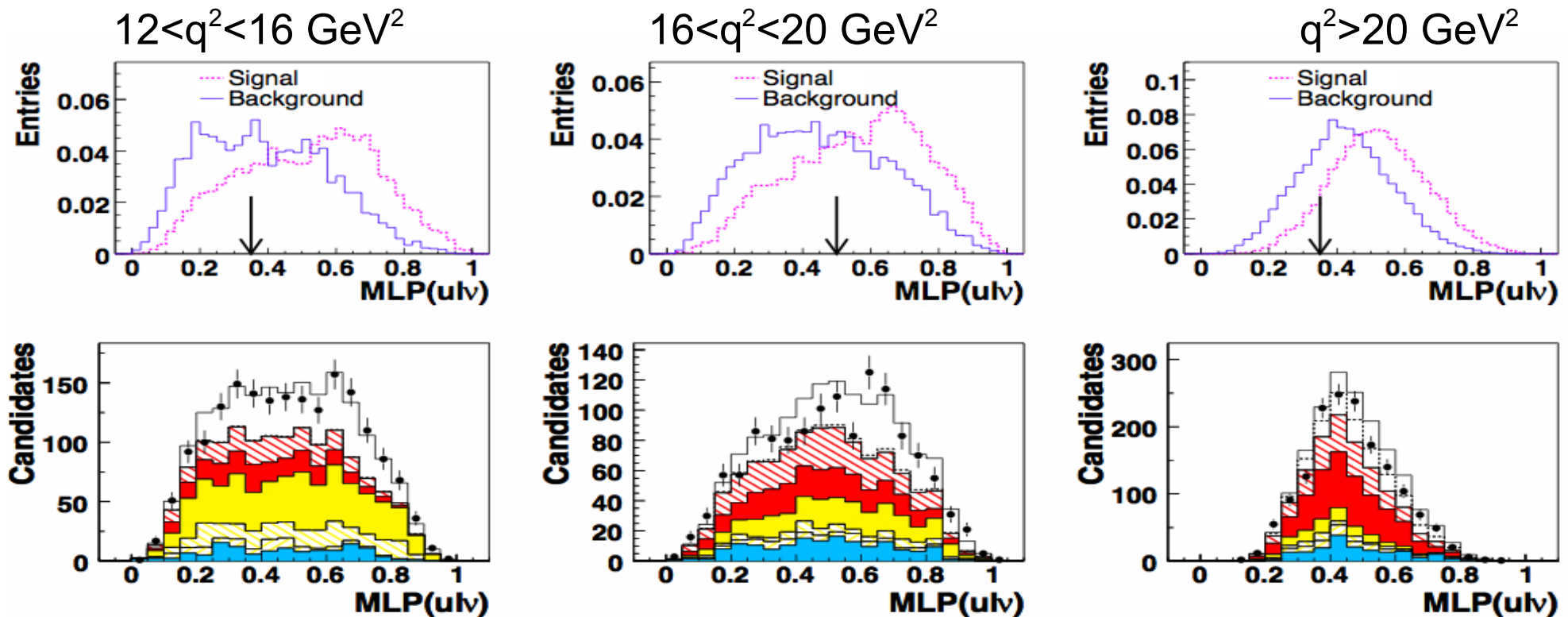
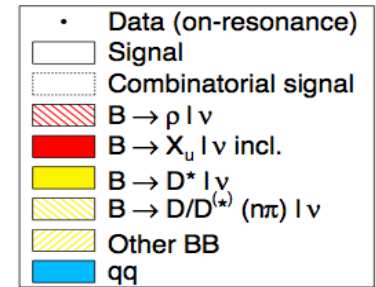
$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Variables included in neural-network discriminators, trained for each background class and q^2 range
- Fox-Wolfram moments, thrust, missing mass, $\cos\theta_{BY}, \dots$

$B \rightarrow Xu l \nu$ NN discriminator:

$$B^0 \rightarrow \pi^- l^+ \nu$$



$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Event yields determined in 12 q^2 bins from a 2D fit to $(\Delta E, m_{ES})$ with signal and background shapes from MC

$$\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$$

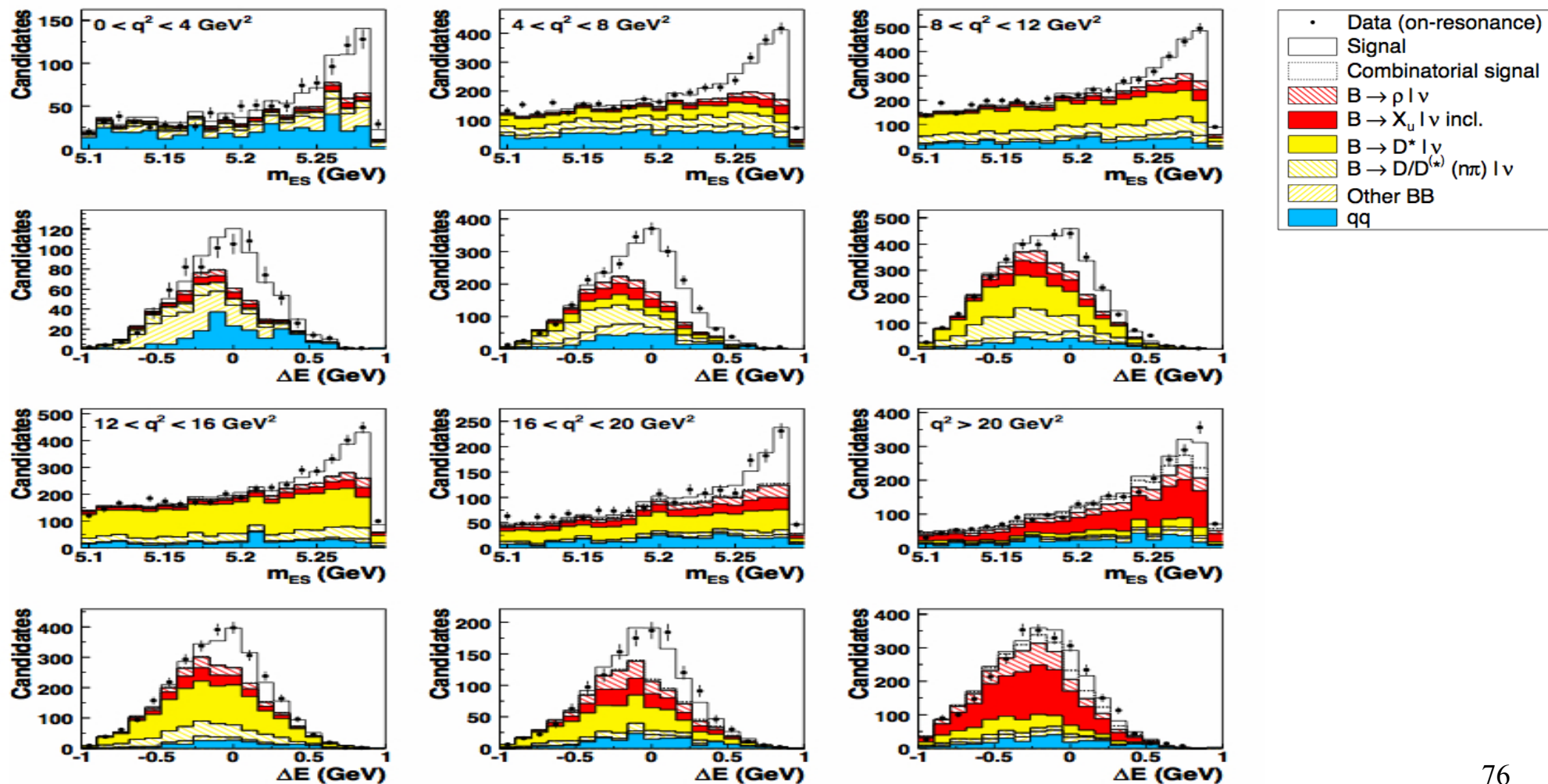
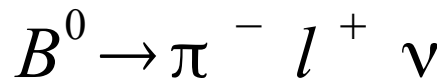
$$\mathcal{B}(B^0 \rightarrow \rho^- \ell^+ \nu) = (1.75 \pm 0.15 \pm 0.27) \times 10^{-4}$$

- Systematics from:
 - K_L^0 spectrum affecting neutrino reconstruction (only a small fraction of K_L^0 energy is deposited in the e.m. Calorimeter),
 - Lepton identification
 - Continuum description
 - $B \rightarrow \rho l \nu$ sensitive to the $B \rightarrow X_u l \nu$ BKG parameterization

$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

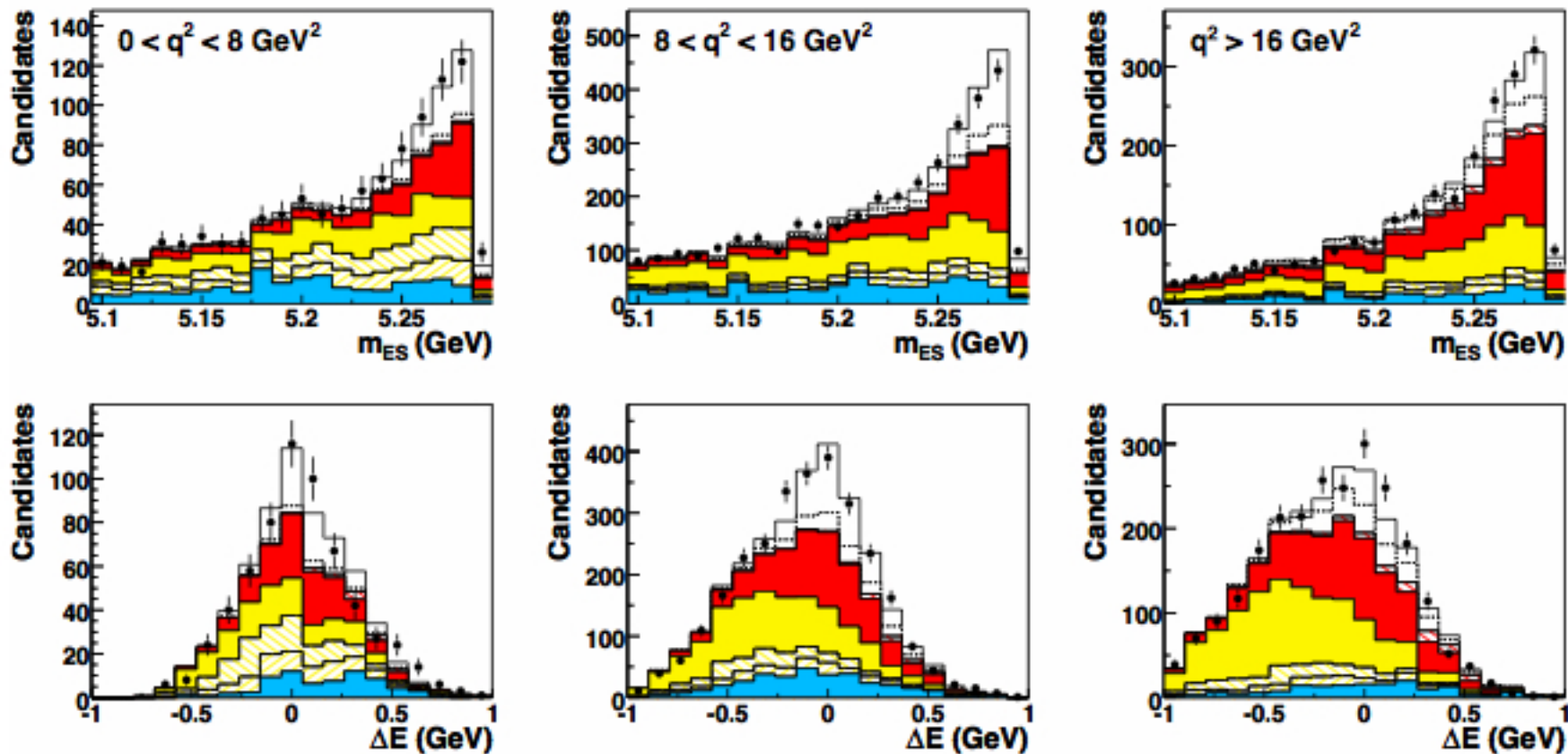
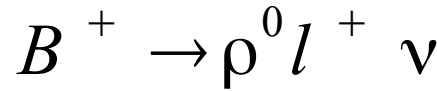
- Event yields determined in 12 q^2 bins from a 2D fit to $(\Delta E, m_{ES})$ with signal and background shapes from MC



$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Event yields determined in 3 q^2 bins from a 2D fit to $(\Delta E, m_{ES})$ with signal and background shapes from MC



$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

Systematics on BR

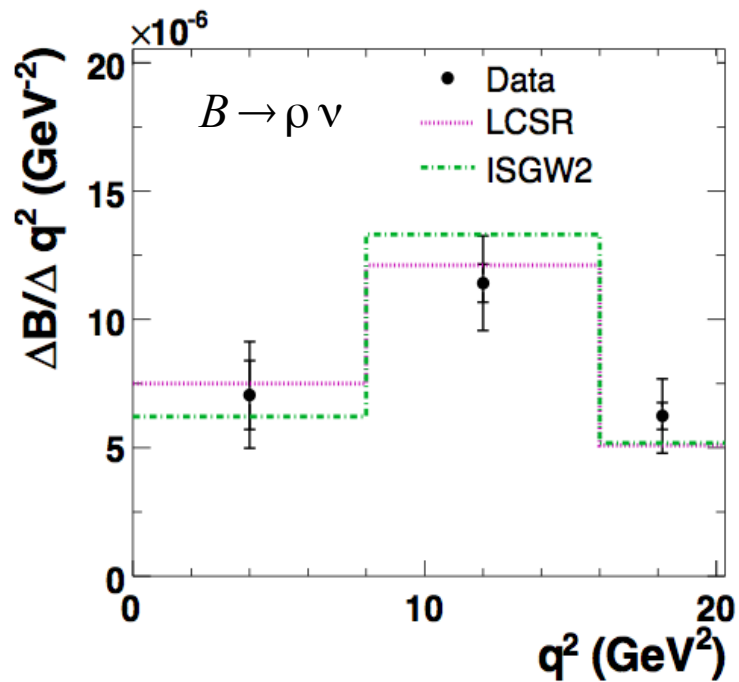
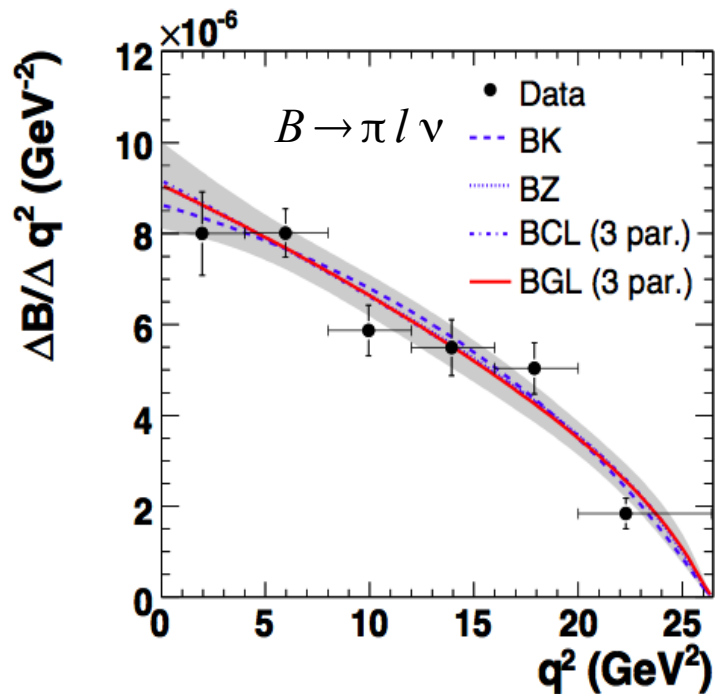
	$B \rightarrow \pi \ell \nu$						
q^2 range (GeV ²)	0-4	4-8	8-12	12-16	16-20	>20	0-26.4
Track efficiency	3.4	1.5	2.3	0.1	1.5	2.8	1.9
Photon efficiency	0.1	1.4	1.0	4.6	2.8	0.3	1.8
Lepton identification	3.8	1.6	1.9	1.8	1.9	3.0	1.8
K_L efficiency	1.0	0.1	0.5	4.5	0.4	2.0	1.4
K_L shower energy	0.1	0.1	0.1	0.8	0.9	3.8	0.7
K_L spectrum	1.6	1.9	2.2	3.1	4.4	2.3	2.5
$B \rightarrow \pi \ell \nu$ FF f_+	0.5	0.5	0.5	0.6	1.0	1.0	0.6
$B \rightarrow \rho \ell \nu$ FF A_1	1.7	1.2	3.4	2.0	0.1	1.6	1.7
$B \rightarrow \rho \ell \nu$ FF A_2	1.3	0.8	2.6	1.0	0.1	0.4	1.1
$B \rightarrow \rho \ell \nu$ FF V	0.2	0.3	0.9	0.7	0.1	0.5	0.5
$\mathcal{B}(B^+ \rightarrow \omega \ell^+ \nu)$	0.1	0.1	0.1	0.2	0.3	1.5	0.2
$\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu)$	0.1	0.1	0.2	0.2	0.2	0.5	0.2
$\mathcal{B}(B^+ \rightarrow \eta' \ell^+ \nu)$	0.1	0.1	0.1	0.1	0.1	0.3	0.1
$\mathcal{B}(B \rightarrow X_u \ell \nu)$	0.2	0.1	0.1	0.1	1.1	1.6	0.4
$B \rightarrow X_u \ell \nu$ SF param.	0.4	0.1	0.2	0.2	0.5	4.2	0.7
$B \rightarrow D \ell \nu$ FF ρ_D^2	0.2	0.1	0.5	0.3	0.2	0.7	0.3
$B \rightarrow D^* \ell \nu$ FF R_1	0.1	0.4	0.8	0.6	0.3	0.6	0.5
$B \rightarrow D^* \ell \nu$ FF R_2	0.5	0.2	0.1	0.2	0.1	0.4	0.2
$B \rightarrow D^* \ell \nu$ FF $\rho_{D^*}^2$	0.7	0.2	0.6	0.8	0.4	1.1	0.6
$\mathcal{B}(B \rightarrow D \ell \nu)$	0.2	0.2	0.3	0.4	0.5	0.5	0.3
$\mathcal{B}(B \rightarrow D^* \ell \nu)$	0.4	0.1	0.3	0.3	0.3	0.7	0.3
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)_{\text{narrow}}$	0.4	0.1	0.1	0.3	0.1	0.5	0.2
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)_{\text{broad}}$	0.1	0.1	0.1	0.5	0.1	0.2	0.2
Secondary leptons	0.5	0.2	0.3	0.2	0.2	0.7	0.3
Continuum	5.3	1.0	2.6	1.8	3.1	6.1	2.0
Bremsstrahlung	0.3	0.1	0.1	0.1	0.1	0.4	0.2
Radiative corrections	0.5	0.1	0.1	0.2	0.2	0.6	0.3
$N_{B\bar{B}}$	1.2	1.0	1.2	1.2	1.1	1.6	1.2
B lifetimes	0.3	0.3	0.3	0.3	0.3	0.7	0.3
f_{\pm}/f_{00}	1.0	0.4	0.8	0.8	0.5	1.3	0.8
Total syst. error	8.2	3.9	6.7	8.3	6.9	10.6	5.0

	$B \rightarrow \rho \ell \nu$			
q^2 range (GeV ²)	0-8	8-16	>16	0-20.3
Track efficiency	3.2	2.9	0.3	2.5
Photon efficiency	2.6	2.0	2.6	2.4
Lepton Identification	5.7	3.0	4.0	3.4
K_L efficiency	10.3	1.2	4.9	4.8
K_L shower energy	1.6	0.8	1.0	1.1
K_L spectrum	4.2	6.1	7.0	5.7
$B \rightarrow \pi \ell \nu$ FF f_+	0.1	0.1	0.7	0.2
$B \rightarrow \rho \ell \nu$ FF A_1	10.7	6.6	4.5	7.5
$B \rightarrow \rho \ell \nu$ FF A_2	8.5	3.8	0.8	4.7
$B \rightarrow \rho \ell \nu$ FF V	3.4	3.0	3.6	3.2
$\mathcal{B}(B^+ \rightarrow \omega \ell^+ \nu)$	0.7	0.7	3.4	1.2
$\mathcal{B}(B^+ \rightarrow \eta \ell^+ \nu)$	0.8	0.1	0.6	0.4
$\mathcal{B}(B^+ \rightarrow \eta' \ell^+ \nu)$	0.8	0.5	1.2	0.7
$\mathcal{B}(B \rightarrow X_u \ell \nu)$	7.4	7.3	10.6	8.0
$B \rightarrow X_u \ell \nu$ SF param.	11.9	7.6	12.8	10.0
$B \rightarrow D \ell \nu$ FF ρ_D^2	0.9	0.2	0.1	0.4
$B \rightarrow D^* \ell \nu$ FF R_1	0.7	0.1	0.3	0.3
$B \rightarrow D^* \ell \nu$ FF R_2	1.7	0.1	0.2	0.6
$B \rightarrow D^* \ell \nu$ FF $\rho_{D^*}^2$	2.0	0.2	0.1	0.7
$\mathcal{B}(B \rightarrow D \ell \nu)$	1.6	0.3	0.1	0.7
$\mathcal{B}(B \rightarrow D^* \ell \nu)$	0.5	0.1	0.3	0.3
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)_{\text{narrow}}$	1.3	0.1	0.1	0.5
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)_{\text{broad}}$	0.7	0.1	0.1	0.3
Secondary leptons	1.5	0.1	0.1	0.5
Continuum	8.9	3.8	5.0	4.0
Bremsstrahlung	0.9	0.1	0.2	0.4
Radiative corrections	1.3	0.1	0.7	0.6
$N_{B\bar{B}}$	2.7	2.0	2.5	2.3
B lifetimes	1.5	0.4	0.4	0.7
f_{\pm}/f_{00}	1.2	0.1	0.1	0.4
Total syst. error	26.1	16.1	21.3	15.7

$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- q^2 spectrum corrected for resolution, radiative effects using an unfolding technique [Hocker, Kartvelishvili, Nucl. Instrum. Meth. A37, 469 (1996)] giving the $\Delta\text{BR}/\Delta q^2$ distributions



- Spectra are fitted to different FF parameterizations & compared with predictions
- Used for the $|V_{ub}|$ extraction

$|V_{ub}|$: Exclusive Measurement

BaBar Untagged Analysis ($L=349 \text{ fb}^{-1}$) [Phys. Rev. D83, 052011 (2011)]

- Spectra are fitted to different FF parameterizations & compared with predictions
- Used for the $|V_{ub}|$ extraction

	q^2 Range (GeV^2)	$\Delta\mathcal{B}$ (10^{-4})	$\Delta\zeta$ (ps^{-1})	$ V_{ub} $ (10^{-3})
<i>B</i> → $\pi l \nu$				
LCSR 1 [15]	0 – 16	1.10 ± 0.07	5.44 ± 1.43	$3.63 \pm 0.12^{+0.59}_{-0.40}$
LCSR 2 [19]	0 – 12	0.88 ± 0.06	$4.00^{+1.01}_{-0.95}$	$3.78 \pm 0.13^{+0.55}_{-0.40}$
HPQCD [23]	16 – 26.4	0.32 ± 0.03	2.02 ± 0.55	$3.21 \pm 0.17^{+0.55}_{-0.36}$
<i>B</i> → $\rho l \nu$				
LCSR [17]	0 – 16.0	1.48 ± 0.28	13.79	2.75 ± 0.24
ISGW2 [14]	0 – 20.3	1.75 ± 0.31	14.20	2.83 ± 0.24

$|V_{ub}|$: Exclusive Measurement

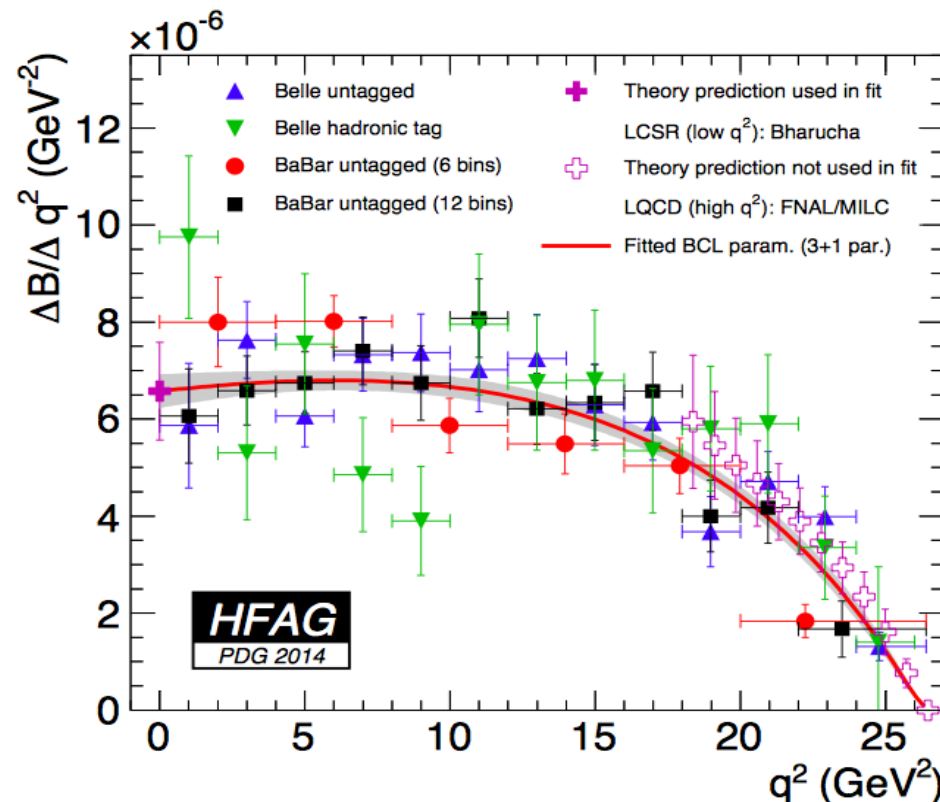
$|V_{ub}|$ Extraction

- Simultaneous fit to the q^2 spectra and the LQCD predictions in the BCL parameterization [Bourenly, Caprini, Lellouch, Phys. Rev. D79, 013008 (2009)]
- Measured partial $\Delta BR(q^2_{\min}, q^2_{\max})$ compared with the normalized partial decay rate $\Delta\zeta(q^2_{\min}, q^2_{\max})$ predicted from FF integration (LQCD: $q^2 > 16 \text{ GeV}^2$, LCSR: $q^2 < 12 \text{ GeV}^2$)

$$|V_{ub}| = \sqrt{\frac{\Delta B(q^2_{\min}, q^2_{\max})}{\tau_0 \Delta\zeta(q^2_{\min}, q^2_{\max})}}$$

$$\tau_0 = (1.519 \pm 0.007) \text{ ps}$$

$$\Delta\zeta \equiv \frac{G_F^2}{24\pi^3} \int_{q_i^2}^{q_f^2} dq^2 \mathbf{p}_\pi^3 |f_+^{B\pi}(q^2)|^2$$



$|V_{ub}|$: Exclusive Measurement

$|V_{ub}|$ Extraction

- Simultaneous fit to the q^2 spectra and the LQCD predictions in the BCL parameterization [Bourely, Caprini, Lellouch, Phys. Rev. D79, 013008 (2009)]
- Measured partial $\Delta\text{BR}(q^2_{\min}, q^2_{\min})$ compared with the normalized partial decay rate $\Delta\zeta(q^2_{\min}, q^2_{\max})$ predicted from FF integration (LQCD: $q^2 > 16 \text{ GeV}^2$, LCSR: $q^2 < 12 \text{ GeV}^2$)

	LCSR	HPQCD	FNAL/MILC	FNAL/MILC fit
$\Delta\zeta$ (ps^{-1})	$4.59^{+1.00}_{-0.85}$	2.02 ± 0.55	$2.21^{+0.47}_{-0.42}$	$2.21^{+0.47}_{-0.42}$
q^2 range (GeV^2)	0 – 12	16 – 26.4	16 – 26.4	16 – 26.4
Experiment	$ V_{ub} $ (10^{-3})			
BABAR (6 bins)	$3.54 \pm 0.12^{+0.38}_{-0.33}$	$3.22 \pm 0.15^{+0.55}_{-0.37}$	$3.08 \pm 0.14^{+0.34}_{-0.28}$	2.98 ± 0.31
BABAR (12 bins)	$3.46 \pm 0.10^{+0.37}_{-0.32}$	$3.26 \pm 0.19^{+0.56}_{-0.37}$	$3.12 \pm 0.18^{+0.35}_{-0.29}$	3.22 ± 0.31
Belle	$3.44 \pm 0.10^{+0.37}_{-0.32}$	$3.60 \pm 0.13^{+0.61}_{-0.41}$	$3.44 \pm 0.13^{+0.38}_{-0.32}$	3.52 ± 0.34
BABAR + Belle	$3.47 \pm 0.06^{+0.37}_{-0.32}$	$3.43 \pm 0.09^{+0.59}_{-0.39}$	$3.27 \pm 0.09^{+0.36}_{-0.30}$	3.23 ± 0.30
Tagged	$3.10 \pm 0.16^{+0.33}_{-0.29}$	$3.47 \pm 0.23^{+0.60}_{-0.39}$	$3.32 \pm 0.22^{+0.37}_{-0.31}$	3.33 ± 0.39

- Error from BR measurements (3%), shapes of the q^2 spectra from data (4%), FF normalization ($f_+(0)$) from theory (8%)

$|V_{ub}|$: Inclusive Measurement

Inclusive Cabibbo-suppressed decays (most precise $|V_{ub}|$ determination)

Theoretical Summary in [Phys. Rept. 494, 197-414 (2010)]

- Theoretical description of inclusive $B \rightarrow X_u \ell \nu$ similar to $B \rightarrow X_c \ell \nu$ used for the V_{cb} determination.
- **Challenge: large BKG from $B \rightarrow X_c \ell \nu$**
 - Highest experimental sensitivity in the region of phase space less impacted by the dominant $B \rightarrow X_c \ell \nu$ background: for $m_X < m_D$ & $p_\ell(b) > p_\ell(c)$
 - Requires extrapolation to the full spectrum using theoretical parameterizations to describe the unmeasured regions of phase space (Shape Functions) reflecting in theory error
- Discriminating variables: lepton energy, $p_X^+ = E_X - |\mathbf{p}_X|$, $p_X^- = E_X + |\mathbf{p}_X|$

- Differential decay rate:

$$\frac{d^3 \Gamma}{dp_X^+ dp_X^- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_X^-, p_X^+, k) F(k) + O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right).$$

- $C(E_\ell, p_X^-, p_X^+, k)$: perturbative description of the b quark decay

$|V_{ub}|$: Inclusive Measurement

Inclusive Cabibbo-suppressed decays (most precise $|V_{ub}|$ determination)

Theoretical Summary in [Phys. Rept. 494, 197-414 (2010)]

- Theoretical description of inclusive $B \rightarrow X_u \ell \nu$ similar to $B \rightarrow X_c \ell \nu$ used for the V_{cb} determination.
- **Challenge: large BKG from $B \rightarrow X_c \ell \nu$**
 - Highest experimental sensitivity in the region of phase space less impacted by the dominant $B \rightarrow X_c \ell \nu$ background: for $m_X < m_D$ & $p_l(b) > p_l(c)$
 - Requires extrapolation to the full spectrum using theoretical parameterizations to describe the unmeasured regions of phase space (Shape Functions) reflecting in theory error
- Discriminating variables: lepton energy, $p_X^+ = E_X - |\mathbf{p}_X|$, $p_X^- = E_X + |\mathbf{p}_X|$

- Differential decay rate:

$$\frac{d^3 \Gamma}{dp_X^+ dp_X^- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_X^-, p_X^+, k) F(k) + O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right).$$

- $F(k)$: non-perturbative shape function describing the momentum distribution of the b quark inside the B meson
[Bigi et al., Int. J. Mod. Phys. A9, 2467-2504 (1994),
Neubert, Phys. Rev. D49, 4623-4633 (1994)]

$|V_{ub}|$: Inclusive Measurement

Untagged Analyses

- First evidence for charmless B semileptonic decays by CLEO [Phys. Rev. Lett. 71, 4111-4115 (1993)] from observation of charged lepton above the kinematical endpoint for $B \rightarrow X_c \ell \nu$ ($p_\ell^* = 2.3$ GeV)
- Inclusive $B \rightarrow X_u \ell \nu$ are described by the sum of three-body decays involving a single light charmless meson (20%) and decays to non-resonant multi-body final states. Two components normalized to reproduce the OPE inclusive prediction, and adjusted to the measured inclusive charmless BR.

Tagged Analyses

- Lepton with $P_\ell > 1$ GeV on the recoil of a reconstructed hadronic decay taken as a signature of B_{SL} : low rate due to $\epsilon(\text{Tag})$, but reduced combinatorial BKG
 - Phase-space restriction covers 90% of the total inclusive rate to reduce error on extrapolation. Theory errors from m_b , higher-order perturbative corrections, signal shape functions used for the efficiency calculation

$|V_{ub}|$: Inclusive Measurement

Untagged Analyses

- First evidence for charmless B semileptonic decays by CLEO [Phys. Rev. Lett. 71, 4111-4115 (1993)] from observation of charged lepton above the kinematical endpoint for $B \rightarrow X_c \ell \nu$ ($p_1^* = 2.3$ GeV)
- Inclusive $B \rightarrow X_u \ell \nu$ are described by the sum of three-body decays involving a single light charmless meson (20%) and decays to non-resonant multi-body final states. Two components normalized to reproduce the OPE inclusive prediction, and adjusted to the measured inclusive charmless BR.

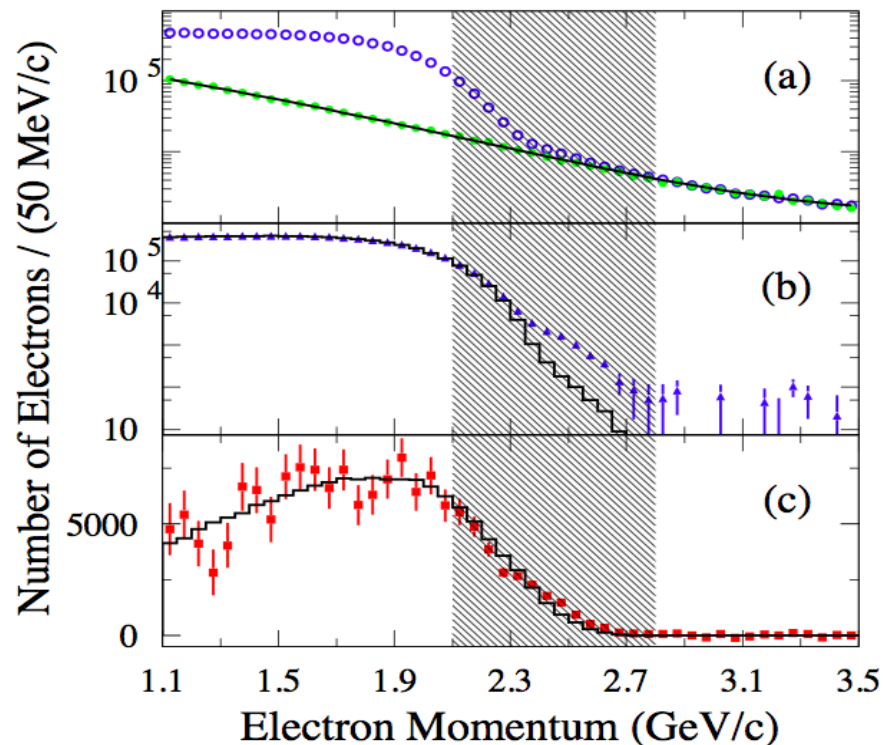
Tagged Analyses

- Observables: q^2 , m_x :
 - $B \rightarrow X_c \ell \nu$ BKG suppressed by vetoing K and slow pions from D decays
 - Events with additional missing particles rejected by cuts on the missing mass squared
 - Systematics from tag efficiency cancels in the ratio of charmless to inclusive SL decays

$|V_{ub}|$: Inclusive Measurement

BaBar Untagged Analysis ($L=80 \text{ fb}^{-1}$) [Phys. Rev. D73, 012006 (2006)]

- Observed spectra of the highest momentum electron in the $Y(4S)$ frame
- Challenge: $B \rightarrow X_c \ell \nu$ subtraction, secondary leptons from D (J/ψ) and fakes.
- Signal yield from a fit using MC shapes for signal and background components
- Sensitivity of the fitted yield from the MC shape of the signal spectrum reduced by combining the data between 2.1 to 2.8 GeV in a single bin



On-resonance

Scaled Off-resonance

Fit to non-BB background

On-resonance after non-BB BKG subtraction

Simulated BB BKG

On-resonance BKG subtracted

Simulated $B \rightarrow X_u \ell \nu$ spectrum

$|V_{ub}|$: Inclusive Measurement

BaBar Untagged Analysis ($L=80 \text{ fb}^{-1}$) [Phys. Rev. D73, 012006 (2006)]

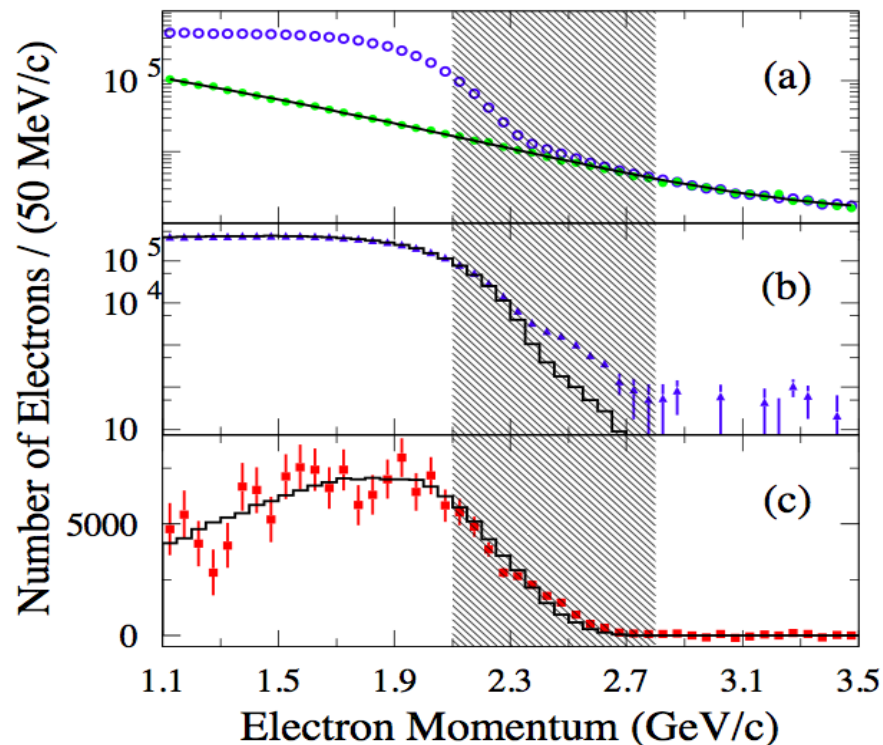
- Observed spectra of the highest momentum electron in the $Y(4S)$ frame
- Challenge: $B \rightarrow X_c l \nu$ subtraction, secondary leptons from D (J/ψ) and fakes.
- Signal yield from a fit using MC shapes for signal and background components
- Sensitivity of the fitted yield from the MC shape of the signal spectrum reduced by combining the data between 2.1 to 2.8 GeV in a single bin

Δp (GeV/c)	2.0–2.6	2.1–2.6	2.2–2.6	2.3–2.6	2.4–2.6
Total sample	609.81 ± 0.78	295.76 ± 0.54	133.59 ± 0.37	65.48 ± 0.26	35.38 ± 0.19
Non- $B\bar{B}$ background	142.38 ± 0.63	105.20 ± 0.48	74.86 ± 0.36	50.13 ± 0.25	29.96 ± 0.16
$X_c e \nu$ background	416.22 ± 2.52	157.17 ± 1.29	38.82 ± 0.47	4.00 ± 0.10	0.09 ± 0.01
J/ψ and $\psi(2S)$	6.17 ± 0.14	4.00 ± 0.10	2.33 ± 0.06	1.17 ± 0.04	0.47 ± 0.02
Other e^\pm background	1.61 ± 0.05	0.62 ± 0.02	0.24 ± 0.01	0.08 ± 0.01	0.03 ± 0.00
π misidentification	1.34 ± 0.04	0.98 ± 0.03	0.64 ± 0.02	0.34 ± 0.02	0.10 ± 0.01
K misidentification	0.47 ± 0.02	0.26 ± 0.01	0.13 ± 0.01	0.05 ± 0.01	0.01 ± 0.00
Other misidentification	0.27 ± 0.01	0.15 ± 0.01	0.08 ± 0.01	0.04 ± 0.01	0.02 ± 0.00
$X_u e \nu$ background	1.62 ± 0.10	0.66 ± 0.05	0.20 ± 0.02	0.03 ± 0.01	0.01 ± 0.00
$X_u e \nu$ signal	39.72 ± 2.70	26.72 ± 1.49	16.31 ± 0.71	9.64 ± 0.38	4.70 ± 0.25
$X_u e \nu$ efficiency (%)	42.1 ± 0.3	41.2 ± 0.4	40.2 ± 0.5	39.5 ± 0.7	37.9 ± 1.0

$|V_{ub}|$: Inclusive Measurement

BaBar Untagged Analysis ($L=80 \text{ fb}^{-1}$) [Phys. Rev. D73, 012006 (2006)]

- Observed spectra of the highest momentum electron in the $Y(4S)$ frame
- Challenge: $B \rightarrow X_c \ell \nu$ subtraction, secondary leptons from D (J/ψ) and fakes.
- Signal yield from a fit using MC shapes for signal and background components
- Sensitivity of the fitted yield from the MC shape of the signal spectrum reduced by combining the data between 2.1 to 2.8 GeV in a single bin



- Fitted yield with $2.0 < p_e^* < 2.6 \text{ GeV}$:

$$N_e = 610 \times 10^3$$

$$N_{\text{signal}} = (39.72 \pm 2.70) \times 10^3$$

$$\Delta\mathcal{B}(B \rightarrow X_u \ell \nu) = (0.572 \pm 0.41 \pm 0.065) \times 10^{-3}$$

$$\Delta\mathcal{B}(\Delta p) = \frac{N_{\text{tot}}(\Delta p) - N_{\text{bg}}(\Delta p)}{2\epsilon(\Delta p)N_{B\bar{B}}} (1 + \delta_{\text{rad}}(\Delta p))$$

Δp = momentum interval

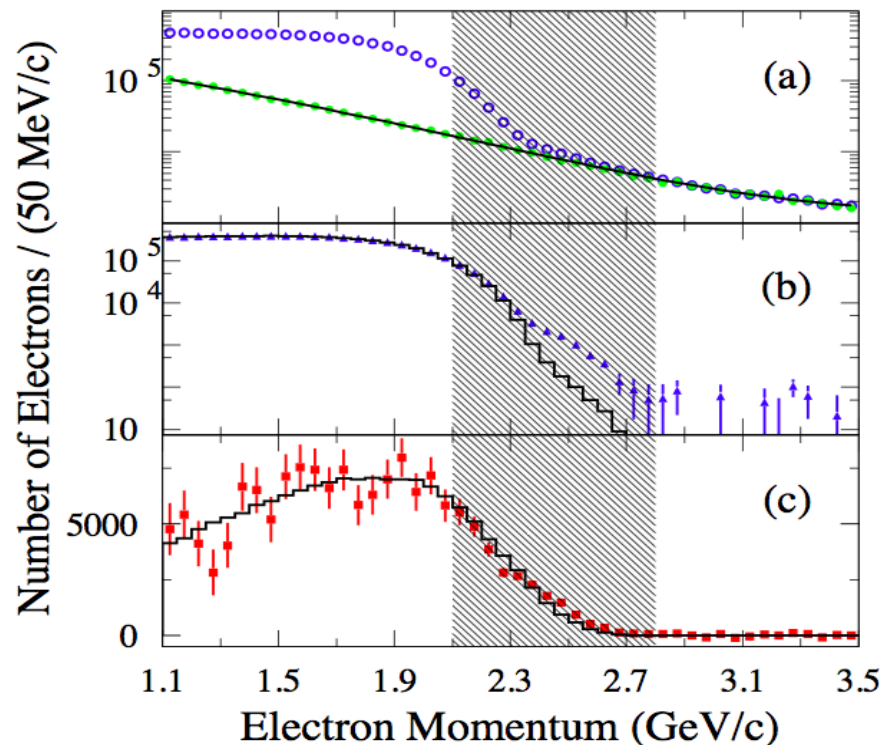
δ_{rad} = accounts for distortion due to Final State

Radiation

$|V_{ub}|$: Inclusive Measurement

BaBar Untagged Analysis ($L=80 \text{ fb}^{-1}$) [Phys. Rev. D73, 012006 (2006)]

- Observed spectra of the highest momentum electron in the $Y(4S)$ frame
- Challenge: $B \rightarrow X_c \ell \nu$ subtraction, secondary leptons from D (J/ψ) and fakes.
- Signal yield from a fit using MC shapes for signal and background components
- Sensitivity of the fitted yield from the MC shape of the signal spectrum reduced by combining the data between 2.1 to 2.8 GeV in a single bin



- Fitted yield with $2.0 < p_e^* < 2.6 \text{ GeV}$:
 $N_e = 610 \times 10^3$
 $N_{\text{signal}} = (39.72 \pm 2.70) \times 10^3$
 $\Delta B(B \rightarrow X_u \ell \nu) = (0.572 \pm 0.41 \pm 0.065) \times 10^{-3}$
- First analysis extending below the $B \rightarrow X_c \ell \nu$ endpoint $p_e^* \sim 2.3 \text{ GeV}$ covering about 25% of the spectrum

$|V_{ub}|$: Inclusive Measurement

BaBar Untagged Analysis ($L=80 \text{ fb}^{-1}$) [Phys. Rev. D73, 012006 (2006)]

p^{\min} (GeV/c)	2.0	2.1	2.2	2.3	2.4
Track finding efficiency	0.7	0.7	0.7	0.7	0.7
Electron identification	1.4	1.4	1.3	0.9	0.8
Event selection efficiency	6.8	6.7	6.1	5.5	7.9
Non- $B\bar{B}$ background	2.4	2.5	2.4	2.5	2.3
J/ψ and $\psi(2S)$ background	0.9	0.8	0.8	0.6	0.5
$B \rightarrow D^* l \nu$ form factor	2.4	2.3	2.0	1.3	0.5
$B \rightarrow D l \nu$ form factor	0.7	0.9	0.8	0.2	0.4
$B \rightarrow D^{**} e \nu$ spectrum	2.8	2.5	2.4	0.9	0.7
Other e^\pm background	0.5	0.3	0.2	0.1	0.1
$B \rightarrow X_u e \nu$ background	1.1	0.6	0.3	0.1	0.0
π misidentification background	0.8	0.9	0.9	0.8	0.5
K misidentification background	0.4	0.4	0.3	0.2	0.1
Other hadron misidentification	0.2	0.2	0.2	0.1	0.1
B movement	1.3	1.7	1.5	0.6	0.1
Bremsstrahlung and FSR	1.0	1.2	1.2	0.9	0.9
$N_{B\bar{B}}$ normalization	1.1	1.1	1.1	1.1	1.1
Total experimental error	8.8	8.6	7.9	6.6	8.5
$B \rightarrow X_u e \nu$ spectrum					
$X_s \gamma$ SF, fit to spectrum	6.0	3.5	1.6	0.3	0.1
$X_s \gamma$ SF, fit to moments	11.3	6.7	3.1	0.6	0.1
$X_c e \nu$ SF, fit to moments	13.3	8.6	4.0	0.8	0.0
SF, combined fit to moments	7.2	4.8	2.3	0.5	0.0
Total systematic error					
$X_s \gamma$ SF, fit to spectrum	10.7	9.3	8.1	6.6	8.5
$X_s \gamma$ SF, fit to moments	14.3	10.9	8.5	6.6	8.5
$X_c e \nu$ SF, fit to moments	15.9	12.2	8.9	6.6	8.5
SF, combined fit to moments	11.4	9.8	8.2	6.6	8.5

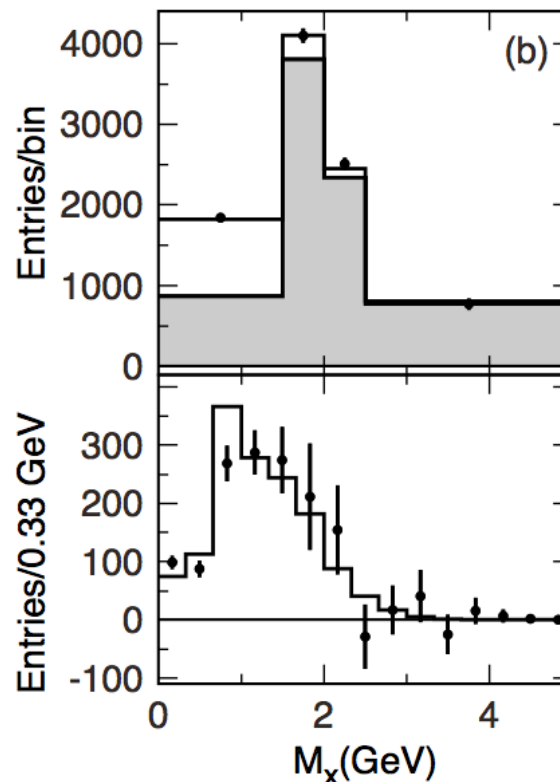
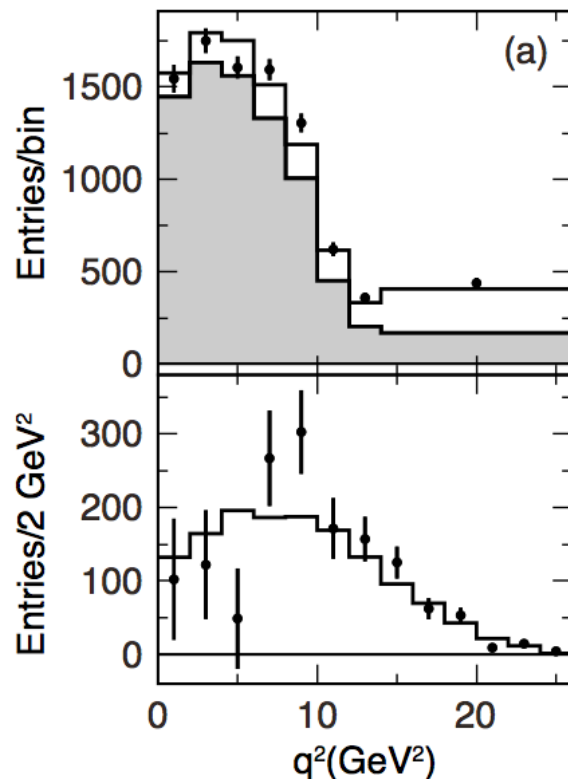
Common Upper limit 2.6 GeV

- Experimental Systematics dominated by efficiency and BKG subtraction
- $B \rightarrow X_u l \nu$ spectrum modeling systematic dominated by signal shape: Shape-Function parameters computed from:
 - Fit to $B \rightarrow X_s \gamma$ spectrum
 - Moments of $B \rightarrow X_s \gamma$
 - Moments of $B \rightarrow X_c l \nu$ inclusive lepton spectrum
 - Combined fit to all the moments

$|V_{ub}|$: Inclusive Measurement

- Tagged Analyses
- Partial BR ($B \rightarrow X_u \ell \nu$) from 2D fit to (m_x, q^2) on the recoil of fully reconstructed hadronic decays
- Systematics from shape of the distribution from simulation and Shape Function for the extrapolation to the full phase space
- From average of Belle [*Phys. Rev. Lett.* 104, 021801 (2010)] & Babar [*arXiv:1112.0702* (2011)] results:

$$\Delta\text{BR}(p^*_{\ell} > 1 \text{ GeV}) = (1.87 \pm 0.10 \pm 0.11) \times 10^{-3}$$



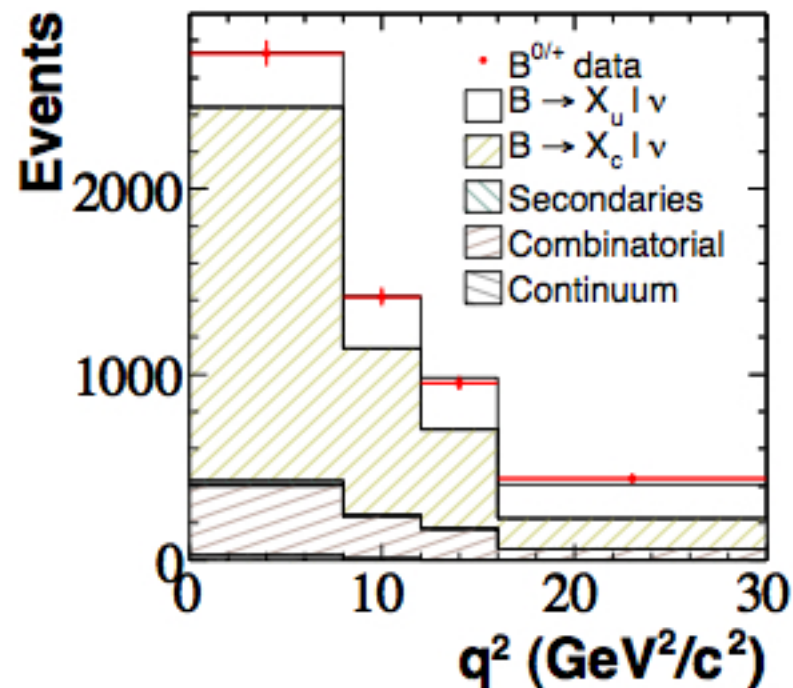
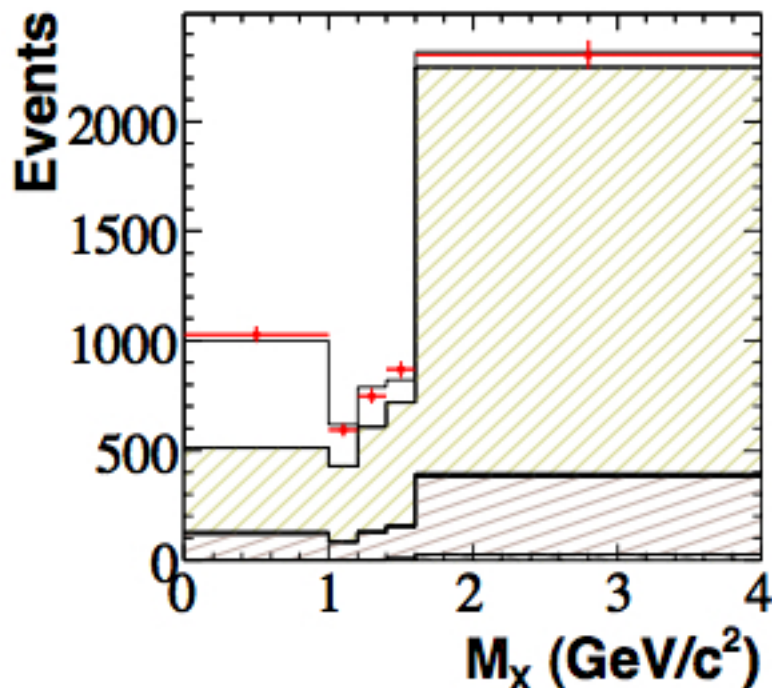
BaBar

Signal BKG subtracted

$|V_{ub}|$: Inclusive Measurement

- Tagged Analyses
- Partial BR ($B \rightarrow X_u \ell \nu$) from 2D fit to (m_X, q^2) on the recoil of fully reconstructed hadronic decays
- Systematics from shape of the distribution from simulation and Shape Function for the extrapolation to the full phase space
- From average of Belle [Phys. Rev. Lett. 104, 021801 (2010)] & Babar [arXiv:1112.0702 (2011)] results:

$$\Delta\text{BR}(p^*_\ell > 1 \text{ GeV}) = (1.87 \pm 0.10 \pm 0.11) \times 10^{-3}$$



Belle

$|V_{ub}|$: Inclusive Measurement

$|V_{ub}|$ Extraction

- Measured partial $\Delta\mathcal{B}(\Delta p)$ (Δp =momentum interval) compared with the predicted partial decay rate $\Delta\Gamma(\Delta p)$ for the selected phase space region

$$|V_{ub}| = \sqrt{\Delta\mathcal{B}/(\tau_B \Delta\Gamma_{\text{theory}})}$$

	BLNP	GGOU	DGE
m_b scheme	SF scheme	Kinetic scheme	$\overline{\text{MS}}$ scheme
m_b (GeV)	$4.588 \pm 0.023 \pm 0.011$	4.560 ± 0.023	4.194 ± 0.043
μ_π^2 (GeV ²)	$0.189^{+0.041}_{-0.040} \pm 0.020$	0.453 ± 0.036	—
Experiment	$ V_{ub} $ (10^{-3})		
CLEO (Bornheim et al., 2002)	$4.19 \pm 0.49^{+0.26}_{-0.34}$	$3.93 \pm 0.46^{+0.22}_{-0.29}$	$3.82 \pm 0.43^{+0.23}_{-0.26}$
Belle (Limosani, 2005)	$4.88 \pm 0.45^{+0.24}_{-0.27}$	$4.75 \pm 0.44^{+0.17}_{-0.22}$	$4.79 \pm 0.44^{+0.21}_{-0.24}$
BABAR (Aubert, 2006x)	$4.48 \pm 0.25^{+0.27}_{-0.28}$	$4.29 \pm 0.24^{+0.18}_{-0.24}$	$4.28 \pm 0.24^{+0.22}_{-0.24}$
BABAR (Aubert, 2005h)	$4.66 \pm 0.31^{+0.31}_{-0.36}$	—	$4.32 \pm 0.29^{+0.24}_{-0.29}$
Average untagged	$4.65 \pm 0.22^{+0.26}_{-0.29}$	$4.39 \pm 0.22^{+0.18}_{-0.24}$	$4.44 \pm 0.21^{+0.21}_{-0.25}$
Belle (Urquijo, 2010)	$4.47 \pm 0.27^{+0.19}_{-0.21}$	$4.54 \pm 0.27^{+0.10}_{-0.11}$	$4.60 \pm 0.27^{+0.11}_{-0.13}$
BABAR (Lees, 2011l)	$4.28 \pm 0.24^{+0.18}_{-0.20}$	$4.35 \pm 0.24^{+0.09}_{-0.11}$	$4.40 \pm 0.24^{+0.12}_{-0.13}$
Average tagged	$4.35 \pm 0.19^{+0.19}_{-0.20}$	$4.43 \pm 0.21^{+0.09}_{-0.11}$	$4.49 \pm 0.21^{+0.13}_{-0.13}$
Average all	$4.40 \pm 0.15^{+0.19}_{-0.21}$	$4.39 \pm 0.15^{+0.12}_{-0.14}$	$4.45 \pm 0.15^{+0.15}_{-0.16}$

$|V_{ub}|$: Inclusive Measurement

$|V_{ub}|$ Extraction [Antonelli et al., Phys. Rept. 494, 197-414 (2010)]

$$\frac{d^3 \Gamma}{dp_X^+ dp_X^- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_X^-, p_X^+, k) F(k) + O\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right).$$

$$p_X^+ = E_X - |\mathbf{p}_X|, \quad p_X^- = E_X + |\mathbf{p}_X|$$

- **BLNP** [Bosch, Lange, Neubert, Paz, Nucl. Phys. B699, 335-386 (2004), Lange, Neubert, Paz, Phys. Rev. D72, 073006 (2005)]: $P_x^+ \ll P_x^-$ (C dependences on P_x^+ , P_x^- factorize). Resummation of double log to NNLL. Full $O(\alpha_s)$ corrections included.
- **GGOU** [Gambino, Giordano, Ossola, Uraltsev, JHEP 0710, 058 (2007)]: $P_x^+ \ll P_x^-$ and $P_x^+ \sim P_x^-$. C computed at fixed order to $O(\alpha_s)$, no resummation at small P_x^+ .
- **DGE** [Anderson, Gardi, JHEP 0601, 097 (2006)]: NNLL resummation and full $O(\alpha_s)$ corrections included. Perturbative model for the leading shape function, non-perturbative corrections included via its moments. Most predictive model.

$|V_{ub}|$: Inclusive Measurement

$|V_{ub}|$ Extraction

- Measured partial $\Delta\mathcal{B}(\Delta p)$ (Δp =momentum interval) compared with the predicted partial decay rate $\Delta\Gamma(\Delta p)$ for the selected phase space region

$$|V_{ub}| = \sqrt{\Delta\mathcal{B}/(\tau_B \Delta\Gamma_{\text{theory}})}$$

$$|V_{ub}|_{\text{incl}} = (4.42 \pm 0.20_{\text{exp}} \pm 0.15_{\text{th}}) \times 10^{-3}$$

- Uncertainty dominated by experimental error. Similar statistical and systematic uncertainties (from particle identification and reconstruction efficiency).
- Theoretical error due to Shape Function for the extrapolation to the full phase space dominated by μ_{π}^2 , m_b depending on the scheme obtained from fits to $B \rightarrow X_c | v$ moments (see before)
- Good agreement between the different QCD calculations

Summary on $|V_{ub}|$

- $|V_{ub}|$:
 - Inclusive fits to $B \rightarrow X_u \ell \nu$ gives comparable experimental and theoretical errors
$$|V_{ub}|_{\text{incl}} = [4.42 (1 \pm 0.045_{\text{exp}} \pm 0.034_{\text{th}})] \times 10^{-3}$$
 - From exclusive decay $B^0 \rightarrow \pi \ell \nu$ using LQCD and the measured spectrum
$$|V_{ub}|_{\text{excl}} = [3.23 (1 \pm 0.05_{\text{exp}} \pm 0.08_{\text{th}})] \times 10^{-3}$$
 - 3 σ difference
- As for V_{cb} , discrepancy between exclusive and inclusive measurements is one of the most long standing tensions in the SM and still an open question