B^0 Mixing

- Neutral weakly-decaying self-conjugate pairs of mesons:

$$K^0/\overline{K}^0;\ \ D^0/\overline{D}^0;\ \ B_q^0/\overline{B}_q^0;\ \ [q=d\ ,s]$$

- Mixing in neutral K, B_d, B_s and D mesons discovered by [Lande et al., Phys. Rev. 103, 1901-1904 (1956)], ARGUS [Phys. Lett. B192, 245 (1987)], CDF/D0 [Phys. Rev. Lett. 97, 242003/021802 (2006)] and Babar [Phys. Rev. Lett. 98, 211802 (2007)]

- Consider for example the B meson system:
  - Two Flavor eigenstates whith definite quark content:

$$B_q^0=\overline{b}q\ ,\ \ \overline{B}_q^0=b\overline{q}$$

    describe particle interaction (production and decays)
  - Two Hamiltonian eigenstates with definite mass and lifetime

$$B_{q\ ,Light}\ ,\ \ B_{q\ ,Heavy}$$

    describe the propagation through space
- Propagation eigenstates are not flavor eigenstates: flavor eigenstates are mixed as they propagate through space
  - The two neutral K propagation eigenstates have very different lifetimes: convenient to define states by the lifetime: K_L, K_S
  - Neutral D mesons have mixing rate much slower than the decay rate: flavor eigenstates $D^0/\overline{D}^0$ are the most convenient basis

B^0 Mixing

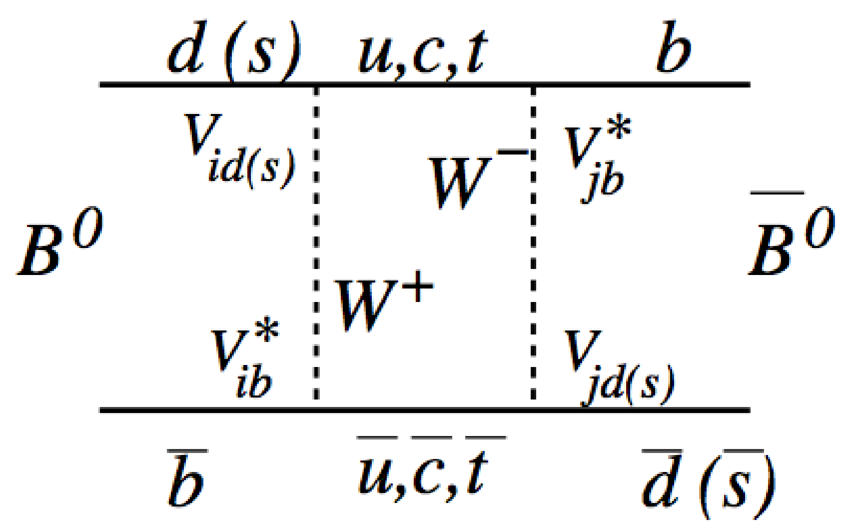
- A linear combination of flavor eigenstates is governed by the time-dependent Schrodinger equation:

$$a|B^0\rangle + b|\overline{B}^0\rangle \qquad\qquad i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv (M - \frac{i}{2}\Gamma)\begin{pmatrix} a \\ b \end{pmatrix}$$

where
$$\mathcal{H}_{\text{eff}} = \mathbf{M} - \frac{i\mathbf{\Gamma}}{2} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}\right]$$

Effective (not Hermitian) Hamiltonian whith M and Γ complex matrices describing the masses and decay rates.

- CPT symmetry: $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$
- Off-diagonal terms related to transition amplitude from B^0 to $\overline{B}^0$ arising from box diagrams with two W exchanges.
- K/D mesons: dominated by long-distance contributions due to common intermediate states
- B mesons: dominated by short-distance contributions (top exchange); long-distance contributions strongly suppressed (off the region of hadronic resonances)



B^0 Mixing

- Eigenvalue problem gives complex eigenvalues and eigenstates represented as an admixture of the flavor eigenstates:

$$|B_{\mathrm{L,H}}\rangle = p|B_q^0\rangle \pm q|\overline{B}_q^0\rangle \qquad |q|^2 + |p|^2 = 1$$

normalization condition

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

- Eigenvalues

$$m_L - \frac{i}{2}\Gamma_L = M_{11} - \frac{i}{2}\Gamma_{11} + \frac{p}{q}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

$$m_H - \frac{i}{2}\Gamma_H = M_{11} - \frac{i}{2}\Gamma_{11} - \frac{p}{q}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

Neglecting $O(m_b^2/m_W^2)$:

$$\Delta m = m_H - m_L \simeq 2|M_{12}| > 0$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|\cos\Phi$$

$$\Phi = arg\left(-M_{12}/\Gamma_{12}\right)$$

$$\Delta m_d = 3.34\times 10^{-10}\ MeV$$

$$\Delta m_s = 1.16\times 10^{-8}\ MeV$$

For comparison:

$$\Delta m_K = 3.48\times 10^{-12}\ MeV$$

$$\Delta m_D = 9.45\times 10^{-12}\ MeV$$

- Sign of Δm by definition
- Sign of ΔΓ determined by experiment. SM predicts Φ ~ few degrees → $\Gamma_H < \Gamma_L$

B^0 Mixing

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$$|B_{\mathrm{L,H}}\rangle = p|B_q^0\rangle \pm q|\overline{B}_q^0\rangle \qquad |q|^2 + |p|^2 = 1$$ normalization condition

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

- Eigenvalues

$$m_L - \frac{i}{2}\Gamma_L = M_{11} - \frac{i}{2}\Gamma_{11} + \frac{p}{q}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

$$m_H - \frac{i}{2}\Gamma_H = M_{11} - \frac{i}{2}\Gamma_{11} - \frac{p}{q}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)$$

Neglecting $O(m^2_b/m^2_W)$:

$$\Delta m = m_H - m_L \simeq 2|M_{12}| > 0$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|\cos\Phi$$

$$\Phi = arg\left(-M_{12}/\Gamma_{12}\right)$$

- In the limit of |q/p|~1 [Lenz, Nierste arXiv 1102.4274 (2011), Beringer et al., Phys. Rev. D 86, 010001 (2012)]:

$$\Delta\Gamma_q/\Delta m_q = |\Gamma_{12}/M_{12}| \sim 5\times 10^{-3}$$ independent of CKM elements (same for B^0_d and B^0_s)

  - $\Delta\Gamma_d/\Gamma_d$=0.42±0.08%, $\Delta\Gamma_s/\Gamma_s$=15±2% width difference caused by the existence of final states to which both B^0_q and $\overline{B}^0_q$ mesons can decay ($b \to c\bar{c}q$ are Cabibbo suppressed (allowed) for q=d(s))

B^0 Mixing

- Time evolved state for an initially pure state at t=0:

$$|B_q^0(t)\rangle = g_+(t)\,|B_q^0\rangle + \frac{q}{p}\,g_-(t)\,|\overline{B}_q^0\rangle$$

$$|\overline{B}_q^0(t)\rangle = g_+(t)\,|\overline{B}_q^0\rangle + \frac{p}{q}\,g_-(t)\,|B_q^0\rangle$$

where

$$g_+(t) = e^{-iMt}e^{-\Gamma t/2}\cos(\Delta m_B\, t/2)$$

$$g_-(t) = e^{-iMt}e^{-\Gamma t/2}\, i\sin(\Delta m_B\, t/2)$$

$$\Gamma = (\Gamma_H + \Gamma_L)/2, \quad M = (M_H + M_L)/2$$

- Time-dependent probability that the flavor states remain unchanged (+) or oscillate (-)

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}\,t\right) \pm \cos(\Delta m_q\, t)\right]$$

B^0 Mixing

- Time integrated mixing probability χ_q

Meson	M/MeV	Δm/MeV	Γ/MeV	$\Delta\Gamma$/MeV
K^0	497.6	3.48×10^{-12}	3.68×10^{-12}	7.34×10^{-12}
D^0	1864.9	9.45×10^{-12}	1.6×10^{-9}	2.57×10^{-11}
B_d	5279.6	3.34×10^{-10}	4.43×10^{-10}	~ 0
B_s	5366.8	1.16×10^{-8}	4.39×10^{-10}	6.58×10^{-11}

$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2+1)} \qquad x_q = \frac{\Delta m_q}{\Gamma_q}, \qquad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}$$

$$\begin{cases} x_d = 0.774 \pm 0.008 & (B_d^0\text{–}\overline{B}_d^0 \text{ system}) \\ x_s = 26.2 \ \ \pm 0.5 & (B_s^0\text{–}\overline{B}_s^0 \text{ system}) \end{cases}$$

$$\chi_d = 0.182 \pm 0.015$$
$$\chi_s = 0.49930 \pm 0.00001$$

$$\bar{\chi} = f_d \chi_d + f_s \chi_s$$

- $\bar{\chi}$ gives informations on B_q production fractions

B^0 Mixing

- Time-dependent evolution depends on mixing parameters

Meson	M/MeV	Δm/MeV	Γ/MeV	$\Delta\Gamma$/MeV
K^0	497.6	3.48×10^{-12}	3.68×10^{-12}	7.34×10^{-12}
D^0	1864.9	9.45×10^{-12}	1.6×10^{-9}	2.57×10^{-11}
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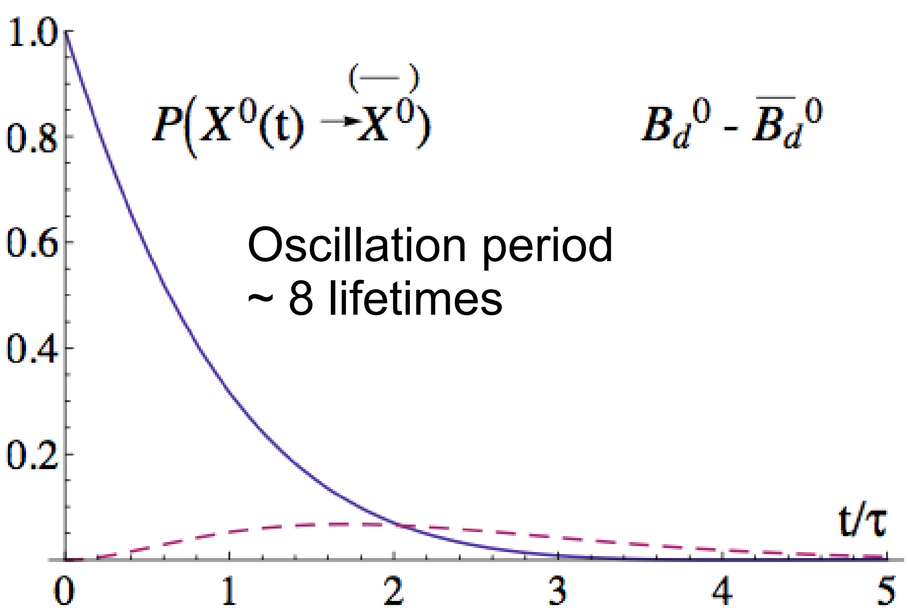
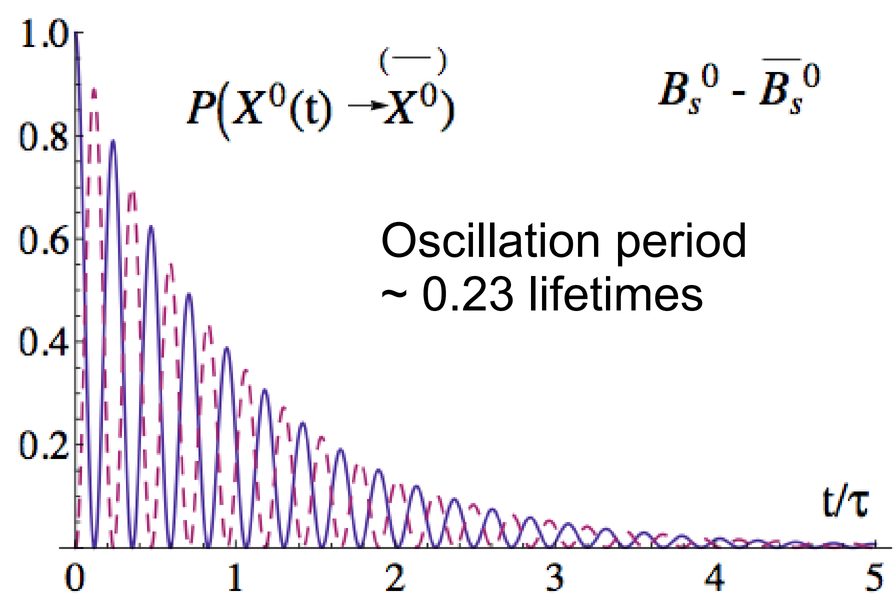
B_s	5366.8	1.16×10^{-8}	4.39×10^{-10}	6.58×10^{-11}

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}\,t\right) \pm \cos(\Delta m_q\, t)\right]$$

Unmixed (+)
Mixed (-)





CPV in Mixing

$$\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| \sim 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\Phi_M - \Phi_\Gamma) \sim 1 - \Im\left(\frac{\Gamma_{12}}{M_{12}}\right); \quad [\Phi_M - \Phi_\Gamma \sim \pi + O(\frac{m_c^2}{m_b^2})]$$

where Φ_M, Φ_Γ are the phases of M_{12} and Γ_{12}

[Lenz, Nierste arXiv 1102.4274 (2011), Beringer et al., Phys. Rev. D 86, 010001 (2012)]

- If CP is conserved (is a symmetry of the Hamiltonian), the mass eigenstates are CP eigenstates. In that case the relative phase between M_{12} and Γ_{12} vanishes:
  - $|q/p| \neq 1$ implies CPV in mixing (so called indirect CPV)

$$|B_{\mathrm{L,H}}\rangle = p|B_q^0\rangle \pm q|\overline{B}_q^0\rangle \qquad |q|^2 + |p|^2 = 1 \qquad [\Phi_q = arg(-M_{12}^q/\Gamma_{12}^q)]$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

$$CP\ |B_{L,H}\rangle = \pm|B_{L,H}\rangle \rightarrow \left|\frac{q}{p}\right| = 1 \rightarrow \Phi_q = 0$$

CPV in Mixing

$$\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| \sim 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\Phi_M - \Phi_\Gamma) \sim 1 - \Im\left(\frac{\Gamma_{12}}{M_{12}}\right); \quad [\Phi_M - \Phi_\Gamma \sim \pi + O(\frac{m_c^2}{m_b^2})]$$

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- If CP is conserved (is a symmetry of the Hamiltonian), the mass eigenstates are CP eigenstates. In that case the relative phase between M_{12} and Γ_{12} vanishes:
  - |q/p| ≠ 1 implies CPV in mixing (so called indirect CPV)

- Effect can be observer through the CP Asymmetry:

$$A_{CP}^q = \frac{Prob(\bar{B}_q^0 \to B_q^0, t) - Prob(B_q^0 \to \bar{B}_q^0, t)}{Prob(\bar{B}_q^0 \to B_q^0, t) + Prob(B_q^0 \to \bar{B}_q^0, t)} = \frac{1 - |q/p|_q^4}{1 + |q/p|_q^4} = \frac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin\phi_q \quad [\Phi_q = arg(-M_{12}^q / \Gamma_{12}^q)]$$

- Independent on t
- Predicted to be very small in the SM [Nierste, arXiv:1212.5805 (2012)]

$$A_{CP}^d = (-4.0 \pm 0.6) \times 10^{-4}; \quad \Phi_d = -4.9^o \pm 1.4^o$$

$$A_{CP}^s = (1.8 \pm 0.3) \times 10^{-5}; \quad \Phi_s = 0.24^o \pm 0.06^o$$

CPV in Mixing

$$\left|\frac{q}{p}\right|^2=\left|\frac{M_{12}^*-\frac{i}{2}\Gamma_{12}^*}{M_{12}-\frac{i}{2}\Gamma_{12}}\right|\sim 1+\left|\frac{\Gamma_{12}}{M_{12}}\right|\sin(\Phi_M-\Phi_\Gamma)\sim 1-\Im\left(\frac{\Gamma_{12}}{M_{12}}\right);\quad [\Phi_M-\Phi_\Gamma\sim\pi+O(\frac{m_c^2}{m_b^2})]$$

where Φ_M, Φ_Γ are the phases of M_{12} and Γ_{12}

[Lenz, Nierste arXiv 1102.4274 (2011), Beringer et al., Phys. Rev. D 86, 010001 (2012)]

- If CP is conserved (is a symmetry of the Hamiltonian), the mass eigenstates are CP eigenstates. In that case the relative phase between M_{12} and Γ_{12} vanishes:
  - $|q/p| \neq 1$ implies CPV in mixing (so called indirect CPV)

- Beyond SM [Lenz, Nierste, JHEP 0706, 072 (2007)]

- New Physics could modify M_{12} by introducing an additional contribution:

$$M_{12}^{NP,q}=M_{12}^{SM,q}\Delta_q;\quad \Delta_q=|\Delta_q|e^{i\phi_q^\Delta}$$

$$A_{SL}^{NP}=\frac{|\Gamma_{12}^q|}{|M_{12}^{SM,q}|}\frac{\sin(\phi_q^{SM}+\phi_q^\Delta)}{|\Delta_q|}$$

New Physics if $\Delta_q=|\Delta_q|e^{i\phi_q^\Delta}\neq 1$

Measurement of Δm_d @ B-Factories

- Assuming $\Delta\Gamma_d$=0 the time-dependent probabilities to have (-) or not (+) flavor oscillation are:

$$h_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [1 \pm \cos(\Delta m_d \Delta t)]$$

- Neutral B^0_d mesons are produced via ϒ(4S) decays. The wave function for the final state is in an anti-symmetric coherent P-wave (L=1) state:

$$\Psi = \frac{1}{\sqrt{2}} \left(|B^0\rangle |\overline{B}^0\rangle - |\overline{B}^0\rangle |B^0\rangle \right)$$

  - The B_d mesons remains in this coherent state with exactly one B^0 and one $\overline{B}^0$ until one of them decays. Then the wave function collapses and the second B meson continues to propagate and oscillate until it also decays.

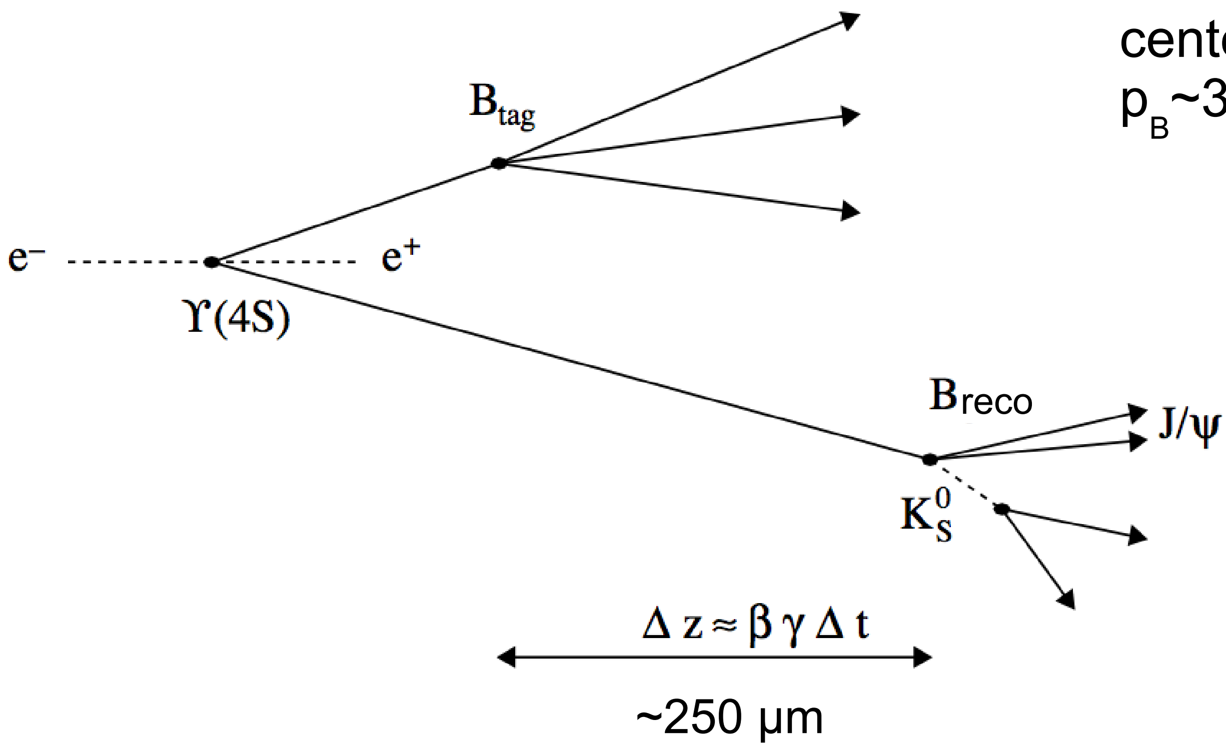
  - If one of the B mesons decays into a flavor-specific eigenstate (B_{tag}), it unambigously determines the flavor of the other (B_{rec}) at this time.

Measurement of Δm_d @ B-Factories

- Asymmetric B-Factories: center-of-mass is boosted forward in the direction of the electron (high energy) beam

- Time evolution described in terms of the time difference between the two B meson decays in the center-of-mass frame, obtained from the distance between the two decay verteces along the beam axis

B mesons almost at rest in the center-of-mass frame
p_B~300 MeV



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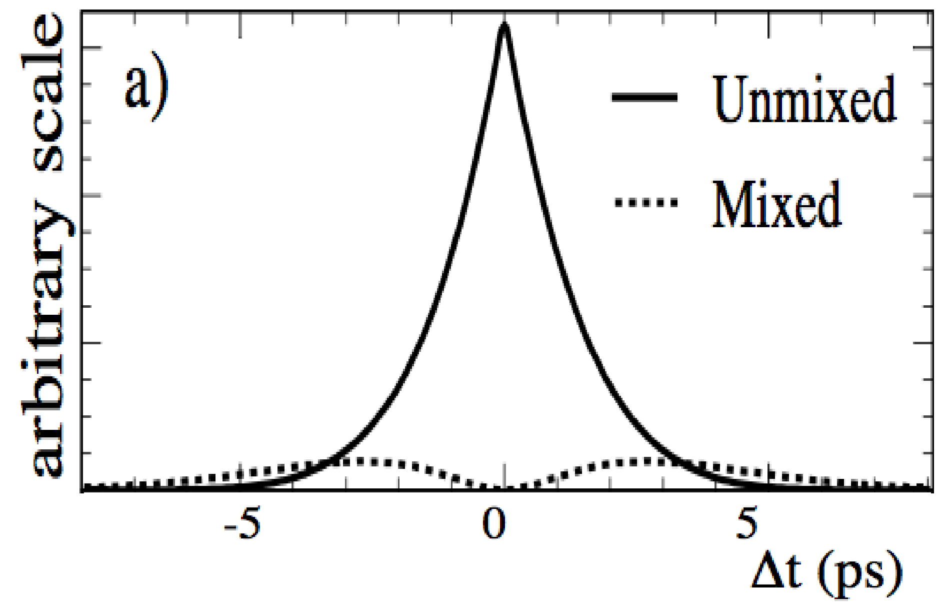
Measurement of Δm_d @ B-Factories

- Δm obtained from a simultaneous fit to the time-dependent asymmetry of unmixed (+) and mixed (-) events in decay channels specific to B^0 or $\bar{B}^0$ mesons (e.g. $B^0 \rightarrow D^{*-} l^+ \nu$)

$$h_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [1 \pm \cos(\Delta m_d \Delta t)]$$

$$A(\Delta t) = \frac{h_+(\Delta t) - h_-(\Delta t)}{h_+(\Delta t) + h_-(\Delta t)} = \cos(\Delta m \Delta t)$$



- Unmixed events: $B^0\bar{B}^0$
- Mixed events: B^0B^0 and $\bar{B}^0\bar{B}^0$
- Measured asymmetry affected by experimental effects:
  - Flavor misidentification
  - Time resolution
- Analysis ingredients: Flavor Tagging of the B_{tag} meson and measurement of Δt

Measurement of Δm_d @ B-Factories

Flavor Tagging

- Purpose: classify the B_{tag} either as a B^0 or a $\overline{B^0}$ at the time of its decay in order to fix the flavor of the other meson (B_{rec}) at the same time
- Signal meson usually fully reconstructed. B meson pairs are produced with no underlying event and pile-up is negligible: tracks not belonging to the B_{rec} are assumed to come from the B_{tag} decay
- A large fraction of B mesons decay to a final state that is flavor specific. Usually inclusive techniques are employed (e.g. Lepton, Kaon, slow Pion charge)
- Two stages algorithms:
  - Analysis of flavor-specific signatures
  - Results combined in a final flavor tag using multivariate methods

- Figure of merit: effective tagging efficiency (tagging power) $Q = \varepsilon_{tag}(1-2w)^2$
  - ε_{tag}: fraction of events with tagging assignment
  - ω:mistag probability
  - $D=1-2\omega$: dilution (factor by which measured CP and mixing asymmetries are reduced from their physical values)

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Measurement of Δm_d @ B-Factories

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- Tagging efficiency and mistag could be different for the two different flavors due to detector performance not charge symmetric (expecially for K tags):

$$\varepsilon_{\rm tag} = \frac{\varepsilon_{B^0} + \varepsilon_{\overline{B}^0}}{2} \qquad \Delta\varepsilon_{\rm tag} = \varepsilon_{B^0} - \varepsilon_{\overline{B}^0}$$

$$w = \frac{w_{B^0} + w_{\overline{B}^0}}{2} \qquad \Delta w = w_{B^0} - w_{\overline{B}^0}$$

Measurement of Δm_d @ B-Factories

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- Two stages algorithms:
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  - Results combined in a final flavor tag using multivariate methods

- Due to the mistag, the measured time dependence of events is now:

$$h_{\pm}^{\text{Phys}}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}[1 \mp \Delta w \pm \langle D\rangle \cos(\Delta m_d \Delta t)]$$

+ : Unmixed
- : Mixed

Measurement of Δm_d @ B-Factories

Flavor Tagging

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- Two stages algorithms:
  - Analysis of flavor-specific signatures
  - Results combined in a final flavor tag using multivariate methods

- Due to the mistag, the measured time dependence of events is now:

$$A(\Delta t) = \frac{h_+(\Delta t) - h_-(\Delta t)}{h_+(\Delta t) + h_-(\Delta t)} = -\Delta\omega + D\cos(\Delta m \Delta t)$$

Unmixed (+), Mixed (-), Asymmetry amplitude reduced by the dilution term D

Measurement of Δm_d @ B-Factories

Tagging Power

- Mixing and CP results come from asymmetries measurements
- Assume N^0 and $\overline{N}^0$ true number of events with a given flavor, the number of reconstructed B decays are

$$N = \varepsilon_{\text{tag}}(1-w)N_0 + \varepsilon_{\text{tag}} w \overline{N}_0$$
$$\overline{N} = \varepsilon_{\text{tag}}(1-w)\overline{N}_0 + \varepsilon_{\text{tag}} w N_0$$

- Measured Asymmetry

$$A^{\text{rec}} = \frac{N-\overline{N}}{N+\overline{N}} = (1-2w)A^0 = DA^0$$

True physical Asymmetry:

$$A^0 = (N_0 - \overline{N}_0)/(N_0 + \overline{N}_0)$$

- Statistical uncertainty:

$$\sigma_{A^0} = \frac{\sigma_{A^{\text{rec}}}}{1-2w}$$

$$\sigma_{A^{\text{rec}}} \propto \frac{1}{\sqrt{N_{\text{tag}}}} \qquad N_{\text{tag}} = N + \overline{N}$$

$$\sigma_{A^0} \propto \frac{1}{\sqrt{\varepsilon_{\text{tag}}}(1-2w)} = \frac{1}{\sqrt{Q}}$$

Measurement of Δm_d @ B-Factories

Sources of Flavor Information

- Leptons: e, μ produced in semileptonic direct B decays $b \rightarrow cl\nu$ tag the flavor of the B meson; cascade decays $b \rightarrow c \rightarrow sl\nu$ provide opposite information, but have softer momentum. Several kinematical variable used (q, p*, θ, p_{miss}, θ(l-p_{miss})...)
- Kaons: produced from $b \rightarrow c \rightarrow s$ transitions have charge correlated to the B flavor (q, K-PID informations, nKs, p*, angles)
- Slow pions: produced from D* decays affected by large background. Pions selected exploiting the small phase space available in the D* decay: D^0 and π emitted almost at rest in the D* rest frame, have direction opposite to the rest of B_{tag} products in the B_{tag} rest frame (q, p*, p, θ, PID informations,...)

BaBar Tagging performance

Category	$\varepsilon_{tag}(\%)$	$\Delta\varepsilon_{tag}(\%)$	$w(\%)$	$\Delta w(\%)$	$Q(\%)$	$\Delta Q(\%)$
Lepton	9.7 ± 0.1	0.2 ± 0.2	2.1 ± 0.2	0.2 ± 0.5	8.9 ± 0.1	0.1 ± 0.4
Kaon I	11.3 ± 0.1	-0.1 ± 0.2	4.1 ± 0.3	0.2 ± 0.6	9.6 ± 0.1	-0.1 ± 0.4
Kaon II	15.9 ± 0.1	-0.1 ± 0.2	13.0 ± 0.3	-0.2 ± 0.6	8.7 ± 0.2	0.0 ± 0.5
Kaon-Pion	13.2 ± 0.1	0.4 ± 0.2	23.0 ± 0.4	-1.3 ± 0.7	3.9 ± 0.1	0.5 ± 0.3
Pion	16.8 ± 0.1	-0.3 ± 0.3	33.3 ± 0.4	-2.7 ± 0.6	1.9 ± 0.1	0.6 ± 0.2
Other	10.6 ± 0.1	-0.5 ± 0.2	41.8 ± 0.5	5.9 ± 0.7	0.28 ± 0.03	-0.4 ± 0.1
Total	77.5 ± 0.1	-0.3 ± 0.5			33.1 ± 0.3	0.7 ± 0.8

Measurement of Δm_d @ B-Factories

Resolution of Δt

- Difference of the proper times of decay of the two B mesons in the center-of-mass frame: $\Delta t = \Delta z / \beta\gamma$
- Experimental error are taken into account by convolving the $R(\delta t, \sigma_{\Delta t})$ resolution function defined in terms of the event-by-event error $\sigma_{\Delta t}$ and $\delta t = \Delta t - \Delta t_{true}$ with the function describing the event rate, $f_{\pm}^{\rm Phys}(\Delta t)$

$$F_{\pm}^{\rm Phys}(\Delta t) = \int_{-\infty}^{\infty} f_{\pm}^{\rm Phys}(\Delta t_{\rm true}) R(\delta t, \sigma_{\Delta t}) {\rm d}\Delta t_{\rm true},$$
$$= f_{\pm}^{\rm Phys}(\Delta t) \otimes R(\delta t, \sigma_{\Delta t}).$$

- Typical Resolution Function:

$$\mathcal{R}_{\rm sig}(\delta t, \sigma_{\Delta t}) = f_{\rm core} G_{\rm core}\left(\delta t, \mu_{\rm core}\sigma_{\Delta t}, s_{\rm core}\sigma_{\Delta t}\right) + f_{\rm tail} G_{\rm tail}\left(\delta t, \mu_{\rm tail}\sigma_{\Delta t}, s_{\rm tail}\sigma_{\Delta t}\right) + f_{\rm outlier} G_{\rm outlier}\left(\delta t, \mu_{\rm outlier}, s_{\rm outlier}\right).$$

$s_{core} = 1.01 \pm 0.04$ (lepton tag)
$s_{core} = 1.20 \pm 0.02$ (non-lepton tag)
$s_{tail} \sim 3$
$S_{outlier} \sim 8$ ps
μ_i, s_i usually determined by the fit

Measurement of Δm_d @ B-Factories

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- Experimental error are taken into account by convolving the $R(\delta t, \sigma_{\Delta t})$ resolution function defined in terms of the event-by-event error $\sigma_{\Delta t}$ and $\delta t = \Delta t - \Delta t_{true}$ with the function describing the event rate, $f_{\pm}^{\rm Phys}(\Delta t)$

$$F_{\pm}^{\rm Phys}(\Delta t) = \int_{-\infty}^{\infty} f_{\pm}^{\rm Phys}(\Delta t_{\rm true}) R(\delta t, \sigma_{\Delta t}) {\rm d}\Delta t_{\rm true},$$

$$= f_{\pm}^{\rm Phys}(\Delta t) \otimes R(\delta t, \sigma_{\Delta t}).$$

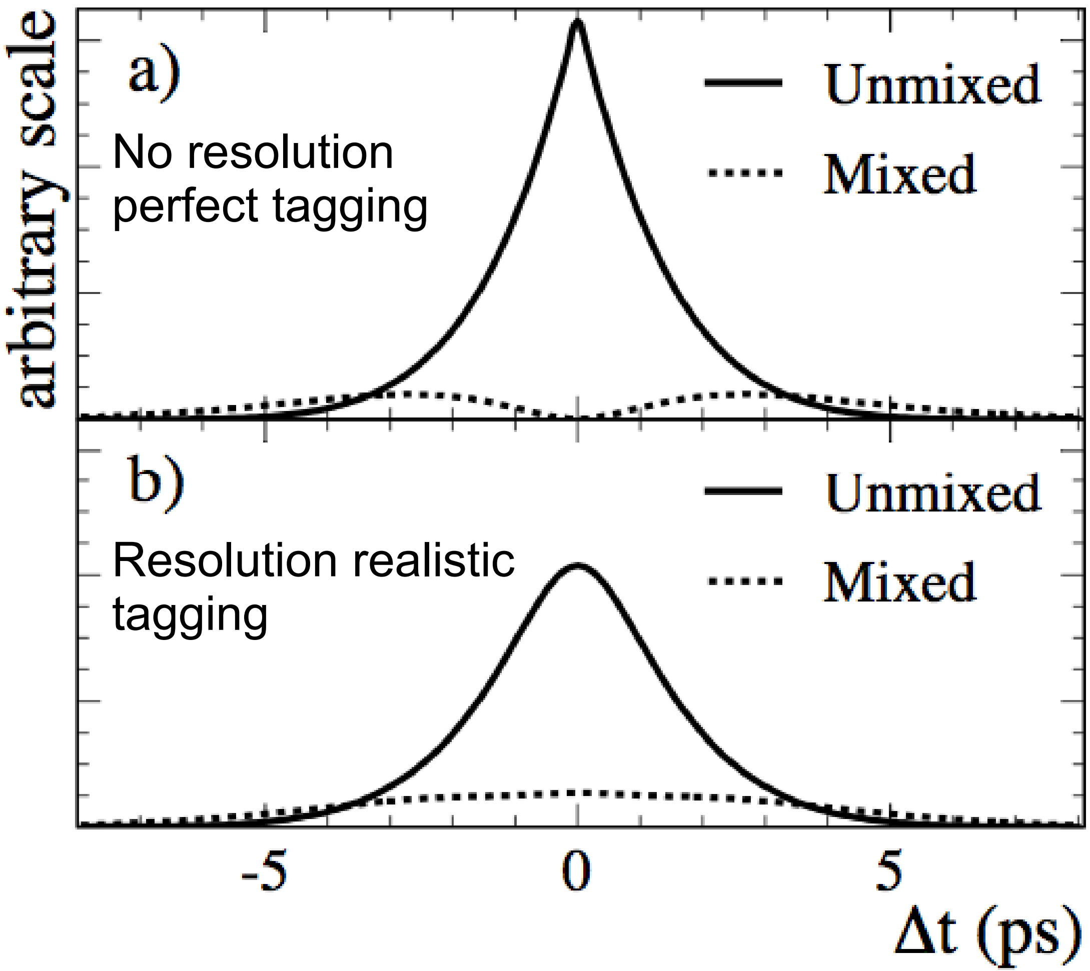
- Measured Asymmetry taking into account experimental effects

$$A(\Delta t) = \frac{F_{+}(\Delta t) - F_{-}(\Delta t)}{F_{+}(\Delta t) + F_{-}(\Delta t)} = F(D, \Delta m_d, \sigma \Delta t)$$

Unmixed (+), Mixed (-)

Measurement of Δm_d @ B-Factories

Resolution of Δt



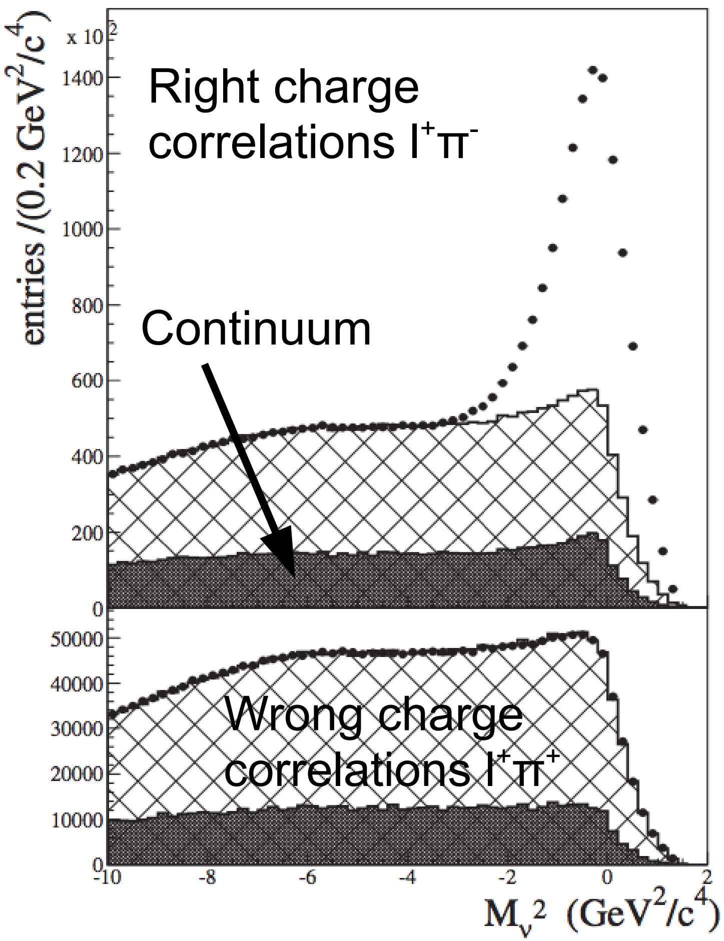
- Factors contributing to the resolution:
  - B_{tag} vertex: tracking, finite lifetime of D mesons
  - B_{rec} vertex: tracking
  - Error on the $\beta\gamma$ boost determined from the beams energy
- Typical $\sigma_{\Delta z}$~ 50-100 μm depending on event reconstruction and dominated by the tag vertex

- Fit performed to extract mistag rate ω, lifetime and Δm

Measurement of Δm_d @ B-Factories

Babar Meas. using Partial $B^0 \rightarrow D^* l \nu$ Reco (L=81 fb^{-1})

[Phys. Rev. D73 012004 (2006)]



$$\mathcal{M}_\nu^2 = \left(\frac{\sqrt{s}}{2} - \tilde{E}_{D^{*+}} - E_{\ell^-}\right)^2 - (\tilde{\mathbf{p}}_{D^{*+}} + \mathbf{p}_{\ell^-})^2$$

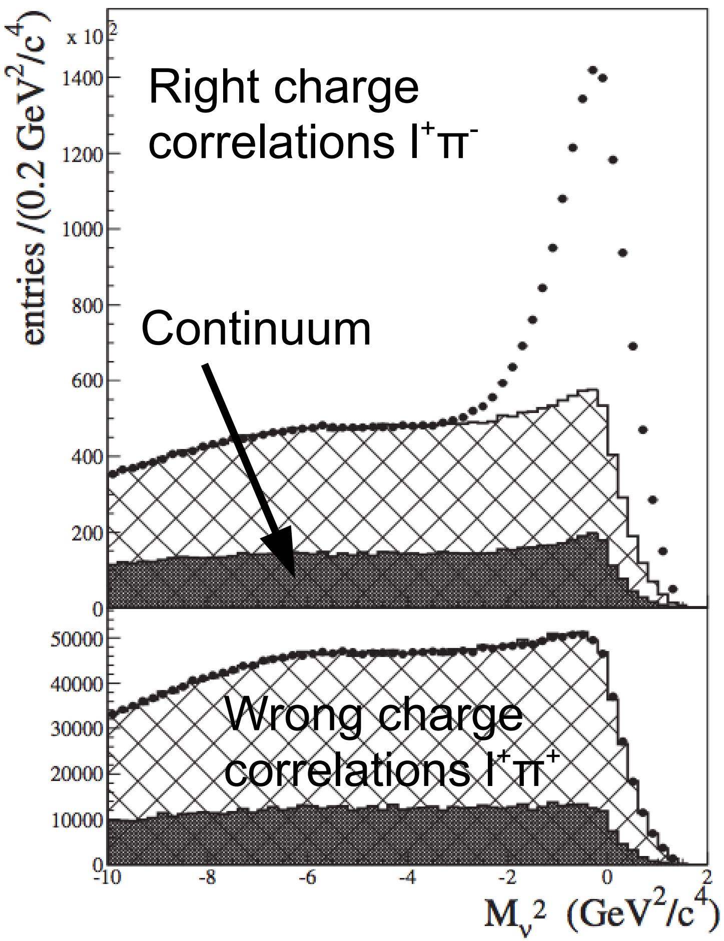
- Only the charged lepton from the B decay and the slow pion from the D* are identified. Due to the limited phase space available (m_{D^*}- m_{D0} ~ 150 MeV), the slow pion is emitted within a one-radian wide cone centered about the D* direction in the Υ(4S) rest frame.
- D* 4-momentum parameterized as a function of the pion momentum
- B^0 assumed at rest
- Combinatorial background studied using wrong charge (same sign) lepton-slow pion correlations

Measurement of Δm_d @ B-Factories

Babar Meas. using Partial $B^0 \rightarrow D^* l \nu$ Reco (L=81 fb^{-1})

[Phys. Rev. D73 012004 (2006)]

$$\mathcal{M}_\nu^2 = \left(\frac{\sqrt{s}}{2} - \tilde{E}_{D^{*+}} - E_{\ell^-}\right)^2 - (\tilde{\mathbf{p}}_{D^{*+}} + \mathbf{p}_{\ell^-})^2$$

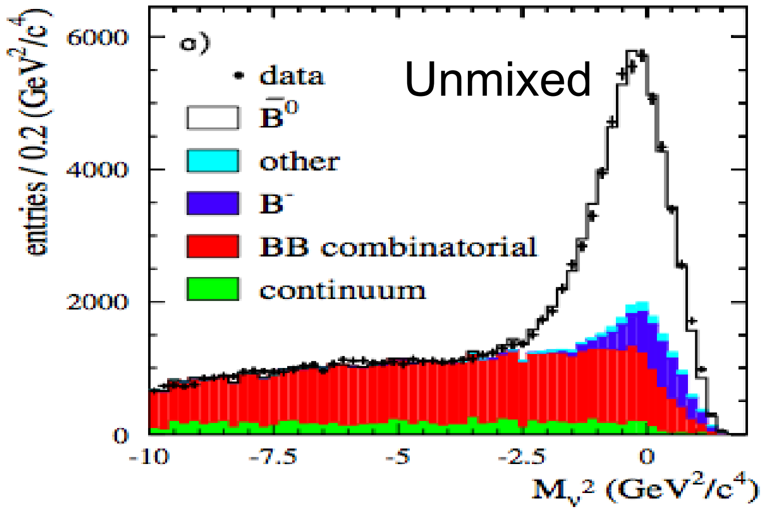


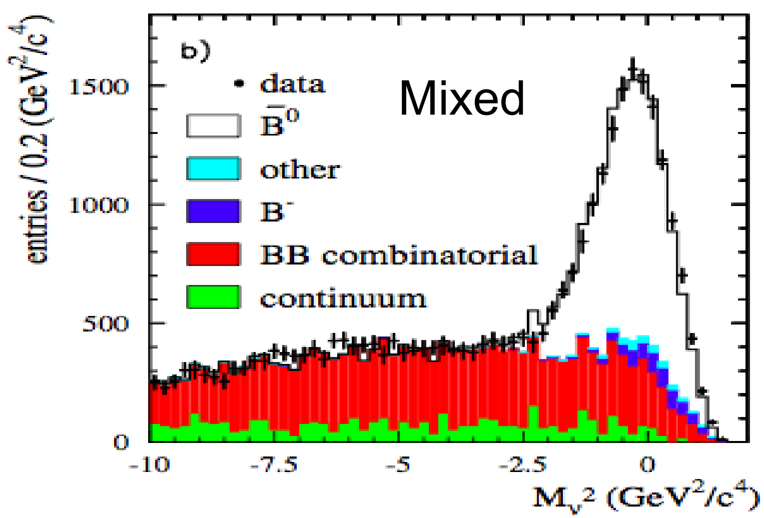
- B_{rec} vertex determined from the lepton and slow pion tracks constrained to the Beam Spot position
- B_{tag} vertex from lepton and BS
- Flavor of B_{rec} from lepton and pion charges, flavor of B_{tag} from lepton charge
- BKG reduced by cut on combined Likelihood ratio using p_l, p_π and vertex probability
- 49K (28K) signal (BKG) events reconstructed

Measurement of Δm_d @ B-Factories

Babar Meas. using Partial $B^0 \to D^* l \nu$ Reco (L=81 fb^{-1})

[Phys. Rev. D73 012004 (2006)]



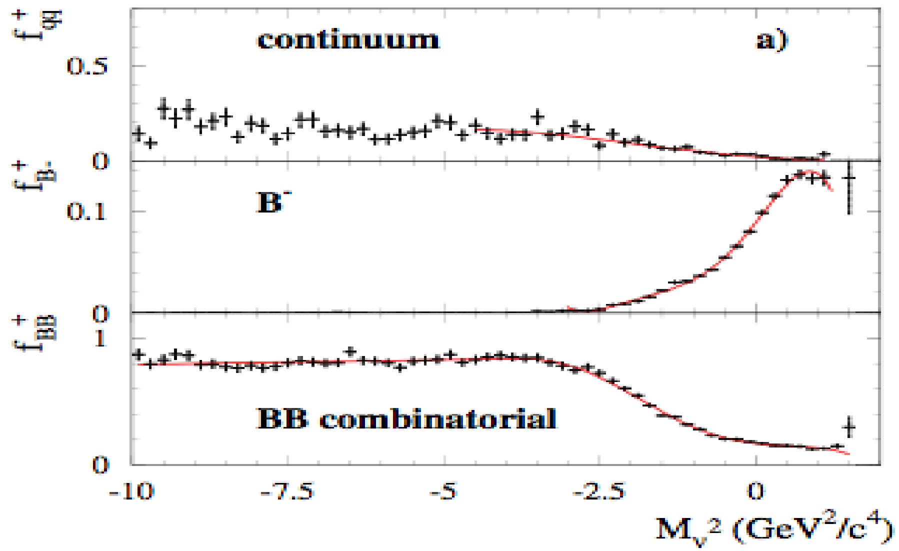


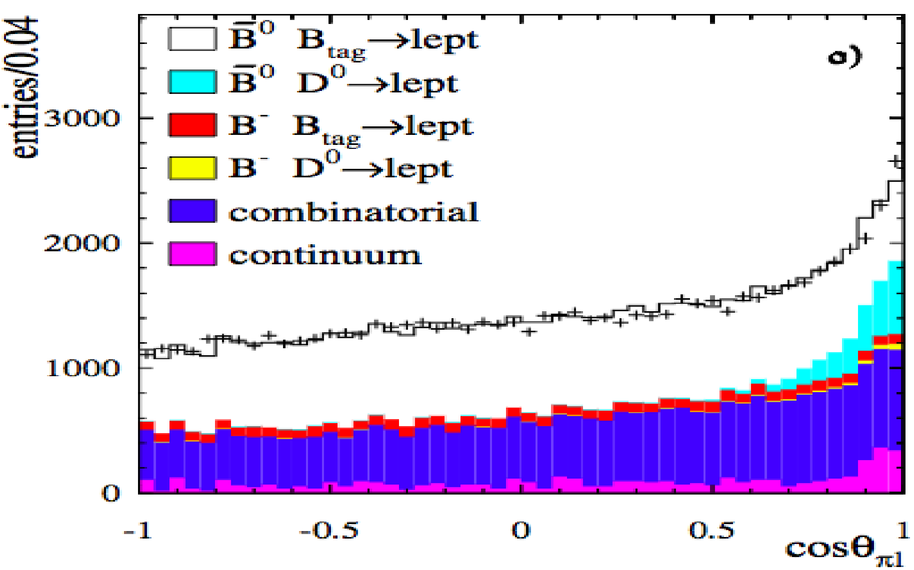
- Signal is any combination of a lepton and a charged D* produced in a single B^0 decay
- BKG from continuum, BB combinatorial, B^- peaking BKG from $B^- \to D^{*+} \pi^- l^- \nu$, $D^{*+} \pi^- X$ with a $\pi \to l$ misidentification
- Leptons divided in: primary, cascade tag-side, decay-side lepton from the unreconstructed D^0 from the B_{rec} decay not carrying any useful information

Measurement of Δm_d @ B-Factories

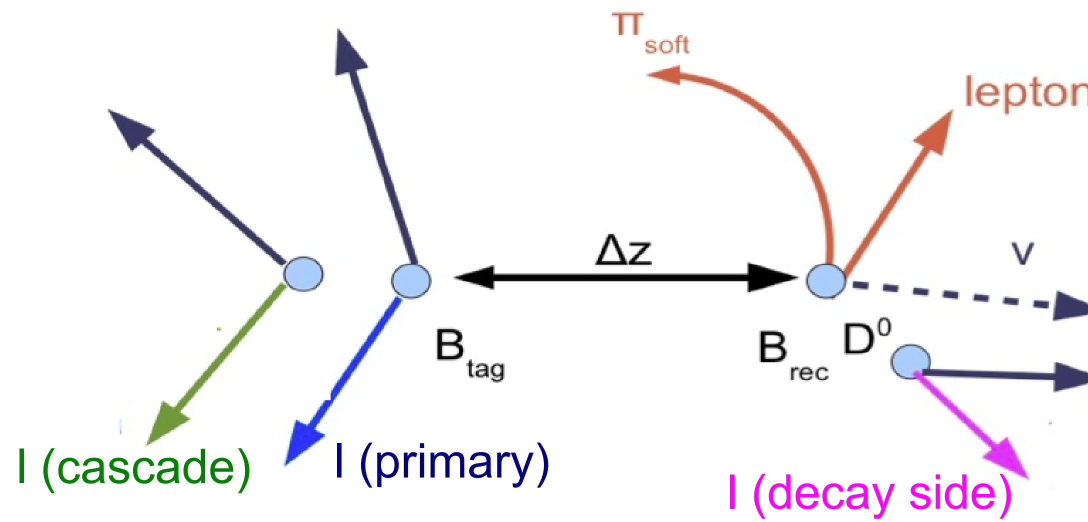
Babar Meas. using Partial $B^0 \rightarrow D^* l \nu$ Reco ($L=81\ fb^{-1}$)

[Phys. Rev. D73 012004 (2006)]





- Fraction of different sample components from a fit to the M^2_ν distribution



- Leptons from B_{tag} and D^0 decay side separated exploiting the angle between the lepton and the slow pion in the center-of-mass frame

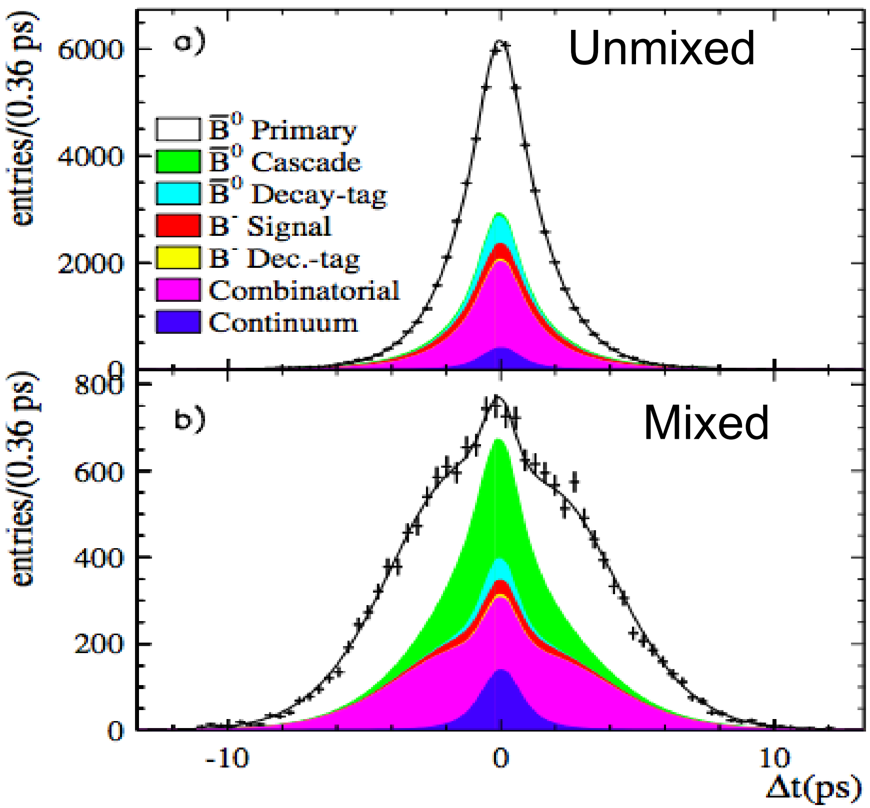
Measurement of Δm_d @ B-Factories

Babar Meas. using Partial $B^0 \rightarrow D^* l \nu$ Reco (L=81 fb^{-1})

[Phys. Rev. D73 012004 (2006)]

- Simultaneous maximum-likelihood fit performed to mixed (same sign leptons) and unmixed (opposite sign leptons)
- Mistag and time resolution determined in the fit

$$\mathcal{F}^{\pm}(\Delta t, \sigma_{\Delta t}, \mathcal{M}_\nu^2|\tau_{B^0}, \Delta m_d) = f^{\pm}_{q\bar{q}}(\mathcal{M}_\nu^2)\cdot\mathcal{F}^{\pm}_{q\bar{q}}(\Delta t, \sigma_{\Delta t}) + f^{\pm}_{B\bar{B}}(\mathcal{M}_\nu^2)\cdot\mathcal{F}^{\pm}_{B\bar{B}}(\Delta t, \sigma_{\Delta t}) + S_{B^-} f^{\pm}_{B^-}(\mathcal{M}_\nu^2)\cdot\mathcal{F}^{\pm}_{B^-}(\Delta t, \sigma_{\Delta t})$$
$$+ [1 - S_{B^-} f^{\pm}_{B^-}(\mathcal{M}_\nu^2) - f^{\pm}_{B\bar{B}}(\mathcal{M}_\nu^2) - f^{\pm}_{q\bar{q}}(\mathcal{M}_\nu^2)]\cdot\mathcal{F}^{\pm}_{\bar{B}^0}(\Delta t, \sigma_{\Delta t}|\tau_{B^0}, \Delta m_d),$$



- Total PDF includes contributions from continuum, BB BKG, charged B BKG and signal
- Δt resolution described by a sum of three Gaussians
- Relative normalization between mixed and unmixed signal events constrained based on the time-integrated mixing rate:

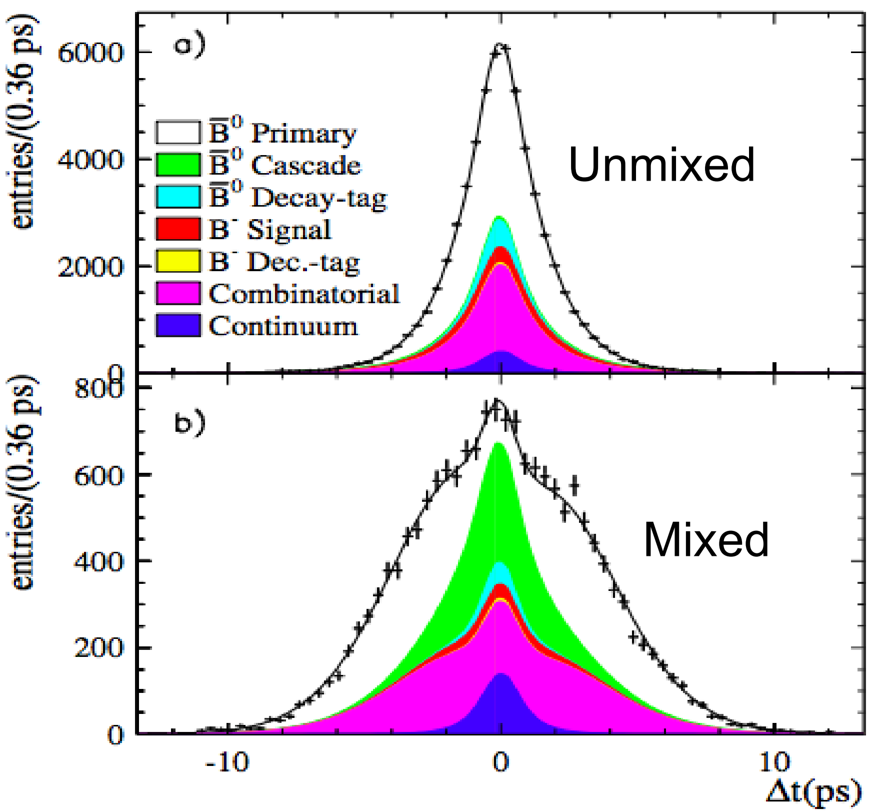
$$\chi_d = \frac{x^2}{2(1+x^2)} \qquad x = \Delta m_d \cdot \bar{\tau}_{B^0}$$

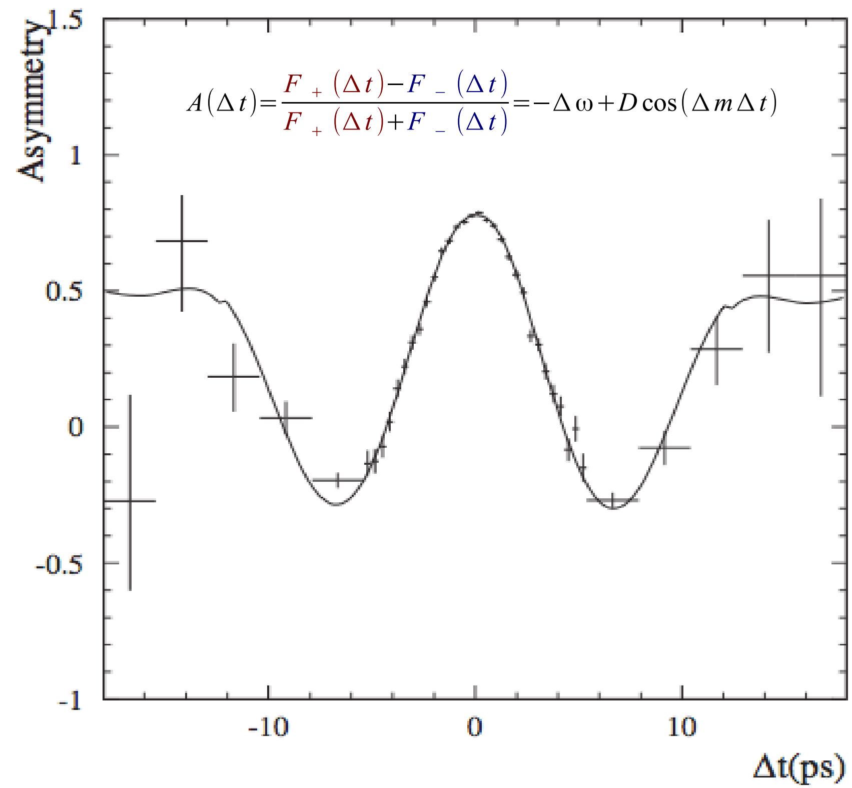
Measurement of Δm_d @ B-Factories

Babar Meas. using Partial $B^0 \rightarrow D^* l \nu$ Reco (L=81 fb^{-1})

[Phys. Rev. D73 012004 (2006)]

$$\tau_{B^0} = \left(1.504 \pm 0.013^{+0.018}_{-0.013}\right) ps\,; \quad \Delta m_d = \left(0.511 \pm 0.007^{+0.007}_{-0.006}\right) ps^{-1}$$





Measurement of Δm_d @ B-Factories

Babar Meas. using Partial $B^0 \rightarrow D^* l \nu$ Reco ($L=81\ fb^{-1}$)

[Phys. Rev. D73 012004 (2006)]

$$\tau_{B^0} = (1.504 \pm 0.013^{+0.018}_{-0.013})\ ps\ ; \quad \Delta m_d = (0.511 \pm 0.007^{+0.007}_{-0.006})\ ps^{-1}$$

- Systematics from vertex detector alignment, z-scale of the detector, fit range and analysis bias (~1 σ) from the MC statistical error

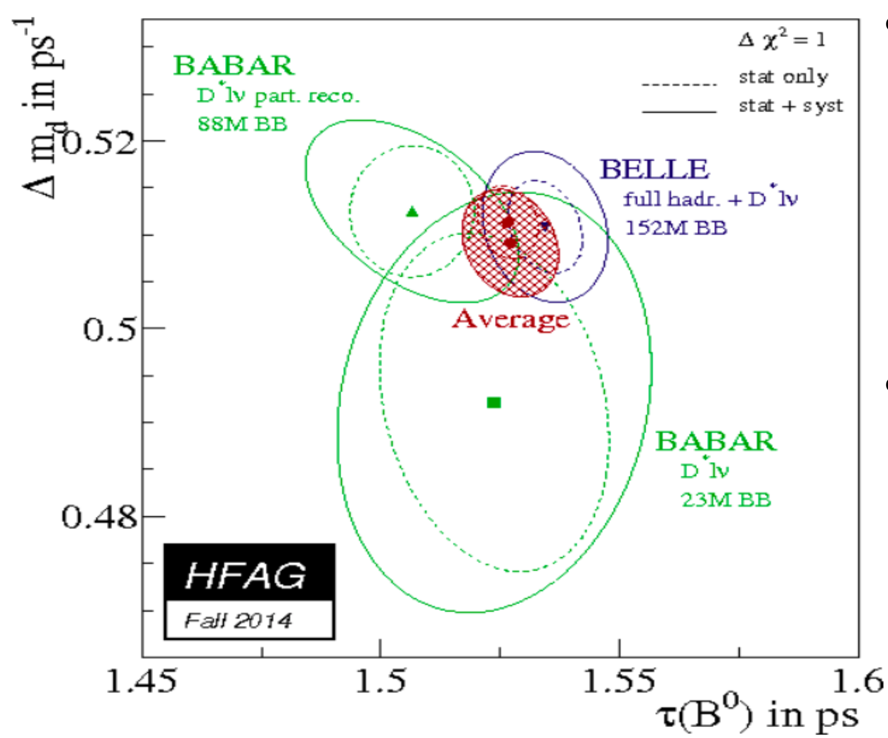
Source	Variation	$\delta\tau_{B^0}$ (ps)	$\delta\Delta m_d$ (ps^{-1})
(a) Sample Composition	$\pm 1.3\%$	± 0.0003	∓ 0.0002
(b) Analysis bias		± 0.0070	∓ 0.0035
(c) τ_{B^-}	1.671 ± 0.018	∓ 0.0014	∓ 0.0008
(d) $\mathcal{D}_{C\ell}$	0.65 ± 0.08	∓ 0.0003	∓ 0.0003
(e) Combinatorial BKG		± 0.0007	∓ 0.0002
(f) z scale		± 0.0070	∓ 0.0020
(g) PEP-II boost		± 0.0020	∓ 0.0003
(h) Beam-spot position		± 0.0050	∓ 0.0010
(i) Alignment		$^{+0.0132}_{-0.0038}$	$^{-0.0038}_{+0.0033}$
(j) Decay-side tags		± 0.0013	
(k) Binning		∓ 0.0021	± 0.0006
(l) Outlier parameters		± 0.0028	± 0.0012
(m) Δt and $\sigma_{\Delta t}$ cut		± 0.0033	∓ 0.0033
(n) GExp model		-0.0016	$+0.0011$
Total		$^{+0.0182}_{-0.0131}$	$^{+0.0068}_{-0.0064}$

MC statistical error

From comparison of beampipe dimention measured using scattered protons and nominal one

From different sets of alignment parameters depending on detector conditions

Measurement of Δm_d



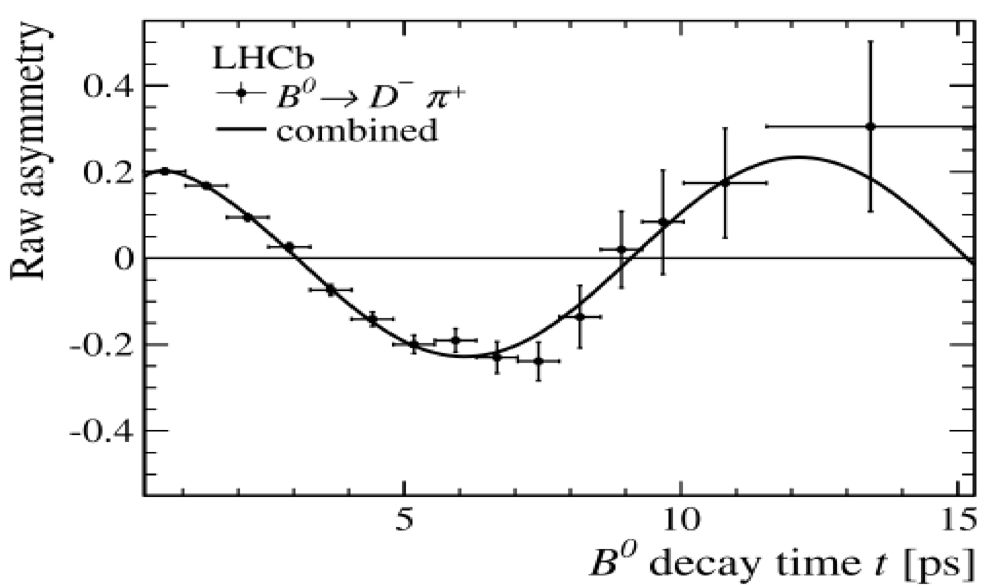
- From a similar analysis using fully reconstructed hadronic and SL B^0 decays Belle found [Phys. Rev. D 71 072003 (2005)]

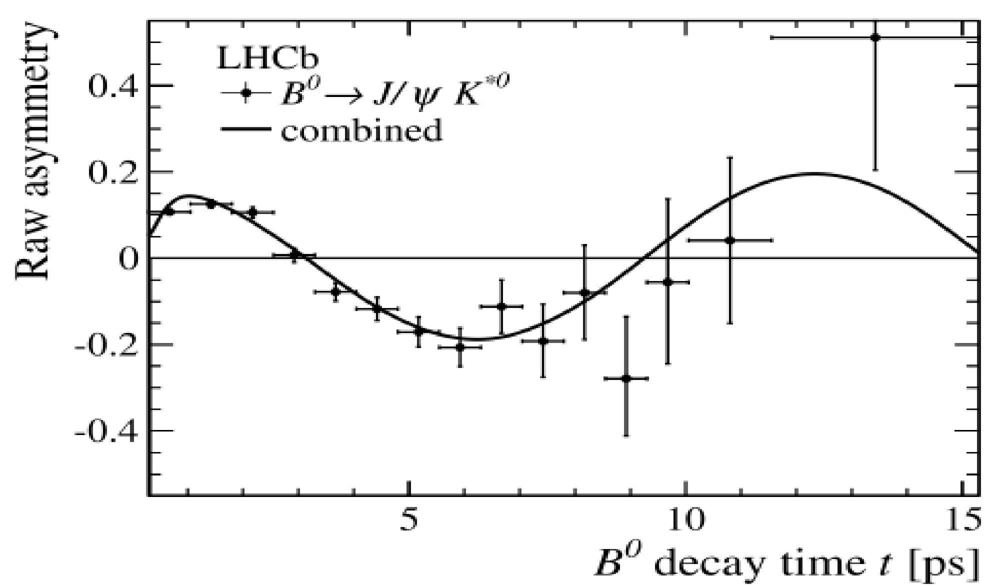
$$\tau_{B^0} = (1.534 \pm 0.008 \pm 0.010)\, ps$$

$$\Delta m_d = (0.511 \pm 0.005 \pm 0.006)\, ps^{-1}$$

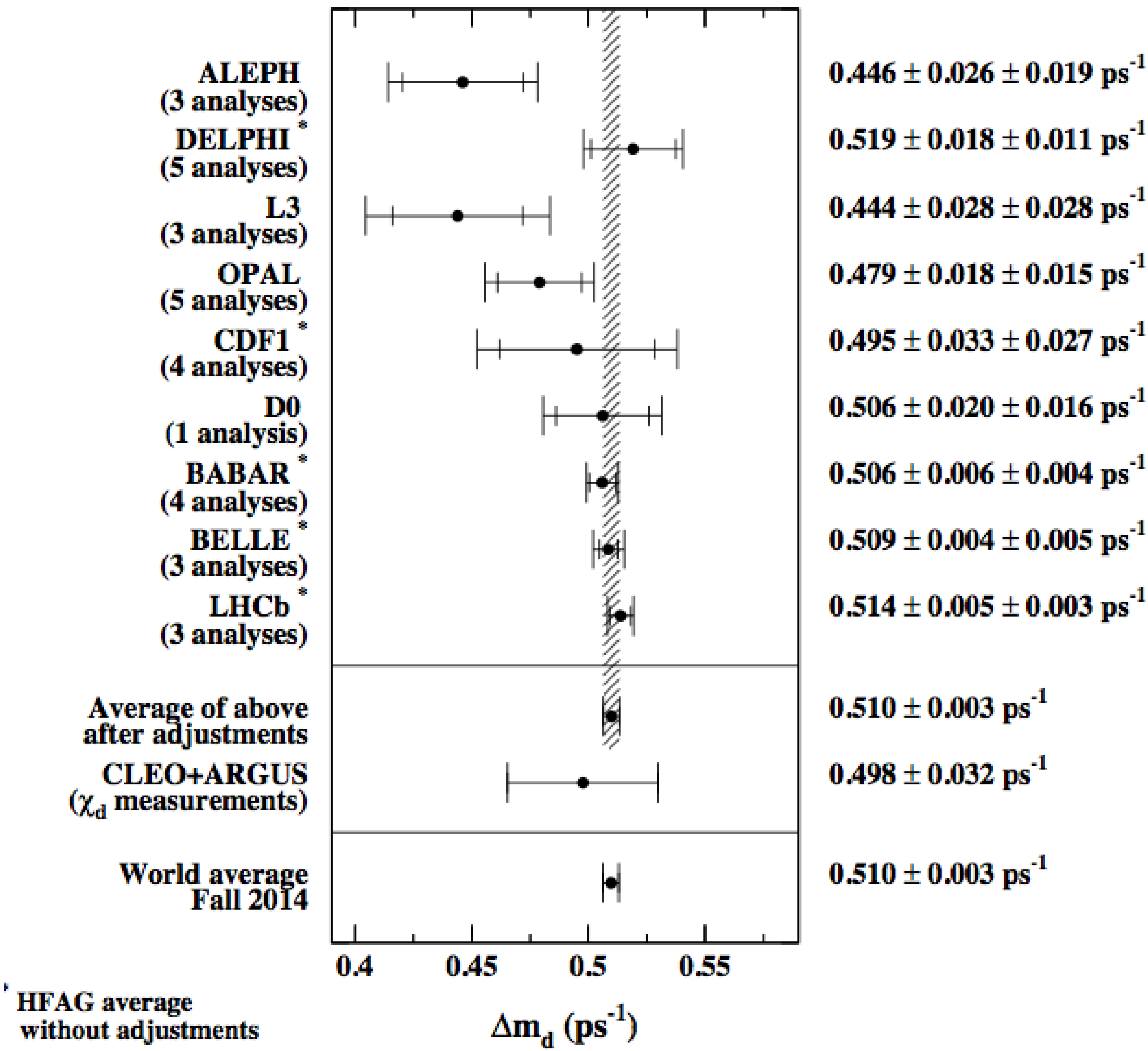
- LHCb from $B^0 \rightarrow D^- \pi^+$, $J/\Psi K^{*0}$ (Most precise) [Phys. Lett. B 719 318-325 (2013)]:

$$\Delta m_d = (0.5156 \pm 0.0051 \pm 0.0033)\, ps^{-1}$$





Measurement of Δm_d



- World Average

$$\Delta m_d = (0.510 \pm 0.003)\, ps^{-1}$$