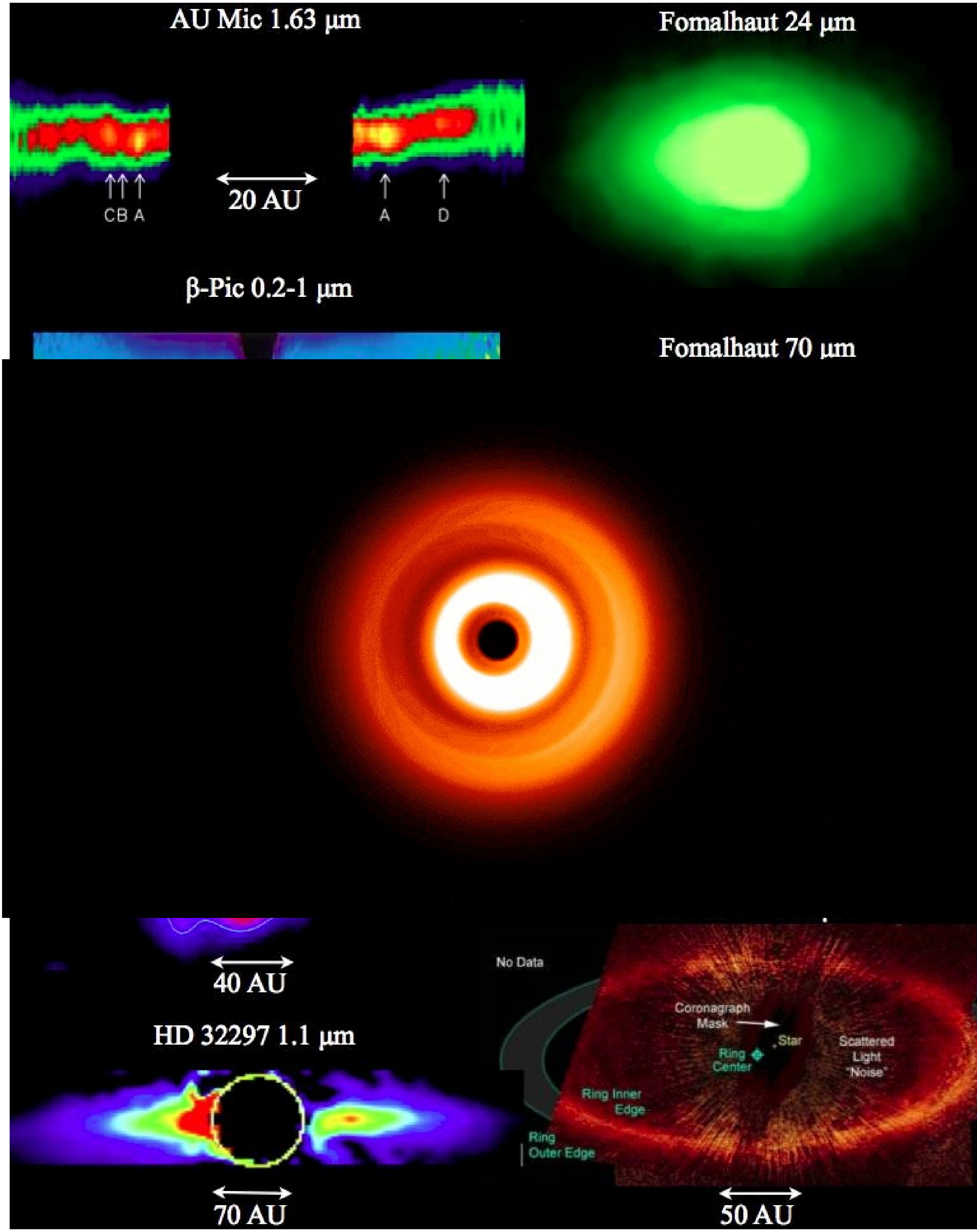
CHAPTER 3: non-gravitational forces

- **3-1 Poynting-Robertson effect:** acting on dust particles populating debris disks.
- **3-2 Yarkovsky effect:** acting on meteoroids and asteroids.
- **3-3 Gas drag:** acting on planetesimals in gaseous protoplanetary disks.

3-1 Poynting-Robertson drag and radiation pressure.

- Emission laws
- Absorption and ri-emission of light by dust grains
- Relativistic approach to the Poynting-Robertson effect (drag)
- How drag affects heliocentric orbits.

(Burns, Lamy, & Soter, Icarus 40, 1-48, 1979)



AU Mic 1.63 μm
C B A
20 AU
A
D
Fomalhaut 24 μm
β-Pic 0.2-1 μm
Fomalhaut 70 μm
40 AU
HD 32297 1.1 μm
70 AU
No Data
Coronagraph Mask
Star
Ring Center
Scattered Light "Noise"
Ring Inner Edge
Ring Outer Edge
50 AU

Emission and absorption laws

An idealized black body emits radiation at different λ depending on the tempearture T according to Planck's formula:

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}$$

h is Planck's constant (6.62606896(33)×10^{-34} J s), c is the speed of light (299792458 m/s) and k is Boltzmann's constant (1.380 6504×10^{-23} J K^{-1}).

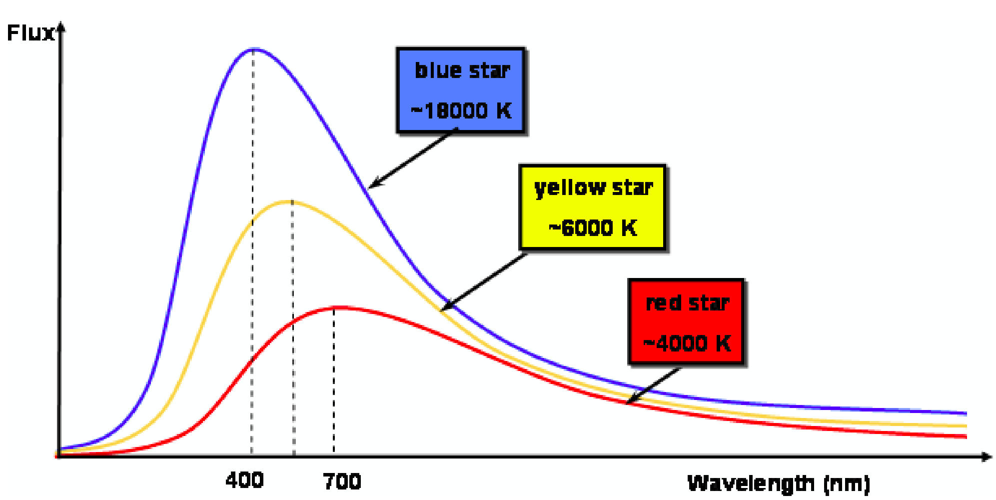
- Stefan-Boltzman's law: integrate Planck's formula over λ and you get the total energy E (σ is the Stefan Boltzmann constant i.e. 5.67 10^{-8} W m^{-2} K^{-4} :

$$E = \sigma T^4$$

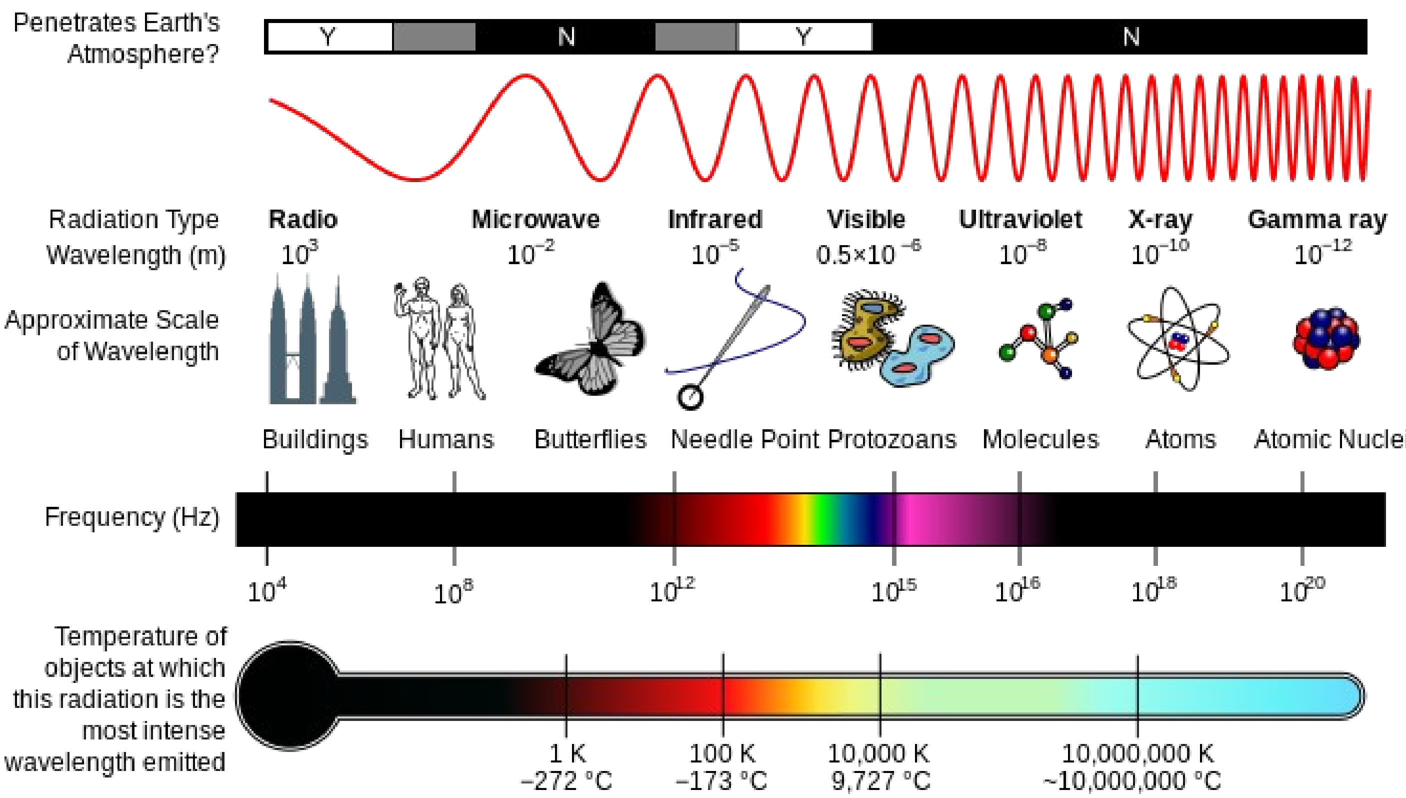
- Wien's law: derive Planck's formula over λ and you get the thewavelenght of maximum energy emission:

$$\lambda_{MAX} = \frac{b}{T}$$

with b= 2.8977685(51)×10^{-3} m K



Spectrum of EM radiation:



A star is a partial blackbody. The surface is a black body but in the atmosphere there are absorptions and emissions at different temperatures.

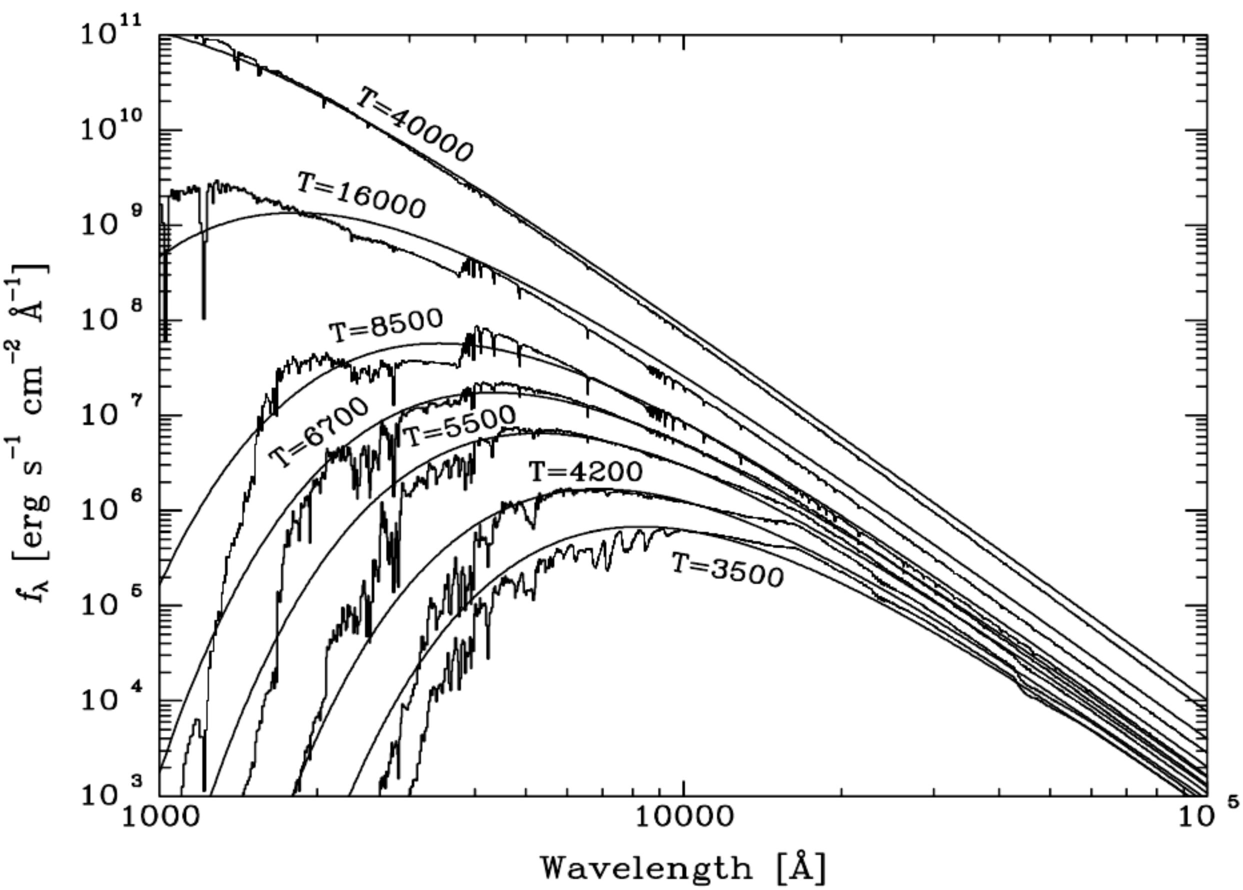
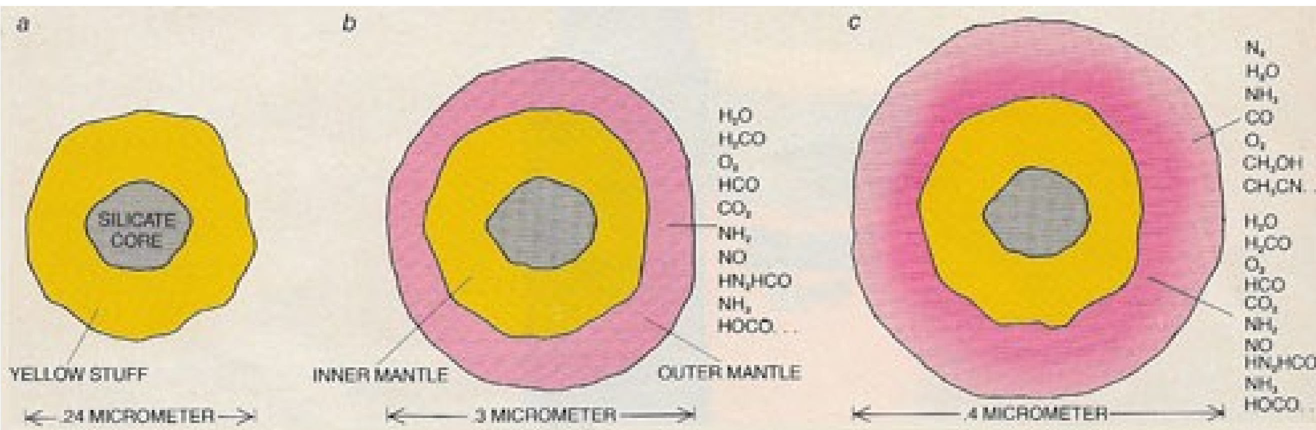


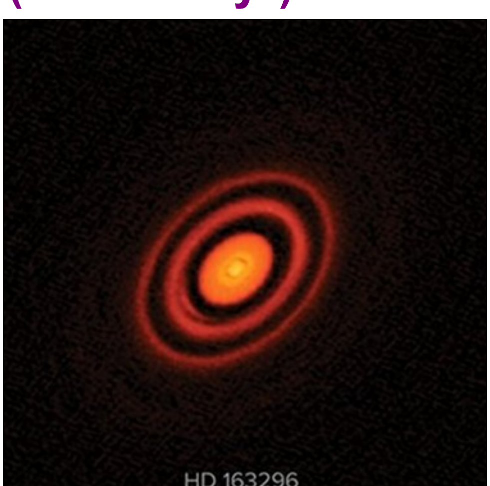
Figure 2.1 Flux per wavelength interval emitted by different types of stars, at their "surfaces", compared to blackbody curves of various temperatures. Each blackbody's temperature is chosen to match the total power (integrated over all wavelengths) under the the corresponding stellar spectrum. The wavelength range shown is from the ultraviolet (1000 Å= 0.1 μm), through the optical range (3200-10,000 Å), and to the mid-infrared (10^5 Å= 10 μm). Data credit: R. Kurucz.

Cosmic dust: produced by the combination of C,O,Si,H,S of late stars. NH_3, CO_2 and H_2S add above them.

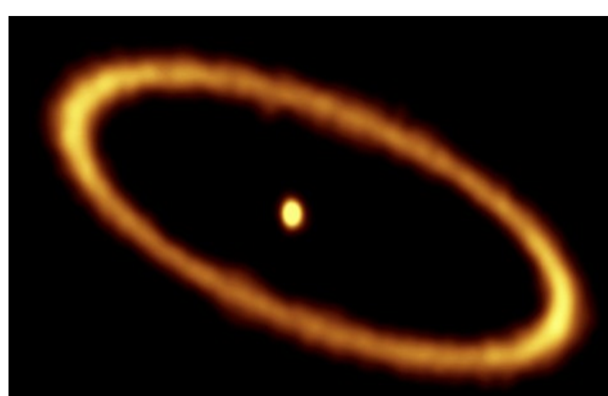


Yellow stuff: organic material processed by radiation like, hydrogenated amorphous carbon, polycyclic aromatic hydrocarbon.

Circumstellar disk (first 10 Myr)



Debris disk (leftover planetesimals)

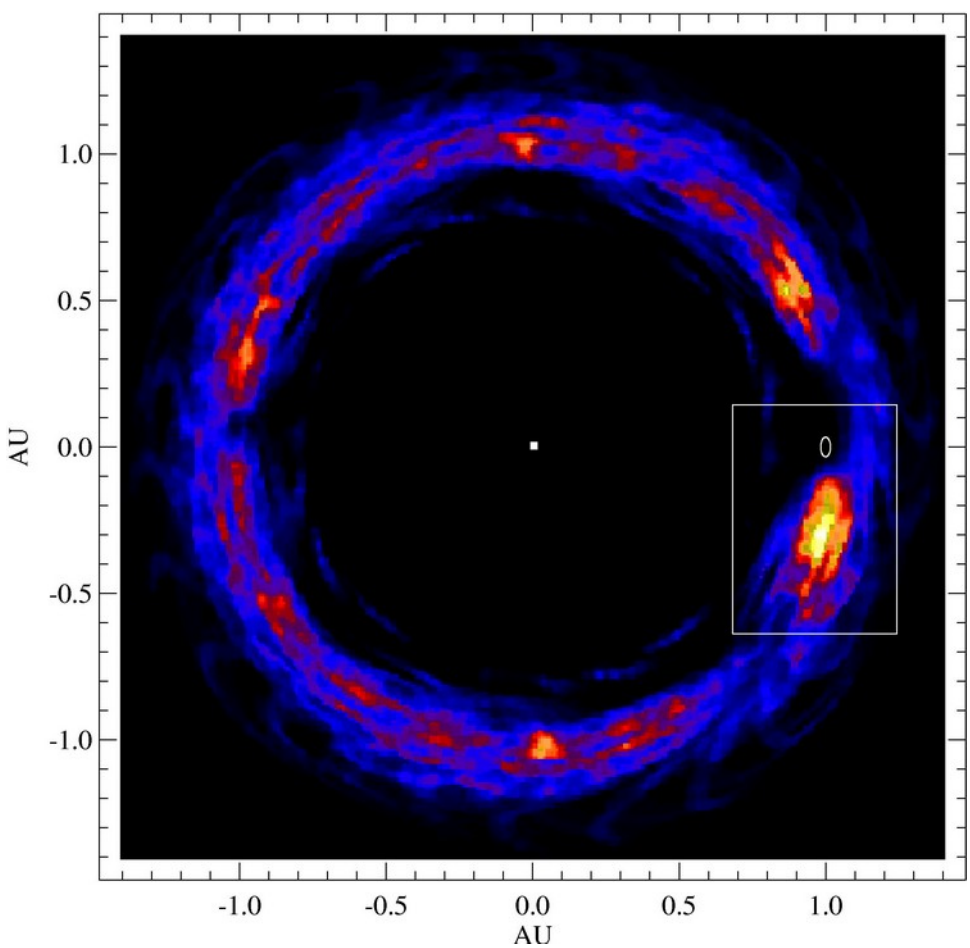


Dust particles

- A protostellar disk surrounding a protostar is made of gas and dust (mainly silicates and CHON). Dust accumulates forming planetesimals and planets.
- At present in the solar system (and in exoplanetary systems) dust is produced by collisions in the asteroid belt and Kuiper belt. The zodiacal dust is produced in this way and populates debris disks.
- Dust is also produced by comets during the perihelion passage.
- Some grains may come from from outside the solar system from galactic dust.

Zodiacal light. →

IRAS and COBE have observed the dust distribution: it is not homogeneous. Ad example, around the Earth (and also Venus) there is a dense ring of dust of particles trapped in resonance. The average size of these particles is 100 μm (the size estimate is obtained by dust grains impacting the Earth).



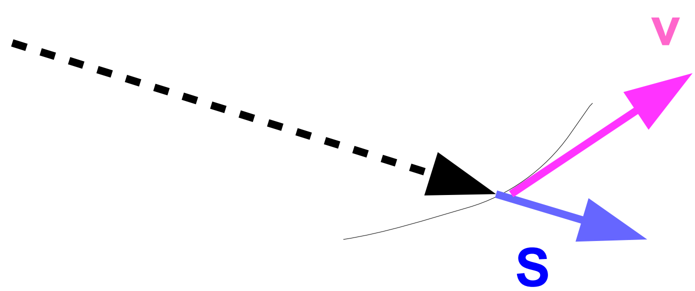
Robertson's formula for perfectly absorbing particles

1) Flux of radiant energy given by Poynting vector:

$$\boldsymbol{S}=\frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0}\simeq 1.35\times10^6\ \frac{erg\cdot s}{cm^2}\qquad at\ \ 1\ \ au$$

2) Relativisyic Doppler's effect for the light due to radial motion

$$S'=S\left(1-\frac{\dot{r}}{c}\right)\qquad where\qquad \dot{r}=\boldsymbol{v}\cdot\boldsymbol{s}$$



Radiation pressure:

$$\boldsymbol{F}_p=\left(\frac{SA}{c}\right)\boldsymbol{s}$$

A is the particle area (cross section)

Poynting-Robertson drag: absorbed radiation (100% in this model) is fully re-emitted in the infrared by the grain to keep constant the energy balance. This is equivalent to the ejection of a 'mass flux ' forward given by:

$$E = mc^2 \quad \Rightarrow \quad m = \frac{S'A}{c^2}$$

The momentum flux, and then the recoil force acting on the particles in the opposite direction, is:

$$\boldsymbol{F}_D = -\left(\frac{S'A}{c^2}\right)\boldsymbol{v}$$

The total force acting on the grain (radiation pressure + Poynting-Robertson drag) is finally:

$$\boldsymbol{F} = \boldsymbol{F}_P + \boldsymbol{F}_D = \left(\frac{S'A}{c}\right)\left(\boldsymbol{S} - \frac{\boldsymbol{v}}{c}\right) = \frac{SA}{c}\left(1 - \frac{\dot{r}}{c}\right)\left(\boldsymbol{S} - \frac{\boldsymbol{v}}{c}\right) \simeq \frac{SA}{c}\left[\left(1 - \frac{\dot{r}}{c}\right)\boldsymbol{S} - \frac{\boldsymbol{v}}{c} + \ldots\right]$$

$$since \quad \frac{\dot{r}\,\boldsymbol{v}}{c^2} \quad is \;\; small$$

If a particle is not perfectly absorbing but it reflects part of the incident radiation the equation is different.

f = fraction of absorbed light
g = fraction of reflected light
f + g = 1
Q_{PR} = 1 +g

$$\boldsymbol{F} = \frac{S' A}{c} Q_{PR} \boldsymbol{S} - \frac{S' A}{c} Q_{PR} \frac{\boldsymbol{v}}{c}$$

When all light is reflected (g=1, f=0, Q_{PR} = 2), ad example for a clean icy particle, the force doubles. This because a reflected photon releases to the dust grain twice the momentum respect to an absorbed photon.

… some special relativity…..

- **Coordinates** $(ct, \boldsymbol{x}) \quad (x^0, x^1, x^2, x^3)$
- **Interval** $ds^2 = -c^2 dt^2 + d\boldsymbol{x}^2$
- **Proper time** $d\tau^2 = -\frac{ds^2}{c^2} = dt^2 - \frac{d\boldsymbol{x}^2}{c^2}$

4-velocity (tangent to the evolution line)

$$\boldsymbol{U} = \frac{dP}{d\tau}$$

$$U^0 = c\frac{dt}{d\tau} = \frac{dx^0}{d\tau} = \frac{c}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma c$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^c}}}$$

$$U^1 = \frac{dx}{d\tau} = \frac{v_x}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma v_x$$

.....

The modulus of the 4-velocity is constant

$$\boldsymbol{U}\cdot\boldsymbol{U} = \eta_{\alpha\beta} U^\alpha U^\beta = -c^2$$

In the proper reference frame, the 4-velocity vector is;

$$\boldsymbol{U} = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

4-momentum

$$\boldsymbol{p} = m_0 \boldsymbol{U} = \begin{pmatrix} m_0 \gamma c \\ m_0 \gamma v_x \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} \frac{E}{c} \\ \gamma p_x \\ \dots \\ \dots \end{pmatrix}$$

Energy

$$E = m_0 \gamma c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \simeq m_0 c^2 + \frac{1}{2} m_0 v^2 \dots.$$

The modulus of the 4-momentum is:

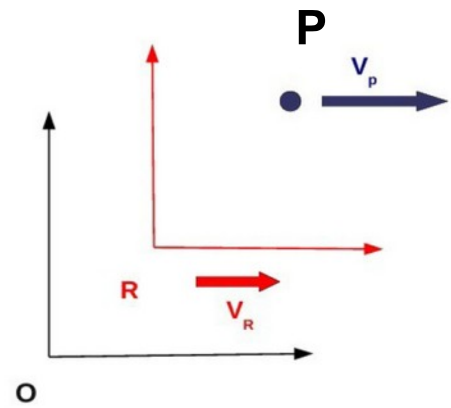
$$\boldsymbol{p} \cdot \boldsymbol{p} = -m_0^2 c^2 = -\frac{E^2}{c^2} + p^2 \qquad (p_x = \gamma p_x)$$

For the photon:

$$p = \frac{E}{c} \quad \Rightarrow \quad \boldsymbol{p} = \begin{pmatrix} \frac{E}{c} \\ \pm \frac{E}{c} \\ \dots \\ \dots \end{pmatrix}$$

Lorentz transformations (..and Tetrads)

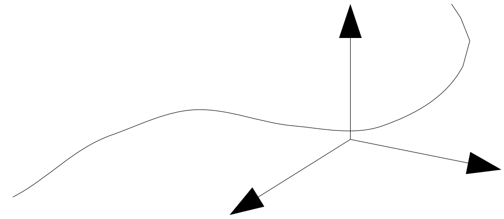
U_O^P is the 4-velocity of P respect to O, U_R^P will be the 4-velocity of P respect to R, v_P is the 3-velocity vector of P measured respect to O, while v_R is the 3-velocity of the reference frame centered in R respect to that centered on O



If we apply the Lorentz transformation Λ to U_O^P we get the components of U_R^P .

$$U_O^P=\begin{pmatrix}\gamma_P c\\ \gamma_P v_P\\ 0\\ 0\end{pmatrix} \quad U_O^R=\begin{pmatrix}\gamma_R c\\ \gamma_R v_R\\ 0\\ 0\end{pmatrix} \quad U_R^P=\Lambda U_O^P=\begin{pmatrix}\gamma_R & -\beta_R\gamma_R & 0 & 0\\ -\beta_R\gamma_R & \gamma_R & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}.\begin{pmatrix}\gamma_P c\\ \gamma_P v_P\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}\gamma_R\gamma_P c-\beta_R\gamma_R\gamma_p v_P\\ -\beta_R\gamma_R\gamma_P c+\gamma_R\gamma_P v_P\\ 0\\ 0\end{pmatrix}$$

An alternative way to compute the components of U_R^P is to use projections over a tetrad. A **tetrad** is a set of axes (usually orthonormal) attached to a point in space-time. We assign to the observer centered in R a tetrad (e_0, e_1, e_2, e_3) so that the components of U_R^P will be computed as projections (scalar product) of U_O^P over $e_0, e_1 \ldots$. **To find the tedrad attached to** R we first note that the 4-velocity U_O^R **is parallel** to e_0 . U_O^R , once computed in the reference frame of R, has component (c,0,0,0) so **it can be related directly to** e_0 trough the identity



$$e_0=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\frac{U_o^R}{c}=\begin{pmatrix}\gamma_R\\ \gamma_R\beta_R\\0\\0\end{pmatrix}$$

From e_0 and the orthogonality relation between versors, we can compute e_1

$e_0 \cdot e_1=0$ and $|e_1|=1$ lead to $e_1=\begin{pmatrix}\gamma_R v_R/c\\ \gamma_R\\0\\0\end{pmatrix}$.

The remaining versors e_2, e_3 can be chosen parallel to those of the observer centered in O. To compute the controvariant component of vector U^P respect to R, we project U^P on the basis vector (e_0, e_1, e_2, e_3) :

$$-U_R^{P,0}=U^P\cdot e_0=\begin{pmatrix}\gamma_P c\\ \gamma_P v_P\\0\\0\end{pmatrix}(-\gamma_R, \gamma_R\beta_r, 0, 0)=-\gamma_R\gamma_P c+\beta_R\gamma_R\gamma_P v_P$$

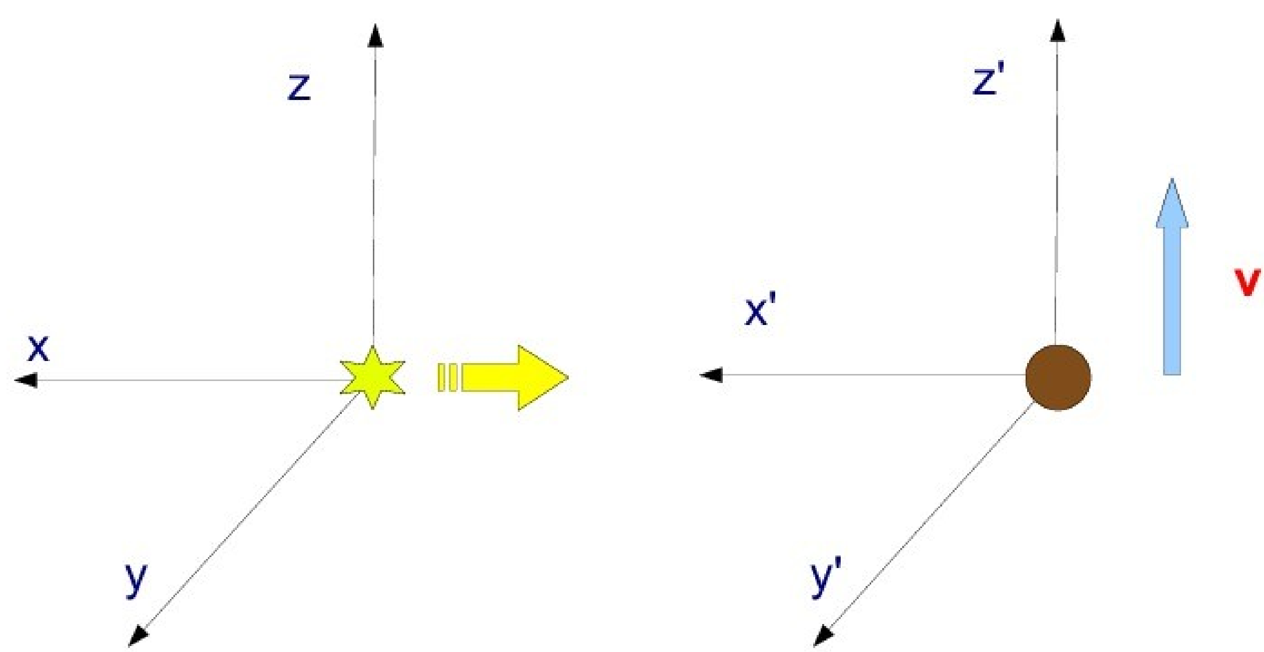
The value of $U_R^{P,0}$ is the same as that obtained with the Lorentz transformation. The minus sign is due to the presence of the metric tensor and the scalar product of any vector **u** with e_0 $u\cdot e_0=u^\alpha e_0^\beta \eta_{\alpha\beta}=-u^0$. The x-component of U_R^P is

$$U_R^{P,1}=U^P\cdot e_1=\begin{pmatrix}\gamma_P c\\ \gamma_P v_P\\0\\0\end{pmatrix}(-\gamma_R v_R/c, \gamma_R, 0, 0)=-\gamma_R\gamma_P c/c\, v_R+\gamma_R\gamma_P v_P=-\beta_R\gamma_R\gamma_P c+\gamma_R\gamma_P v_P$$

This, again, is equal to the value obtained with the Lorentz transformation. With tetrads we can compute the components of a 4-vector in any reference frame attached to an observed if we now his 4-velocity. We compute its tetrad and project the 4-vector of interest on the tetrad.

Relativistic derivation of Robertson's equation for P-R drag

Let's assume that a dust grain is moving respect to the star on a circular orbit (no radial velocity component) with velocity *v*. A flux of photons leaves the star radially and part of the flux will meet the particle after traveling in the antisolar direction -x. At any instant of time we can assume that the reference frames are inertial with the frame attached to the grain moving with constant velocity. As a first approximation we assume that the reference frame centered on the sun is inertial (it is not due to the planetary perturbations, but the baricenter is close to the center of the sun).



The radiation flux S of photons from the sun is described by the 4-momentum:

$$\boldsymbol{p} = \begin{pmatrix} S/c \\ -S/c \\ 0 \\ 0 \end{pmatrix}$$

Transforming to the reference frame of the grain where absorption and reflection occurs requires a Lorentz transformation along z, direction perpendicular to the grain orbit.

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\boldsymbol{p}' = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \cdot \begin{pmatrix} S/c \\ -S/c \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma S/c \\ -S/c \\ 0 \\ -\beta\gamma S/c \end{pmatrix}$$

In the reference frame of the grain **p' A** radiation is absorbed per unit of time and the amount reflected is:

$$\boldsymbol{p}'_R = \begin{pmatrix} \gamma A S/c \\ -AS/c(1-Q_{pr}) \\ 0 \\ -\beta\gamma A S/c(1-Q_{pr}) \end{pmatrix}$$

..negative sign because of reflection. From now on **S A = E**: intercepted energy.

In the reflected 4-momentum the energy term is not multipied by 1- Q_{PR}. This because it includes the contribution from the re-emitted radiation. It is isotropically emitted, so it does not contribute to the momentum since it averages to 0, but it is included in the energy term

$$E_f = f\,E = (1-Q_{pr}+1)E = (2-Q_{pr})E \quad \textit{absorbed} \Rightarrow \textit{isotropically reemitted}$$
$$E_g = g\,E = (Q_{pr}-1)E \quad \textit{reflected}$$
$$E_R = (f+g)E = (2-Q_{pr}+Q_{pr}-1)E = E \quad \Rightarrow \quad E_0$$

To get back to the inertial sun-centered reference frame (from now on E=S/c):

$$\boldsymbol{p}_R = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \cdot \begin{pmatrix} \gamma E/c \\ -E/c(1-Q_{PR}) \\ 0 \\ -\beta\gamma E/c(1-Q_{PR}) \end{pmatrix} =$$

$$\begin{pmatrix} \gamma^2 E/c - \beta^2\gamma^2 E/c(1-Q_{PR}) \\ -E/c(1-Q_{PR}) \\ 0 \\ \beta\gamma^2 E/c - \beta\gamma^2 E/c(1-Q_{PR}) \end{pmatrix} =$$

$$\begin{pmatrix} E/c(\gamma^2-\beta^2\gamma^2+\beta^2\gamma^2 Q_{PR}) \\ -E/c(1-Q_{PR}) \\ 0 \\ \beta\gamma^2 E/c\,Q_{PR} \end{pmatrix} = \begin{pmatrix} E/c(1+\beta^2\gamma^2 Q_{PR}) \\ -E/c(1-Q_{PR}) \\ 0 \\ \beta\gamma^2 E/c\,Q_{PR} \end{pmatrix}$$

The recoil force on the dust grain is then given by:

$$\boldsymbol{F}=-(\boldsymbol{p_R}-\boldsymbol{p})=\begin{pmatrix} E/c \\ -E/c \\ 0 \\ 0 \end{pmatrix}-\begin{pmatrix} E/c\left(1+\beta^2\gamma^2 Q_{PR}\right) \\ -E/c\left(1-Q_{PR}\right) \\ 0 \\ \beta\gamma^2 A\, E/c\, Q_{PR} \end{pmatrix}=$$

$$\begin{pmatrix} -E/c\,\beta^2\gamma^2 Q_{PR} \\ -E/c\,Q_{PR} \\ 0 \\ -\beta\gamma^2 E/c\, Q_{PR} \end{pmatrix}$$

The radiation pressure and P-R drag forces are then:

$$\boldsymbol{F_P}=-E/c\,Q_{PR}\,\boldsymbol{s}$$

$$\boldsymbol{F_D}=-\beta\gamma^2 E/c\,Q_{PR}\,\boldsymbol{u_v}\approx-\beta\, E/c\, Q_{PR}\,\boldsymbol{u_v}=-E/c^2 Q_{PR}\,\boldsymbol{v}$$

Effects of P-R drag on heliocentric orbits: average values of semimajor axis and eccentricity derivatives (average over orbital period).

$$\overline{\frac{da}{dt}} = -\left(\frac{\eta}{a}\right) Q_{PR} \frac{\left(2+3e^2\right)}{\left(1-e^2\right)^{3/2}}$$

$$\overline{\frac{de}{dt}} = -\frac{5}{2}\left(\frac{\eta}{a^2}\right) Q_{PR}\, e \frac{1}{\left(1-e^2\right)^{1/2}}$$

$$\eta = \frac{2.53 \times 10^{11}}{\rho S}$$

Solar wind contributes to radiation pressure and P-R drag

$$\boldsymbol{F}_{rad} = \frac{SA}{c} Q_{PR}\left(1-\frac{\dot{r}}{c}\right)\frac{\left(\boldsymbol{c}-\boldsymbol{v}\right)}{c}$$

$$\boldsymbol{F}_{sw} = \left(n\, m \frac{u^2}{2}\right) A\, C_D \frac{\left(\boldsymbol{w}-\boldsymbol{v}\right)}{w}$$

$$\boldsymbol{F}_{sw} = \sum_j \boldsymbol{F}_{sw}^{j}$$

$\boldsymbol{w}$: *solar wind velocity*

$\boldsymbol{u} = \boldsymbol{w} - \boldsymbol{v}$

n: *number density*

m: *mass of SW−particles*

j: *different SW particles*

Esample of dynamical evolution of dust grains

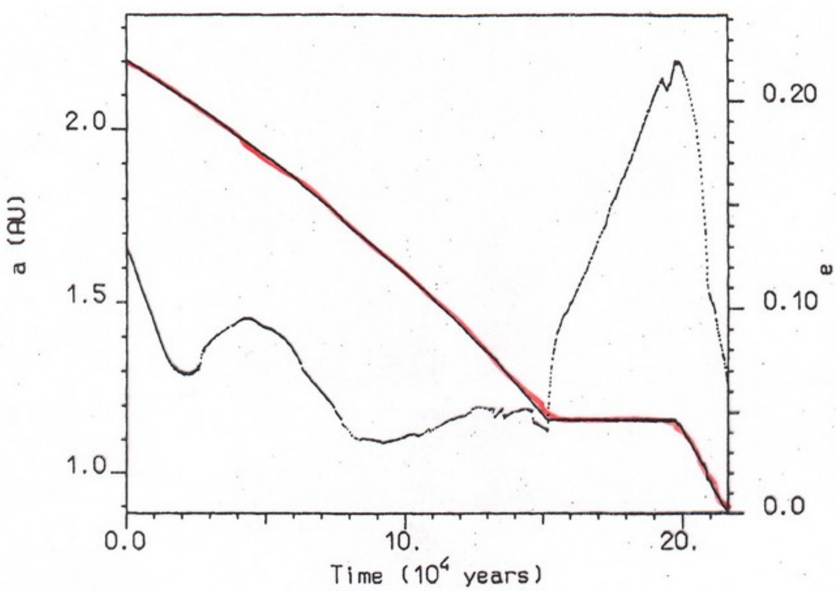
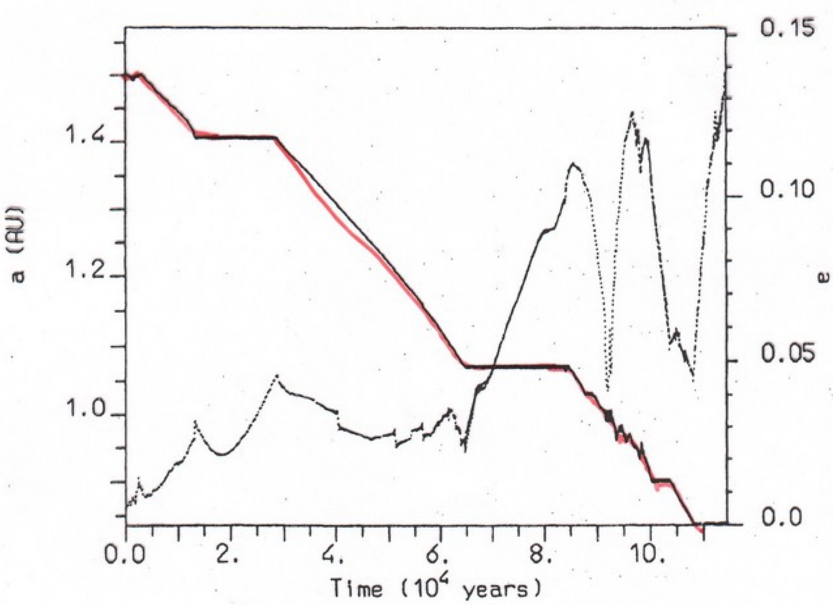


Fig. 3. Combined view of the semimajor axis and eccentricity evolution for a 30 μm dust particle started at the inner edge of the main belt. The initial conditions are $a = 2.2$ AU, $e = 0.14$ and $i = 5°$ rad. The orbital decay is interrupted at $a = 1.1588$ AU, where the particle is trapped for ~ 45 000 yr in the 4/5 outer resonance with the Earth. The orbital eccentricity grows larger than 0.2 during the resonance trapping



The orbital decay due to P-R drag (da/dt < 0) is temporarely halted by mean motion resonances with the planets. In this case it is the Earth.

Radiation pressure on cometary particles: ejection from the solar system and injection in the galactic dust particle population.

$$r_p = a(1-e) \quad v^2 = \frac{\mu}{a}\frac{(1+e)}{(1-e)}$$

$$E = \frac{1}{2}v^2 - \frac{\mu}{r_p} = \frac{\mu}{2a}\frac{(1+e)}{(1-e)} - \frac{\mu}{a}\frac{1}{(1-e)} \qquad at \;\; perihelion$$

When the particle is ejected, it feels the radiation pressure that counteracts the gravity force (both radial but opposite in direction):

$$\beta = \frac{S' A Q_{PR}}{c} / \frac{G M_s m}{r^2} = \left(\frac{S'_0 Q_{PR}}{r^2}\right) r^2 \frac{A/c}{G M_s m} = \frac{3}{4}\frac{S_0 Q_{PR}}{G M_s c \rho s}$$

Where *s* is the radius of the dust particle and S_0 is the solar radiation flux at 1 au (solar constant).

After the ejection from the comet body, the orbital velocity is approximately the same (the ejection velocity is small compared to the orbital velocity) and the orbital energy becomes:

$$E = \frac{1}{2}v^2 - \frac{\mu'}{r_p} = \frac{\mu}{2a}\frac{(1+e)}{(1-e)} - \frac{\mu(1-\beta)}{a}\frac{1}{(1-e)} = \frac{\mu(e-1+2\beta)}{2a(1-e)}$$

$$if \;\; e-1+2\beta \geq 0$$

The orbital energy becomes positive and the body is injected on a hyperbolic trajectory

Perihelion

$$\beta \geq \frac{(1-e)}{2}$$

Aphelion

$$\beta \geq \frac{(1+e)}{2}$$

It is much easier to have ejction at perihelion for particles ejected from comets on highly eccentric orbits.

Example:

$$e = 0.9 \qquad \beta \geq \frac{0.1}{2} \geq 0.05 \qquad \beta = 5.7 \times 10^{-5} \frac{Q_{PR}}{\rho s} \quad \Rightarrow$$

$$s \leq 5.7 \times 10^{-5} \frac{Q_{PR}}{\rho \cdot 0.05}$$

Assuming that $\rho \simeq 3 \ \frac{gr}{cm^3} \qquad Q_{PR} \simeq 1.5$

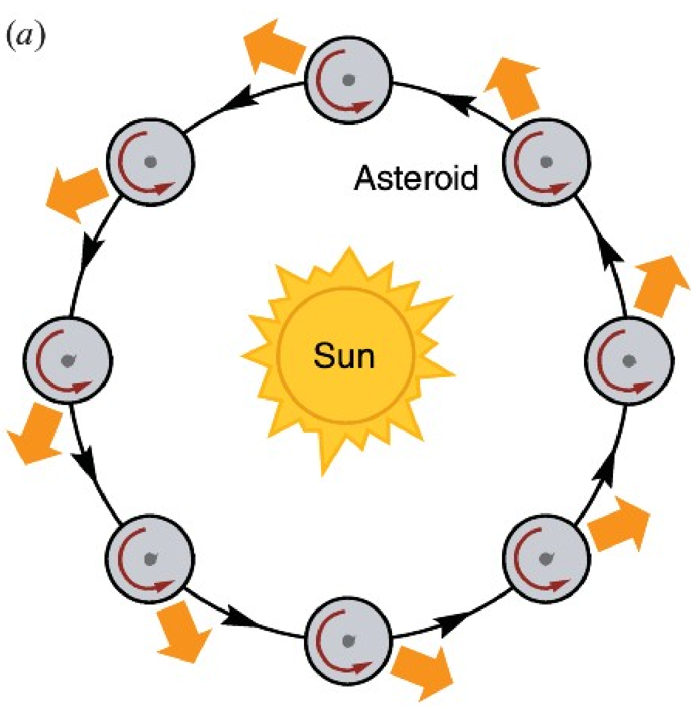
We get an estimate of the maximum size of particles that will be ejected at perihelion:

$$s \leq 5.7 \times 10^{-4} \ cm$$

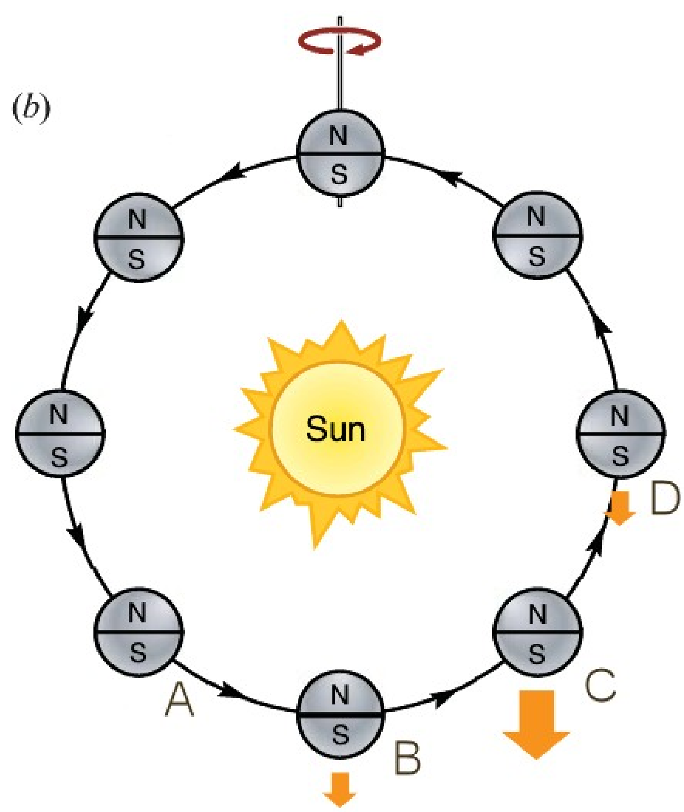
3-2 Yarkovsky effect

Bodies larger than dust grains (from some cm and beyond) have thermal inertia that implies an anysotropic ditribution of the superficial temperature. The sunlit surface is hotter and emits more energy and at shorter wavelenght while the shaded surface will be colder and emits less energy at lower wavelenght. The net dynamical effect depends on the orientation of the body spin axis.

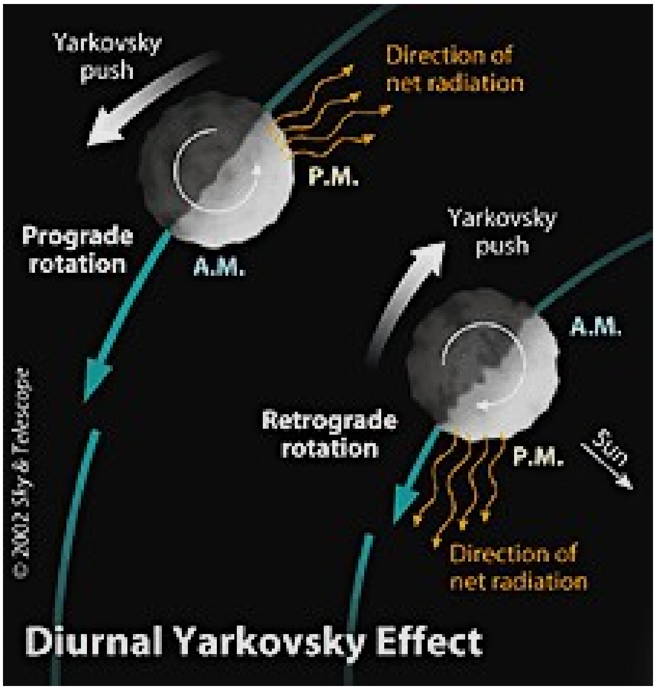
Diurnal

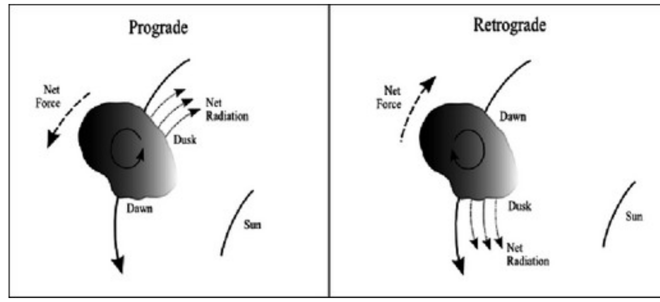


Seasonal



Diurnal component: due to the change in the temperature distribution due to the body rotation.





$$E = \sigma T^4$$
$$\vec{p} = E/c\,\vec{n}$$
$$\vec{f} = \frac{\vec{F}}{m}$$

Force per mass unit of the body, σ is the Stefan-Boltzmann constant

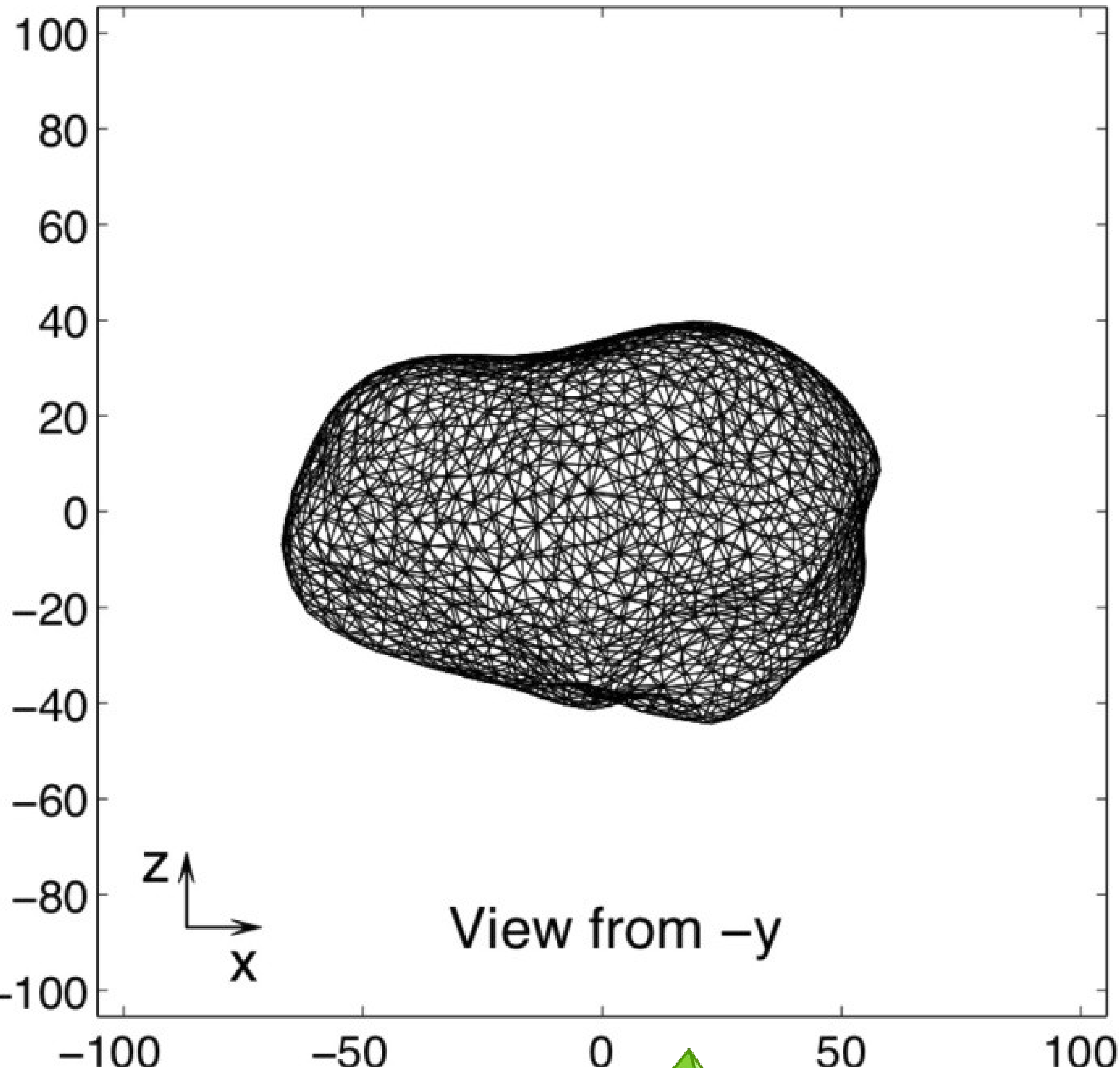
$$\vec{f} = -\frac{2}{3}\frac{\epsilon\sigma}{mc}\int_S ds\, T^4\,\vec{n}$$

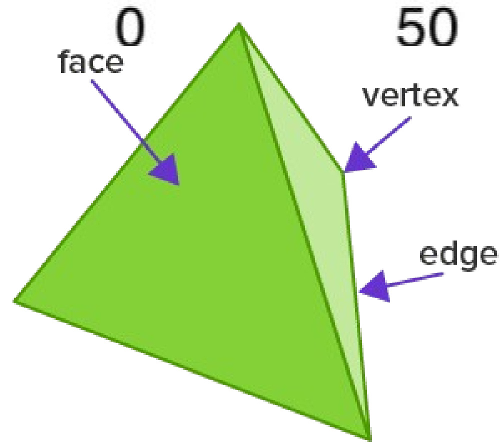
ε is the infrared emissivity.

Prograde rotation: outward drift
Retrograde rotation: inward drift

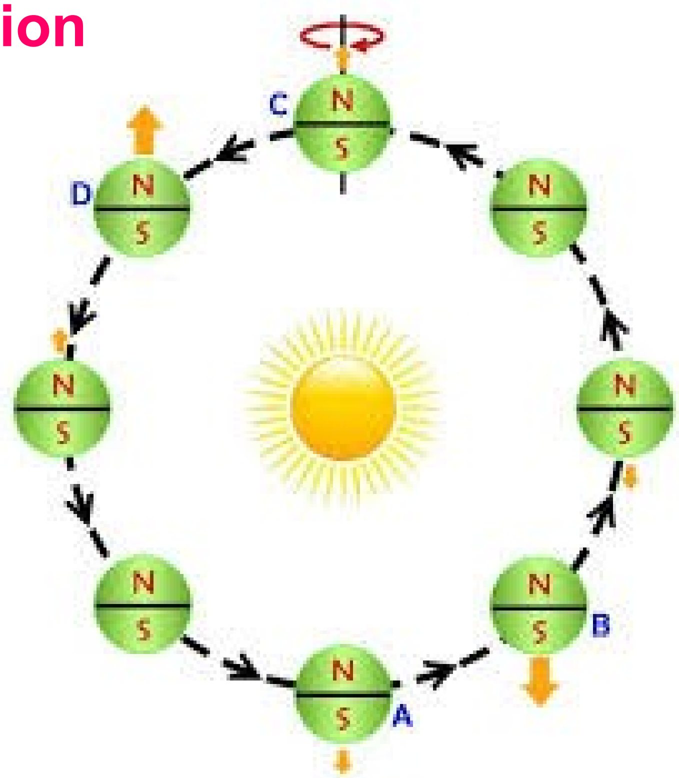
PROBLEM: to calculate the temperature T(x,y,z) all over the surface of the body. The Fourier equation for the heat transfer must be solved, but it requires a deep knowledge of the internal structure and composition of the body and the insolation pattern.

Asteroid Lutetia polyhedron model derived from images of ROSETTA S/C





Seasonal component: related to the change in the temperature distribution due to the orbital motion



Always negative drift because the warm side (S) is always pushing against the orbital motion.

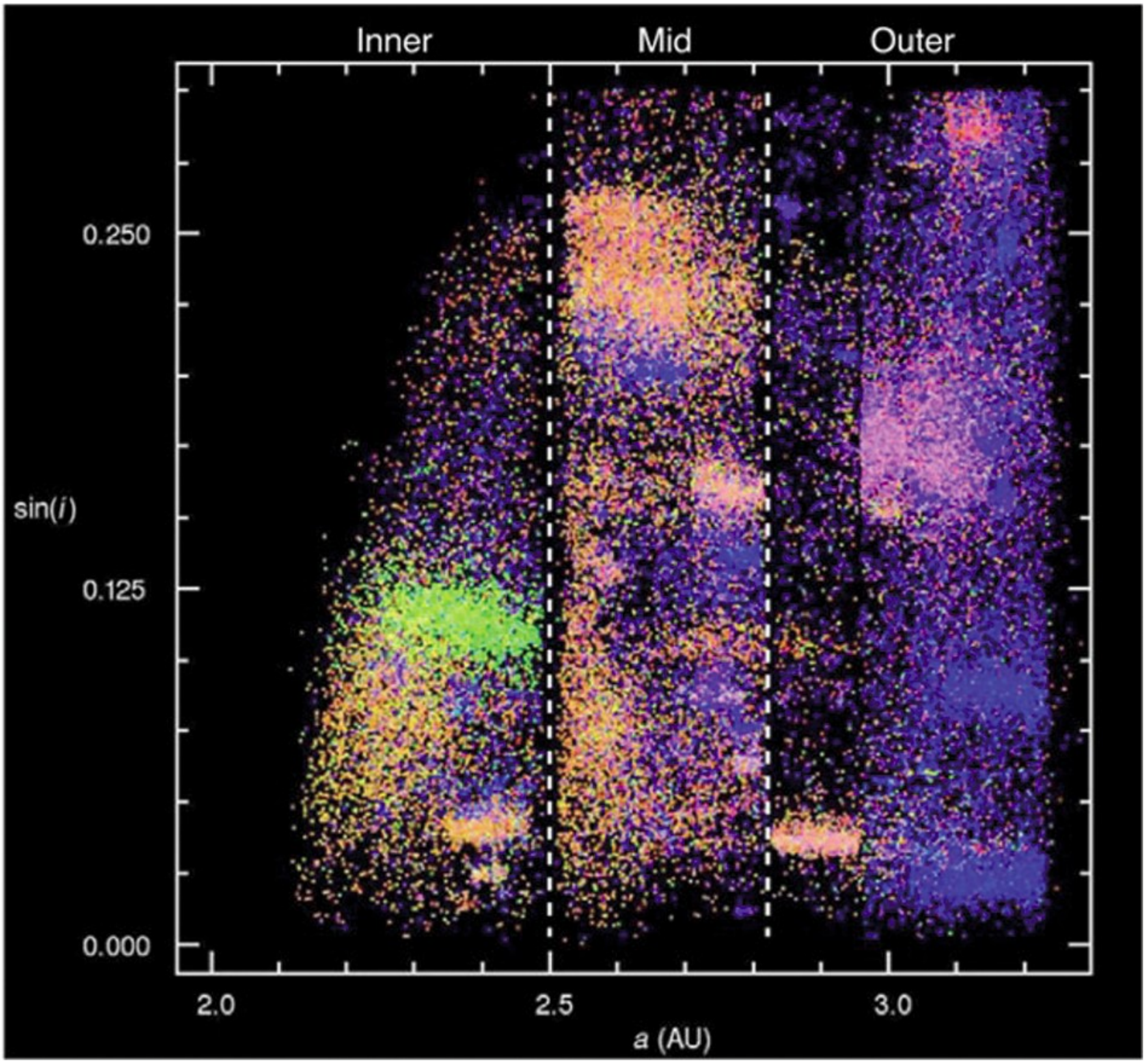
$$\left.\frac{da}{dt}\right|_{diurnal} = \frac{8\alpha}{9n}\frac{\pi R^2 \epsilon_0}{mc} F_\omega(R,\theta)\cos\gamma$$

$$\left.\frac{da}{dt}\right|_{seasonal} = \frac{4\alpha}{9n}\frac{\pi R^2 \epsilon_0}{mc} F_n(R,\theta)\sin^2\gamma$$

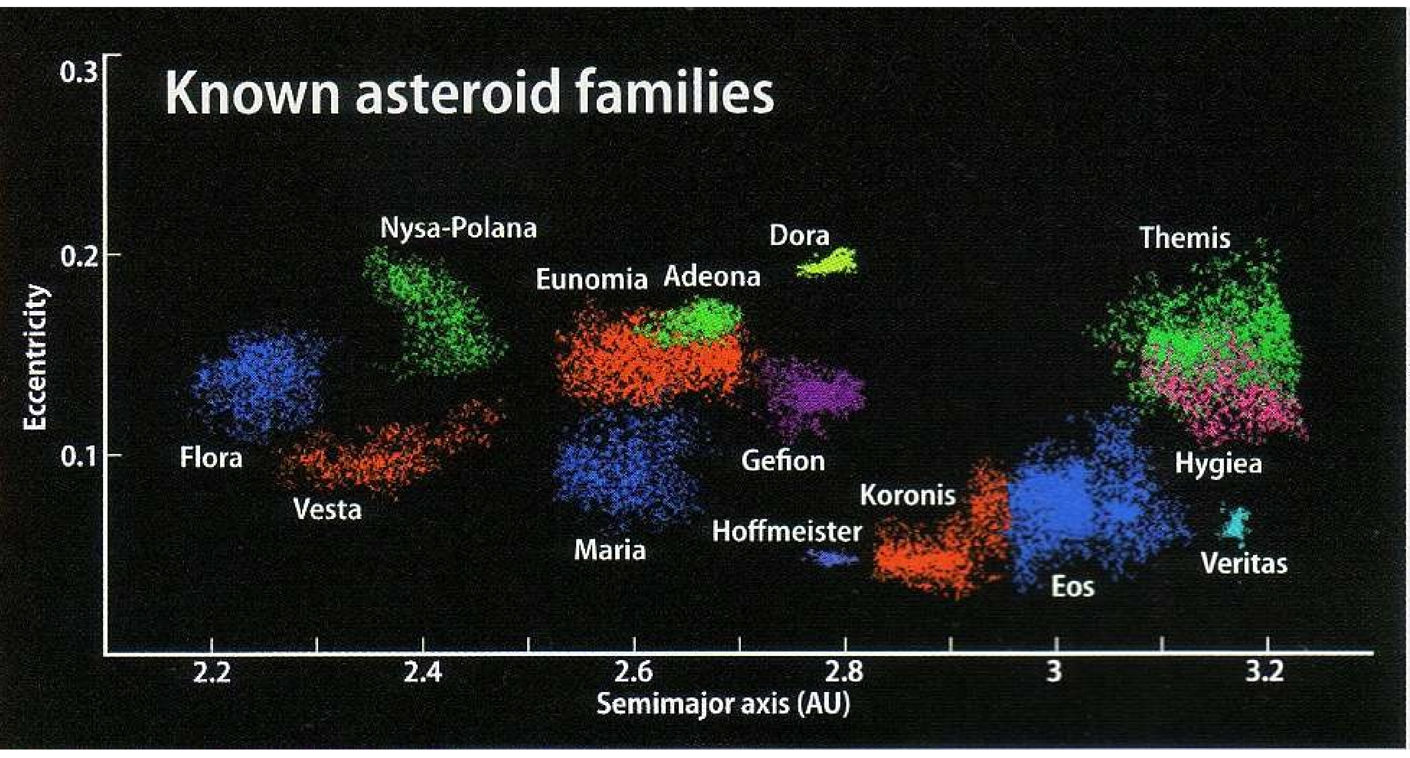
n=mean motion
ω=rotation rate
α=albedo
γ=obliquity
R=radius of the body
θ=thermal parameter
$F_\omega(R,\theta)$ is a positive function
$F_n(R,\theta)$ is a NEGATIVE function

Bertotti, Farinella, Vokrouhlicky, Physics of the solar system, pg. 499

Asteroid families and their evolution with time.



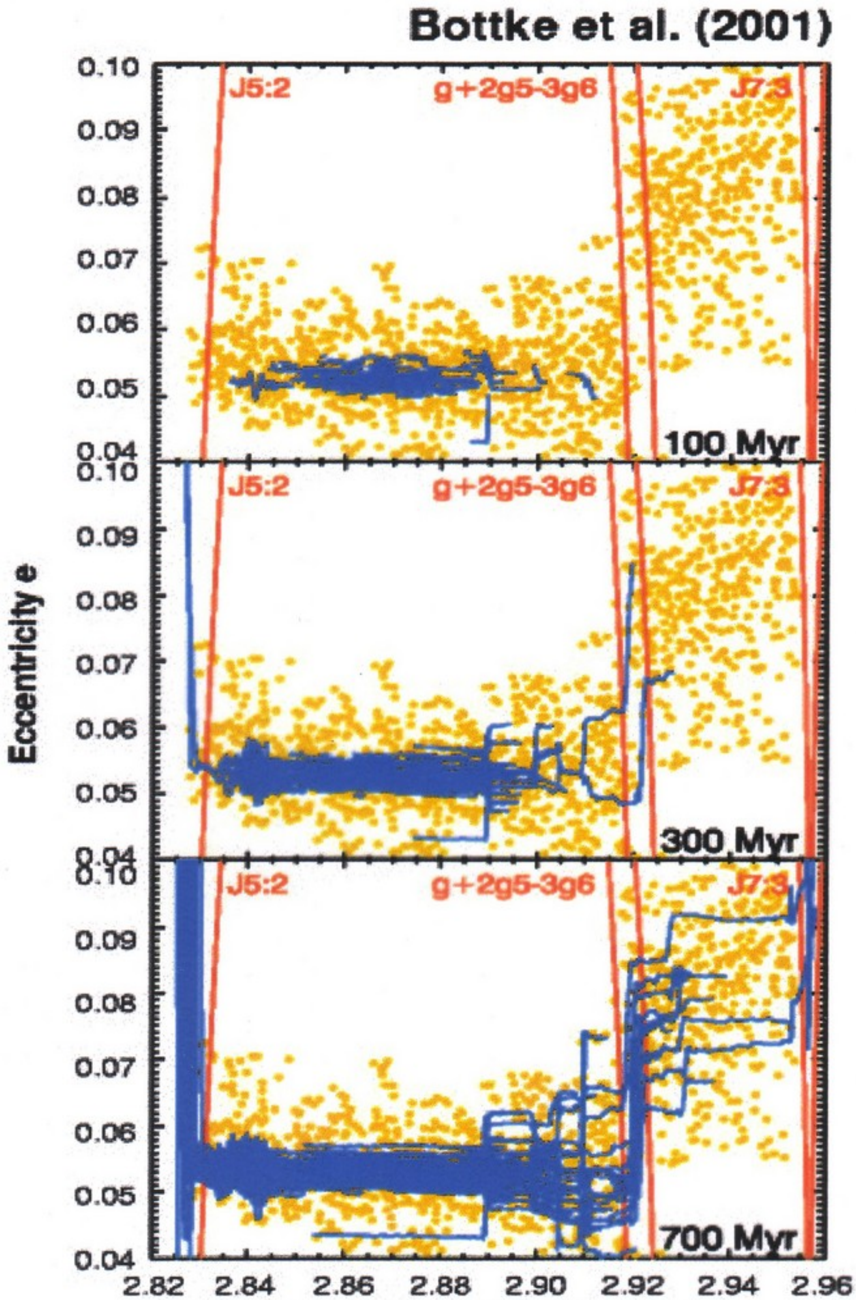
Asteroid families are the outcome of collisions between asteroids in the Main Belt. They appear as clusters in the proper orbital element planes. Proper orbital elements are filtered of planetary perturbations and reflect the orbtial distribution, due to the collisional dv, just after the collision. They are almost constant with time.



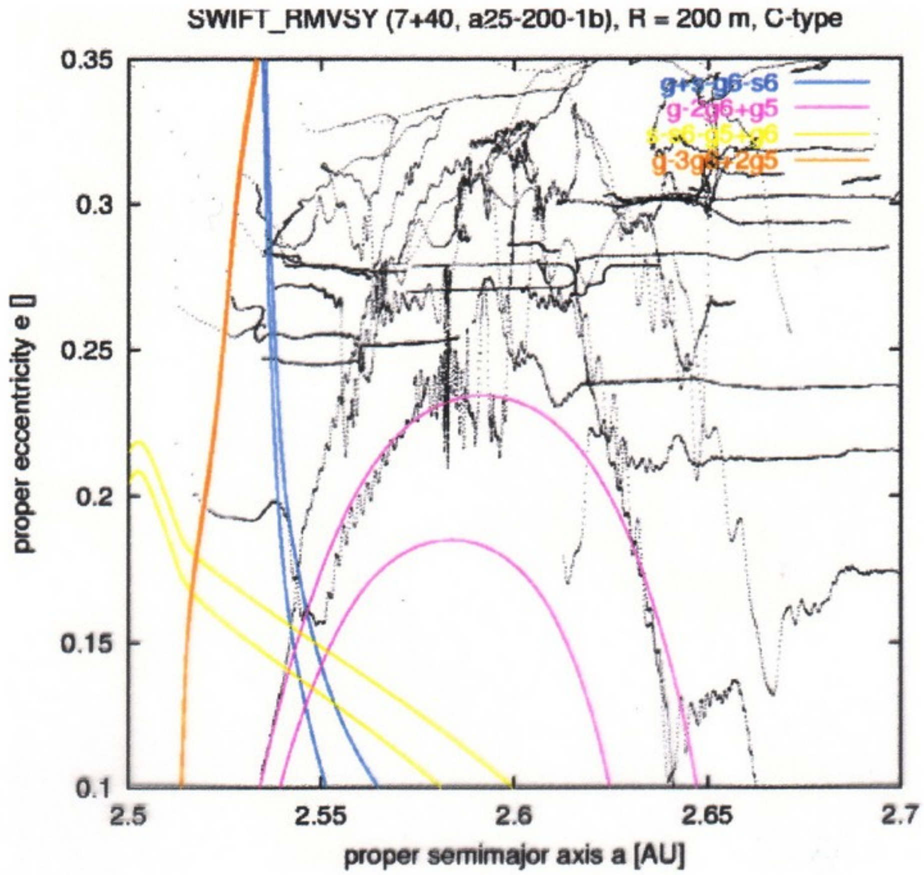
ASTRONOMY: ROEN KELLY, AFTER M. JURIĆ, Ž. IVEZIĆ, R. LUPTON ET AL., 2002

Proper elements can change because of the Yarkovsky effect (both diurnal and seasonal).

Any asteroid family is named after its bigger member. In figure it is represented the Koronis family.



In blue are illustrated the trajectories in the orbital element space of small asteroids (< 20 km) in a family which drift either inward or outward due to Yarkovsky.



The Yarkosky effect refills the population of NEO asteroids (Near Earth Objects). After a collision, a family forms, and because of the Yarkovsky effect some of its members migrate on inner orbits. They find secular resonances which pump up their eccentricities until they have close encounters with Mars. After that they become planet crossing and can have close approaches with all terrestrial planets until, after about 10 Myr (Gladman 2000) either they impact a planet or the sun or are ejected out of the solar system.


BEAUFORT SEA
Banks Island
Victoria Islands
BAFFIN SEA
Baffin Islands
YUKON
NORTHWEST TERRITORIES
NUNAVUT
Hudson Bay
LABRADOR SEA
Gulf of Alaska
BRITISH COLUMBIA
ALBERTA
SASKATCHEWAN
MANITOBA
ONTARIO
QUEBEC
NEWFOUNDLAND AND LABRADOR
Whitehorse
Yellowknife
Iqaluit
Edmonton
Calgary
Vancouver
Victoria
Regina
Winnipeg
Quebec
OTTAWA
Toronto
NEW BRUNSWICK
NOVA SCOTIA
PACIFIC
OCEAN
ATLANTIC OCEAN
UNITED STATES OF AMERICA
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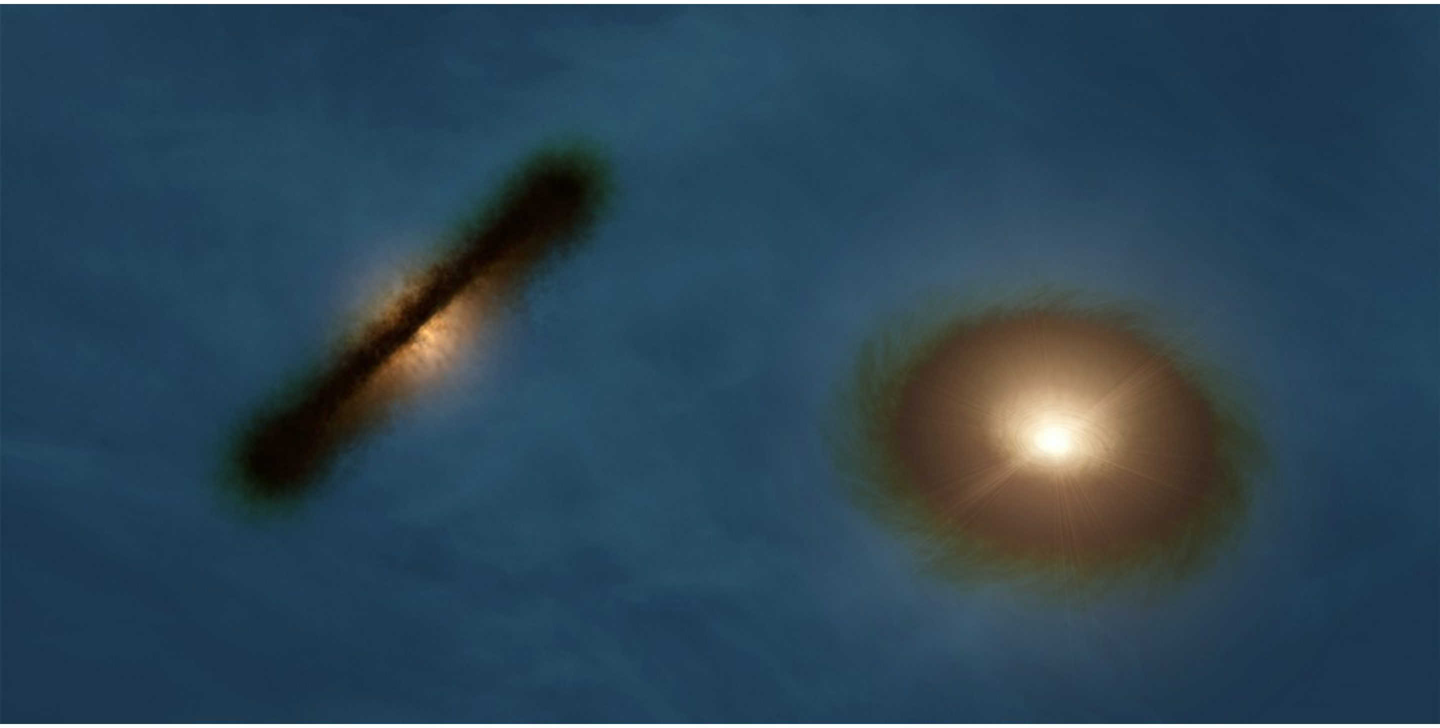
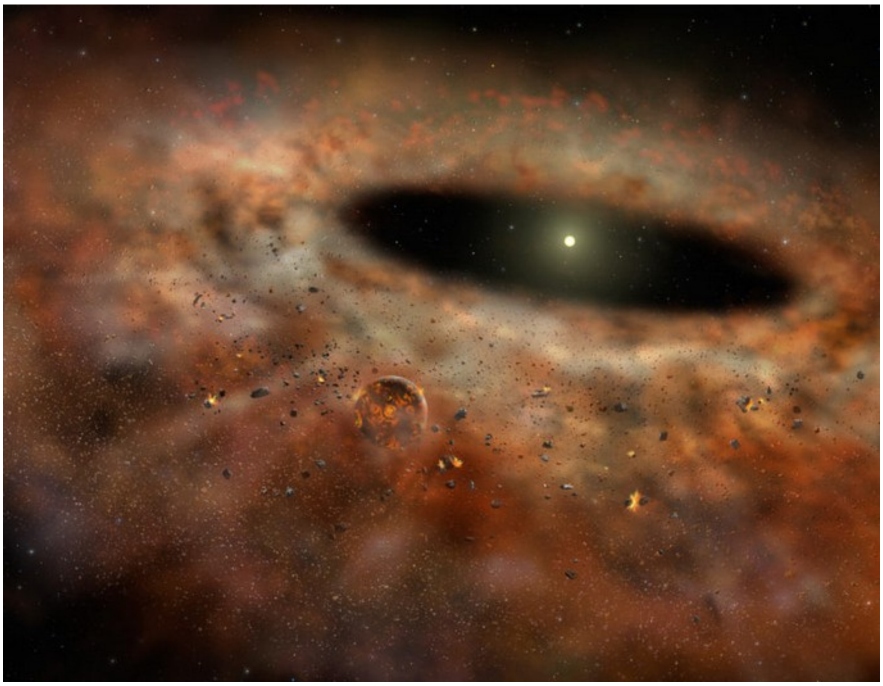
3-3 Gas drag

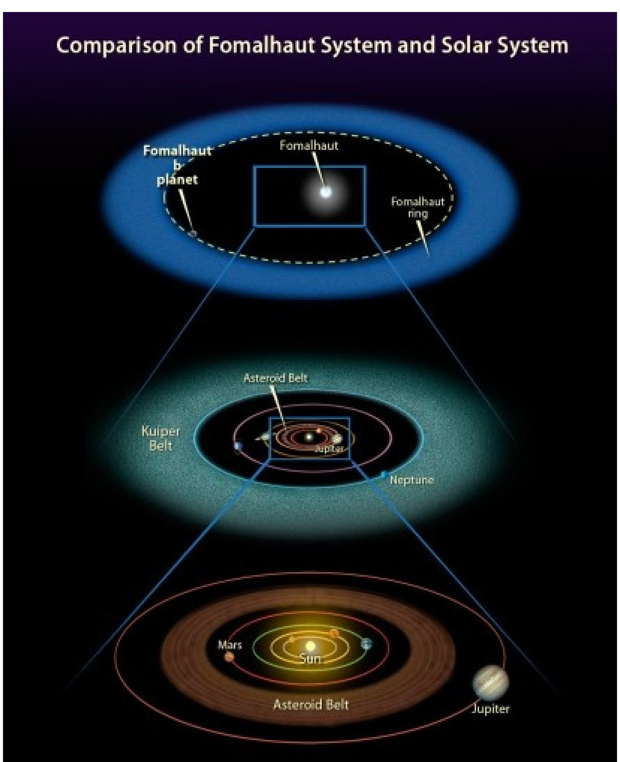


Circumplanetary disks (or accretion or circumstellar disks) are made of gases (mostly H and He) and dust. They form during the last phases of the gravitational collapse of a protostar and their shape is dictated by the conservation of angular momentum. The gas acts on dust particles and planetesimals orbiting around the protostar altering their Keplerian motion.

n.b: there is a significant difference between debris disks and circumplanetary disks.

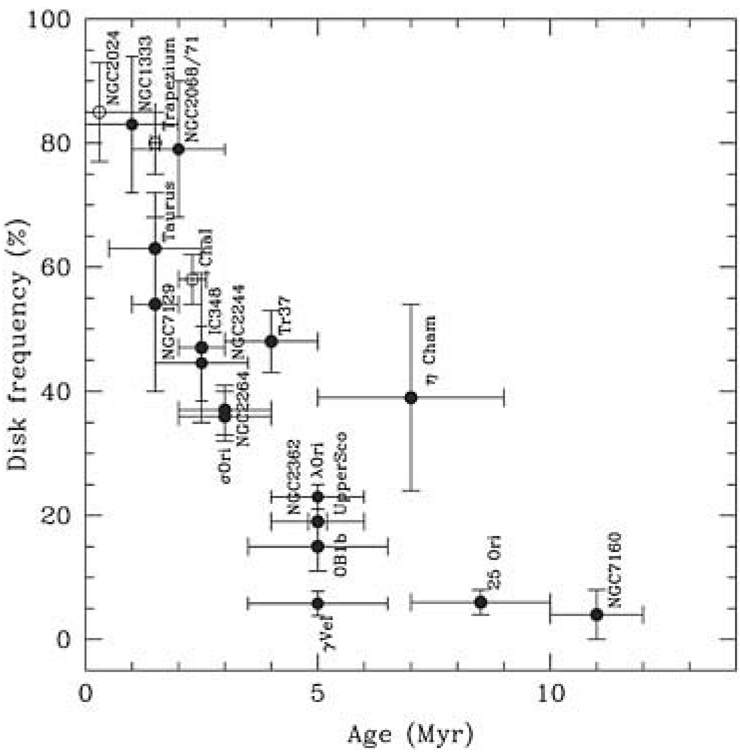


Circumstellar disk: gas+dust. Ongoing accretion process (dust, planetesimals, planets)



Debris disk: made of dust produced by collisions in a leftover planetesimal belt. No more gas that dissipated after about 10 Myr from the formation of the star.

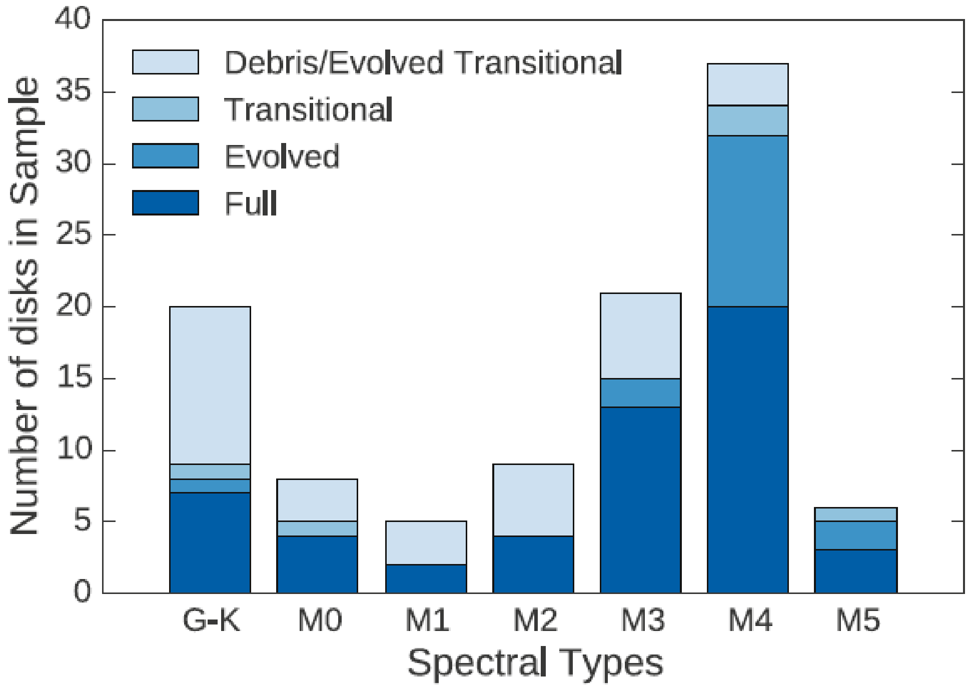
What is the **lifetime** of a protoplanetary disk? (time available for giant planets to form...)



Focus on stellar clusters where the ages of the stars are approximately the same. The fraction of stars with disks give an idea of the dissipation process. Lifetime around 5-10 Myr.

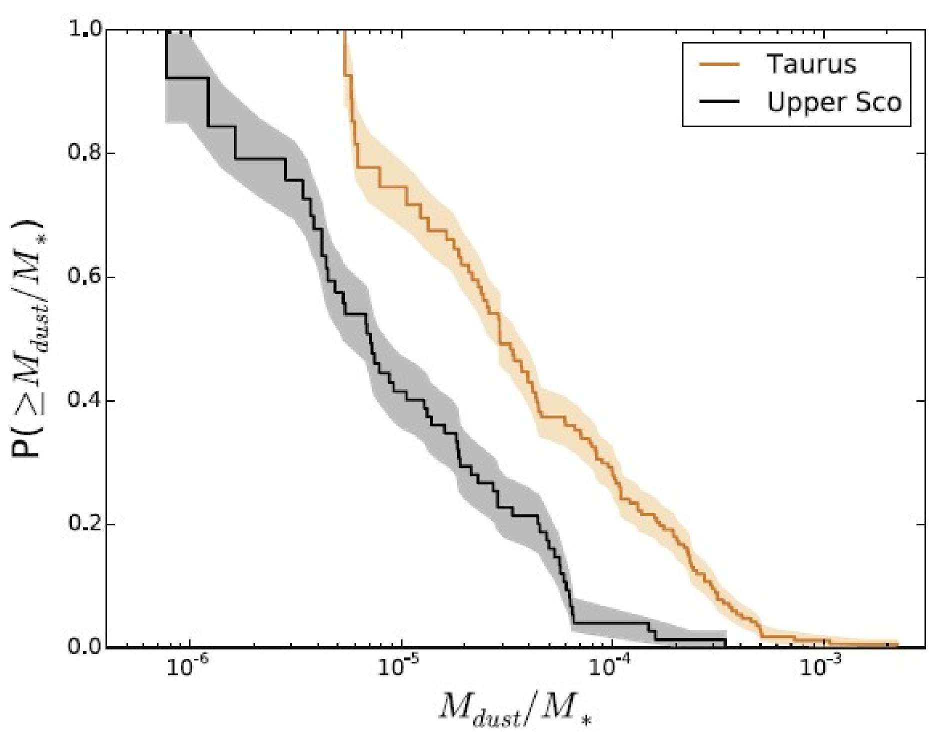
...other images of protoplanetary disks..

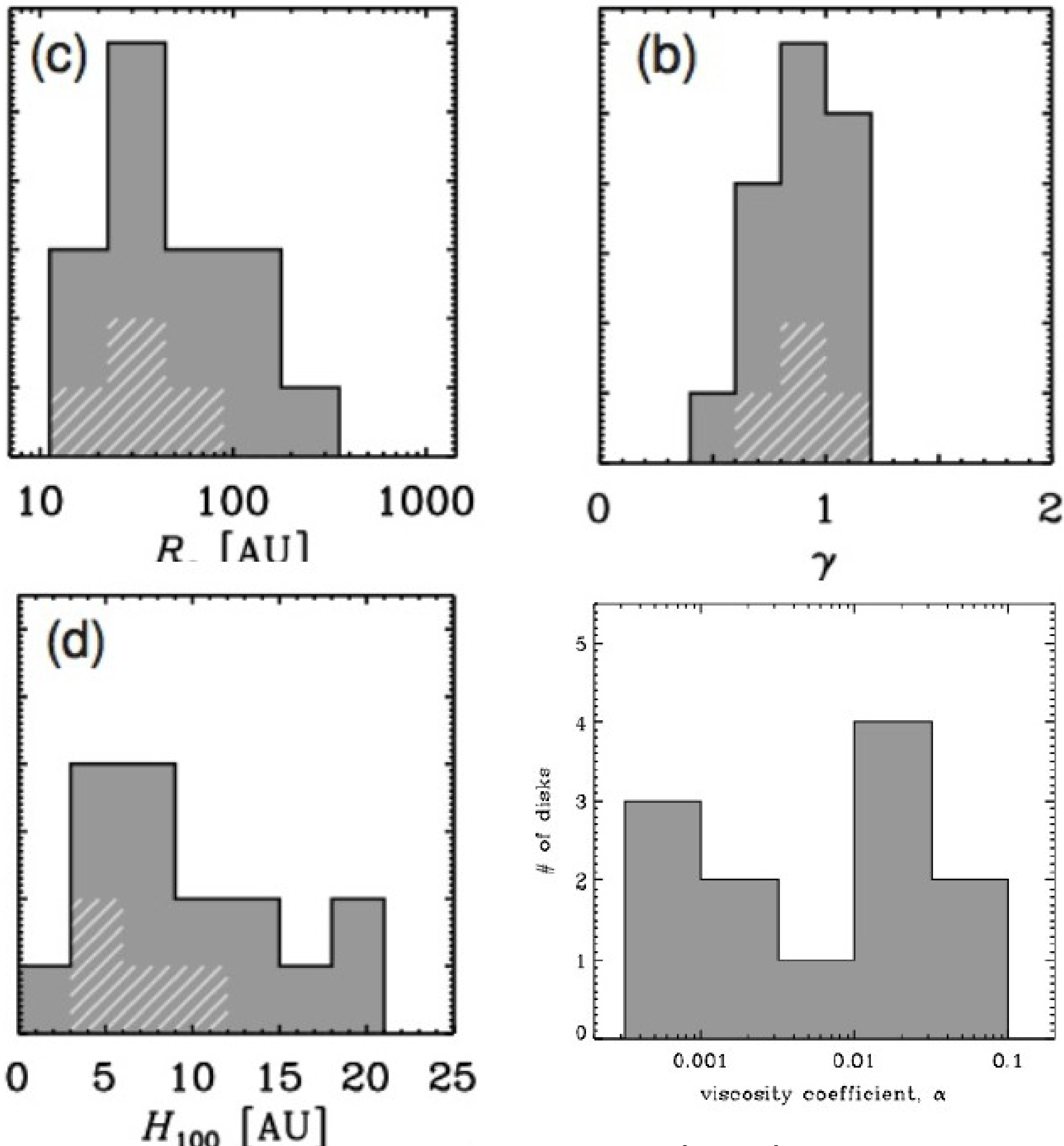
Statistics ….statistics…..



Upper Scorpius OB association (~ 5-11 Myr)

Comparison with the Taurus region (1-2 Myr). Younger region has more massive disks… (Barenfeld et al. 2016)

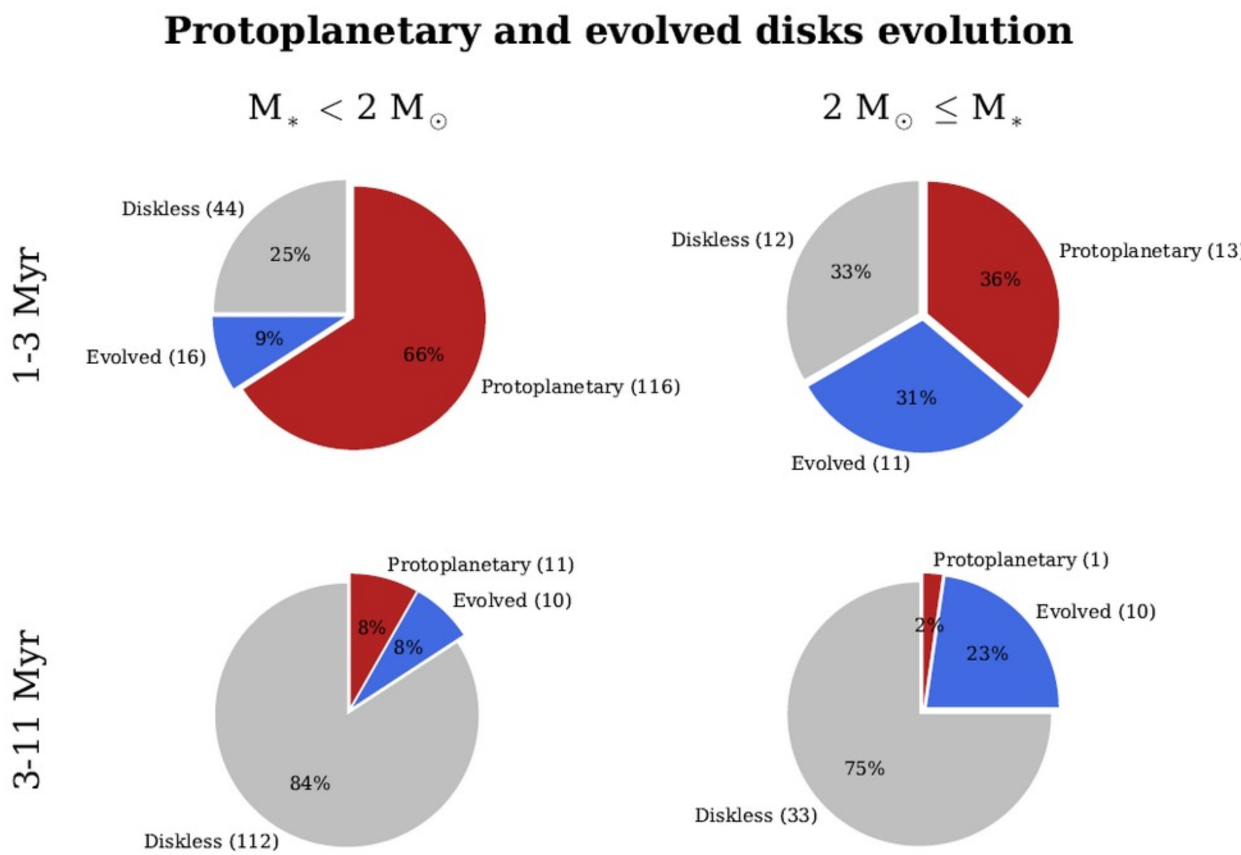




Viscosity α

Andrews et al. (2010)

Pre-ALMA statistics to be improved. Due to disk evolution is also difficult to interpret these data. Do radius, radial surface density coefficient γ, scale height at 100 au and viscosity change with time?



From Ribas et al. 2015, results from 22 nearby young star forming regions. The lifetime of the disks depend on the star mass. Is photoevaporation relevant in the dissipation of the disk since more massive stars lose their disks more quickly?

Physical properties of circumplanetary disks from observations (infrared excess, direct imaging...)

$$M \sim 0.01 - 0.1\, M_{sun}$$
$$98\% \quad H, He$$
$$1{,}2\% \quad metals$$

$$\rho(r) = \rho_0 r^{-\alpha}$$
$$T(r) = T_0 r^{-\beta}$$
$$\sigma_d(r) = \sigma_{d0} r^{-\gamma}$$

$$\rho_0 \sim 2 - 3 \times 10^{-9} \quad g/cm^3$$
$$T_0 \sim 200 - 400\, K$$
$$\sigma_{d0} \sim 7 - 10 \quad g/cm^2$$

At r= 1 au
σ_d is the dust superficial density

Adopting these profiles we now study the gas motion around the star and how it is affected by the gas pressure

Laminar gas motion (no turbulence)

$$r\,\omega_g^2 = \frac{GM}{r^2} - F(P)$$

Centripetal acceleration

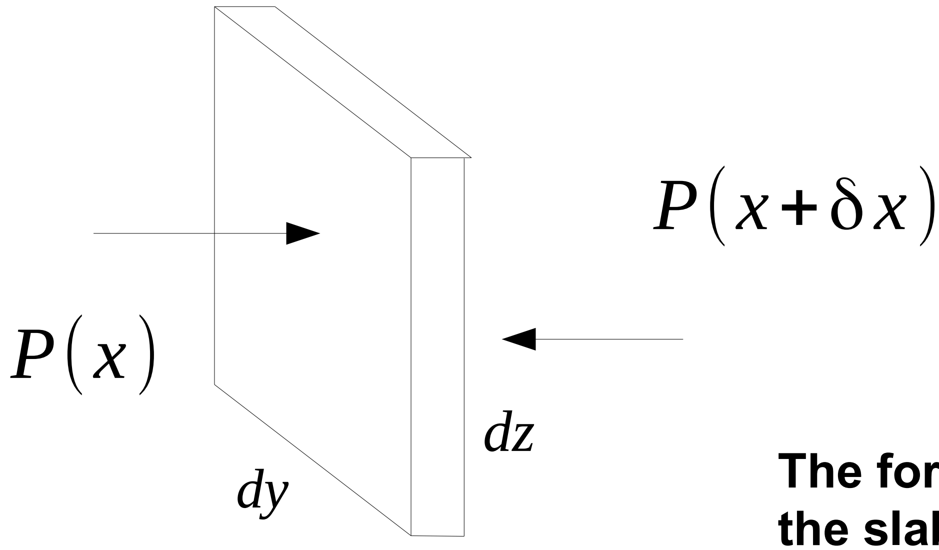
Gravity

Force due to pressure gradient

$$\omega_K = \left(\frac{GM}{r^3}\right)^{\frac{1}{2}}$$

Keplerian angular velocity

$$r\,\omega_g^2 = r\,\omega_K^2 - F(P)$$



The force acting on the slab is:

$$F=[P(x)-P(x+\delta x)]\cdot dy\,dz$$

The force per volume unit is:

$$F_V=\frac{F}{V}=\frac{[P(x)-P(x+\delta x)]\cdot dy\,dz}{dx\,dy\,dz}=-\frac{dP}{dx}$$

$$F_V=\rho\,a=-\frac{dP}{dx}\quad\Rightarrow\quad a=-\frac{1}{\rho}\frac{dP}{dx}$$

Where a is the acceleration of the volume element and ρ its density. Finally, changing x with the radial distance from the star r, we get:

$$r\,\omega_g^2=r\,\omega_K^2+\frac{1}{\rho}\frac{dP}{dr}$$

To compute the pressure term dP/dr in a disk the equation of state is used

$$PV = nRT = n\frac{N_A}{N_A}\frac{m}{m}RT \Rightarrow P = \frac{n N_A m}{V N_a m}RT$$

$$P = \rho\frac{KT}{m} \quad m = \text{mean molecular mass} = \mu \cdot m_p$$

$$\frac{dP}{dr} = \frac{d}{dr}\left(\rho_0 r^{-\alpha}\frac{K}{m}T_0 r^{-\beta}\right) = \frac{K}{m}(\alpha+\beta)\rho_0 T_0 r^{-\alpha-\beta-1}$$

$$\omega_g^2 = \omega_K^2 - \frac{K}{mr}(\alpha+\beta)\rho_0\frac{T_0 r^{-(\alpha+\beta+1)}}{\rho_0 r^{-\alpha}} =$$

$$= \frac{GM}{r^3}\left(1 - \frac{K}{m}(\alpha+\beta)\frac{1}{r^2}T_0 r^{-\beta}\frac{r^3}{GM}\right) =$$

$$= \frac{GM}{r^3}[1-2\eta(r)] = \omega_K^2(r)[1-2\eta(r)]$$

$$\eta(r) = \frac{\pi}{16}(\alpha+\beta)\frac{c_m^2}{v_K^2} \qquad c_m = \sqrt{\frac{8KT}{\pi m}}$$

c_m mean thermal speed of gas

$$\omega_g(r) = \omega_K(r)[1-2\eta(r)]^{1/2}$$

$$\eta \sim 10^{-5} \times 10^{-3}$$

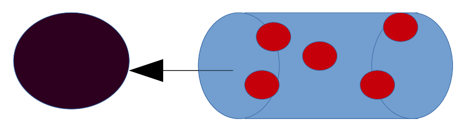
Slower than purely Keplerian!

Drag force on dust particles embedded in a circumstellar disk. We assume that the particle be spherical with radius s smaller than the mean free path of the gas particles and move respect to the gas with velocity v. The thermal velocity of the gas is:

$$c_m = v_{th} = \sqrt{\frac{8 k_B T}{\pi \mu m_H}}$$

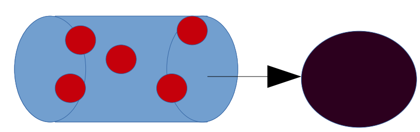
μm_H = average molecular weight in terms of the hydrogen atom mass. The frequency with which the molecules collide head-on is

$$f = \pi s^2 \cdot \left(\frac{1}{3} v_{th} + v\right) \cdot \frac{\rho_g}{\mu m_H}$$



$v \Delta t N \cdot \pi s^2$

$$b = \pi s^2 \cdot \left(\frac{1}{3} v_{th} - v\right) \cdot \frac{\rho_g}{\mu m_H}$$



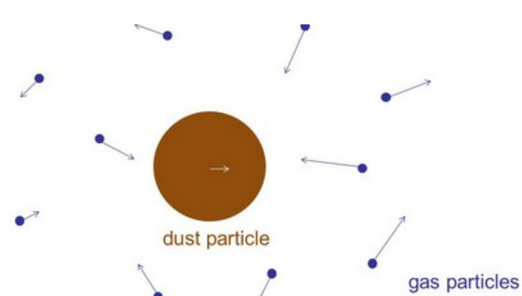
$v \Delta t N \cdot \pi s^2$

$$N = \frac{M}{\mu m_H} = \frac{V \rho}{\mu M_H}$$

Each collision releases (assuming v_{th} >> v) a momentum: $\Delta p = 2 \mu m_H \frac{1}{3} v_{th}$

$$\boldsymbol{F_D} = -(f - b)\, \Delta p = -\frac{4}{3} \pi s^2 \rho_g v_{th} \boldsymbol{v}$$

Epstein drag law:



Details…..

$$f=\frac{v\,\Delta t\cdot N\cdot As}{\Delta t}=v\,\frac{\rho}{\mu\, m_H}\pi\, s^2$$

$$\Delta\, p=2\mu\, m_H\frac{1}{3}v_{th}$$

$$\frac{\Delta\, p}{\Delta\, t}=dp\cdot f-dp\cdot b=\pi\, s^2\cdot\left(\frac{1}{3}v_{th}+v-\frac{1}{3}v_{th}-v\right)\cdot\frac{\rho_g}{\mu\, m_H}\cdot dp$$

$$\frac{\Delta\, p}{\Delta\, t}=2\pi\, s^2\cdot v\cdot\frac{\rho_g}{\mu\, m_H}2\,\mu\, m_H\frac{1}{3}v_{th}$$

Stopping time

$$t_{stop}=\sqrt{\frac{\pi}{8}}\frac{\rho_s}{\rho_g}\frac{a}{c_s}$$

With a particle radius and c_s isothermal sound speed

$$c_s=\sqrt{\frac{K_B T}{\mu\, m_H}}$$

Stokes number

$$St=t_{stop}\Omega_k$$

In 2D

$$St=\frac{\pi}{2}\frac{a\,\rho_s}{\Sigma_g}$$

If St << 1 dust particles are strongly coupled to the gas, for St ~ 1 they feel strong headwind and drift inwards.

Motion in a viscous regime:

$$ma = F - f\,v \quad \Rightarrow \quad v(t) = \frac{F}{f}\left(1 - \exp\left(-\frac{f}{m}t\right)\right)$$

$$Per \quad t \rightarrow \infty \quad v = \frac{F}{f}$$

The sedimentation speed in the median plane of a circumstellar disk is then:

$$v_{settle} = \frac{G\,M\,m\,z}{r^3} / \frac{4}{3}\pi\,s^2\,v_{th}\,\rho_g = \frac{G\,M\,\rho_d\,z\,s}{r^3\,v_{th}\,\rho_g}$$

If we express the gravity force in function of the Keplerian frequency, we get:

$$\Omega_K^2 = \frac{GM}{r^3} \quad \Rightarrow \quad v_{settle} = \frac{\Omega_K^2\,z\,\rho_d\,s}{v_{th}\,\rho_g}$$

The time required by the dust to sediment on the median plane is then:

$$t_{settle} = \frac{z}{v_{settle}} = \frac{v_{th}\,\rho_g}{\Omega_K^2\,\rho_d\,s} \sim 10^5\,yr \qquad a \;\; 1 \;\; AU$$

Stokes drag law:

It applies when the free mean path is shorter than the size of the particle. In this case the gas is treated as a fluid acting on the particle and the result is similar to the solar wind pressure i.e. ρv^2 (Ram pressure). The force F_D becomes:

$$\vec{F}_D = -\frac{C_D}{2}\pi s^2 \rho_g v \vec{v}$$

Where C_D depends on the Reynolds number (typical length is 2s)

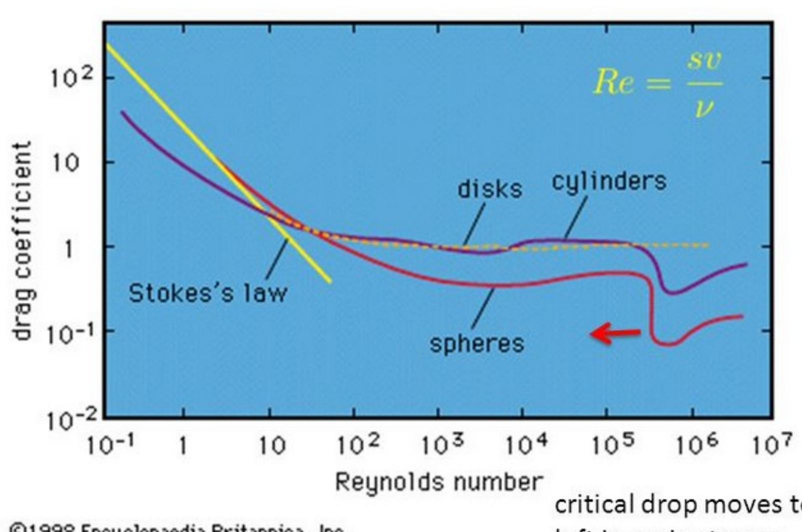
$$\Re = \frac{2 s v}{\nu_m}$$

ν_m *viscosity*

A typical value R at 1 au is > 800 so C_D at 1 au is

$$C_D \approx 0.44$$

Drag Coefficient



critical drop moves to the left in main stream turbulence or if the surface is rough

Note: we do not used turbulent viscosity to calculate drag coefficients

Explicit computation of the drag force on planetsimals:

$$\vec{f}_D = -\frac{1}{2} C_D (\pi s^2) \rho_g \vec{u} \cdot u$$

The vector u is the relative velocity between the body and the gas. Ad example, assuming that i=0 we can compute u as:

$$\dot{x}_p = \frac{-na}{\sqrt{1-e^2}} \sin f$$

$$\dot{y}_p = \frac{na}{\sqrt{1-e^2}} (e + \cos f)$$

For the body

$$\dot{x}_g = r \sin f \, \omega_g = -\frac{a(1-e^2)}{1+e\cos f} \sin f \, \omega_g$$

$$\dot{y}_g = r \cos f \, \omega_g = \frac{a(1-e^2)}{1+e\cos f} \cos f \, \omega_g$$

For the gas

$$\vec{u} = \begin{pmatrix} \dot{x}_p - \dot{x}_g \\ \dot{y}_p - \dot{y}_g \\ 0 \end{pmatrix}$$

Planetesimal orbtial element variations due to gas drag

The drag force is decomposed along the radial, tangent and vertical directions so that Guass equations can be used to compute the effect of the perturbation.

$$\frac{da}{dt} = \frac{2}{n(1-e^2)^{1/2}}\left[F_R e \sin f + F_T \frac{a(1-e^2)}{r}\right]$$

$$\frac{de}{dt} = \frac{(1-e^2)^{1/2}}{na}\left[F_R \sin f + F_T(\cos E + \cos f)\right]$$

$$\frac{di}{dt} = \frac{1}{n(1-e^2)^{1/2}} F_i \frac{r}{a} \cos(f+\omega)$$

These equations are averaged over an orbital period

$$\overline{\frac{da}{dt}} = \int_0^{2\pi} \frac{da}{dt}\, dM$$

$$\overline{\frac{da}{dt}} = -\frac{2a}{\tau_0}\left[(0.97e\, 0.64i + \eta)\eta + 0.35e^3 + 0.61i^3\right]$$

$$\overline{\frac{de}{dt}} = -\frac{e}{\tau_0}(0.77e + 0.64i + \eta)$$

$$\overline{\frac{di}{dt}} = -\frac{i}{\tau_0}(0.77e + 0.85i + \eta)$$

$$\tau_0 = 365.25 \frac{1}{C_D}\left(\frac{m}{10^{21}}\right)^{1/3}\left(\frac{1}{1.5\times 10^{13}}\right)^{-1}\left(\frac{\rho}{10^{-9}}\right)^{-1}\frac{2\pi}{\Omega_K}$$

Numerical integration of a planetesimal orbit perturbed by gas drag and the gravity of Jupiter. The semimajor axis decreases due to the gas friction.

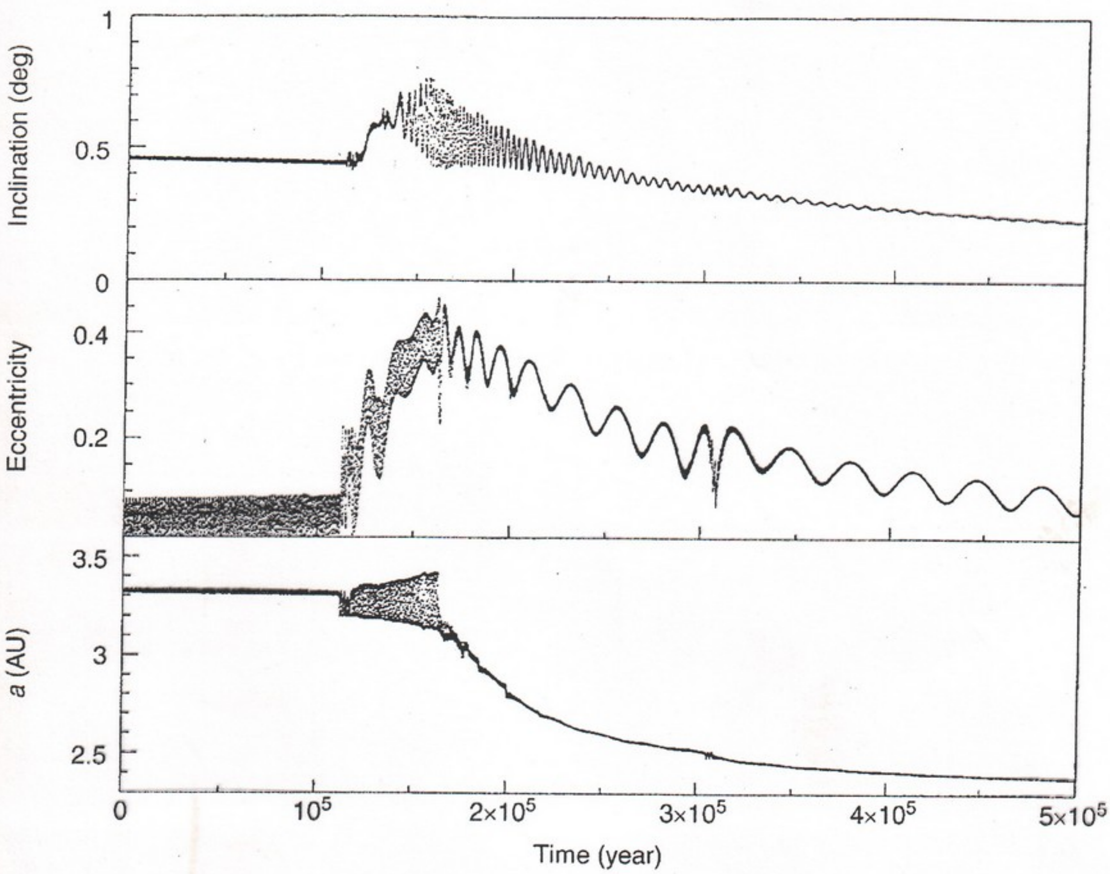


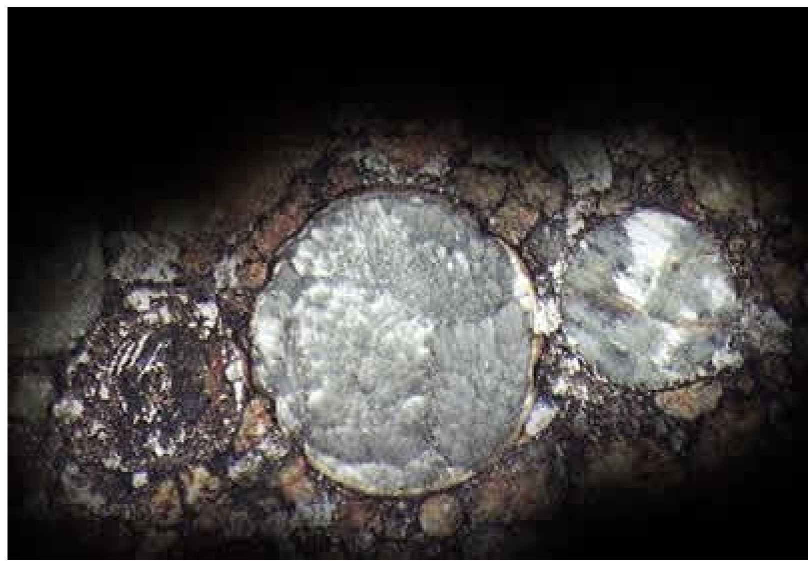
Figure 3. Temporary trapping in the 2:1 resonance of a 300 km size planetesimal.

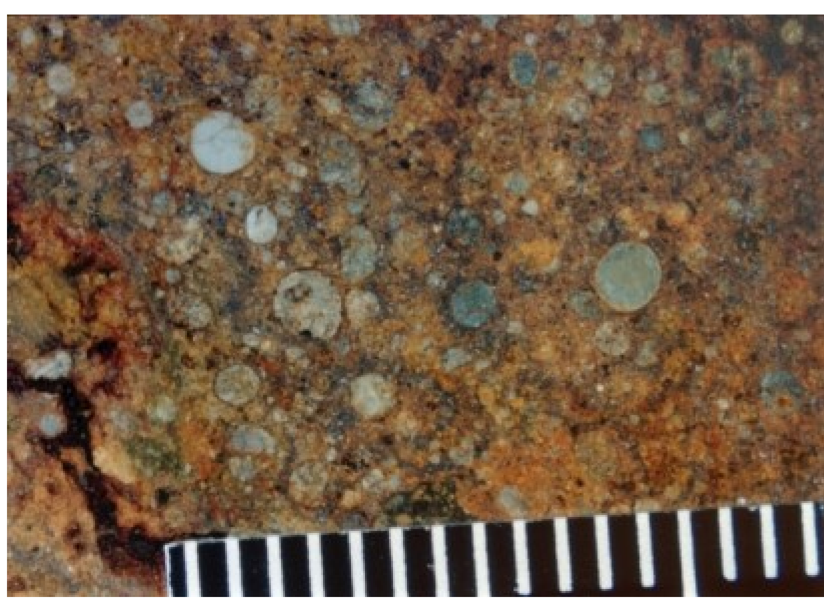
When the planetesimal crosses the MM resonance 2:1 with Jupiter, its eccentricity is pumped up and the inward drift rate is increased. After the resonance crossing, the gas drag plays the major role and the eccentricity is damped again towards 0. The drifting rate is reduced.

Relevance of gas drag in the initial phases of planet formation

- It keeps low the relative velocities between planetesimals favoring accreting impacts and speeding up planet formation.
- The combination of gas drag and MM resonances may drive local planetesimals on eccentric orbits → formation of shock waves and chondrules.

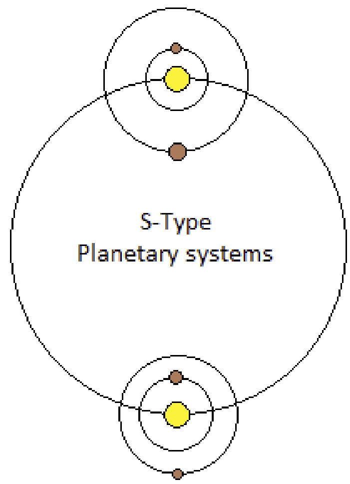
Formation of chondrules (silicate inclusions) in chondrites (most common meteorites) can be due to supersonic planetesimals creating shock waves in the protoplanetary disk with melting and quick solidification when the wave has passed.



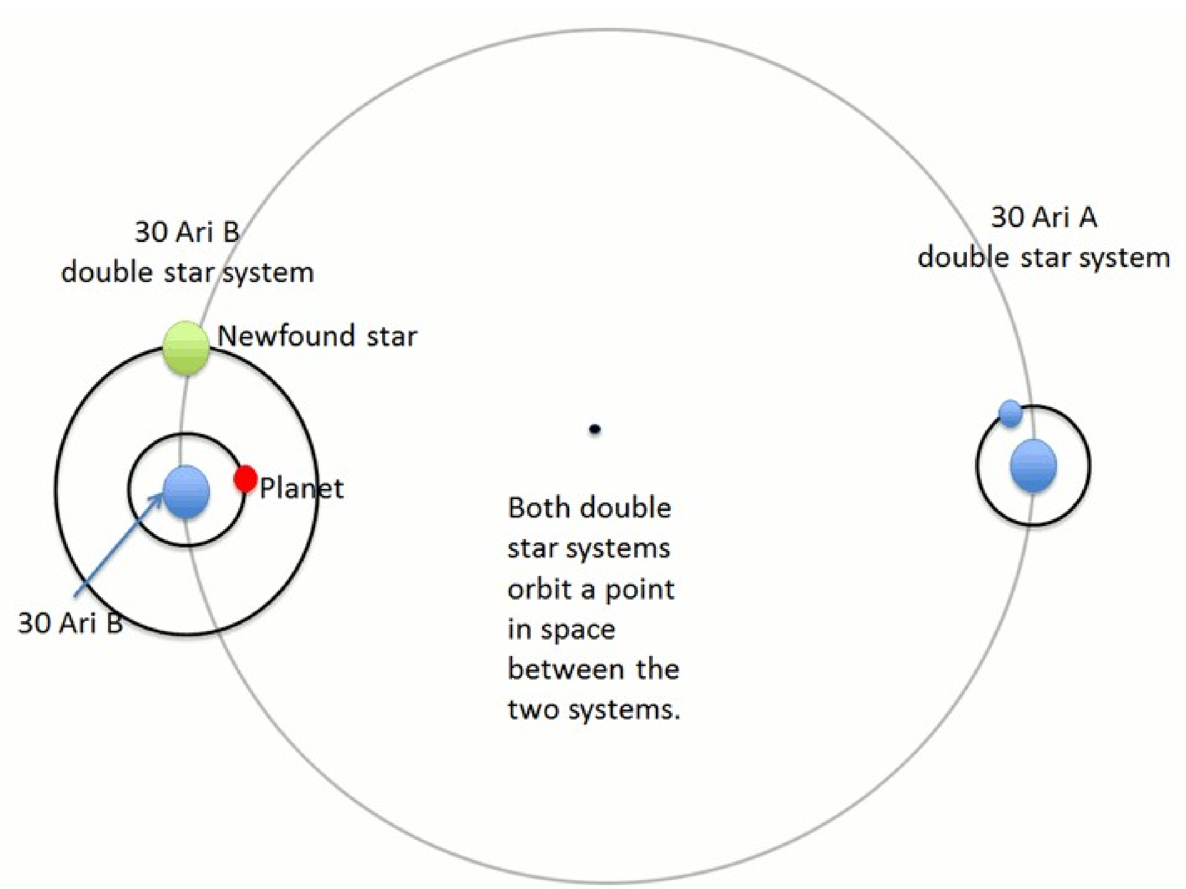


Planet formation in binary star systems.

S-type orbits

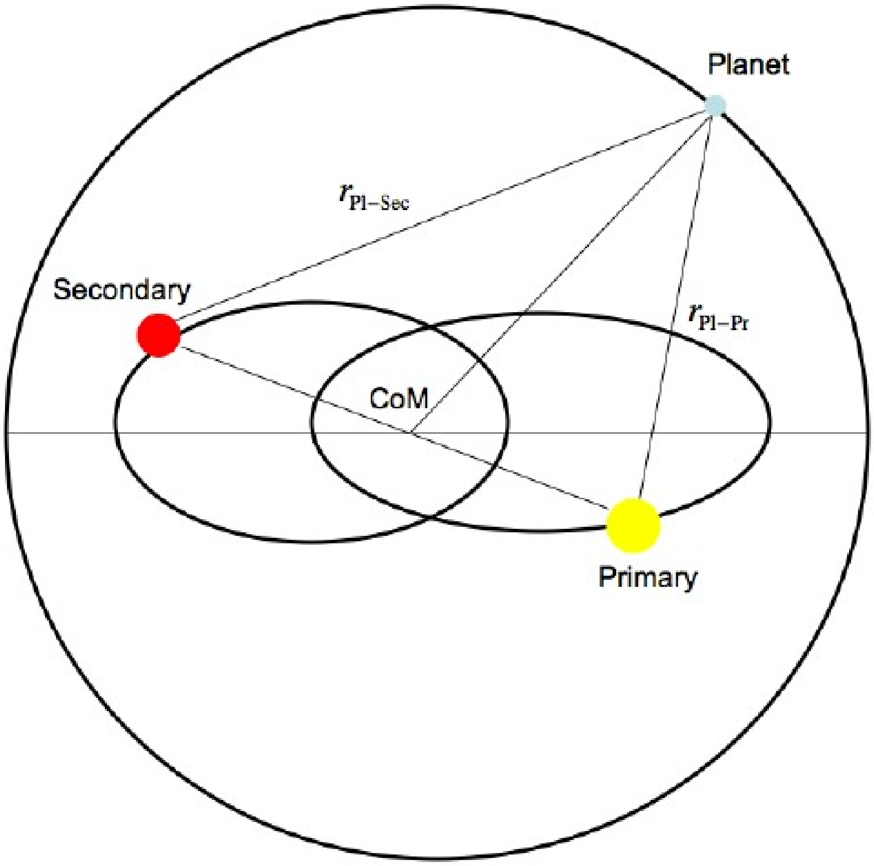


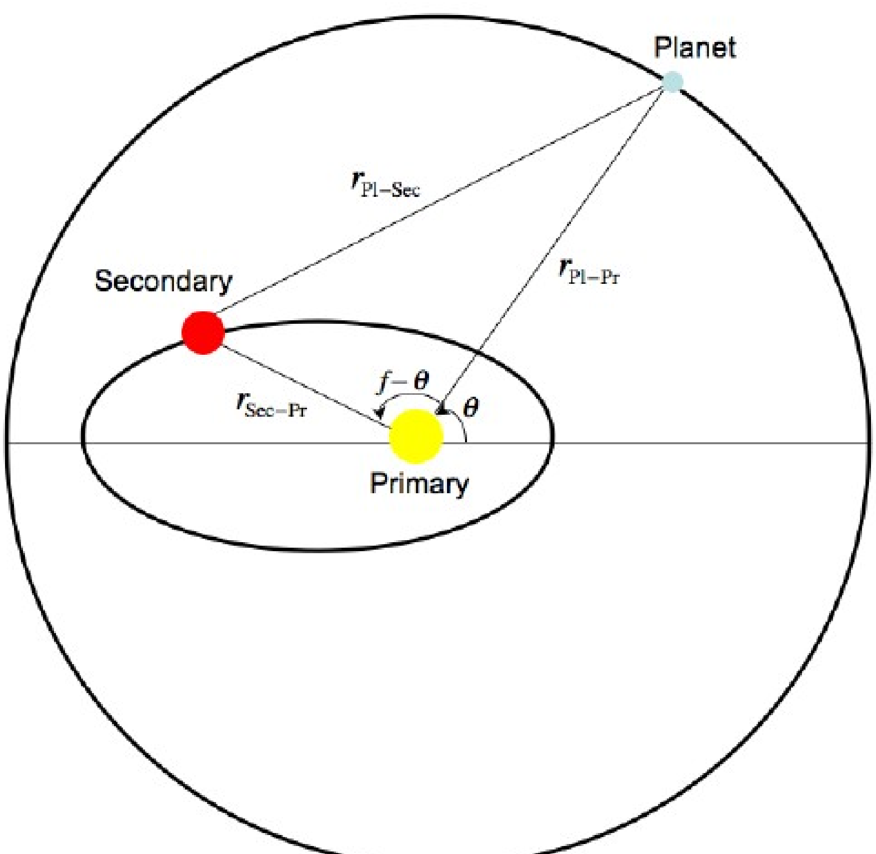
More complex configurations….

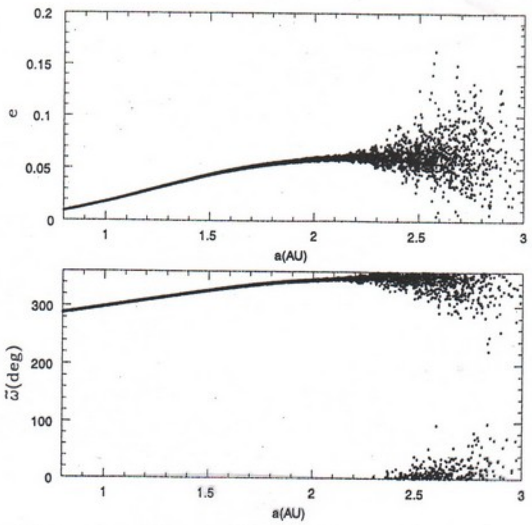


Quadruple Star System - 30 Ari

P-type orbit.


Planet
$r_{\mathrm{Pl-Sec}}$
Secondary
$r_{\mathrm{Pl-Pr}}$
CoM
Primary


Planet
$r_{\mathrm{Pl-Sec}}$
$r_{\mathrm{Pl-Pr}}$
Secondary
$f-\theta$
$r_{\mathrm{Sec-Pr}}$
θ
Primary



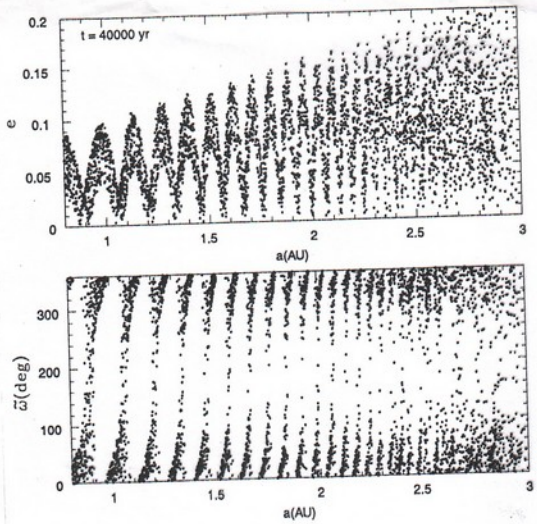


FIG. 1c

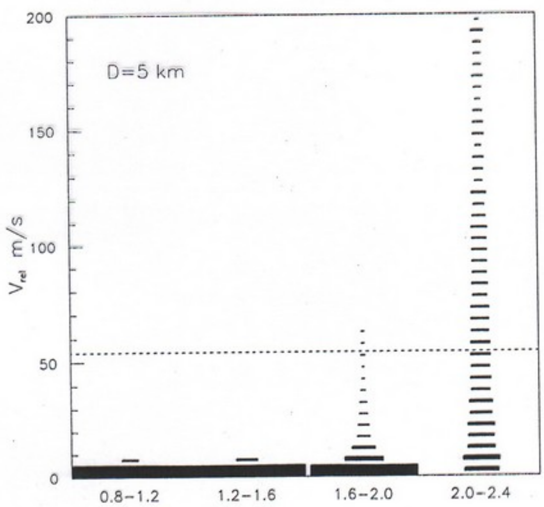


FIG. 8a

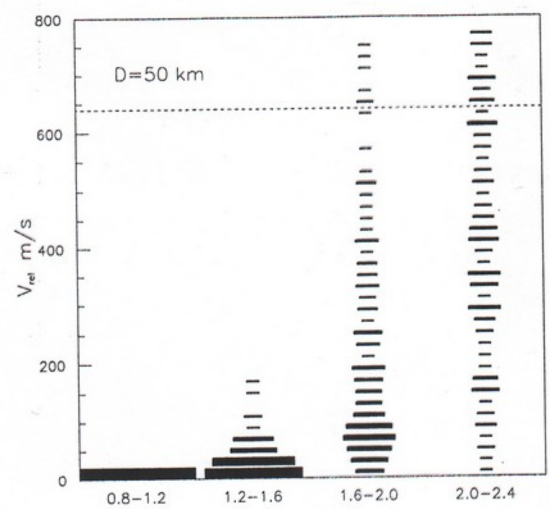


FIG. 8b

The gravitational perturbations of the companion star increase the relative impact velocities but………

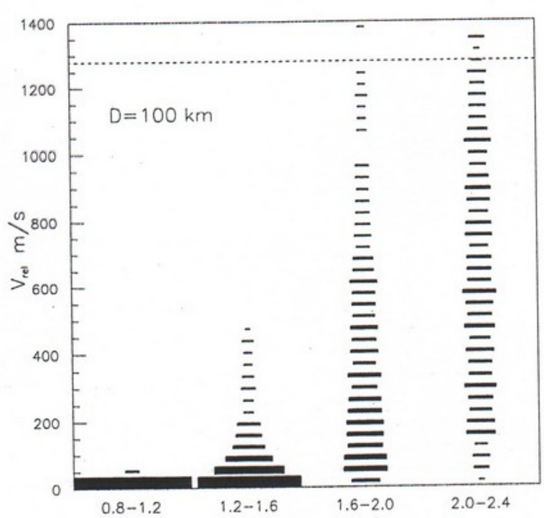


FIG. 8c

Gas drag combined to secular perturbations lead to apsidal alignment of planetesimal orbits and reduction of impact velocities favoring planet accretion.

Doppler effect for the light:

$$\lambda_O/\lambda_S = \frac{c\cdot T + v\cdot T}{c\cdot T} = 1+\frac{v}{c}$$ **where T=1/ν.**

Because of the length relativistic change:

$$\lambda_O = \lambda_S\left(1+\frac{v}{c}\right)\frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \lambda_S\frac{\sqrt{\left(1+\frac{v}{c}\right)}}{\sqrt{\left(1-\frac{v}{c}\right)}}$$

$$\frac{\lambda_O}{\lambda_S} = \frac{\nu_S}{\nu_O} \qquad \nu_O = \nu_S\frac{\sqrt{\left(1-\frac{v}{c}\right)}}{\sqrt{\left(1+\frac{v}{c}\right)}}$$

$$\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = 1-\frac{v}{c}+\frac{1}{2}\left(\frac{v}{c}\right)^2-\frac{1}{2}\left(\frac{v}{c}\right)^3+\dots \qquad \nu_O = \nu_S\left(1-\frac{v}{c}\right)$$

$$E_O = h\nu_S\left(1-\frac{v}{c}\right) = E_S\left(1-\frac{v}{c}\right)$$

ESEMPIO: MOTO UNIFORMEMENTE ACCELERATO

Passeggero su un missile con accelerazione costante $= g$

Nel sistema di riferimento COMOVENTE

$a \perp U$ perché $a = \frac{dU}{d\tau} \Rightarrow$

$$a \cdot U = \frac{dU}{d\tau} \cdot U = \frac{d}{d\tau}\left(\frac{1}{2} U \cdot U\right) = \frac{d}{d\tau}\left(-\frac{1}{2}c^2\right) = 0$$

Nel sistema comovente $U = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix} \to a^0 = 0$ [Per COMODITÀ $c = 1$]

Inoltre le accelerazioni sono $a^i = \frac{d^2 x^i}{d\tau^2} = \frac{d^2 x^i}{dt^2} = g$ (per $i = 1$)

quindi $a^2 = g^2$

Quindi: $\boxed{a \cdot U = 0, \quad a^2 = g^2, \quad U \cdot U = -c^2 = -1}$

$$\begin{cases} -u^0 u^0 + u^1 u^1 = -1 \\ -u^0 a^0 + u^1 a^1 = 0 \\ -a^0 a^0 + a^1 a^1 = g^2 \end{cases}$$

ANCHE nel SISTEMA INERZIALE

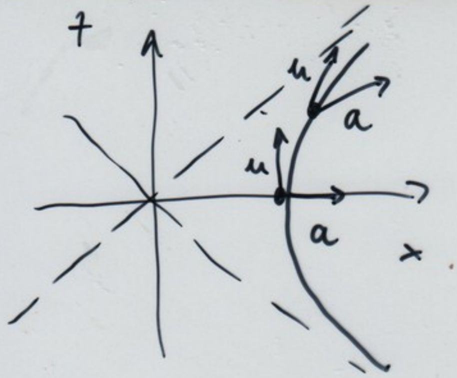
$\Downarrow$

$$a^0 = \frac{du^0}{d\tau} = g u^1 \qquad \Rightarrow \qquad \frac{d^2 t}{d\tau^2} = g \frac{dx}{d\tau}$$

$$a^1 = \frac{du^1}{d\tau} = g u^0 \qquad \qquad \frac{d^2 x}{d\tau^2} = g \frac{dt}{d\tau} \Rightarrow$$

$$t = \frac{\sinh g\tau}{g} \qquad x = \frac{\cosh g\tau}{g} \Rightarrow x^2 - t^2 = \frac{1}{g^2}$$

TRAIETTORIA PARAMETRIZZATA IN τ



COSTRUIAMO LA TETRADE

$e_{0'} = U \qquad e_{2'} = e_2 \qquad e_{3'} = e_3$

il terzo vettore $\perp$ a $e_{0'}$ $e_{2'}$ $e_{3'}$ $\Rightarrow$ // a

$$e_{1'} = \frac{a}{g}$$

$$(e_{0'})^{\mu} = (\cosh g\tau, \sinh g\tau, 0, 0)$$

$$(e_{1'})^{\mu} = (\sinh g\tau, \cosh g\tau, 0, 0)$$

$$(e_{2'})^{\mu} = (0, 0, 1, 0)$$

$$(e_{3'})^{\mu} = (0, 0, 0, 1)$$